## LEARNING MODULE

Mathematics G10 | Q2

## Geometry



## NOTICE TO THE SCHOOLS

This learning module (LM) was developed by the Private Education Assistance Committee under the GASTPE Program of the Department of Education. The learning modules were written by the PEAC Junior High School (JHS) Trainers and were used as exemplars either as a sample for presentation or for workshop purposes in the JHS InService Training (INSET) program for teachers in private schools.

The LM is designed for online learning and can also be used for blended learning and remote learning modalities. The year indicated on the cover of this LM refers to the year when the LM was used as an exemplar in the JHS INSET and the year it was written or revised. For instance, 2017 means the LM was written in SY 2016-2017 and was used in the 2017 Summer JHS INSET. The quarter indicated on the cover refers to the quarter of the current curriculum guide at the time the LM was written. The most recently revised LMs were in 2018 and 2019.

The LM is also designed such that it encourages independent and self-regulated learning among the students and develops their 21st century skills. It is written in such a way that the teacher is communicating directly to the learner. Participants in the JHS INSET are trained how to unpack the standards and competencies from the K-12 curriculum guides to identify desired results and design standards-based assessment and instruction. Hence, the teachers are trained how to write their own standards-based learning plan.

The parts or stages of this LM include Explore, Firm Up, Deepen and Transfer. It is possible that some links or online resources in some parts of this LM may no longer be available, thus, teachers are urged to provide alternative learning resources or reading materials they deem fit for their students which are aligned with the standards and competencies. Teachers are encouraged to write their own standards-based learning plan or learning module with respect to attainment of their school's vision and mission.

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## Module 2: Geometry



Have you thought of why Geometry is difficult to understand and often perceived to have confusing set figures like circles, squares, rectangles and etc. Have you at a certain time asked yourself how can challenging problems involving geometric figures be analyzed and solved?

A very famous mathematician called Rene Descartes lay in bed one night. As he lay there, he looked up at the ceiling in his bedroom. He noticed a fly was asleep on the ceiling. Descartes, being a mathematician wondered if he could figure out a way of stating where exactly the fly was on the ceiling. Obviously it has to be a precise description he thought.

The discovery of Rene Descartes serves now as one of the foundations of geometry which was used as a tool for more discoveries which lead to modernization.

In this module, you will pay attention to circle and distance formula where the knowledge and skills learned in the previous lessons can be applied. You will find out the different concepts involved in distance formula. As you go over the exercises, you will be able to answer the questions: How can problems where two quantities bounded by conditions are solved? How do related quantities affect each other? For example, how do you determine the strategic position of a camera/ video recorder in a specific location and finally you need to be able to answer

In this module, you will examine this question when you take the following lessons:

Lesson 1 - Chords, Arcs, and Angles, Secants, Tangents, Segments and Sectors
Lesson 2 - Distance Formula
Lesson 3 - Equation of a Circle
In these lessons, you will learn the following:

| Lesson 1 | - Derives inductively the relations among chords, arcs, central angles, and inscribed angles. <br> - Proves theorems related to chords, arcs, central angles, and inscribed angles. <br> - Illustrates secants, tangents, segments, and sectors of a circle. <br> - Proves theorems on secants, tangents, and segments. <br> - Solves problems on circles. |
| :---: | :---: |
| Lesson 2 | - Derives the distance formula <br> - Applies the distance formula to prove some geometric properties <br> - Graphs other geometric figures on the coordinate plane <br> - Solves problems involving geometric figures on the coordinate plane |
| Lesson 3 | - Illustrates the center-radius form of the equation of a circle. <br> - Determines the center and radius of a circle given its equation and vice versa. <br> - Graphs a circle and other geometric figures on the coordinate plane. |

■ MODULE MAP:
Here is a simple map of the above lessons you will cover:

$\boxtimes$ EXPECTED SKILLS:

To do well in this module, you need to remember and do the following:

1. Look up the meaning of words you do not know.
2. Complete all activities and exercises.
3. Take note on the proper modeling of situations using oral, written, graphical and algebraic methods to solve problems.
4. Use the checklist and rubric provided to evaluate your work before submission.
5. Be mindful of the meaning of unfamiliar words you encounter in this module. A glossary of terms is provided in the last part of this module.
6. Maximize the use of online resources in each lesson. Online resources can be accessed multiple times. The summary of online resources is provided in the end of the module.

## Lesson 1. Chords, Arcs, Angles, Tangents and Secants

## PRE-ASSESSMENT:

Let's find out how much you already know about this module. Click on the letter that you think best answers the question. Please answer all items. After taking this short test, you will see your score. Take note of the items that you were not able to correctly answer and look for the right answer as you go through this module.

1. Which of the following defines a chord?
A. A line that passes through the circle at exactly one point.
B. A line that passes through the circle at exactly two points.
C. A segment that passes through the circle at exactly one point.
D. A segment whose endpoints are on the circle.
2. If the measure of the central angle is $45^{\circ}$, what is the measurement of the intercepted arc?
A. $\quad 22.5^{\circ}$
B. $45^{\circ}$
C. $\quad 90^{\circ}$
D. $135^{\circ}$
3. You are a professional photographer. Your camera has a 90 field of vision and you want to photograph the front of a statue. You move to a spot where the statue is the only thing captured in your picture, as shown. You want to change your position. Where else can you stand so that the statue
is perfectly framed?

A. You may stand to any point on the semi- circle in front of the statue.
B. You may stand to any point equidistant from your distance to the statue.
C. You may move to the other side and stand to any point equidistant from your distance to the statue.
D. There is no other place where you can capture the whole picture of the statue but your original position.
4. A surveillance camera is mounted on a corner of a building. It rotates clockwise and counterclockwise continuously between Wall A and Wall B at a rate of $10^{\circ}$ per minute as illustrated in the figure. How long does it take the camera to survey the entire area once? Justify your answer.
A. 4 minutes and 30 seconds, since the measure of the angle is half of the intercepted arc.

