LEARNING MODULE

Mathematics G10 | Q1.1

Sequences and Series

NOTICE TO THE SCHOOLS

This learning module (LM) was developed by the Private Education Assistance Committee under the GASTPE Program of the Department of Education. The learning modules were written by the PEAC Junior High School (JHS) Trainers and were used as exemplars either as a sample for presentation or for workshop purposes in the JHS In-Service Training (INSET) program for teachers in private schools.

The LM is designed for online learning and can also be used for blended learning and remote learning modalities. The year indicated on the cover of this LM refers to the year when the LM was used as an exemplar in the JHS INSET and the year it was written or revised. For instance, 2017 means the LM was written in SY 2016-2017 and was used in the 2017 Summer JHS INSET. The quarter indicated on the cover refers to the quarter of the current curriculum guide at the time the LM was written. The most recently revised LMs were in 2018 and 2019.

The LM is also designed such that it encourages independent and self-regulated learning among the students and develops their 21st century skills. It is written in such a way that the teacher is communicating directly to the learner. Participants in the JHS INSET are trained how to unpack the standards and competencies from the K-12 curriculum guides to identify desired results and design standards-based assessment and instruction. Hence, the teachers are trained how to write their own standards-based learning plan.

The parts or stages of this LM include Explore, Firm Up, Deepen and Transfer. It is possible that some links or online resources in some parts of this LM may no longer be available, thus, teachers are urged to provide alternative learning resources or reading materials they deem fit for their students which are aligned with the standards and competencies. Teachers are encouraged to write their own standards-based learning plan or learning module with respect to attainment of their school's vision and mission.

The learning modules developed by PEAC are aligned with the K to 12 Basic Education Curriculum of the Department of Education. Public school teachers may also download and use the learning modules.

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MATHEMATICS 10

Module 1.1: Sequences And Series

☑ **INTRODUCTION AND FOCUS QUESTION(S):**

Have you ever wondered how outcomes in certain real-life could be solved mathematically? What do businessmen use to predict future sales? How do bank executives deal with fluctuating rates such that the business does not go bankrupt? Have you every asked yourself how even epidemics and nature use the tools of math? Have you at some point in your life used numbers to be able to make sound and logical decisions?

In this module, you will discover how important it is to utilize essential mathematical skills to be able to understand these questions that arise in various real-life situations that we encounter everyday and use these skills wisely to be able to come up with the desired output.

As you go through this module, think of this question: **How can the outcomes of certain real life problems be predicted?**

☑ **MODULE LESSONS AND COVERAGE:**

In this module, you will examine this question when you take the following lessons:

Lesson 1 – Arithmetic Sequences and Series
Lesson 2 –Geometric Sequences and Series and Other Types

In these lessons, you will learn the following:

Lesson 1	Arithmetic Sequences and Series • generate patterns • illustrate arithmetic sequence • determines arithmetic means and nth term of an arithmetic sequence
Lesson 2	Geometric Sequences and Series and Other Types • illustrate geometric sequence • differentiates a geometric sequence from an arithmetic sequence • differentiates a finite geometric sequence from an infinite geometric sequence • determines geometric means and nth terms of a geometric sequence • finds the sum of the terms of a given finite and infinite geometric sequence • illustrates other types of sequences (e.g. harmonic, Fibonacci)

☑ **MODULE MAP:**

Here is a simple map of the above lessons you will cover:
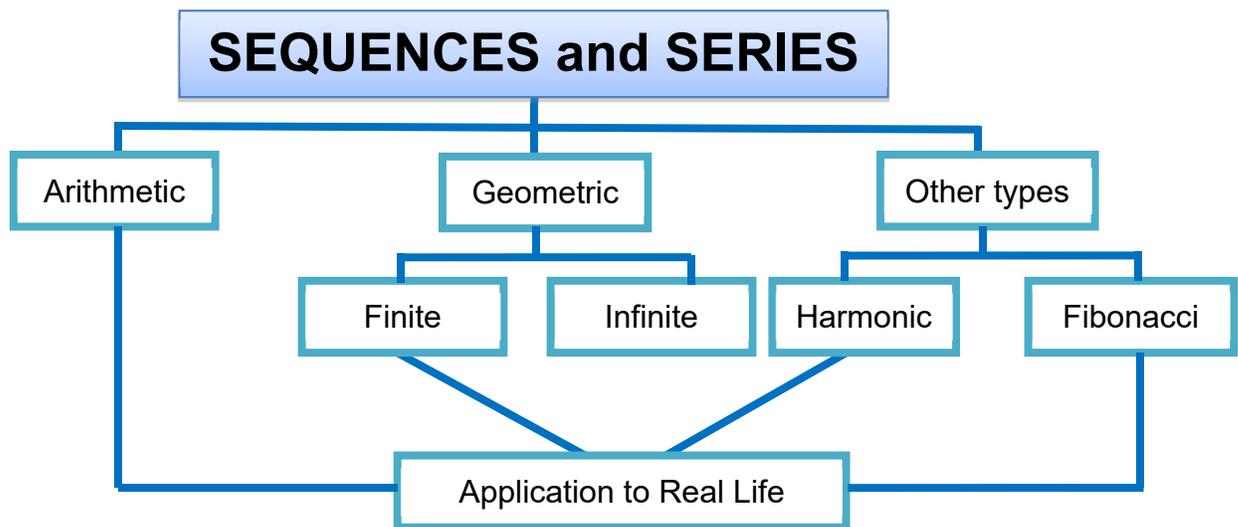
☑ **EXPECTED SKILLS:**

To do well in this module, you need to remember and do the following:
1. follow the directions carefully
2. master the prerequisite skills such as formulating and solving equations and observing patterns
3. solve with speed and accuracy

PRE-ASSESSMENT

Let's find out how much you already know about this module. Click on the letter that you think best answers the question. Please answer all items. After taking this short test, you will see your score. Take note of the items that you were not able to correctly answer and look for the right answer as you go through this module.

1. Which is the next term in the sequence 3, 9, 15, 21, 27, . . . ?

   A. 30
   B. 33
   C. 36
   D. 39

2. Which of the following is a geometric sequence?
   A 1, 2, 4, 8, 16, . . .
   B. -3, 1, 5, 9, . . .
   C. 4, 8, 24, 96, 480, .
   D . -5, 0, 10, 25, 45,

3. The sequence (1, 2, 3, 4) in terms of the number of elements is called ____.
   A. an infinite sequence
   B. a finite sequence
   C. an infinite set
   D. an infinite series

4. What are the next four terms of the arithmetic sequence 122, 111, 100, . . . ?
   A. 94, 88, 82, 76
   B. 95, 90, 85, 80
   C. 92, 88, 84, 80
   D. 89, 78, 67, 56

5. Find the 18th term of the sequence where $a_1 = 3$ and $d = 7$.
    A. 129
    B. 136
    C. 119
    D. 122

6. What is the fifth term of a geometric sequence if $a_1 = 7$ and $r = 0.2$?
    A. 1.4
    B. 0.28
    C. 0.0112
    D. 5.6

7. Which is the rule for the nth term of the arithmetic sequence with $a_8 = 21$ and $a_{14} = 45$?
    A. $a_n = 4n - 11$
    B. $a_n = 4n - 8$
    C. $a_n = 4n + 1$
    D. $a_n = 4n + 7$

8. Find the first three terms of the arithmetic series in which $a_1 = 6$, $a_n = 201$, and $S_n = 4140$.
    A. 6, 11, 16
    B. 5, 10, 15
    C. 6, 15, 40
    D. 6, 46, 86

9. The table shows the predicted growth of a particular bacteria after various number of hours. Write an explicit formula for the sequence of the number of bacteria.

Hours (h)	1	2	3	4	5
Number of Bacteria	19	38	57	76	95

    A. $a_n = 19n + 19$
    B. $a_n = n + 19$
    C. $a_n = 1/19\ n$
    D. $a_n = 19n$

10. A ball is dropped from a height of 7 feet. Each time it hits the ground, it bounces half of its previous height. What is the total distance traveled by the ball?
    A. 7 feet
    B. 14 feet
    C. 21 feet

D. 28 feet

11. Viola makes gift baskets for Valentine's Day. She has 13 baskets over the last year, and she plans to make 12 more each day. If there are 15 work days until the day she begins to sell the baskets, how many baskets will she have to sell?
A. 193 baskets
B. 156 baskets
C. 181 baskets
D. 205 baskets

12. Justine earned $17 000 during the first year of her job at the city hall. After each year, she received a 4% raise. Find her total earnings during the first five years on the job.
A. $92 077.48
B. $72 189.89
C. $517 077.48
D. $85 541.44

13. The sum of the interior angles of a triangle is 180°, of a quadrilateral is 360°, and of a pentagon is 540°. Assuming this pattern continues, find the sum of the interior angles of a dodecagon.
A. 1800°
B. 1980°
C. 1620°
D. 900°

14. A culture of bacteria doubles every 2 hours. If there are 500 bacteria at the beginning, how many bacteria will there be after 24 hours?
A. 204 800
B. 204 080
C. 2 048 000
D. 20 048 000

15. Your brother puts ₱10.00 into his piggy bank. The next day, he puts ₱15.00. Two days after, he puts ₱20.00. He intends to save enough money to be able to raise ₱1000.00 so he can buy wireless speakers. He needs to come up with the amount in two weeks' time so that he will still be able to avail of a 15% discount. If his daily allowance during school days is ₱150.00, what recommendation can you give him so that he will be able to meet his target in two weeks' time given that his daily expenses is ₱50.00 at a minimum for food and needs to allot money for class projects?
A. Increase your savings by ten pesos per day.
B. Start saving ₱60.00 and increase your savings by five pesos per day.
C. Save ₱100.00 everyday for 9 days.
D. Save ₱150.00 for six days.

16. A small snowplowing company makes a profit of $8000 during its first year. The owner of the company sets a goal of reaching a total profit of $20000 after five years. After two years of operation, profits have increased by only $1500. If the trend continues, the company might not be able to reach the goal it has set. What recommendations can you give to the owner of the company?
   A. The company can offer promos in order to attract more customers.
   B. The company cannot attain the goal unless it will increase its price.
   C. The company need to retrench some workers to be able to lessen its expenses.
   D. The company needs to add equipment to be able to cater to more customers.

17. The architect of an amphitheater in your school submitted a design to your school president. In his proposal, there are 26 seats in the front row and each successive row contains two more seats than the previous row. There are 24 rows. Upon checking, the president considered the population of the students in each grade level and recommends increasing the number of seats such that a maximum of 1200 students could be accommodated at one time without changing the number of rows given that the first row can accommodate a maximum of 32 seats. What recommendations can you give to the architect?

   A. Increase the additional seats per row by 4.
   B. Increase the first row by two seats and maintain the same interval.
   C. Increase the number of seats in the first row to 32 and maintain the same interval.
   D. Increase the number of seats in the first row to 30 and maintain the same interval.

18. You are a marathon athlete and you will be competing in the national level 5 months from now. Your coach told you to start running 5 kilometers for the first week and increase by 4 more kilometers for each of the following weeks for 12 weeks. On the 7th, 9th, and 11th weeks, you failed to continue the pattern. What will you suggest to your coach in order to avoid this kind of situation?
   A. Propose a new program which is attainable.
   B. Tell him to accompany you during practice.
   C. Think of another strategy in order to advance with the athletes without undergoing a rigorous practice.
   D. Tell him to search for a vitamin which will boost your performance so that you will not be spending many days for practice.

19. You are running a successful business. Because of this, your sister would also like to open her own business too. She is seeking for advice from you

on the standards she needs to consider in maximizing her sales. Which of the following standards will you suggest?

A. Feasibility, Location, Nature of consumers
B. Feasibility, Location, Aesthetic Appearance of the store
C. Practicality of the price, Nature of consumers, location
D. Feasibility, Practicality of the price, Mathematical Formula

20. You are a finance consultant. A neophyte businesswoman seeks your advice on the mathematical formula that she will use in order to maximize her sales. In which of the following sequences will you derive the formula?
A. Arithmetic
B. Fibonacci
C. Geometric
D. Harmonic

LESSON 1: ARITHMETIC SEQUENCE AND SERIES

EXPLORE

Have you been to a theater in your school or in a mall? What have you noticed about the way the seats are arranged? How are the arrangements determined? Is it possible to use mathematical tools to make the work faster and easier? Let us begin this lesson by looking at the problem presented below.

ACTIVITY 1.   What's Next!

There are 20 seats in the front row of a theater. Each successive row contains two more seats than the previous row. If there are 18 rows, how many seats are there in the last row of the theater?

1. What pattern do the seats of the theater make?

2. How can you find the number of seats in the succeeding rows?

3. How many seats are there in next 4 rows?

4. Can you expect this pattern to continue infinitely? Explain.

5. *How can outcomes of certain real life problems be predicted?*

ACTIVITY 2.   Anticipation-Reaction Guide

Write your response for each statement in the Before Lesson Column only.

Write **A** you if you agree with the statement.
Write **B** you if you disagree with the statement.

Before Lesson	Statements	After Lesson
	1. A pattern is formed when a set of shapes, numbers or designs are repeated over and over again.	
	2. To complete a continuing pattern, identify the pattern rule.	
	3. A sequence is arithmetic when it has a common ratio.	
	4. To continue an continuing pattern, determine the pattern rule.	
	5. A sequence is finite when the pattern will not continue.	
	6. Real-life patterns such as in money, sports, etc. can continue infinitely.	
	7. To get the nth term of an arithmetic sequence, use the formula $a_n = a_1 + (n-1)d$.	
	8. Use the formula $s_n = n\dfrac{(a_1 + a_n)}{2}$ to get the sum of an arithmetic series when the first term and last terms are given.	

End of EXPLORE:

You gave your initial answers to the questions and statements given above.

Let's find out how others would answer the above and compare their ideas to our own. As you compare, you will find out if your ideas are in line with the standard. You will also learn other concepts which will help you complete a required project found at the end. This project is about making an educational poster.

FIRM-UP

Your goal in this section is to learn and understand key concepts of arithmetic sequence, mean, and series. You will also apply these concepts in solving real-life problems.

GENERATING PATTERNS

ACTIVITY 3. Patterns in Numbers and Shapes

You will watch a video that will explain pattern rules, number patterns and symmetrical patterns. Read first the **Process Questions** before watching it. Do not forget to take down notes in EverNote or in your notebook. Click https://www.youtube.com/watch?v=0HzXapUeL5Y to watch the video now.

Process Questions:

1. How are the designs from around the world formed?
2. How are patterns formed?
3. Where can we see patterns?
4. How will you complete a shape or a number pattern?

Post your answers to these questions in the Discussion Forum.

Sum-up what you have learned by completing the following statements.

A pattern is formed when_____

_____.

To complete a continuing pattern, just identify the _____.

ACTIVITY 4. Patterns are Everywhere

Go to www.pinterest.com . Create your own account. Select pictures that show patterns and Pint them. Then think of day-to-day patterns in natural phenomenon that you have observed and experienced. Take pictures of them. Post it in Pinterest. Make sure to add a caption describing the pattern before posting them.

Process Question:

How can outcomes of certain real life problems be predicted?

You have just learned that patterns can be seen in numbers, shapes, and in nature. In the next lessons, you will be learning the types of sequences. As you learn each, try to compare the pattern in generating the numbers in each type of sequence.

Defining a Sequence

ACTIVITY 5. What is a sequence?

You will watch a video defining a sequence and its types. Take note of the process questions before watching it. Click the link https://www.youtube.com/watch?v=p-rc5mTDt9E to watch the video.

Process Questions:

1. When is a list of numbers called a sequence?
2. When is a sequence finite or infinite?
3. In what ways can you express a sequence?
4. When is a sequence arithmetic? Explain.

Post your answers to these questions in the Discussion Forum.

Sum-up what you have learned by completing the following statements.

A sequence is _____.

A sequence is finite when_____.
It is infinite when _____.

A sequence can be expressed using a _____or a _____.

A sequence is arithmetic when _____.

Arithmetic Sequence

ACTIVITY 6.   Arithmetic or not?

Watch videos from
1. http://www.virtualnerd.com/algebra-2/sequences-series/arithmetic/arithmetic-sequences/arithmetic-sequence-definition
2. http://my.hrw.com/math06_07/nsmedia/lesson_videos/alg2/player.html?contentSrc=6498/6498.xml

Then complete the Frayer Model below.

Definition	Facts/Characteristics
Examples	Non-

(In the center: **Arithmetic Sequence**)
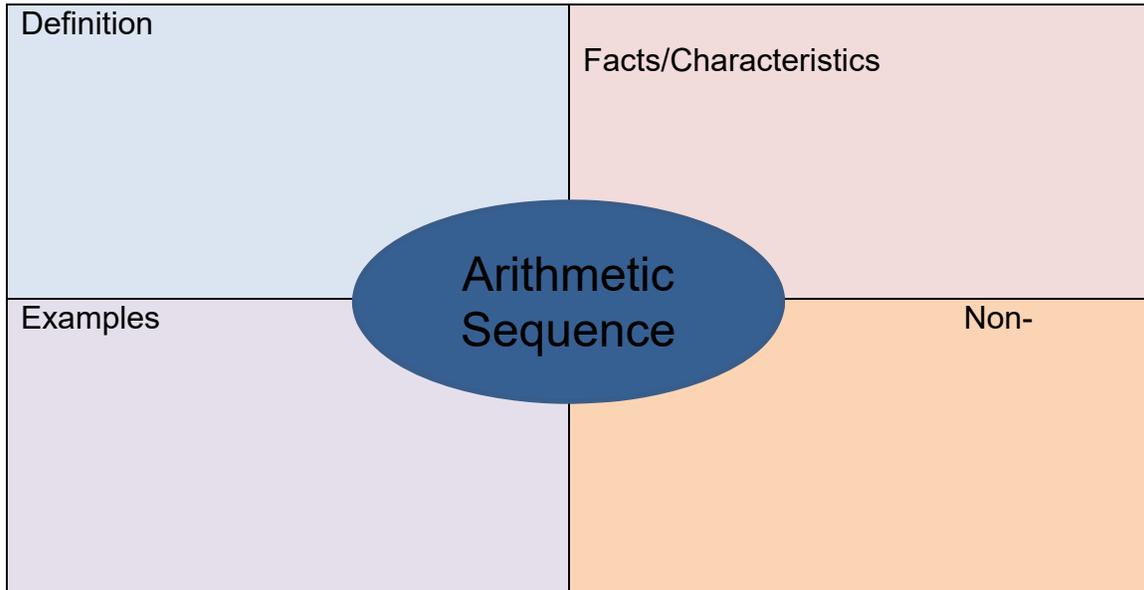
PROCESS QUESTIONS:

1. How did you determine the examples and non-examples of an arithmetic sequence?

2. How will you know that a given sequence is arithmetic or not?

ACTIVITY 7.   Skills Practice in Identifying Arithmetic Sequences

Tell whether each sequence is arithmetic or not. If it is arithmetic check on the **Yes** column then give the common difference; if it is not, check the **No** column.

Sequence	Yes	No	Common difference
1.12, 16, 20, 24,. ..			
2. 35, 32, 29, 26, …			
3. 40, 45, 50, 55, …			
4. -3, -23, -43, -63, ...			
5. 4, 9, 13, 17, …			
6. -34, -64, -94, -124, …			
7. -3-, -40, -50, -60, …			
8. -8, -3, 1, 4, …			

9. -7, -9, -11, -13, …			
10. 9, 14, 19, 24, ...			

Process Questions:
1. How many items were correctly answered?
2. Did you encounter any difficulty in identifying which is arithmetic and which is not? If yes, why? If no, why?

*Post your answers to these questions in the **Discussion Forum**. If it happens that your score is lower than 7, you may ask any of your classmates for help.*

Answer Key

Sequence	Yes	No	Common difference
1.12, 16, 20, 24,. ..	/		4
2. 35, 32, 29, 26, …	/		-3
3. 40, 45, 50, 55, …	/		5
4. -3, -23, -43, -63, ...	/		-20
5. 4, 9, 13, 17, …		/	
6. -34, -64, -94, -124, …	/		-30
7. -3-, -40, -50, -60, …	/		-10
8. -8, -3, 1, 4, …		/	
9. -7, -9, -11, -13, …	/		-2
10. 9, 14, 19, 24, ...	/		5

ACTIVITY 8.   Finding the terms of an arithmetic sequence

In this activity, you will be taught how to find the next terms in a sequence; how the formula for the nth term is derived; how to determine the nth term of a sequence using the formula; and how to determine the missing term/s. Just click on the videos below.

After watching each video, complete each statement found in every box to sum-up what you have learned. Do not forget to take down notes in your Evernote or notebook.

1. Finding the next term

http://www.virtualnerd.com/algebra-2/sequences-series/arithmetic/arithmetic-sequences/sequence-next-terms-example

> To get the next term of an arithmetic sequence,
>
> _____.
>
> of an arithmetic sequence,

2. Formula for the nth term and Finding the nth Term of a Sequence

a. http://www.virtualnerd.com/algebra-2/sequences-series/arithmetic/arithmetic-sequences/arithmetic-sequence-definition

b. http://www.virtualnerd.com/algebra-2/sequences-series/arithmetic/arithmetic-sequences/sequence-nth-term-example

c. http://my.hrw.com/math06_07/nsmedia/lesson_videos/alg2/player.html?contentSrc=7224/7224.xml

> To determine the nth term of an arithmetic sequence, use the formula_____ where a_n is the _____; a_1 is the _____;n is the _____; and d is the _____.
>
> To determine the nth term of an arithmetic sequence, use the formula_____ where a_n is the _____; a_1 is the _____;n is the _____; and d is the _____.

3. Finding the Missing term/s in the Arithmetic Sequence

a. https://www.youtube.com/watch?v=bM4bsgEh7p4

b. http://my.hrw.com/math06_07/nsmedia/lesson_videos/alg2/player.html?contentSrc=7225/7225.xml

> To determine the missing term/s of an arithmetic sequence given its first and last terms, determine the _____ by using the formula _____and substitute the first and last terms to it. After finding the_____,add it to the first term to get the second term, add the _____ to the second term to get the third term and so on to To

Process Questions:

1. How will you determine the next term of an arithmetic sequence?

2. How will you determine the nth term of an arithmetic sequence?

3. How will you determine the in between missing terms of an arithmetic sequence if the given are its first and last terms?

Post your answers to these questions in the Discussion Forum and compare your answer to your classmates' and do not hesitate to ask them if their answers are different from yours.

From the ideas you have shared and have gained from your classmates, complete each statement as an evidence of mastering the skill in determining the next term/s, the missing term/s, and the nth term of an arithmetic sequence.

- To get the next term of an arithmetic sequence,

  _____.

- To determine the nth term of an arithmetic sequence, use the
  formula_____

From the ideas you have shared and have gained from your classmates, complete each statement as an evidence of mastering the skill in determining the next term/s and the nth term of an arithmetic sequence.

- To get the next term of an arithmetic sequence, <u>determine the common difference and add it to the number before it.</u>
- To determine the nth term of an arithmetic sequence, use the formula
  $a_n = a_1 + (n-1)d$, where

  a_n is the <u>nth term;</u>
  a_1 is the <u>first term;</u>
  n is the <u>number of terms;</u> and
  d is the <u>common difference</u>.

- To determine the missing term/s of an arithmetic sequence given its first and last terms, find the<u> common difference </u>by using the formula $a_n = a_1 + (n-1)d$ and

  substitute the first and last terms to it. After finding the<u> common difference, </u>add it to the first term to get the second term, add the<u> common difference </u>the second term to get the third term and so on to complete the sequence.