B. 9 minutes, since the angle and arc have equal measures.
C. 18 minutes, since the measure of the angle is twice the measure of the intercepted arc.
D. 27 minutes, the camera needs to survey about $135^{\circ}$ of the circle.
5. You are a professional photographer, a company requested you to submit a set of pictures of beautiful sceneries. Which among the following should you consider most to in selecting your pictures?
A. Pictures are very clear but some important parts are not captured.
B. All parts are captured but no highlighted subject.
C. You have a focus subject and important parts are visible.
D. All parts are captured but some parts are not that clear.
6. If rays WE and WI are both tangents to the circle P below, then which of the following is NOT necessarily true?
a. $E W=I W$
b. $\mathrm{m} \angle \mathrm{PEW}=\mathrm{m} \angle \mathrm{PIV}$
c. $\angle \mathrm{EWP} \cong \angle \mathrm{IWP}$
d. $\angle \mathrm{PEW}$ and $\angle \mathrm{EWI}$

7. If ray $m$ and $n$ are tangents to circle $P$ below, then which of the following is the value of $w$ ?
A.
$30^{\circ}$
B. $40^{\circ}$
C. $50^{\circ}$
D. $60^{\circ}$


If ray $m$ and $n$ are tangents to circle $P$ below, then which of the following is the value of $w$ ?
a. $30^{\circ}$
b. $40^{\circ}$
c. $50^{\circ}$
d. $60^{\circ}$
8. In the figure below, if segment MT is tangent to the circle, segments NT and PT are secants, PS is a semicircle and the measure of minor arc MP is $140^{\circ}$, then what is the measure of $\angle \mathrm{MTP}$ ?
A. $50^{\circ}$
B. $60^{\circ}$
C. $90^{\circ}$
D. $100^{\circ}$

9. If $R S=V W$ in the figure below, then which of the following will justify that $\triangle \mathrm{RVX}$ is congruent to $\triangle \mathrm{WSX}$ ?

A. SAA Congruence Theorem
B. SAS Congruence Postulate
C. SSS Congruence Postulate
D. Hypotenuse-Leg Theorem
10. As an Archeologist, you volunteer to help a group of students to determine the center of a circular object that they have found. Which of the following will help you explain how the T-square will be an effective instrument to do so?

a. Tangents are perpendicular to the radii at the points of tangency.
b. The tangent segments intersecting at a point outside a circle are congruent.
c. The measure of an angle formed by two secants intersecting outside a circle is half the measures of the difference of the intercepted arcs of the circle.
d. The measure of an angle formed by a secant and a tangent to a circle, intersecting outside a circle is half the measures of the difference of the intercepted arcs of the circle.
11. What is the distance between the points $(3,3)$ and $(6,7)$ ?
a. 3
b. 5
c. 7
d. 9
12. The point $(5,4)$ lies on a circle. What is the length of the radius of this circle if the center is located at $(3,2)$ ?
a. $\sqrt{8}$
b. $\sqrt{3}$
c. $\sqrt{18}$
d. 8
13. If plotted in the coordinate plane, which of the following geometric figures best describes the coordinates $A(-6,4), B(-6,-4), C(0,4)$ and $D(0,-4)$ ?
a. Square
b. Rectangle
c. Rhombus
d. Kite
14. On a map's coordinate grid, Brgy. Malaya is located at $(2,4)$ and Brgy. Pasong Tamo is located at (2, 2). Brgy. Katapan is the in the middle of the two barangay. What is the distance from Brgy. Malaya to Brgy. Katapatan?
a. 2 km
b. 4 km
c. 6 km
d. 8 km
15. You are a city planner who is requested by the city mayor to provide a vicinity map to be used in the municipal strategic planning. Which of the following standards will be assessed your work?
a. Accuracy, Clarity, application of the concept of distance formula
b. Accuracy, Clarity, practicality
c. Practicality, authenticity, use of grid lines
d. Practicality, Accuracy, presentation
16. Low - Earth orbit satellites occupies a region of space from 180-2000 kilometers above Earth. If the satellite is moving in a circular path with Earth as the point of origin, which circular equation represents the lowest path of the satellite?
A. $x^{2}+y^{2}=180$
B. $x^{2}+y^{2}=2000$
C. $x^{2}+y^{2}=32,400$
D. $x^{2}+y^{2}=4,000,000$
17. Based on the figure below, what equation that represents the circle?

I. $(x-3)^{2}+(y-1)^{2}=5 \quad$ II. $x^{2}+y^{2}-6 x-2 y+5=0$ III. $x^{2}+y^{2}-6 x-2 y$
$-5=0$
A. I only
B. II only
C. I and II only
D. I, II and III
18. To determine the radius and coordinates of the center of the circle $x^{2}+y^{2}-6 x$ $+2 y+8=0$, Nestor and Andy came up with the following solutions:

| Nestor's Solution | Andy's Solution |
| :--- | :--- |
| $x^{2}+y^{2}-6 x+2 y+8=0$ | $x^{2}+y^{2}-6 x+2 y+8=0$ |
| $x^{2}-6 x+\_+y^{2}+2 y+-$ | $x^{2}-6 x+\ldots+y^{2}+2 y+\ldots=$ |
| $=-8$ | -8 |
| $x^{2}-6 x+(-3)^{2}+y^{2}+2 y+$ | $x^{2}-6 x+(-3)^{2}+y^{2}+2 y+(1)^{2}$ |
| $(1)^{2}=-8$ | $=-8+9+1$ |
| $(x-3)^{2}+(y+1)^{2}=-8$ | $(x-3)^{2}+(y+1)^{2}=2$ |
| Radius $=\sqrt{-8 ;}$ center: $(3$, | Radius $=\sqrt{2}$; center: $(3,-1)$ |
| $-1)$ |  |
|  |  |

Which statement best describes the solutions of Nestor and Andy?
A. The solution of Nestor is correct. Andy committed an error in when he added 9 and 1 on the right side of the equation.
B. The solution of Andy is correct. Nestor committed an error when he failed to add 9 and 1 on the right side of the equation.
C. The solution of Andy and Nestor are both correct.
D. The solution of Andy and Nestor are both incorrect.
19. When delivering weather forecast during typhoons, weather forecasters uses diagrams and charts to illustrate the movement of the typhoon. The circles indicate the affected area of the typhoon at a particular time. If you are the forecaster, which statement best describes typhoon Henry based on the illustration below?