ACTIVITY 9.   Skills Practice Using an Interactive Website

Click on the link

http://go.hrw.com/math/midma/gradecontent/loadlesson.html?course=c3&chapter=12&lesson=1&SE=1&sz_audio=1&calc=1&state=xx&actCourse=4to
practice your skills in: identifying whether a given sequence is arithmetic or not;

determining the common difference; determining the missing term in a sequence; determine the nth term; solving real-life problem. Click to your chosen answer then click **Select your answer** button

If you find it difficult to answer the problem, click the See Example button to view a tutorial on how to answer it.

Skills Readiness Check: Reflect on the level of your performance for this lesson.

Check the first column if you **need more practice** or you are now **ready to move on to the next activity**.

I Need more practice (if most of your answers in the interactive quiz were wrong)	I am ready to move on to the next activity (if you have answered all the items correctly or have committed only one or two wrong answers)
Answer the next interactive quiz at http://www.ixl.com/math/algebra-1/arithmetic-sequences .	You may proceed to the next activity.

Process Questions:
1. How did you find the activity?

2. Did you encounter any difficulty? If yes, what did you do in order to surpass it?

Post your answers to these questions in the Discussion Forum.

Make a summary of the steps that you have used to answer all the items correctly by completing each statement below.

- A sequence is arithmetic when_____.

- To get the common difference of an arithmetic sequence, _____.

- To determine the nth term of an arithmetic sequence, use the formula _____.

- To determine the missing term/s of an arithmetic sequence given its first and last terms, _____ _____ _____

Make a summary of the steps that you have used to answer all the items correctly by completing each statement below.

- A sequence is arithmetic when <u>the terms have a common difference</u>.

- To get the common difference of an arithmetic sequence, <u>subtract the term from the number preceding it/before it.</u>

- To determine the nth term of an arithmetic sequence, use the formula <u>$a_n = a_1 + (n-1)d$</u>

- To determine the missing term/s of an arithmetic sequence given its first and last terms, <u>find the common difference by using the formula $a_n = a_1 + (n-1)d$ and substitute the first and last terms to it. After finding the common difference, add it to the first term to get the second term, add the common difference the second term to get the third term and so on to complete the sequence.</u>

ACTIVITY 10. Graphing Arithmetic Sequence: An Investigation

Solve the problem below, complete the table and graph it.
Auntie Sally sells brownies for 15 pesos each. For every brownies sold, an additional of 15 pesos is added to the income. If she sells 50 pieces of brownies, how much money will she have at the end of the day?

x (Number of Brownies)	y (Income)
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
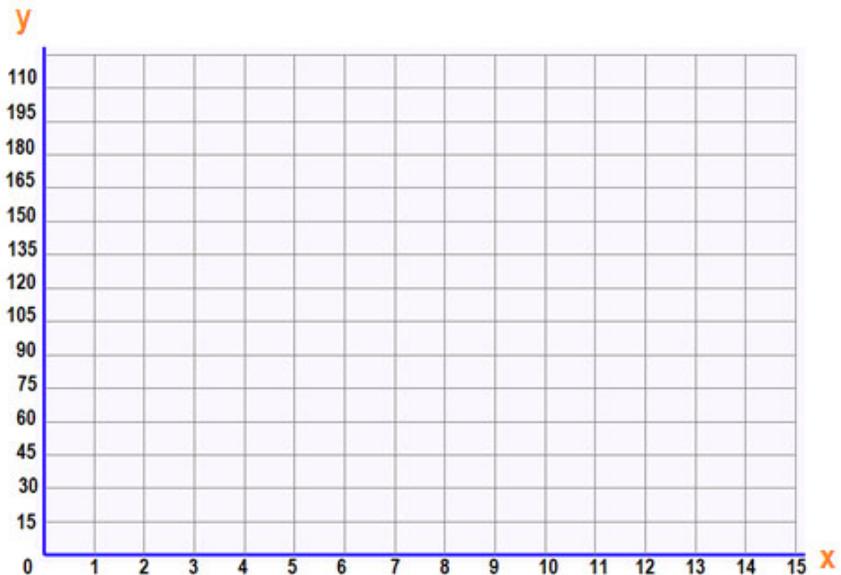
Process Questions:

1. What is the resulting graph?

2. What kind of function is an arithmetic sequence?

3. Write the equation that represents Auntie Sally's income. Can you also write it as a formula of an arithmetic sequence $a_n = a_1 + (n - 1) d$? If so, then write its equivalent equation.

4. How will you relate the slope of the line to the common difference of an arithmetic sequence?

5. ***How can outcomes of certain real life problems be predicted?***

Conclusion:

An arithmetic sequence is a _____ function in which the term ____ is the independent variable (x), the term ____ is the dependent variable (y), and the ____ is the slope.

Answer Key:

Sum-up the things you have found out in the investigation by competing each statement below.

An arithmetic sequence is a <u>linear</u> function in which the term <u>n</u> is the independent variable (x), the term <u>a_n</u> is the dependent variable (y), and the <u>common difference (d)</u> is the slope.

Sum-up the things you have found out in the investigation by competing each statement below.

---

x (Number of Brownies)	y Income
1	15
2	30
3	45
4	60
5	75
6	90
7	105
8	120
9	135
10	150
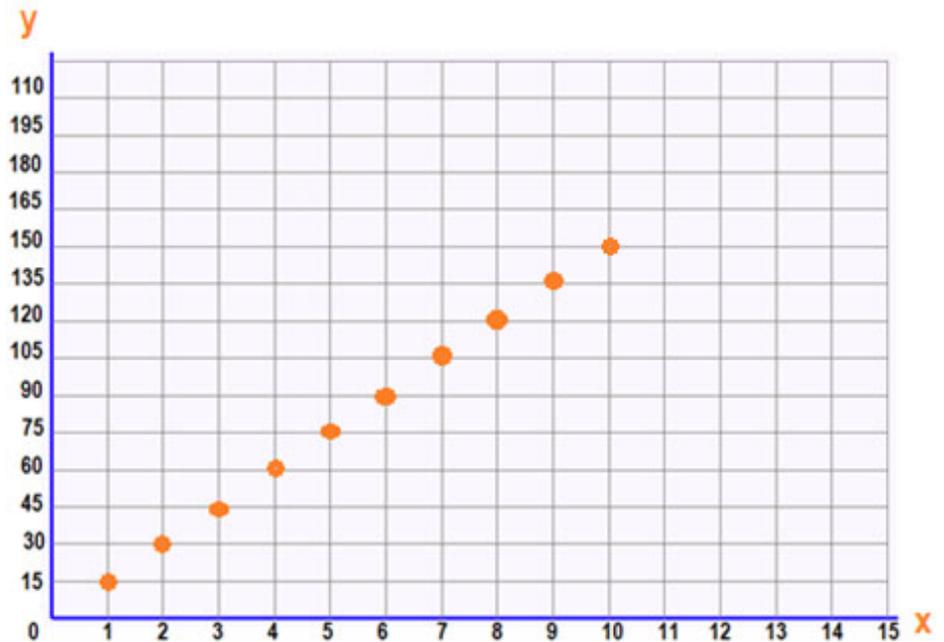
ACTIVITY 11.   Revisiting Activity 1: What's Next!

Revisit your answers to this problem, and revise those answers which you think is incorrect.

There are 20 seats in the front row of a theater. Each successive row contains two more seats than the previous row. If there are 18 rows, how many seats are there in the last row of the theater?

1. What pattern do the seats of the theater make?

2. How can you find the number of seats in the succeeding rows?

3. How many seats are there in next 4 rows?

4. Can you expect this pattern to continue infinitely? Explain.

5. *How can outcomes of certain real life problems be predicted?*

ACTIVITY 12.   I can solve more real-world problems

1. Kelly is saving her money to buy a car. She has Php 50 000, and she plans to save Php 3 750 per week from her job as a call center manager.
   a. How much will Kelly have saved after 8 weeks?

   b. If the car's down payment costs Php 270 000, how long will it take her to save money at this rate?

2. There are 28 seats in the front row of a theater. Each successive row contains two more seats than the previous row. If there are 24 rows, how many seats are there in the last row?

3. Mario began an exercise program to get back in shape. He plans to row 5 minutes on his rowing machine the first day and increase his rowing time by one minute and thirty seconds each day.
   a. How long will he row on the 18th day?

   b. On what day will Mari first row an hour and more?

Process Questions:
   1. How are the answers to each problem solved?

2. Will the patterns of the above problems continue indefinitely? Explain.

3. *How can outcomes of certain real life problems be predicted?*

Answer Key:
    1. a. 30 000
b. 9 weeks
    2. 74
    3. a. 30 minutes and 30 seconds
b. on the 38th day
c. No. It is unreasonable because there are only so many hours in the day that can be dedicated to rowing.

Harmonic Sequence

Another sequence which is similar to an arithmetic sequence is the harmonic sequence. Find out how they differ and similar from each other.

ACTIVITY 13. Identifying A Harmonic Sequence

Watch a video from https://www.youtube.com/watch?v=CR8Aoy8fsvc . This video will teach you how to identify harmonic sequence, solve for its missing and nth term/s. Do not forget to take down notes and write everything you have learned in the box below.

I"ve learned that …

ACTIVITY 14.   Skills Practice

Download a pdf worksheet from
http://lhsblogs.typepad.com/files/am3-harmonic_fibonacci-ws.pdf   and   answer
items 1-6 only.

Pass your output to your teacher on your next face-to-face meeting.

Arithmetic Series

You have just learned how arithmetic sequences can help us predict outcomes in some real-life problems. You have also compared arithmetic sequence to that of a harmonic sequence. In the next part of the lesson, you will learn about arithmetic series and its application. How do arithmetic series differ from arithmetic sequence? Find the answer to this question by doing the next set of activities.

ACTIVITY 15.   Gauss's Problem and Arithmetic Series

Click on the given link below.
http://coolmath.com/algebra/19-sequences-series/06-gauss-problem-arithmetic-series-01.htm
This 7-page article
1. tells the story of young Karl Friedrich Gauss who astonishingly got the sum of the numbers from 1 to 100 in few seconds only;
2. shows Karl's technique in getting the sum of numbers 1 to 100;
3. explains how the formula for the sum of arithmetic series is derived;
4. allows you to practice getting the sum of a given sequence.

Process Questions:

1. What was young Karl's trick in getting the sum of the numbers 1 to 100?
2. What formula can be derived from his trick?
3. By applying his trick and the formula, were you able to get the sum of a sequence in a lesser time?
4. What is your insight in this activity?

Post your answers to these questions in the Discussion Forum and discuss with your classmates. After discussing with your classmates, fill-in the box below of the ideas you have learned from the given pages and have gained from your classmates.

I've learned that…..

ACTIVITY 16.   What is an arithmetic series?

Watch the following videos to learn the concepts of series and arithmetic series:

1. http://www.virtualnerd.com/algebra-2/sequences-series/series-definition.php defining a series

2. https://www.youtube.com/watch?v=Dj1JZIdIwwo   shows how to find the sum of an arithmetic series

3. http://my.hrw.com/math06_07/nsmedia/lesson_videos/alg2/player.html?contentSrc=7227/7227.xml another way using only the first formula

Process Questions:

1. What is the difference between a sequence and a series?

2. When is a series definite and infinite?

3. When will you use the formula $s_n = n\dfrac{(a_1 + a_n)}{2}$ and $s_n = \dfrac{n\left[2a_1 + (n-1)d\right]}{2}$ in finding the nth partial sum of an arithmetic series?

Post your answers to these questions in the Discussion Forum and compare your answer to your classmates' and do not hesitate to ask them if their answers are different form yours.

From the ideas you have shared and have gained from your classmates, complete each statement.

A   sequence   is   _____while   a   series   is   the

_____.

A   series   is   definite   if   _____.   It   is   infinite   if

_____.

Use the formula $s_n = n\dfrac{(a_1 + a_n)}{2}$ when _____

and use the formula $s_n = \dfrac{n[2a_1 + (n-1)d]}{2}$ when _____.

ACTIVITY 17.   Solving Real-life Problems using Arithmetic Sequence and Series

Watch a video from http://my.hrw.com/math06_07/nsmedia/lesson_videos/alg2/player.html?contentSrc=7228/7228.xml as sample problem. This video will show you how to solve problems applying both arithmetic sequence and series. Then revisit Activity No. 1 's "What's next!" and answer the specified problem.

> There are 20 seats in the front row of a theater. Each successive row contains two more seats than the previous row. If there are 18 rows, what is the total number of seats?

There are a total of _____ seats.

Process Question:

1. How did you solve for the answer?

2. *How can outcomes of certain real life problems be predicted?*

Answer Key:

There are a total of <u>666</u> seats.

Skills Check
Check the level of your skills that you have already learned and mastered.

Skill	Advanced	Proficient	Progressing	Beginning
I know how to determine whether a sequence is arithmetic or not.				
I know how to determine the common difference of an arithmetic sequence.				
I know how to solve for the unknown term in an arithmetic sequence				
I know how to solve for the sum of an arithmetic series.				
I know how to solve real-life problems involving arithmetic sequence and series				

If you are still in the Beginning and Progressing Level, do not hesitate to ask assistance from any person who is knowledgeable of the lesson or seek the help of your teacher in your next face-to-face meeting. You may also post your questions in the discussion room. Reflect on the reasons why you succeeded or still have problems in learning the lesson. Fill-up the box below.

Fill up the appropriate box below.

Reasons why I suceeded in learning Arithmetic Sequence and Series

Reasons why I had problems in learning Arithmetic Sequence and Series

Resolution: What will I do in order to master the skills?

END OF FIRM-UP:

> In this section, the discussion was about arithmetic sequences and series. Here, you were asked to determine the missing term/s, the nth term, and the sum of the indicated arithmetic series. You also solve real-life problems which calls for arithmetic sequence ad series as a means of solving it.
>
> Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?
>
> Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.

DEEPEN

> Your goal in this section is to relate arithmetic sequences and series to real-life problems or situations. Before proceeding to the next section, consider the following questions: ***How can outcomes of certain real life problems be predicted?***
> Let us do the next activity to find out how.

ACTIVITY 18.   Error Analysis

1. Claire wrote the first 3 terms of an arithmetic sequence: 3, 6, 9,…

    When she asked Robert to extend the sequence to the first 10 terms, he wrote: 3,6,9,3,6,9,3,6,9,3,…

    a. Is Robert correct? Explain.

    b. What fact did Robert ignore when he extended the sequence?

    c. What is the correct sequence?

2. Joseph and Jean are determining the formula for the nth term for the sequence -11, -2, 7, 16,… . Is either of them correct? Explain.

Joseph	Jean
$d = 16-7$ or 9, $a_1 = -11$ $a_n = -11 + (n-1)9$ $= 9n - 20$	$d = 16-7$ or 9, $a_1 = -11$ $a_n = 9n - 11$

My Answer:

ACTIVITY 19.   Extra Challenge!

Download a pdf from

https://docs.google.com/file/d/0BzdsQ0MvngT0ZmE1ODJkMmEtMDl2My00

MThmLThiZDMtNjMzMmE1YWQ4MmQz/edit?authkey=CPn-

2egB&ddrp=1&hl=en . Print a copy of this file and answer all the items.

Process Questions:

1. How did you find the activity?
2. What concepts did you apply in order to answer all items?
3. ***How can outcomes of certain real life problems be predicted?***

Pass your answers to your teacher in your next face-to-face meeting. Then post your answers to the process questions in the Discussion Forum.

ACTIVITY 20.   Test of Understanding

Instruction: In the previous section, we looked at different problems about arithmetic sequence and series. Let's put together in the table below our answers to the essential question that we asked for each problem.

Essential Question	**PROBLEM 1: Seating Capacity**	**PROBLEM 2: Arrangement**	**PROBLEM 3: Financial Literacy**
How can outcomes of certain real life problems be predicted?	A new football stadium is to be built. The bottom row will have 250 seats, and each row above the bottom row will have 40 seats more than the row in front of it. The design calls for 30 rows. How	Students created a trapezoid from the cans they had collected for the food bank. There were 10 rows in the trapezoid. The bottom row had 100 cans. The 10th row has 136 cans.	Daniela borrowed some money from her parents. She agreed to pay Php 2000 at the end of the first month and Php 500 more for each additional month for 12 months. How much does she pay in total after 12 months?

	many seats will there in the 30th row? The problem was solved using …	How many cans were in the 8th row? The problem was solved using …	The problem was solved using …

Process Questions:
1. Look at your answers to the essential question in the above table. What do all the answers have in common?
2. When will you know if a problem calls for the application of an arithmetic sequence, an arithmetic series, or the application of both?
3. Complete the following statement and support your answer with the examples from the above problems.

Outcomes of certain real life problems can be predicted …

Supporting reasons and examples:

ACTIVITY 21.   Revisiting Activity No. 2: A/R Guide

Go back to Activity No.2 and answer the **After Lesson** Column.
Write **A** you if you agree with the statement
Write **B** you if you disagree with the statement

Before Lesson	Statements	After Lesson
	A pattern is formed when a set of shapes, numbers or designs are repeated over and over again.	
	1. To complete a continuing pattern, identify the pattern rule.	
	2. A sequence is arithmetic when it has a common ratio.	
	3. To continue an continuing pattern, determine the pattern rule.	
	4. A sequence is finite when the pattern will not continue.	
	5. Real-life patterns such as in money, sports, etc. can continue infinitely.	

	6. To get the nth term of an arithmetic sequence, use the formula $a_n = a_1 + (n-1)d$.	
	7. Use the formula $s_n = n\dfrac{(a_1 + a_n)}{2}$ to get the sum of an arithmetic series when the first term and last terms are given.	

ACTIVITY 22.   Synthesis Journal

Complete the table below.

Synthesis Journal			
Lesson	What I did.	What I learned.	How I can use it
Arithmetic Sequence			
Arithmetic Series			

End of DEEPEN:

In this section, the discussion was about the real world use of the arithmetic sequences and series.

What new realizations do you have about the topic? What new connections have you made for yourself? What helped you make these connections?

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.

TRANSFER

Your goal in this section is apply your learning to real life situations. You will be given a practical task which will demonstrate your understanding.

ACTIVITY 23.   Arithmetic Sequence in the Real-World

1. Describe a real-life situation that can be represented by an arithmetic sequence with a common difference of

a. 8

b. 12

c. 20

Process Question:

How can outcomes of certain real life problems be predicted?

End of TRANSFER:
You have just completed lesson 1. As you proceed to the next lesson, you will be given the chance to learn more kinds of sequences that can help you predict the outcomes of some real life problems.

LESSON NO. 2: GEOMETRIC SEQUENCE AND SERIES

EXPLORE

In the previous lesson, you learned how arithmetic sequences and series can help you predict outcomes of certain real life problems. In this section, you will be able to discover a new sequence and series that you can use to help predict the outcomes of more real life problems. Be reminded to take each activity seriously. Make a conscious effort to highlight the new concepts that you will be learning.

Let us begin this lesson by reading a short story about a king in India who loved to play games. Discover how this story led to the invention of a very popular game which most students today find challenging and mentally engaging.

ACTIVITY 1. The Math In Chess

Hundreds and hundreds of years ago, there was a King in India who loved to play games. But he had gotten bored of the games that were present at the time and wanted a new game that was much more challenging. He commissioned a poor mathematician who lived in his kingdom to come up with a new game.

After months of struggling with all kinds of ideas, the mathematician came up with the game of Chaturanga. The game had two armies each led by a King who commanded the army to defeat the other by capturing the enemy King. It was played on a simple 8×8 square board. The King loved this game so much that he offered to give the poor mathematician anything he wished for. "I would like one grain of rice for the first square of the board, two grains for the second, four grains for the third and so on doubled for each of the 64 squares of the game board." said the mathematician.

"Is that all? Why don't you ask for gold or silver coins instead of rice grains?,"asked the King. "The rice is sufficient for me," replied the mathematician. The King ordered his staff to lay down the grains of rice and soon learned that all the wealth in his kingdom would not be enough to buy because the amount of rice was exhausted before the 30th square was reached. You have provided me with such a great game and yet I cannot fulfill your simple wish. You are indeed a

genius!" said the King and offered to make the mathematician his top most adviser then.

Are you wondering to know exactly how many grains of rice would be needed on the 64th square and what is the number of rice grains would be needed for all 64 squares? Let us try to discover the answer by doing the next activity.

http://tugba.ozcan.bilkent.edu.tr/ozcan_geometric%20sequences

PROCESS QUESTIONS:
1. Complete the table below:

First Square	Second Square	Third Square	Fourth Square	Fifth Square	Sixth Square
1					

2. Find the ratio between two consecutive terms.

3. What kind of relationship did you find between the ratios?

4. How can you find a way to find the number of grains inside the 20th square?

5. How can one find a way to solve for the sum of the terms in the first twenty squares?

The story and the activity show you one of the many situations where we can encounter sequences in real-life. As you go through all the activities in this module, think of this question: **How can the outcomes of certain real life problems be predicted?**

ACTIVITY 2.   IRF Sheet

Let's begin by answering the "I" portion of the IRF Worksheet that you see below. Fill it up by writing your initial answer to the topical focus question. Click the save button after typing your answer.

IRF Worksheet
Initial Answer
Revised Answer
Final Answer

Well, those are your thoughts and ideas about our lesson. Before you proceed with the next activity, let us first find out whether the focus question is clear to you by doing some windshield check in the next activity.

This formative assessment will check on your own understanding as far as the focus question is concerned. Using the analogy of a windshield, you will now decide which of the following best describes what your understanding of the focus question.

How many bugs do you have on your windshield? What is making it hard to see clearly?

CLEAR = I get it! I thoroughly understand the concept.
BUGGY = I understand it for the most part, but a few things are still unclear.
MUDDY = I don't get it at all.

After making an assessment, write a short description for your choice and email your response to your teacher.

END OF EXPLORE:

You just tried finding out how sequences and series can be used to model and solve real-life problems. It is now time to learn more about geometric sequences and series. What you will learn in the next sections will also enable you to do the final project which involves looking at real-life situations that involve decision making and coming up with sound recommendations.

FIRM-UP

Your goal in this section is to learn and understand key concepts of geometric sequences and series. You will learn the important skills that are necessary in order to understand the succeeding lessons in this module. It is advised that for every new term that you encounter, be sure to define it in your own words aside from the definitions presented to you.

We will start by doing the next activity.

ACTIVITY 3.   What's Up Geometric?

In this activity, you will try to find out what a geometric sequence is. As you read the article, try to highlight the important words that you encounter. If you do not know the meaning of these words, try looking for the meaning in the dictionary. Click on the link to read the article entitled, "Geometric Sequences:"

https://share.ehs.uen.org/sites/default/files/Unit06Lesson03_0.pdf. Please read only the first five pages of the article.

After reading the article, complete the Frayer's model below:

Definition	Facts/Characteristics
Examples	Non-examples

Geometric Sequence (center oval)
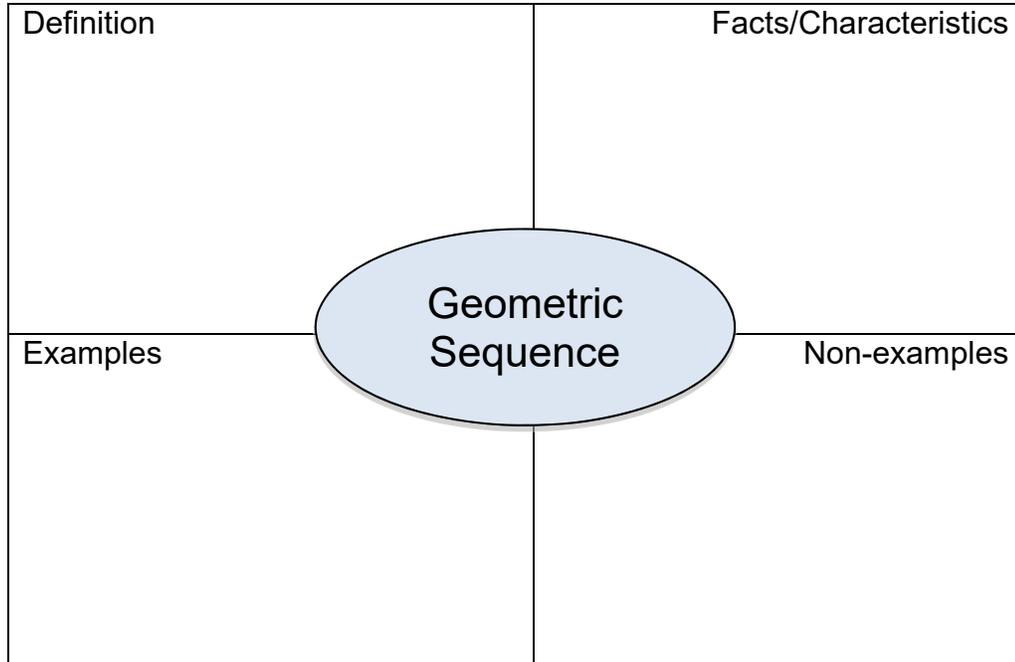
PROCESS QUESTIONS:
1. How did you come up with the list of facts and characteristics of the geometric sequence?