A. Typhoon Henry grew stronger from July 20 - 23, 2014.
B. Typhoon Henry move faster from July 20 - 23, 2014.
C. Typhoon Henry's coverage increased from July $20-23,2014$.
D. Typhoon Henry's coverage did not change from July $20-23,2014$.
20.The barangay officials recently reported alarming news that the number of crimes in your community is drastically increasing. The barangay council decided to put up a CCTV camera in the area with the most number of crime incidents. You are a marketing officer of a CCTV company that will participate in the bidding to provide the barangay with the CCTV unit. You need to identify the best location of the CCTV unit in the area that is identified in your barangay. You will present your recommendation and justification to the barangay officials. Which are the appropriate standards that your work should be evaluated?
A. organization, neatness, practical recommendations, application of the concepts of the properties of circles
B. neatness, presentation, practical recommendations, application of the concepts of the properties of circles
C. creativity, accurate computation, practical recommendations, application of the concepts of the properties of circles
D. authentic data, accurate computation, practical recommendations, application of the concepts of the properties of circles
$\boxtimes$ In this lesson you will learn the following:

1. Describes a chord, arc and angle.
2. Illustrates chords, arcs, and angles.
3. Describes the relationship between the central angle and the intercepted arc.
4. Describes the relationship between the inscribed angle and the intercepted arc.
5. Derives inductively the relations among chords, arcs, central angles, and inscribed angles.
6. Proves theorems related to chords, arcs, central angles, and inscribed angles.
7. Solves problems on circles.
8. Proves theorems on secants, tangents, and segments.
9. Solves problems on circles.

## EXPLORE

You learned from the previous level some geometric figures involving circles. You also encountered some problems and terms about circle which are important in learning Geometry.

In this lesson, you will learn the concepts of chords, arcs, and angles which are used in many situations. You will also learn different relationships and theorems related to these lessons which are useful in solving real world problems. You will also gather ideas to answer the question "How can challenging problems involving geometric figures be analyzed and solved?" These concepts will also help you visualize situations and create solutions to the problems that you encounter. Answers to the question above will also help you do your performance task.

In this section you need to analyze pictures by answering different questions for you to discover important concepts. You will also do selfmonitoring activity as you fill up the map of conceptual change.

Let's us start the lesson by analyzing the pictures and answering the questions that follow.

## A. Circle

How can challenging problems involving geometric figures be analyzed and solved?"
Let's answer these questions by doing the activities below.

Activity 1a: Picture Analysis (Eliciting of prior Knowledge, Motivation, Hook) Observe the pictures below and answer the questions.


1. What geometric concepts can you associate with the pictures?
$\square$
2. How are these concepts used in different situations?

3. Can you determine any purpose why these geometric concepts are present in the pictures? Please specify.
$\square$
4. Can you cite any problem which can be answered through these geometric concepts? Describe at least one.
$\square$
5. How can challenging problems involving geometric figures be analyzed and solved?
$\square$

Now, try to write your initial answer about the essential question presented for you to monitor your understanding.

## Activity 2a: CONCEPTUAL UNDERSTANDING CHECK

In the table below, write your answers on the initial part for the question how can challenging problems involving geometric figures be analyzed and solved?

| INITIAL ANSWER |
| :---: |
| REVISED ANSWER |
|  |
| FINAL ANSWER |

## SUBMIT

## End of EXPLORE:

You just have tried to find out how mathematics can help you determine how can challenging problems involving geometric figures be analyzed and solved. Let us now strengthen that insight by doing the succeeding activities. What you will be learning in this section will help you perform well in your final performance task to formulate and solve challenging problems related to angles and distances by making recommendation and justification.

Now move to the next activity to learn the knowledge and skills you need to be a good problem solver and respond to different situations accurately.

## FIRM-UP

Your goal in this section is to learn and understand key concepts of arcs, chords, angles, secant and tangent which are important in solving problems involving circles. In this section there are activities which will help you discover and understand the different theorems and terms which are useful tools in solving real life problem related to arcs, chords and angles of a circle.

To help us appreciate the important of the lessons, take a look at the situation in the next activity.

## Activity 3a: GARDENING

In the previous activity, we looked at different pictures where the lessons can be used. Let us see what concepts can be used to answer the situation below.

You are a landscape designer. Three bushes are arranged in a garden as shown. Where should you place a sprinkler so that it is the same distance from each bush?


Process Questions:

1. How did you answer the problem above?
2. What concepts did you use to answer the problem?
3. What might happen if you can't be able to respond to the given situation?
4. How can challenging problems involving geometric figures be analyzed and solved?

Write your answers here:

## Activity 4a: Let's do an experiment

In the previous activity you are task to solve a real life problem concerning circle. Now, let's see if you can be able to develop the concepts that you learned by doing the next activity. For the online students you may accomplish the activity on your own or click the interactive website below before you answer the questions but for the non-online students you may do it with your classmates.
http://www.mathwarehouse.com/geometry/circle/inscribed-angle.php
This is an interactive website showing the relationships between the central angles and the intercepted arcs.

Directions: Follow the step by step procedure and then answer the questions.
Materials: compass, protractor, ruler, scissor

1. Draw a circle.
2. Divide the circle into equal parts, minimum of six and maximum of twelve.
3. Cut a part of the circle.
4. Measure the angle of that part together with the arc.
5. What part of the angle is cut from the whole?
$\square$
6. What part of the arc is cut from the total circumference?
$\square$
7. What relationship did you notice between the central angle and its intercepted arc?
$\square$

In the previous activity you learned the relationships between the central angles and chords. Now, you will improve your knowledge in this lesson by doing the next activity.

## Activity 5a: Let's Consult the Expert

Directions: Click the video below which explains further the concepts of central angle and the intercepted arc. After watching the video do the exercises below.
https://www.youtube.com/watch?v=xI2BZ5R PrM

This site contains video explaining the relationships between central angle and its intercepted arc.

Directions: Name the arc made by the given angle. Write your answers on the table provided below.