2. Do all the examples that you have given have all the essential characteristics?

3. How did you come up with the non-examples? Why do you consider these as non-examples? Support your answer.

4. How did the listing down of non-examples help you understand fully the concept?

ACTIVITY 4.   The Big Difference

In the previous lesson, you learned what arithmetic sequences are. In this activity, you will now be asked to read an article entitled "Comparison of Arithmetic and Geometric Sequences."

Please bear in mind the following reminders are you read the article:

1. If there are words which are unfamiliar to you, look for the meaning in the dictionary.
2. While you pay attention on the differences between the two sequences, try to find some similarities as well.

Click on the link below to be able to read the article:https://share.ehs.uen.org/sites/default/files/Unit06Lesson04_0.pdf

Now that you have finished reading the article, let us find out if you are able to complete the Venn Diagram below completely.
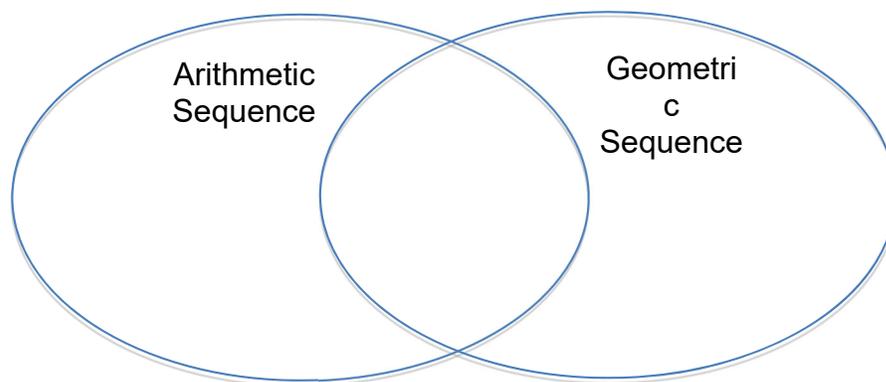
After completing the Venn diagram, complete the following statements below:

1. An arithmetic sequence and a geometric sequence are similar because they both _____
_____.

2. They are different because _____
_____
_____
_____.

PROCESS QUESTIONS:

1. What were presented in the article to help you compare the two sequences?

2. If you looked at only numbers, can you easily detect the difference? Why? Why not?

3. When you were asked to use formulas, were you able to see the differences easily? Why? Why not?

4. Did the graphs help you visualize the difference between the two? How?

5. Which among the three methods presented in the article help you contrast the two sequences? Why?

ACTIVITY NO. 6: Can You Tell?

After finding out the difference between arithmetic and geometric sequences, you will now be asked to watch a video that will help you fully understand how to determine whether the sequence is arithmetic or geometric. Click on this link: https://www.youtube.com/watch?v=ri767qbKLc4.

PROCESS QUESTIONS:
1. How does one distinguish an arithmetic sequence from a geometric sequence?

2. How did the video help you fully understand the difference between the two?

3. Can you find another way to make the distinction apart from the one presented in the video? Explain the process.

ACTIVITY 5.   Magic List Of Terms

As you go through the lesson, it is important that you gain correct understanding of the terms that you will be needing as the lesson progresses. Try completing the table below by defining some of the terms in the first column and try to use each term in a meaningful sentence.

Terms	Own definition	Sentence
1. sequence		
2. common ratio		
3. recursive formula		
4. term		
5. general formula		

ACTIVITY 6.   3-2-1 Chart

After reading the two articles, let us see if you can summarize your insights using the journal below. In this activity, you will be asked to complete the 3-2-1 Chart regarding the special products that you have discovered.

3-2-1 Chart

Three things you found out:
   1. _____
      _____
   2. _____
      _____
   3. _____
      _____

Two interesting things:
   1. _____
      _____
   2. _____
      _____

One question I still have:
   1. _____
      _____

Please email your question to your teacher so the needed clarification will be addressed.

ACTIVITY 7.   The Search Is On

Now that you have learned what a geometric sequence is, it is now time to find out how the terms of a geometric sequence are derived. By clicking this link, http://web.psjaisd.us/auston.cron/ABCronPortal/.../ppt/lat04_1103.ppt, you will be able to view the Powerpoint presentation below that will help you understand how to find the terms of a geometric sequence.

nth Term of a Geometric Sequence

In a geometric sequence with first term a_1 and common ratio r, the nth term, a_n, is given by

$$a_n = a_1 r^{n-1}.$$

▶ Example 1   **FINDING THE nth TERM OF A GEOMETRIC SEQUENCE**

Use the formula for the nth term of a geometric sequence to answer the first question posed at the beginning of this section. How much will be earned on day 20 if daily wages follow the sequence 1, 2, 4, 8, 16,...cents?

Solution

$$a_{20} = a_1 r^{19} = 1(2)^{19} = 524,288 \text{ cents, or } \$5242.88$$

Example 2 — FINDING TERMS OF A GEOMETRIC SEQUENCE

Find a_5 and a_n for the geometric sequence 4, 12, 36, 108,….

Solution

The first term, a_1, is 4. Find r by choosing any term after the first and dividing it by the preceding term. For example,
$$r = \frac{36}{12} = 3.$$

$$a_5 = 4\ g(3)^{5-1} = 4\ g3^4 = 324$$

Example 3 — FINDING TERMS OF A GEOMETRIC SEQUENCE

Find r and a_1 for the geometric sequence with third term 20 and sixth term 160.

Solution

Since $a_1 r^2 = 20$, $a_1 = \dfrac{20}{r^2}$.

Substitute this value for a_1 in the second equation.

$a_1 r^5 = 160$	$20r^3 = 160$	Quotient rule for exponents
$\left(\dfrac{20}{r^2}\right) r^5 = 160$	$r^3 = 8$	Divide by 20.
Substitute.	$r = 2$	Take cube roots.

7.3 - 12

Example 3 — FINDING TERMS OF A GEOMETRIC SEQUENCE

Find r and a_1 for the geometric sequence with third term 20 and sixth term 160.

Solution

Use the formula for the nth term of a geometric sequence.

For $n = 3$, $a_3 = a_1 r^2 = 20$.

For $n = 6$, $a_6 = a_1 r^5 = 160$.

Example 3 — FINDING TERMS OF A GEOMETRIC SEQUENCE

Find r and a_1 for the geometric sequence with third term 20 and sixth term 160.

Solution

Since $a_1 r^2 = 20$, and $r = 2$,

$$a_1(2)^2 = 20 \qquad \text{Substitute.}$$

$$4a_1 = 20$$

$$a_1 = 5. \qquad \text{Divide by 4.}$$

7.3 - 13

Example 4 — MODELING A POPULATION OF FRUIT FLIES

A population of fruit flies is growing in such a way that each generation is 1.5 times as large as the last generation. Suppose there were 100 insects in the first generation. How many would there be in the fourth generation?

7.3 - 14

Example 4 — MODELING A POPULATION OF FRUIT FLIES

Solution

Write the population of each generation as a geometric sequence with a_1 as the first-generation population, a_2, the second-generation population, and so on. Then the fourth-generation population is a_4. Using the formula for a_n, with $n = 4$, $r = 1.5$, and $a_1 = 100$, gives

$$a_4 = a_1 r^3 = 100(1.5)^3 = 100(3.375) = 337.5.$$

In the fourth generation, the population will number about 338 insects.

7.3 - 15

PROCESS QUESTIONS:

1. How do you find the nth term of a geometric sequence?

2. How do you think was the formula arrived at?

3. Is the formula presented applicable to the examples presented? How?

4. **How can the outcomes of certain real life problems be predicted?**

ACTIVITY 8. Skill Builder

After learning preliminary concepts on geometric sequences, it is now time to help you build your skills. In this activity, you will answer a worksheet that contains important skills that you need to practice.

Determine if the sequence is geometric. If it is, find the common ratio.
1) −1, 6, −36, 216,...
2) −1, 1, 4, 8,...
3) 4, 16, 36, 64, ...
4) −3, −15, −75, −375, ...
5) −2, −4, −8, −16, ...
6) 1, −5, 25, −125, ...

Given the explicit formula for a geometric sequence find the first five terms and the 8th term.
7) $a_n = 3^{n-1}$

$a_n = 2 \cdot \left(\frac{1}{4}\right)^{n-1}$ 8)

9) $a_n = -2.5 \cdot 4^{n-1}$

10) $a_n = -4 \cdot 3^{n-1}$

Given the recursive formula for a geometric sequence find the common ratio, the first five terms, and the explicit formula.

11) $a_n = a_{n-1} \cdot 2$
$a_1 = 2$

$a_n = a_{n-1} \cdot -3$ 12)
$a_1 = -3$

13) $a_n = a_{n-1} \cdot 5$
$a_1 = 2$

14) $a_n = a_{n-1} \cdot 3$
$a_1 = -3$

Given the first term and the common ratio of a geometric sequence find the first five terms and the explicit formula.

15) $a_1 = 0.8$, r = −5
16) $a_1 = 1$, r = 2

Given the first term and the common ratio of a geometric sequence find the recursive formula and the three terms in the sequence after the last one given.

17) a_1 − , r
18) a_1 , r
19) a_1 , r
20) a_1 − , r 4

Given a term in a geometric sequence and the common ratio find the first five terms, the explicit formula, and the recursive formula.

21) a_4 , r −
22) a_1 , r

Given two terms in a geometric sequence find the 8th term and the recursive formula.

23) a_4 −   and a_5 −
24) a_5 and a_2

25) a_1 – and a_5 –
26) a_5 and a_3

Determine if the sequence is geometric. If it is, find the common ratio.
1) −1, 6, −36, 216,...   r = -6
2) −1, 1, 4, 8,...   not geometric
3) 4, 16, 36, 64, ...   not geometric
4) −3, −15, −75, −375, ...   r = 5
5) −2, −4, −8, −16, ...   r = 2
6) 1, −5, 25, −125, ...   r = -5

Given the explicit formula for a geometric sequence find the first five terms and the 8th term.

7) $a_n = 3^{n-1}$ First five terms: 1, 3, 9, 27, 81 $a_8 = 2187$

$a_n = 2 \cdot \left(\frac{1}{4}\right)^{n-1}$ First five terms: 2, 1/2, 1/8, 1/32, 1/128 $a_8 = 1/192$

8)

9) $a_n = -2.5 \cdot 4^{n-1}$ First five terms: -2.5, -10, -40, -160, -640 $a_8 = -40960$

10) $a_n = -4 \cdot 3^{n-1}$ First five terms: -4, -12, -36, -108, -324 $a_8 = -8748$

Given the recursive formula for a geometric sequence find the common ratio, the first five terms, and the explicit formula.

11) $a_n = a_{n-1} \cdot 2$
$a_1 = 2$
Common ratio: 2
First five terms: 2, 4, 8, 16, 32
Explicit: $a_n = 2 (2^{n-1})$

12) $a_n = a_{n-1} \cdot -3$
$a_1 = -3$
Common ratio: -3
First five terms: -3, 9, -27, 81, -243
Explicit: $a_n = -3 (-3)^{n-1}$

13) $a_n = a_{n-1} \cdot 5$
$a_1 = 2$
Common ratio: 5
First five terms: 2, 10, 50, 250, 1250
Explicit: $a_n = 2 (5^{n-1})$

14) $a_n = a_{n-1} \cdot 3$
$a_1 = -3$
Common ratio: 3
First five terms: -3, -9, -27, -81, -243
Explicit: $a_n = -3 (3^{n-1})$

Given the first term and the common ratio of a geometric sequence find the first five terms and the explicit formula.

15) $a_1 = 0.8$, r =−5

First five terms: 0.8, -4, 20, -100, 500
Explicit: $a_n = 0.8 (-5)^{n-1}$

16) $a_1 = 1$, r =2

First five terms: 1, 2, 4, 8, 16
Explicit: $a_n = 2^{n-1}$

Given the first term and the common ratio of a geometric sequence find the recursive formula and the three terms in the sequence after the last one given.

17) a_1 − , r

Next three terms: -24, -144, -864
Recursive: $a_n = a_{n-1} (6)$; $a_1 = -4$

18) a_1 , r

Next three terms: 24, 144, 864
Recursive: $a_n = a_{n-1} (6)$; $a_1 = 4$

19) a_1 , r

Next three terms: 12, 72, 432
Recursive: $a_n = a_{n-1} (6)$; $a_1 = 2$

20) a_1 − , r 4

Next three terms: -16, -64, -256
Recursive: $a_n = a_{n-1} (4)$; $a_1 = -4$

Given a term in a geometric sequence and the common ratio find the first five terms, the explicit formula, and the recursive formula.

21) a_4 , r −

First five terms: -0.2, 1, -5, 25, -125
Explicit: $a_n = -0.2(-5)^{n-1}$
Recursive: $a_n = a_{n-1} (-5)$; $a_1 = -0.2$

22) a_1 , r

First five terms: 4, 20, 100, 500, 2500
Explicit: $a_n = 4(5)^{n-1}$
Recursive: $a_n = a_{n-1} (5)$; $a_1 = 4$

Given two terms in a geometric sequence find the 8th term and the recursive formula.

23) a_4 −   and
a_5 −

a_8 = -3/4
Recursive: $a_n = a_{n-1} (0.5)$; $a_1 = -96$

24) a_5 and a_2 $a_8 = 49152$
Recursive: $a_n = a_{n-1} (4); a_1 = 3$

25) a_1 –   and a_5 – $a_8 = 32768$
Recursive: $a_n = a_{n-1} (-4); a_1 = -2$

26) a_5 and a_3 $a_8 = 839808$
Recursive: $a_n = a_{n-1} (6); a_1 = 3$

ACTIVITY 9.   Muddiest Point

After practicing the different skills, you will now complete the journal below.

> The part of the lesson that I still find confusing is _____
> because _____.

ACTIVITY 10.   Interactive Website

Let us try to see if you have mastered the skill of finding the missing term in a geometric sequence.   Click on this link to do the activity:
http://www.ixl.com/math/algebra-1/geometric-sequences

PROCESS QUESTIONS:
1. How did you find the activity?

2. How many correct answers did you get?

3. What are the problems that you encountered?

ACTIVITY 11.   Geometric Series

It is now time to discover what a geometric series is all about. To find out, kindly read the article found in this website: http://wwwf.imperial.ac.uk/metric/metric_public/algebra/series/geometric_series.html. Make sure to take note of the significant difference between a geometric sequence and a geometric series. Also, bear in mind some formulas that are needed and the purpose of these.

PROCESS QUESTIONS:
1. What is a geometric series?

2. How is it different from a geometric sequence?

3. What are the two formulas that can be used to solve for geometric series?

4. What must be the reason why two formulas are provided?

ACTIVITY 12.   Finite And Infinite Geometric Series

A geometric series can be finite or infinite. It is now time to find out the difference between the two by watching two videos through this link:
VIDEO 1:
https://www.youtube.com/watch?v=XJ0oVPoEm60&list=PL8880EEBC26894DF4&index=100
VIDEO 2:
https://www.youtube.com/watch?v=lIYc8TYxbY8&index=101&list=PL8880EEBC26894DF4

Now that you have finished viewing the videos, let us find out if you are able to complete the Venn Diagram below completely.

Finite Geometri c Series   Infinite Geometri c Series
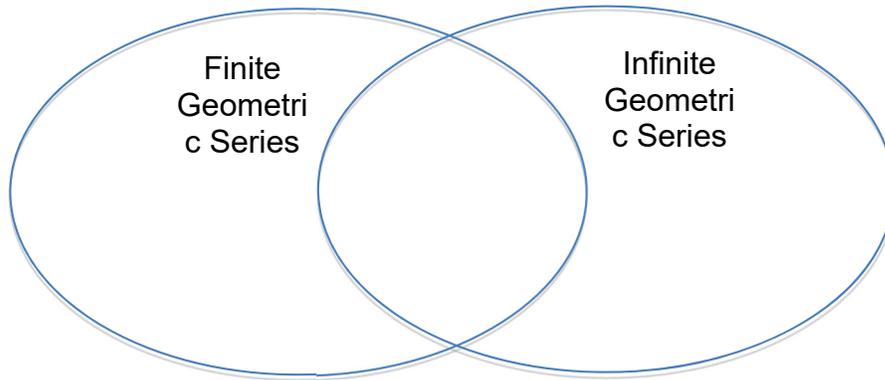
After completing the Venn diagram, complete the following statements below:

1. A finite geometric series and an infinite geometric series are similar because they _____ _____ _____.

2. They are different because _____ _____ _____ _____.

PROCESS QUESTIONS:
1. What were presented in the videos to help you compare the two ways of solving for a geometric series?

2. How do the formulas reveal the difference between the two?

_____

3. When will you know which type to use? How can you tell?

_____

ACTIVITY 13.   The Practicality Of Infinity

This activity will help you look at more examples of real life problems that make use of infinite geometric series. Watch the video through this link: https://www.youtube.com/watch?v=T989afJB0gM.

PROCESS QUESTIONS:
1. What kind of real life problems were presented in the video?

_____

2. Did the video help you see clearly the use of infinite geometric series? How?

_____

3. When can the concept of infinite geometric series be used in real life? Cite specific examples.

_____

ACTIVITY 14.   Skill Booster

After finding out the difference between finite and infinite geometric series, try to visit this link: http://www.shmoop.com/series/word-problem-exercises.html

and find out problems that use one or both types of geometric series. After reading each problem, try solving it on your own before checking for the answer.

PROCESS QUESTIONS:
1. How did you find the activity?

2. Can certain problems be solved using only one type of geometric series? Why?

3. How can you easily distinguish which type to use?

ACTIVITY 15. The Problem With The Ball

In this activity, you will now be asked to look at more examples of problems that make use of geometric series. Watch the video through this link: https://www.youtube.com/watch?v=Nx2hp_1MTuQ.

PROCESS QUESTIONS:
1. What kind of problem was presented in the video?

2. Why does the problem make use of geometric series?

3. What other things did you learn from the video?

4. Did you make use of finite or infinite geometric series? Why?

ACTIVITY 16.   Finding Meaning In A Geometric Mean?

Another important concept that you need to learn is the geometric mean. Find out what this is by watching the video through this link: https://www.youtube.com/watch?v=_UdGUULKN-E.   Be able to note its differences with the arithmetic mean which you learned in the previous lesson.

PROCESS QUESTIONS:

1. Define a geometric mean.

2. How is it solved?

3. How is it different from an arithmetic mean?

4. Why do you need to learn it?

ACTIVITY 17.   Mind Map

After finding out the different concepts which are important in understanding geometric sequences and series, you are now supposed to put the concepts that you have learned by creating a mind map of your own. You will use mindomo.

Click on this link to create your own mind map: https://www.mindomo.com/signup.htm. After creating the mind map, email it to your teacher.

PROCESS QUESTIONS:

1. How did you make your mind map?

    ┌───┐
    │   │
    │   │
    └───┘

2. Why did you choose to arrange it the way you did?

    ┌───┐
    │   │
    │   │
    └───┘

3. How did the mind map help you put together all the concepts that you learned?

    ┌───┐
    │   │
    │   │
    └───┘

4. What did you learn by making the mind map?

    ┌───┐
    │   │
    │   │
    └───┘

ACTIVITY 18. The Nature Of Numbers

After learning geometric sequences and series, it is now time to find out a new kind of sequence through this video: https://www.youtube.com/watch?v=P0tLbI5LrJ8.

PROCESS QUESTIONS:

1. What is the name of the sequence presented in the video?

    ┌───┐
    │   │
    │   │
    └───┘

2. Can you describe the new sequence that you just found out?

[ ]

3. How do you think is this new sequence related with nature?

[ ]

4. In what way can it be seen in the things we see around us?

[ ]

ACTIVITY 19.   Re-Search

You might be wondering how the sequence got its name. Allow yourself first to know the man behind this new sequence – Fibonacci. Use the resources on the Internet to do some research about his life. Use the graphic organizer below as your guide in doing this activity.

Name:

Biography Research

Persons Name:

Picture:

Early Life:

Family Life:

Major Accomplishments:

3 Interesting Facts:

ACTIVITY 20.   What's It All About?

To find out more about this sequence, click on this link and watch the video: https://www.youtube.com/watch?v=SjSHVDfXHQ4.

PROCESS QUESTIONS:

1. Discuss the Fibonacci sequence.

2. How does one identify Fibonacci numbers?

3. What new things did you find out in the video?

4. What other real life examples make use of the Fibonacci sequence?

ACTIVITY 21.   Investigative Mathematics

After finding out what this new sequence is all about, you will now be asked to do some investigation regarding the Fibonacci sequence. Please do the activity below.

Take any Fibonacci number and square it. Next find the product of the Fibonacci number before and after it. Compare this product with the square of the number. Copy and complete this table:

Fibonacci Number	Square of Fibonacci number	Product of Fibonacci Numbers before and after	Difference
2	4	1x3=3	1
3		2x5=...	
5			
8			
13			
21			
34			
55			

PROCESS QUESTIONS:

1. What do you notice in the table?

2. Look below to spot a pattern that occurs when the squares of Fibonacci numbers area added. Copy and complete the sequence:
   $1^2 = 1 \times 1$
   $1^2 + 1^2 = 1 \times 2$
   $1^2 + 1^2 + 2^2 = 2 \times 3$
   $1^2 + 1^2 + 2^2 + 3^2 = ... \times ......$
   $1^2 + 1^2 + 2^2 + 3^2 + 5^2 = ... \times ......$
   $1^2 + 1^2 + 2^2 + 3^2 + 5^2 + ... = ... \times ......$
   $1^2 + 1^2 + 2^2 + 3^2 + 5^2 + ... + ... = ... \times ......$

3. Continue the sequence for the next five sums.

4. What generalization can you make?

Now, let us look at the problem below and see how the Fibonacci sequence can be used to analyze real life problems.

Suppose a newly-born pair of rabbits, one male, one female, are put in a field. Rabbits are able to mate at the age of one month so that at the end of its second month a female can produce another pair of rabbits. Suppose that our rabbits never die and that the female always produces one new pair (one male, one female) every month from the second month on.

How many pairs of rabbits will there be after six months?