1) $\angle 1$

A) $\overline{C E D}$
B) $\overline{C D}$
2) $\angle 1$

A) $\overrightarrow{U W V}$
B) $\widehat{v V}$
3) $\angle 1$

A) $\overrightarrow{J I}$
B) $\overline{J H I}$
4) $\angle 1$

A) $\widehat{D E}$
B) $\overline{D F E}$
5) $\angle 1$

A) $\overline{W U V}$
B) $\overline{W V}$
6) $\angle 1$

A) $\overline{S U T}$
B) $\widetilde{S T}$
7) $\angle 1$

A) $\widehat{G H}$
B) $\overline{G I H}$
8) $\angle 1$

A) $\overparen{H J I}$
B) $\overparen{H I}$

| 1. |  | 5. |  |
| :--- | :--- | :--- | :--- |
| 2. |  | 6. |  |
| 3. |  | 7. |  |
| 4. |  | 8. |  |

Answer key:

1. $B$
2. $B$
3. B
4. B
5. A
6. A 7.A
7. A

## Questions

1. How did you answer the questions above?
$\square$
2. What relationship did you notice about the central angle and the intercepted arc?
$\square$
3. Why is it important for you understand that relationship?

Now that you have enough exercises about the concepts of central angle and intercepted arc, you are ready for a short quiz.

## Activity 6a: Let's fill this up (QUIZ)

Directions: Given the value of the central angle and the intercepted arc, find the value of $x$ which will make each statement correct.

|  | Central Angle <br> (degrees) | Intercepted <br> Arc(degrees) | Value of (x) |
| :--- | :--- | :--- | :--- |
| 1. | 60 | $x+20$ |  |
| 2. | $2 x+20$ | 100 |  |
| 3. | $3 x+20$ | $10-2 x$ |  |


| 4. | $4 x-30$ | $2 x+30$ |  |
| :--- | :--- | :--- | :--- |
| 5. | $100-2 x$ | $3 x-30$ |  |

After learning the concepts of central angle and intercepted arc you are now ready to learn other concepts. To start your journey, take a look at the situation below and answer the questions.

## Activity 7a: PHOTOGRAPH

In the previous section, we encountered problems involving the central angle and the intercepted arc. Let us take a look at the situation below to learn other concepts about circle.

You are a professional photographer. Your camera has a 90 field of vision and you want to photograph the front of a statue. You move to a spot where the statue is the only thing captured in your picture, as shown. You want to change your position. Where else can you stand so that the statue is perfectly framed?


Process Questions:

1. How did you answer the problem above?
2. What concepts did you use to answer the problem?
3. What might happen if you can't be able to respond to the given situation?
4. How can challenging problems involving geometric figures be analyzed and solved?

Write your answers here:

To verify the concepts you used in the situation above, let us do the next activity to discover other ways we can use in solving the problem above.

## Activity 8a: LET’S DISCOVER!

Directions: For the online students please do the left side while the non-online students do the activity on the right side. You may also click the interactive website below before answering the questions.
http://www.mathwarehouse.com/geometry/circle/interactive-central-angle-ofcircle.php

This website contains interactive activities showing the relationships between arcs and angles of the circle. You may also discover the angles formed by three points using the diameter of the circle.

| ON | NON-ONLINE LEARNER |
| :---: | :---: |
| 1. Draw a circle similar to what is illustrated below. <br> 2. Get a point on the circle and then connect it to the end points of the diameter. <br> 3. Measure the angle formed. <br> 4. Repeat the process until you got five different points. | 1. Group yourselves with 8 or 10 members. <br> 2. Form a circle similar to the points on the circle below. <br> 3. Using your arms (one by one) try to point your arms to the persons which you consider as the endpoints of the diameter of a circle. <br> 4. Try to estimate the angle formed by your arms. |
| Questions: <br> 1. What did you discover from the activity? <br> 2. What conjecture or conclusion can you give from what you learned? <br> 3. How will you validate your answer? |  |


| 4. Be ready to share what you discovered? |
| :--- |
| Answers: |
|  |
|  |

Now let us check if your generalization in the previous activity is correct by answering a worksheet related to your answers.

## Activity 9a: PROVE IT!

Directions: Analyze the figures below and then answer the questions. However, if you are not yet ready to answer the problem below, you may open the video below to help you strengthen your understanding of the topic.
https://www.youtube.com/watch?v=Ybzcm2-rU4M
This video contains discussion about the concept of inscribed right angle.
If you want practice exercises before answering the problem below you may open the interactive quiz below.
http://www.ixl.com/math/geometry/angles-in-inscribed-right-triangles
This contains interactive quiz related to the problem below.
Given:
EG - diameter

Now that you learned the concepts of inscribed right triangles it now time for you to learn other angles. The next activity will help you understand other relationships of arcs and angles which are very important for you to do your performance task.

## Activity 10a: Watch and Learn

Directions: Click the website below and listen to the video then answer the worksheets below. Write your answer on the table provided.
https://www.youtube.com/watch?v=-QBJtmEVg9s
This video contains detailed lecture and explanation how other angles are related to their intercepted arcs.

Find the value of $x$
4.

5.


6. In the figure, $O$ is the center of the circle. What is $\mathrm{m} \angle A B C$, if $\mathrm{m} \angle \mathrm{OBC}=$

67 ?


Write your answers here:

| 1. |  | 3. |  | 5. |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 2. |  | 4. |  | 6. |  |

## Questions:

1. How did you answer each question above? What concepts did you use?
2. What did you learn about the relationships between the inscribed angles and the intercepted arcs?
3. How can challenging problems involving geometric figures be analyzed and solved?

## Answers:

You are now ready for more challenging activities. Since you have already acquired some knowledge and skills let's see if you can be able to use to respond to the challenge.

## Activity 11a: Let's us ask the expert

Directions: Click the videos below. These videos will help you answer the activity below. However, if you want to challenge yourself you may go directly to the activity but you need to complete each part. https://www.youtube.com/watch?v=z-YxfG42P2M This site contains video about tangent line.
https://www.youtube.com/watch?v=z-YxfG42P2M
This site contains video about common tangent to two circles.
https://www.youtube.com/watch?v=CJNAO6LQmAw
This site contains video about secants, tangents and angles measure.
Process Questions:

1. What did you notice about the videos above?
2. What relationships did you observe about the angles and the intercepted arcs?
3. How can you use those relationships in real life problems?
4. How can challenging problems involving geometric figures be analyzed and solved?

Write your answers here:

To check if you can be able to use the concepts you learned from the videos; let us solve together the problem below.

## Activity 12a: Let's model

Directions: Complete the graphic organizer below to answer the problem below.
Find the measurement of arc $C D$ in the figure.
Given: $\mathrm{mAE}=80^{\circ}$
$M \angle C=20^{\circ}$


## FORMULATE:

The longer arc subtracted by the shorter are within the angle divided by two equals the measurement of the angle. Hence,
$\operatorname{arc} \mathrm{AE}-\operatorname{arc} B D$
$=$ $\qquad$
2
-

## VALIDATE:

If arc AE is $100^{\circ}$ and $\operatorname{arc} \mathrm{BD}$ is $35^{\circ}$, what will be the measure of $\angle \mathrm{C}$ ?

COMPUTE:
m AE - m BD
------------------- = $\qquad$
2
$80^{\circ}-\mathrm{m} \mathrm{BD}$
---------------- = 20
2

Continue solve for BD.

INTERPRET:
Therefore, the measure of arc BD if arc $\mathrm{AE}=80^{\circ}$ and angle $\mathrm{C}=$ $20^{\circ}$ is $\qquad$ .

Now that you have used the modelling technique, let us see if you can be able to do it on your own.

## Activity 13a: Let's make it real

Directions: Read the problem carefully and then complete the boxes.

## PROBLEM:

GLOBAL POSITIONING SYSTEM (GPS) GPS satellites orbit about 11,000 miles above Earth. The mean radius of Earth is about 3959 miles. Because GPS signals cannot travel through Earth, a satellite can transmit signals only as far as points $A$ and $C$ from point $B$, as shown. Find $B A$ and $B C$ to the nearest mile.


FORMULATE: (Write the concepts or formula you need to answer the problem)

COMPUTE: (Write your complete solution)

INTERPRET: (Write a paragraph about the interpretation of your answer)

VALIDATE: (Provide one more example to validate your answer)

## REPORT:

NON-ONLINE: Be ready to report your work in front of the class.
ONLINE: Use voki.com to be able to report your work. You may also send your written report through email or using the student dash board. Just attached vour file or send it the discussion forum.

You learned a lot of concepts and theorems from the previous activity. To test if you can use it correctly, do the next activity.

## Activity 14a: Let's use it (Quiz)

Directions: Answer the problems below.

1. Describe what it means to bisect an arc.
2. Two chords of a circle are perpendicular and congruent. Does one of them have to be a diameter? Explain your reasoning.

Find the value of the red arc or chord in the circle $C$.
3.

4.

5.


Find the value of $x$ in circle $Q$.
6.

7.

8.

9.

10.

11.


Write your answers here:

| 1. |  |  |
| :--- | :--- | :--- |
| 2. | 4. | 5. |
| 3. | 7. | 8. |
| 6. | 10. | 11. |
| 9. |  |  |

Questions:

1. What concepts in geometry did you use to answer each problem?
$\square$
2. What are the difficulties they you encounter?
$\square$
3. How do geometric concepts related to one another?
$\square$
4. How can challenging problems involving geometric figures be analyzed and solved?
$\square$

## Activity 15a: 3-2-1

You encountered a lot of concepts related to circle. Now it's time to pause for a while and reflect to your learning process by doing the 3-2-1.

What are the 3 most important things you learned?

What are the 2 things you are not sure about?

What is 1 thing that you want to clarify immediately?

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To help you summarize and remember important concepts that you learned about triangle similarity, try to complete the graphic organizer below.

## Activity 16a: Do the Map

Directions: Observe the diagram below and complete the missing parts.


## End of FIRM UP:

In this section, the discussion was about circle and other related concepts. It also involves theorems relating the angles and the intercepted arcs.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision? What new learning goal should you now try to achieve?

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.


## DEEPEN

Your goal in this section is to take a closer look at some aspects of the topic. With the activities that you have accomplished, do you think you are now ready for more challenging situations? Can you assess now how can challenging problems involving geometric figures be analyzed and solved?

To test your analysis and understanding of the concepts let's see if you can be able to identify mistakes and be able to correct it with justification. This activity will also help you check your understanding and possible misconceptions you absorb from the previous activities.

## Activity 17a: Concept Analysis

Directions: Analyze each situation carefully and write your answer, solution and justification.

Note: Only one of the two circles BELOW includes the intersection of a tangent and a secant. Which one is it? Explain your answer.


Before you continue the learning process, it will be better if you stop and reflect. After taking different activities, what happened to your initial answers? You may now answer the R part of you IRF worksheet.

## Activity 18a: CONCEPTUAL UNDERSTANDING CHECK

In the table below, write your answers on the revised answer for the question how can challenging problems involving geometric figures be analyzed and solved?

| INITIAL ANSWER |
| :---: |
| REVISED ANSWER |
|  |
| FINAL ANSWER |
|  |

To validate your understanding about the concepts of circles, answer the next activity.

Activity 19a: TRUE OR FALSE AND WHY?
Analyze each statement carefully whether it is right or wrong and then explain your answer.

| STATEMENT | ANSWER <br> (TRUE/ <br> FALSE) | EXPLANATION |
| :--- | :--- | :--- |
| 1. If the measure of the <br> central angle is $120^{\circ}$ then <br> its intercepted arc is $60^{\circ}$. |  |  |
| 2. If the measure of the |  |  |
| inscribed angle is $78^{\circ}$ then |  |  |
| the measure of its |  |  |
| intercepted arc is $39^{\circ}$. |  |  |$\quad$ $\quad$ | 3. If two chords intersect on |
| :--- |
| the circle then the sum of |
| two intercepted arcs equals |
| to the measure of the angle |
| formed. |


| 4.Some tangent lines are <br> perpendicular to the radius. |  |  |
| :--- | :--- | :--- |
| 5. If two secant lines intersect |  |  |
| outside the circle, then the |  |  |
| measure of the angle |  |  |
| formed is the measure of |  |  |
| the major arc minus the |  |  |
| minor arc. |  |  |

Now that you validate your understanding, you are ready for more challenging activities. The next three activities will help you develop your understanding of the concepts and importance of the lesson.