SOLUTION:

1. At the end of the first month, they mate, but there is still one only 1 pair.
2. At the end of the second month the female produces a new pair, so now there are 2 pairs of rabbits in the field.
3. At the end of the third month, the original female produces a second pair, making 3 pairs in all in the field.
4. At the end of the fourth month, the original female has produced yet another new pair, the female born two months ago produces her first pair also, making 5 pairs.
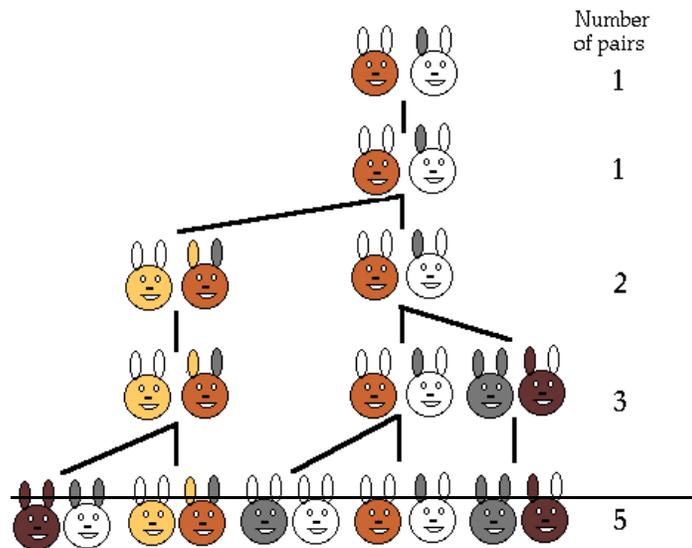
At the end of six months, there will be 12 pairs of rabbits.

ACTIVITY 22. Summing It Up (Tabular Comparison)

After learning all the sequences and series, you will now compare the different sequences that you have learned by completing the table below. Open Microsoft Word, then copy the table that you see below. Complete the table with the appropriate information needed. Save your work and send the file to your teacher.

Sequences	Description	Formulas	Real Life Examples	Significant Notes
Arithmetic				
Harmonic				
Geometric				
Fibonacci				

ACTIVITY 23.   IRF Revisit

It is now time to complete the "R" portion of the IRF Worksheet that you see below. Fill it up by writing your revised answer to the topical focus question. Click the save button after typing your answer.

IRF Worksheet
Initial Answer
Revised Answer
Final Answer

ACTIVITY 24.   Learning Log

After learning the preliminary concepts on geometric sequences and series and the Fibonacci sequence, write a one paragraph essay on your progress in this module. What are the things that you need more assistance on?

ACTIVITY 25.   Skills Check

Before you proceed with the next part of lesson, let us first determine whether you have already learned and mastered the different skills listed in the table below. Check the appropriate column/s.

Skills	Learned	Mastered

Illustrating a geometric sequence		
Differentiating a geometric sequence from an arithmetic sequence		
Differentiating a finite geometric series from an infinite geometric series		
Determining geometric means and nth terms of geometric sequences		
Finding the sum of the terms of a given finite and infinite geometric series		
Illustrating Fibonacci sequences		
Solving problems involving sequences		

If you have clarifications regarding the skills that you think you have not mastered yet, please do not hesitate to ask help from somebody or from your teacher. Feel free to post questions in the discussion forum as well.

End of FIRM UP:

In this section, the discussion was about the different sequences that you can use to predict outcomes of certain real life problems. You were also given the opportunity to learn how series can facilitate ease in solving real life problems.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision? What new learning goal should you now try to achieve?

Now that you know the important ideas about this topic, let us go deeper by moving on to the next section.

DEEPEN

Now that you have learned the important skills needed to understand sequences and series, your goals in this section is to look at some real-life situations where we can apply the concepts that you have learned.

In the next three activities, you will now be asked to solve real life problems that make use of geometric sequences and series. Show the solution to each problem and complete the statement below.

ACTIVITY 26.   The Mathematics In Banking

Suppose that Miguel saves P100 in January and that each amount thereafter he manages to save one-half more than what he has saved in the previous month. How much is Miguel's total savings after 10 months?

The   outcome   of   the   real   life   problem   is   easily   predicted   by
_____.
The   choice   of   the   type   of   series   to   be   used   depends   on
_____.

ACTIVITY 27.   The Mathematical Economy

A state government decides to stimulate the economy by giving P500 to every adult. The government assumes that everyone who receives the money will spend 80% on consumer goods and that the producers of these goods will in turn reinvest 80% on consumer goods. How much money is generated for the economy for every P500 that the government provides?

The outcome of the real life problem is easily predicted by
_____.
The choice of the type of series to be used depends on
_____.

ACTIVITY 28.   The Music of Math

Julian sees a new band at the concert. He emails a link for the band's website to five of his friends. They each forward the link to five of their friends. The link is forwarded again following the same pattern. What is the total number of emails sent if there are eight rounds?

The outcome of the real life problem is easily predicted by
_____.
The choice of the type of series to be used depends on
_____.

ACTIVITY 29.   Putting It All Together

You will now revisit the different situations that you have used to apply the concepts of geometric sequences and series. What are the common things that you notice among the various situations presented?

> From the various real-life connections presented in the different activities, I noticed that the situations have the following things in common:
> 1. _____ ___
> 2. _____ ___
> 3. _____

ACTIVITY 30.   Processing

Look at the answers to the statements found after each problem.

PROCESS QUESTIONS:
1. Were you able to observe a pattern in each problem?

2. How did you come up with each equation?

3. Was there only one way to solve it? If yes, explain. If not, why are there many ways to solve the problem? Discuss and cite examples.

4. How is each equation useful in solving the problem?

5. If the equations will differ, how do you know which one to use? Explain.

6. Complete the statements below and support your answer with examples from the above problems.

The outcome of the real life problem is easily predicted by
_____.
The choice of the type of series to be used depends on
_____.

ACTIVITY 31.   One More Time

After summing up everything, try solving the problem below. Try to see if the generalizations that you have made can also be used in the new situation.

Nate has P15000 in credit card debt when he graduated from college. The balance increased by 2% each month due to interest, and he could only make payments of P400 per month. Determine the balance after five months.

The outcome of the real life problem is easily predicted by _____.
The choice of the type of series to be used depends on _____.

End of DEEPEN:

In this section, the discussion was about using the concepts of geometric sequences and series to be able to solve real-life problems. Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.

TRANSFER

Your goal in this section is to apply your learning to real life situations by doing scaffold activities which will help you in the making of final project.   You will be given a practical task which will allow you to demonstrate your understanding of the basic concepts you have learned in this lesson.

ACTIVITY 32.   Scaffold for Transfer 3

Due to the implementation of the K to 12 Program, your school decided to open a Senior High School that will not only cater to the graduating students of your Junior High School but also to the other students coming from nearby schools that will not have Grades 11 and 12. Because of this, the administration and the Board of Trustees deem it possible to transfer the basic education unit to a new campus. However, there are financial constraints that the school needs to deal with.

As the head of the ad hoc committee that is tasked to help the school look for its needed financial resources, you are tasked to examine the loans offered by BPI, Banco de Oro and Metrobank. The school intends to borrow P30,000,000.00 and plans to pay this in five years. Your task is to make a comparison of the various offers, and come up with recommendations afterwards.   Your recommendations will be presented to the administration and the Board of Trustees. It will be graded according to the following criteria: accuracy of representation and computations, practicality of the recommendation, authenticity of data and applications of sequences.

ACTIVITY 33.   THE FINAL TASK

You have now reached the final task for this module. Presented below are three tasks you can choose from. These tasks will allow you to consider various situations in real life that make use of the concepts of sequences and series. You will only choose one task which you prefer to do. The same standards will be used to assess your work.

TASK 1:
You are working as one of the financial advisers of a leading firm in your city. A newlywed couple comes to your office and asks you to assist them in deciding which among the housing loans offered by BPI, Banco de Oro and Metrobank will allow them to save money and pay off the loan within 15 years. You are tasked to make a housing loan comparison and come up with

recommendations after wards. Your recommendations will be presented to the head of your firm and the couple. It will be graded according to the following criteria: accuracy of representation and computations, practicality of the recommendation, authenticity of data and applications of sequences.

TASK 2:
      You are a manager of a certain rural bank in your province. Since your bank is new to the community, the owner requested you to come up with schemes that will entice potential clients to deposit their money in the bank. You are expected to come up with a brochure that shows the possible schemes that clients could choose from. Your recommendations will be presented to the head of your firm and the couple. You work will be graded according to the following criteria: accuracy of representation and computations, practicality of the recommendation, authenticity of data and applications of sequences.

TASK 3:
      You are a finance officer of a certain advertising company. The Chief Operating officer wants to know the future salary schemes that the company could offer to its employees in the next five years following the existing fixed percentage increase per year. You are expected to come up with a five-year salary scale scheme for the employees of the company. You work will be graded according to the following criteria: accuracy of representation and computations, practicality of the recommendation, authenticity of data and applications of sequences.

Scoring Rubric

Criteria	4 Excellent	3 Proficient	2 Progressing	1 Beginning
Authenticity of Data	The data used are authentic and updated. Data are taken from reliable resources.	The data used are authentic. Data are taken from reliable resources.	Some of the data used are not authentic.	Data are not useful and relevant.
Accuracy of the Computations	Computations are accurate and supported with correct and detailed interpretation.	Computations are accurate and supported with correct interpretation	Computations are correct but interpretation is incorrect.	Most of the computations and interpretations are erroneous.
Practicality of Recommen dation	The recommendation suggests a bank which offers the	The recommendation suggests a bank which offers a	The recommendation suggests a bank which offers a	The recommendation suggests a bank which offers the

	lowest loan interest which will be paid for 15 years.	reasonable loan interest which will be paid for 15 years.	bigger loan interest which will be paid for 15 years.	biggest loan interest which will be paid for 15 years.
Application of the Concepts of Sequences	The formulation of mathematical equation derived from sequences and series is correct with additional model to support it.	The formulation of mathematical equation derived from sequences and series is correct.	The formulation of mathematical equation derived from sequences and series does not use the defined variables.	The formulation of mathematical equation derived from sequences and series is incorrect.

ACTIVITY 34.   IRF Revisit

It is now time to complete the "F" portion of the IRF Worksheet that you see below. Fill it up by writing your revised answer to the topical focus question. Click the save button after typing your answer.

IRF Worksheet
Initial Answer
Revised Answer
Final Answer

ACTIVITY 35.   Reflection on The Performance Task

After doing the final task, look back at your experience as you complete the log below:

    1. What have I learned?

    2. How do I feel with what I've learned?

    3. What can I do with what I've learned?

Should you have any clarifications on this lesson, type your question and email this to the teacher or post it to the discussion forum.

ACTIVITY 36.   Synthesis Journal

The unit's lesson was on _____.
One key idea was _____. This is important because _____.
Another key idea was _____.
This is also important because _____.
In summary, the unit's lesson _____
_____
_____.

End of TRANSFER:

You have completed this lesson. Before you go to the next module, you have to answer the following post-assessment.

MODULE: POST-ASSESSMENT

> *It's now time to evaluate your learning. Click on the letter of the answer that you think best answers the question. Your score will only appear after you answer all items. If you do well, you may move on to the next module. If your score is not at the expected level, you have to go back and take the module again.*

1.   Which is the next term in the sequence 4, 7, 10, 13, 16, . . . ?

    A. 19
    B. 21
    C. 23
    D. 25

2.   Which of the following sequences is NOT arithmetic?

    A. 2, 6, 10, 14, ...
    B. −60, −45, −30, −15, ...
    C. 54, 51, 48, 45, ...
    D. $\frac{1}{2}$, 1, 2, 4, ...

3.   Find the next three terms of the geometric sequence below.
                          4, 12, 36, ___, ___, ___

    A. 104, 312, 936
    B. 108, 324, 972
    C. 72, 108, 324
    D. 72, 216, 648

4.   Which is the third term of the sequence defined by $a_n = 4n + 6$?

    A. 6
    B. 10
    C. 14
    D. 18

5.   What is the rule for the nth term of the arithmetic sequence with $a_{21} = 147$ and common difference d = 11?

    A. $a_n = 11n - 21$

B. $a_n = 11n - 42$
C. $a_n = 11n + 21$
D. $a_n = 11n - 84$

6. Which is the rule for the nth term of the geometric sequence -3, -6, -12, -24, -48, . . . ?

A. $a_n = 2(-3)^{n-1}$
B. $a_n = -3(2)^{n-1}$
C. $a_n = 3(-2)^{n-1}$
D. $a_n = -3(-2)^{n-1}$

7. Which is the sum of the first twelve terms of the geometric series 1 + 2 + 4 + 8 + 16 + . . ?

A. 4095
B. 1060
C. 880
D. 1850

8. What is the common ratio of an infinite geometric series whose sum is 30 and the first term is 6?

A. 2/3
B. 1/5
C. 3/4
D. 4/5

9. Joan started a new job with an annual salary of P150 000 in 1997. If she receives a P12 000 raise each year, how much will her salary be in 2007?