## Activity 16a: GARDENING

In the previous section, we looked at different problems about the application of circle and its properties. Let us put together in the table below our answers to the essential question that we asked for each problem.

You are a landscape designer. Three bushes are arranged in a garden as shown. Where should you place a sprinkler so that it is the same distance from each bush?

## Activity 17a: PHOTOGRAPH



What property was used to solve the given problem? Why?

Show your solution/justification:

## Complete the statement:

Problems in real life involving gardening c
you stand so that the statue is perfectly framed?

## What property was used to solve the given problem? Why?

Show your solution/justification:

Complete the statement:
Problems in real life involving photography can be analyzed and solved by

The choice of properties to be used depends on

## Activity 18a: DO THE SURVEY

In the previous section, we looked at different problems about the application of circle and its properties. Let us put together in the table below our answers to the essential question that we asked for each problem.

A surveillance camera is mounted on a corner of a building. It rotates clockwise and counterclockwise continuously between Wall A and Wall B at a rate of 10 per minute as illustrated in the figure. How long does it take the camera to survey the entire area once? Justify your
 answer.

What property was used to solve the given problem? Why?

Show your solution/justification:

Complete the statement:
Problems in real life involving monitoring places can be analyzed and solved by

The choice of properties to be used depends on

Activity 19a: SUM IT UP
Based from your answer on the three previous activities, answer the following.

What are the common words or phrase among your answers?

Using the words/phrases above, write a one paragraph statement summarizing what you learned from this activity?

## PROCESS QUESTIONS:

1. Look at your answers above; what do all the solutions have in common?
2. Were you able to answer all the problems?
3. How did you come up with correct property for each situation?
4. Have you encountered problems which can be solved with two or more ways? What made you choose the solution that you used?
5. How is each property useful in solving the problem?
6. If the situations will differ, how do you know which one to use?

Explain.
7. Complete the statements below.

Answer to the process questions:

In general, problems in real life can be analyzed and solved by.....

In general, the choice of properties to be used depends on ......

How can challenging problems involving geometric figures be analyzed and solved?

After summing up everything, try solving the problem below. Try to see if the generalization that you have made can also be used in the new situation.

## Activity 20a: TRY IT TO OTHERS

Directions: Read the problem below and answer each question.
You are an accident investigator, you learned from research that the speed of the car can be modelled by the equation $S=$ $3.86 \sqrt{f r}$ where $\mathrm{S}=$ car's speed, $\mathrm{f}=$ coefficient of friction, and $r=$ radius of the circle in feet. If a car goes around a turn too quickly, it can leave tracks that form an arc of a circle, what is the estimated speed of the car? You may refer to the given illustration.


## Process Questions:

1. What information would help you solve the given problem?
2. What property can be used to solve the problem below? Why?
3. Show your solution and justification.
4. How can challenging problems involving geometric figures be analyzed and solved?

Answers:

## End of DEEPEN:

In this section, the discussion was about circle and other related concepts.
What new realizations do you have about the topic? What new connections have you made for yourself? What helped you make these connections?

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.


## TRANSFER

Your goal in this section is apply your learning to real life situations. You will be given a practical task which will demonstrate your understanding.

To help you summarize everything that you learned in this lesson, try to make a problem posing and solving activities with the use of mathematical modelling framework.

## Activity 21a: LET'S INVESTIGATE

Directions: Read the procedure carefully and follow. You may do your work using MS Word and send it to your teacher. For your explanation and justification you may do it face to face or you may try another web 2.0 present.me. Here you may record your explanation, justification and generalization of the lessons. In your presentation, do not forget to answer the questions, how can challenging problems involving geometric figures be analyzed and solved?

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## PROCEDURE:

1. Problem: Think of a problem or situation where the concepts of the lesson are very important. Create your own word problem.
2. Formulate: Write the concept, theorem or formula you need to be able to solve the problem.
3. Compute: Write your complete solution (step by step).
4. Interpret: Write a paragraph about the interpretation of your answer.
5. Validate: Provide more examples about your answer and concepts you used.
6. Report: Report the result of your investigation.

After doing your own investigation you make take a long quiz to assess more the knowledge and skills you need before doing your transfer task.

Activity 22a: LONG TEST
Directions: Read each problem carefully and answer each question.

| 1. What is the value of x in the |
| :--- |
| problem on below? |
| 2. What is the measure of mAC ? |
| $\overline{A E}$ is a diameter |
| What is $\mathbf{m} \angle T ?$ |



| 5. What is the value of $x$ in the problem below? | 6. What is the measure of m CH ? |
| :---: | :---: |
|  |  |
| Answer: | Answer: |
| 7. What is the value of $k$ in the problem below? | 8) The diagram on the right is not to sca $\mathrm{ABC}: \mathrm{XYZ}=3: 2$, arc $\mathrm{AX}=80$ and $\operatorname{arc} C Z=170$. What is k ? |



Now you may have enough knowledge, skills and understanding to do your transfer task. To assess what you learned and understand, you will apply the concepts of the lesson in an actual situation by doing the transfer task of this lesson Scaffold 1.

## Activity 23a: SCAFFOLD 1

Directions: Read the procedure in the box and do what is asked.

Students will be asked to conduct an experiment involving different situations. Students may choose among the three situations below.

In a theater, what will be the best locations of the actors so that they will always be visible to their audience?

For a cameraman, what will be his best locations so that he can take good shots to the actors performing on the stage?

For a gardener, what should be the best position of the lawn sprinkler to maximize its use?

Students will submit a written output containing explanation and justification. Students will be using the knowledge they learned about the concepts of circle and other related terms.

Note: To make your written output appealing and accurate you may use the geogebra. To do it, download geogebra then you may use it for free.

For your presentation and explanation, you may use voki.com. This will help you record your presentation in the most exciting way.

After doing your transfer task (Scaffold 1) it's time again to reflect on the learning process to check if there are ideas which you need to change, to revise or improve. You may now complete you IRF worksheet by writing your ideas on the F part.

## Activity 24a: CONCEPTUAL UNDERSTANDING CHECK

In the table below, write your answers on the final answer for the question how can challenging problems involving geometric figures be analyzed and solved?

| INITIAL ANSWER |
| :---: |
| REVISED ANSWER |
|  |
| FINAL ANSWER |

To complete the learning process, reflect again and complete the table below. This will also check if you have absorbed some misunderstandings which need to be corrected.

## Activity 25a: Let's Reflect!

| CORNELL' S NOTES |  |
| :--- | :--- |
|  |  |
| TOPICS |  |
|  |  |
| Questions I want to be answered: |  |
|  |  |
|  |  |

To summarize what you learned, you may complete the synthesis journal below.

## Activity 26a: Synthesis Journal

The lesson was on $\qquad$ . One key idea was $\qquad$ . This is important because
$\qquad$ . This is also important because
lesson $\qquad$
$\qquad$ .

## End of TRANSFER:

In this section, your task was to make a written explanation and justification about the situation that you encountered.

How did you find the performance task? How did the task help you see the real world use of the topic?

You have completed this lesson. But you have two more lessons before you finish this module. You need to learn more about circles and coordinate geometry to complete what you need in doing your performance task.

## POST ASSESSMENT

1. Which of the following is true about the measures of central angles in relation to the intercepted arc?
A. The measure of the intercepted arc is half of the measure of the central angle.
B. The measure of the intercepted arc is twice the measure of the central angle.
C. The measure of the central angle is equal to the measure of the intercepted arc.
D. The measure of the central angle is half of the measure of the intercepted arc.
2. Name two pairs of congruent angles using the figure below.
A. $\quad \angle \mathrm{JKM} \cong \angle \mathrm{KJL} \& \angle \mathrm{JLM} \cong \angle \mathrm{KML}$
B. $\quad \angle \mathrm{JLM} \cong \angle \mathrm{KJL} \& \angle \mathrm{JKM} \cong \angle \mathrm{KML}$
C. $\quad \angle \mathrm{JKM} \cong \angle \mathrm{JLM} \& \angle \mathrm{KJL} \cong \angle \mathrm{KML}$
D. $\quad \angle \mathrm{JLM} \cong \angle \mathrm{KJL} \& \angle \mathrm{JLM} \cong \angle \mathrm{JKM}$

3. A professional photographer has a camera that has a 90 ${ }^{0}$ field of vision and he wants to photograph the front of a statue. He moves to a spot where the statue is the only thing captured in your picture, as shown. After seeing this, you are also encouraged to capture the photograph of the statue but your camera has less than $90^{\circ}$ field of vision. Which of the following should be your strategy to capture the whole picture?
A. You may move closer to the statue.
B. You may move farther from the statue.
C. You may move to the other side of the statue.
D. Look for an elevated place to capture the statue.
4. You are a member of the board who will assess the different proposals who are applying for bidding for CCTV cameras to be installed in different places. Which among the following criteria should you consider most?
A. Authenticity of data, accuracy, practicality of proposal
B. Accuracy, organization, practicality of proposal
C. Practicality, neatness, accuracy of proposal
D. Organization, practicality, neatness of proposal
5. A surveillance camera is mounted on a corner of a building. It rotates clockwise and counterclockwise continuously between Wall A and Wall B at a rate of $15^{\circ}$ per minute as illustrated in the figure. How long does it take the camera to survey the entire area once? Justify your answer.
A. 3 minutes, since the measure of the angle is half of the intercepted arc.
B. 6 minutes, since the angle and arc have equal measures.
C. 12 minutes, since the measure of the angle is twice the measure of the intercepted arc.
D. 18 minutes, the camera needs to survey about $135^{\circ}$ of the circle.
6. If rays WE and WI are both tangents to the circle P below, then which of the following is NOT necessarily true?
A. $A O=O B$
B. $\mathrm{m} \angle \mathrm{PAO}=\mathrm{m} \angle \mathrm{PBO}$
C. $\angle A O B \cong \angle A P B$
D. $\mathrm{AP}=\mathrm{BP}$

7. If ray $m$ and $n$ are tangents to circle $P$ below, then which of the following is the value of $w$ ?
A. $30^{\circ}$
B. $40^{\circ}$
C. $50^{\circ}$
D. $60^{\circ}$

8. In the figure below, if segment MT is tangent to the circle, segments NT and PT are secants. If MP is $140^{\circ}$ and the measure of $\angle \mathrm{MTP}=50^{\circ}$, what is the measure of arc MS?
A. $30^{\circ}$
B. $40^{\circ}$
C. $50^{\circ}$
D. $60^{\circ}$

9. If $\mathrm{RS}=\mathrm{VW}$ in the figure below, then which of the following will justify that $\Delta$ RTS is congruent to $\Delta \mathrm{WTV}$ ?

A. SAA Congruence Theorem
B. SAS Congruence Postulate
C. SSS Congruence Postulate
D. Hypotenuse-Leg Theorem
10. What is the distance between the points $(2,2)$ and $(5,6)$ ?
a. 3
b. 5
c. 7
d. 9
11. The point $(5,4)$ lies on a circle. What is the length of the radius of this circle if the center is located at the origin?
a. $\sqrt{8}$
b. $\sqrt{3}$
c. $\sqrt{18}$
d. $\sqrt{41}$
12. Which of the following coordinates must added to $A(2,3), B(4,4), D(-1,-1)$ to prove that the given figure is a kite?
a. $C(3,2)$
b. $\mathrm{C}(3,5)$
c. $\mathrm{C}(3,7)$
d. $\mathrm{C}(2,5)$
13. On a map's coordinate grid, Brgy. Malaya is located at $(2,4)$ and Brgy. Pasong Tamo islocated at (2, 2). Brgy. Katapan is the in the middle of the two barangay. What is the distance from Brgy. Malaya to Brgy. Katapatan?
a. 2 km
b. 4 km
c. 6 km
d. 8 km
14. The JDC Cargo ship is sailing from Zhanjiang China to Honolulu and then to Long Beach. The amount of diesel used of the cargo sheep in travelling is 100 gallons for every 10 miles. Using the information in the figure below, determine the number of gallons used by the cargo from Zhanjiang to Long Beach.