A. P250 000
B. P270 000
C. P290 000
D. P302 000
The first term is 150000 and the common difference is 12000. Using the formula,
a_{10}= *150000 + (10) 12000 = 270000*

10. In June, Cory begins to save money for a video game and a TV he wants to buy in December. He starts with $20. Each month, he plans to save 10% more than the previous month. How much money will he have at the end of December?

A. $154.31
B. $251.59

C. $228.72
D. $189.74

11.   A large asteroid crashed into a moon of a planet, causing several boulders from the moon to be propelled into space toward the planet. Astronomers were able to measure the speed of one of the projectiles. The distance (in feet) that the projectile traveled each second, starting with the first second, was given by the arithmetic sequence 26, 44, 62, 80, … Find the total distance that the projectile traveled in seven seconds.

A. 434 feet
B. 560 feet
C. 212 feet
D. 426 feet

12.   A mine worker discovers an ore sample containing 500 mg of radioactive material. It is discovered that the radioactive material has a half life of 1 day. Find the amount of radioactive material in the sample at the beginning of the 7th day.

A. 6 mg
B. 7.8125 mg
C. 8.025 mg
D. 9.012 mg

13.   You visit the Grand Canyon and drop a penny off the edge of a cliff. The distance the penny will fall is 16 feet the first second, 48 feet the next second, 80 feet the third second, and so on in an arithmetic sequence. What is the total distance the object will fall in 6 seconds?

A. 176 ft.
B. 300 ft.
C. 576 ft.
D. 1250 ft.

14.   After knee surgery, your trainer tells you to return to your jogging program slowly. He suggests jogging for 12 minutes each day for the first week. Each week thereafter, he suggests that you increase that time by 6 minutes per day. How many weeks will it be before you are up to jogging 60 minutes per day?

A. 9 weeks
B. 8 weeks
C. 6 weeks
D. 12 weeks

15. Your brother wishes to save ₱300.00 for ten days. He plans to save ₱20.00 on the first day from his allowance of ₱100.00.   On the second day, he plans to increase his savings by ₱5.00 and so on. Given that he needs to spend a minimum of ₱50.00 a day for food and allot some amount for class projects, what recommendations can you suggest so he will be able to target his goal?

   A. Start saving ₱10.00 on the first day and increase his savings by ₱5.00 per
       day.
   B. Save ₱50.00 a day for seven days.
   C. Increase the interval per day.
   D. Save ₱20.00 per day for the first seven days and divide the remaining amount equally for the last three days.

16. You are an accomplished architect in your city. Since your school is transferring to another campus, you are hired to design its theater. You decide to design the theater with 15 seats in the first row, 18 in the second, 21 in the third, and so on. If the school requires that the theater should have a seating capacity of 870, how many rows must you use in the design?

   A. 20
   B. 24
   C. 26
   D. 28

17. Consider the sequence: 16, -8, 4, -2, 1
   Which is the best description of the pattern formed?

   A. Multiply the previous term by 1/2.
   B. Divide the previous term by -2.
   C. Multiply the previous term by -1/4.
   D. Subtract from the previous term 8.

18. A recovering heart attack patient is told to get on a regular walking program. The patient is told to walk a distance of 5 km the first week, 8 km the second week, 11 km the third week and so on for a period of 10 weeks. At that point the patient is to maintain the distance walked during the 10th week. How far will the patient walk during the 10th week?

   A. 32
   B. 35
   C. 36
   D. 39

19. You are working as one of the financial advisers of a leading firm in your city. A client comes to your office and asks you to assist them in deciding which among the housing loans offered by BPI, Banco de Oro and Metrobank will allow them to save money and pay off the loan within 15 years. You are tasked to make a housing loan comparison and come up with recommendations afterwards.

    Which should be the characteristics of you report?

    A. Profile of the Bank and Authenticity of Data
    B. Accuracy of Computations and Explanation of the Schemes
    C. Practicality of the Recommendation and Neatness of the Report
    D. Practicality of the Recommendation and Authenticity of Data

20. You are tasked to forward an announcement to all alumni in your batch regarding the batch's upcoming reunion. Which of the following strategy will you use in order to connect to the majority of your batch mates?

    A. Post posters around the city.
    B. Make announcements in radio.
    C. Make announcements in TV.
    D. Email 4 batch mates and ask each batch mate to email the information to another 4 batch mates, and so on.

GLOSSARY OF TERMS USED IN THIS LESSON:

ARITHMETIC MEAN - is the sum of a collection of numbers divided by the number of numbers in the collection.

ARITHMETIC SEQUENCE - is one in which the same number is added or subtracted from each element to get the next element in the sequence.

FIBONACCI NUMBERS - isthe integer sequence whose elements are the sum of the previous two elements

GEOMETRIC MEAN–is the central number in a geometric progression (e.g., *9* in *3, 9, 27*), also calculable as the *n* th root of a product of *n* numbers.

GEOMETRIC SEQUENCE - is a sequence of numbers where each term after the first is found by multiplying the previous one by a fixed, non-zero number called the common ratio.

HARMONIC MEAN - is the reciprocal of the arithmetic mean of the reciprocals.

HARMONIC SEQUENCE - is a progression formed by taking the reciprocals of an arithmetic progression.

INFINITE GEOMETRIC SEQUENCE –is a sequence which is infinite in one direction, and finite in the other—the sequence has a first element, but no final element.

SEQUENCE –is a list of elements with a particular order.

SERIES –is the sum of the terms of a sequence.

REFERENCES AND WEBSITE LINKS USED IN THIS LESSON:

LESSON 1: Arithmetic Sequences and Series

Videos:
https://www.youtube.com/watch?v=0HzXapUeL5YRetrieved September 15, 2014. This video explains pattern rules, number patterns and symmetrical patterns
https://www.youtube.com/watch?v=p-rc5mTDt9E Retrieved September 15, 2014. This video defines a sequence and its types.
http://www.virtualnerd.com/algebra-2/sequences-series/arithmetic/arithmetic-sequences/arithmetic-sequence-definition Retrieved Nov. 4, 2014. This video defines an arithmetic sequence and its examples.

http://my.hrw.com/math06_07/nsmedia/lesson_videos/alg2/player.html?contentSrc=6498/6498.xmlRetrieved Nov. 4, 2014. This video describes the characteristics of an arithmetic sequence.

http://www.virtualnerd.com/algebra-2/sequences-series/arithmetic/arithmetic-sequences/sequence-next-terms-exampleRetrieved Nov. 4, 2014. This video shows how to find the next term of an arithmetic sequence.
http://www.virtualnerd.com/algebra-2/sequences-series/arithmetic/arithmetic-sequences/arithmetic-sequence-definition Retrieved Nov. 4, 2014. This video shows how the formula for the nth term of an arithmetic sequence was derived.
http://www.virtualnerd.com/algebra-2/sequences-series/arithmetic/arithmetic-sequences/sequence-nth-term-example Retrieved Nov. 4, 2014. This video shows how to find the nth term of an arithmetic sequence using the formula.

http://my.hrw.com/math06_07/nsmedia/lesson_videos/alg2/player.html?contentSrc=7224/7224.xml Retrieved Nov. 4, 2014. This video shows how to find the nth term of an arithmetic sequence using the formula.

https://www.youtube.com/watch?v=bM4bsgEh7p4 Retrieved Nov. 4, 2014. This video shows how to determine the missing term of an arithmetic sequence given the first and last terms.

http://my.hrw.com/math06_07/nsmedia/lesson_videos/alg2/player.html?contentSrc=7225/7225.xml Retrieved Nov. 4, 2014. This video shows how to determine the missing term of an arithmetic sequence given the first and last terms.

https://www.youtube.com/watch?v=CR8Aoy8fsvc . Retrieved November 4, 2014. This video shows how to identify harmonic sequence, solve for its missing and nth term/s.

http://www.virtualnerd.com/algebra-2/sequences-series/series-definition.php RetrievedNovember 4, 2014.This video defines a series.

https://www.youtube.com/watch?v=Dj1JZIdIwwo This video shows how to find the sum of an arithmetic series

http://my.hrw.com/math06_07/nsmedia/lesson_videos/alg2/player.html?contentSrc=7227/7227.xml This video shows another way using only the first formula of an arithmetic series.

http://my.hrw.com/math06_07/nsmedia/lesson_videos/alg2/player.html?contentSrc=7228/7228.xml as sample problem. This video will show you how to solve problems applying both arithmetic sequence and series.

Interactive Website
http://go.hrw.com/math/midma/gradecontent/loadlesson.html?course=c3&chapter=12&lesson=1&SE=1&sz_audio=1&calc=1&state=xx&actCourse=4 Retrieved August 25, 2024. This interactive website provide skills practice in identifying whether a given sequence is arithmetic or not; determining the common difference; determining the missing term in a sequence; determine the nth term; solving real-life problem.

http://www.ixl.com/math/algebra-1/arithmetic-sequenceshttp://www.ixl.com/math/algebra-1/arithmetic-sequencesRetrieved August 25, 2024. This interactive website provide skills practice in identifying whether a given sequence is arithmetic or not; determining the common difference; determining the missing term in a sequence; determine the nth term; solving real-life problem.

PDF
Download a pdf worksheet from http://lhsblogs.typepad.com/files/am3-harmonic_fibonacci-ws.pdfRetrieved Nov. 4, 2014. It a worksheet in identifying harmonic sequence.

Articles
http://coolmath.com/algebra/19-sequences-series/06-gauss-problem-arithmetic-series-01.htmRetrieved Nov. 4, 2014. This is 7-page article which tells the story of young Karl Friedrich Gauss who astonishingly get the sum of the numbers from 1 to 100 in few seconds only; shows Karl's technique in getting the sum of numbers 1 to 100; explains how the formula for the sum of arithmetic series is derived; allows you to practice getting the sum of a given sequence.

LESSON 2: Geometric Sequences and Series

Articles
https://share.ehs.uen.org/sites/default/files/Unit06Lesson03_0.pdf.
       This is an article about geometric sequences.

https://share.ehs.uen.org/sites/default/files/Unit06Lesson04_0.pdf
       This article compares arithmetic and geometric sequences.

http://www.kutasoftware.com/FreeWorksheets/Alg2Worksheets/Geometric%20Sequences.pdf
       This worksheet contains questions about geometric sequences.

http://wwwf.imperial.ac.uk/metric/metric_public/algebra/series/geometric_series.html.
       This article is about geometric series.

http://www.shmoop.com/series/word-problem-exercises.html
       This will enable the student to look at word problems involving geometric series.

Videos
https://www.youtube.com/watch?v=ri767qbKLc4.
       This will assist the students in determining whether the sequence is arithmetic or geometric.
https://www.youtube.com/watch?v=XJ0oVPoEm60&list=PL8880EEBC26894DF4&index=100
       This is about the difference between finite and infinite geometric series.

https://www.youtube.com/watch?v=IIYc8TYxbY8&index=101&list=PL8880EEBC26894DF4
       This is about the difference between finite and infinite geometric series.

https://www.youtube.com/watch?v=T989afJB0gM.
       This video discusses problems on infinite geometric series.

https://www.youtube.com/watch?v=Nx2hp_1MTuQ.
       This will allow the student to look at more examples of problems that make use of geometric series

https://www.youtube.com/watch?v=_UdGUULKN-E.
       This video is about getting the geometric mean.

https://www.youtube.com/watch?v=P0tLbI5LrJ8.
       This video introduces the Fibonacci sequence.

Powerpoint Presentation
http://web.psjaisd.us/auston.cron/ABCronPortal/.../ppt/lat04_1103.ppt
This will enable the student to know how to get the terms of a geometric sequence

Interactive Website
http://www.ixl.com/math/algebra-1/geometric-sequences
This is all about finding the terms in a geometric sequence