a. 616.56 gallons
b. 600 gallons
c. 700 gallons
d. 716.65 gallons
15. In the municipal vicinity map the coordinate of your barangay hall is $(2,-1)$ and the school is $(-2,2)$. How long will it take you to go to school from the barangay hall if the tricycle that you are riding is at $10 \mathrm{~km} / \mathrm{hr}$ ?
a. 30 minutes
b. 45 minutes
c. 1 hour
d. 1 hr and 10 minutes
16. Low - Earth orbit satellites occupies a region of space from 180-2000 kilometers above Earth. If the satellite is moving in a circular path with Earth as the point of origin, which circular equation represents the maximum path of the satellite?
A. $x^{2}+y^{2}=180$
B. $x^{2}+y^{2}=2000$
C. $x^{2}+y^{2}=32,400$
D. $x^{2}+y^{2}=4,000,000$
17. Based on the figure below, what equation that represents the circle?

I. $(x+1)^{2}+(y-3)^{2}=9$
II. II. $(x-1)^{2}+(y+3)^{2}=9$
III. $x^{2}+y^{2}+2 x-6 y+1=0$
A. I only
B. II and III only
C. I and III only
D. I, II and III
18. To determine the radius and coordinates of the center of the circle $x^{2}+y^{2}+$ $4 x-8 y+4=0$, Jerry and Akzie came up with the following solutions:

| Jerry's Solution | Akzie's Solution |
| :--- | :--- |
| $x^{2}+y^{2}+4 x-8 y+4=0$ | $x^{2}+y^{2}+4 x-8 y+4=0$ |
| $x^{2}+4 x+-+y^{2}-8 y+-$ | $x^{2}+6 x+\ldots+y^{2}-8 y+\ldots=$ |
| $=-4$ | -4 |
| $x^{2}+4 x+2^{2}+y^{2}-8 y+(-$ | $x^{2}+4 x+2^{2}+y^{2}-8 y+(-4)^{2}=$ |
| $4)^{2}=-4$ | $-4+4+16$ |
| $(x+2)^{2}+(y-4)^{2}=-4$ | $(x+2)^{2}+(y-4)^{2}=16$ |
| Radius $=\sqrt{-4} ;$ center: $(-$ | Radius $=4 ;$ center: $(-2,4)$ |
| $2,4)$ |  |
|  |  |

Which statement best describes the solutions of Jerry and Akzie?
A. The solution of Jerry is correct.
B. The solution of Akzie is correct.
C. The solution of Jerry and Akzie are both correct.
D. The solution of Jerry and Akzie are both incorrect.
19. When delivering weather forecast during typhoons, weather forecasters uses diagrams and charts to illustrate the movement of the typhoon. The multiple colored circles indicate the effect of the typhoon in the area at a given time at. If you are the weather forecaster, which statement best describes typhoon Haiyan based on the illustration below?

A. The area affected by 128 kph winds of Haiyan decreased from Nov. 7 - 10.
B. The area affected by 95 kph winds of Haiyan decreased from Nov. 7 10.
C. The area affected by $95-128 \mathrm{kph}$ winds of Haiyan was maintained from Nov. 7-10.
D. The area affected by Haiyan decreased from Nov. 7 - 10.
20. The barangay officials recently reported alarming news that the number of crimes in your community is drastically increasing. The barangay council decided to put up a CCTV camera in the area with the most number of crime incidents. You are a marketing officer of a CCTV company that will participate in the bidding to provide the barangay with the CCTV unit. You need to identify the best location of the CCTV unit in the area that is identified in your barangay. Your work should have authentic data, accurate computation, should have practical recommendations, and with correct application of the concepts of the properties of circles. What should you do to convince the barangay officials?
A. You will present the CCTV to the barangay officials.
B. You will present the CCTV and layout to the barangay officials.
C. You will present your recommendation to the barangay officials.
D. You will present your recommendation and justification to the barangay

## GLOSSARY OF TERMS USED IN THIS LESSON:

Circle - A circle is a plane figure bounded by one line, and such that all right lines drawn from a certain point within it to the bounding line, are equal.

Arc - Any connected part of the circle.
Center - It is a point equidistant from the points on the circle.
Radius - It is a line segment joining the center of the circle to any point on the circle itself; or the length of such a segment, which is half a diameter.

Sector - It is a region bounded by two radii and an arc lying between the radii.
Secant - It is an extended chord, a coplanar straight line cutting the circle at two points.

Tangent - It is a coplanar straight line that touches the circle at a single point.
Central Angle - It is the angle subtended by the arc to the center of the circle of which it is a part

Inscribed Angle - It is the angle subtended by the arc to any point on the circumference of the circle of which it is a part.

## REFERENCES AND WEBSITE LINKS USED IN THIS LESSON:

https://www.youtube.com/watch?v=xl2BZ5R PrM
This site contains video explaining the relationships between central angle and its intercepted arc.
https://www.youtube.com/watch?v=Ybzcm2-rU4M
This video contains discussion about the concept of inscribed right angle.
http://www.ixl.com/math/geometry/angles-in-inscribed-right-triangles
This contains interactive quiz related to the problem below.
https://www.youtube.com/watch?v=-QBJtmEVg9s
This video contains detailed lecture and explanation how other angles are related to their intercepted arcs.
https://www.youtube.com/watch?v=z-YxfG42P2M
This site contains video about tangent line.
https://www.youtube.com/watch?v=z-YxfG42P2M
This site contains video about common tangent to two circles.
https://www.youtube.com/watch?v=CJNAO6LQmAw
This site contains video about secants, tangents and angles measure.
http://www.mathwarehouse.com/classroom/worksheets/circles/secants-tangents-arcs-angle.pdf
This site contains worksheets about secants, tangents and angles measure.

