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PEAC
PRIVATE EDUCATION ASSISTANCE COMMITTEE
Trustee of the Fund for Assistance to Private Education

DepED
DEPARTMENT OF EDUCATION

LEARNING MODULE

Mathematics G7 | Q2

Algebra

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NOTICE TO THE SCHOOLS

This learning module (LM) was developed by the Private Education Assistance Committee under the GASTPE Program of the Department of Education. The learning modules were written by the PEAC Junior High School (JHS) Trainers and were used as exemplars either as a sample for presentation or for workshop purposes in the JHS In-Service Training (INSET) program for teachers in private schools.

The LM is designed for online learning and can also be used for blended learning and remote learning modalities. The year indicated on the cover of this LM refers to the year when the LM was used as an exemplar in the JHS INSET and the year it was written or revised. For instance, 2017 means the LM was written in SY 2016-2017 and was used in the 2017 Summer JHS INSET. The quarter indicated on the cover refers to the quarter of the current curriculum guide at the time the LM was written. The most recently revised LMs were in 2018 and 2019.

The LM is also designed such that it encourages independent and self-regulated learning among the students and develops their 21st century skills. It is written in such a way that the teacher is communicating directly to the learner. Participants in the JHS INSET are trained how to unpack the standards and competencies from the K-12 curriculum guides to identify desired results and design standards-based assessment and instruction. Hence, the teachers are trained how to write their own standards-based learning plan.

The parts or stages of this LM include Explore, Firm Up, Deepen and Transfer. It is possible that some links or online resources in some parts of this LM may no longer be available, thus, teachers are urged to provide alternative learning resources or reading materials they deem fit for their students which are aligned with the standards and competencies. Teachers are encouraged to write their own standards-based learning plan or learning module with respect to attainment of their school's vision and mission.

The learning modules developed by PEAC are aligned with the K to 12 Basic Education Curriculum of the Department of Education. Public school teachers may also download and use the learning modules.

MATHEMATICS 7

Module 2: Algebra

☑ Introduction and Focus Questions

Algebra is one of the oldest branches of mathematics. It has its own language and structure that allows discovery of relationships between quantities, promotes critical thinking, communicates ideas and solves real – life problems.

However, for most students, Algebra is often perceived as a confusing set of letters, numbers and symbols which are difficult to understand.

In this module, you will explore and perform different activities that will help you acquire the necessary competencies and skills, develop a deeper understanding of the application of algebra and improve your critical thinking. In the end of the module, you should be able to answer the question:

How can real – life problems involving one variable be modeled and solved?

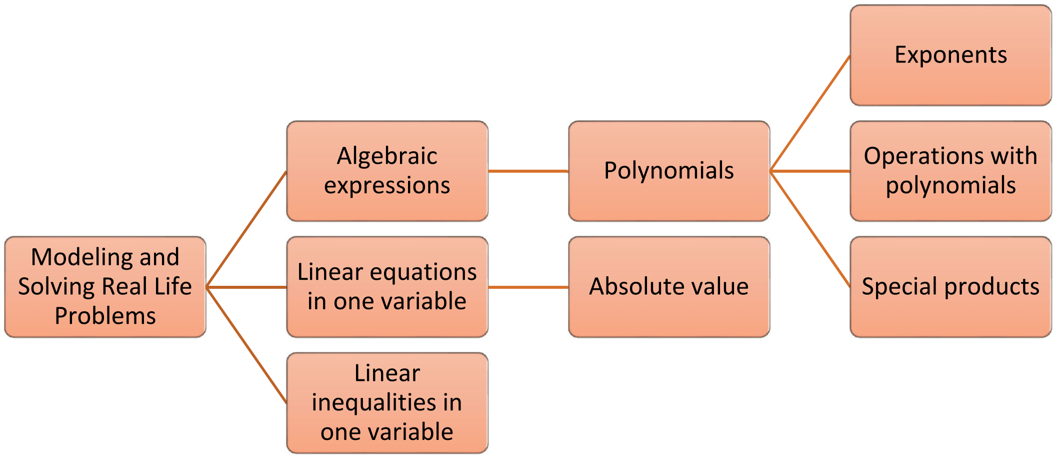
☑ MODULE COVERAGE

Lesson No.	Title	You'll learn to…	Estimated Time
Lesson 1	Algebraic Expressions	a. differentiate between constants and variables in a given algebraic expression. b. evaluate algebraic expressions for given values of the variables. c. translate English phrases to mathematical phrases and vice versa. d. solve problems involving algebraic expressions	5 hours

Lesson 2	Polynomials	a. classify algebraic expressions which are polynomials according to degree and number of terms b. add and subtract polynomials. c. interpret the meaning of a^n where n is a positive integer. d. derive the laws of exponents. e. multiply and divide polynomials g. use models and algebraic methods to find the: (i) product of two binomials; (ii) product of the sum and difference of two terms; (iii) square of a binomial; (iv) cube of a binomial; (v) product of a binomial and a trinomial. h. solves problems involving special products	18 hours
Lesson 3	Linear Equations	a. differentiate between algebraic expressions and equations. b. translate English sentences to mathematical sentences and vice versa. c. illustrate linear equation in one variable d. find the solution of linear equation in one variable e. solve linear equations in one variable involving absolute value by: (i) graphing; and (ii) algebraic methods f. solve problems involving equations in one variable	
Lesson 4	Linear Inequalities	a. differentiate between equations and inequalities b. illustrate linear inequality in one variable c. find the solution of linear inequality in one variable d. solve linear inequality in one variable involving absolute value by: (i) graphing; and (ii) algebraic methods e. solve problems involving inequalities in one variable	

☑ Concept Map of the Module

Here is a simple map of the above lessons you will cover:



☑ Expected Skills

To do well in this module, you need to remember and do the following:

1. Follow the instructions provided for each activity.
2. Complete all activities and exercises.
3. Review and evaluate your work using the rubric provided before submission.
4. Be mindful of the meaning of unfamiliar words you encounter in this module. A glossary of terms is provided in the last part of this module.
5. Maximize the use of online resources in each lesson. Online resources can be accessed multiple times. The summary of online resources is provided in the end of the module.

PRE-ASSESSMENT:

Let's find out how much you already know about this module.
Answer the pre-test below.

Click on the letter that you think best answers the question. Please answer all items. After taking this short test, you will see your score. Take note of the items that you were not able to correctly answer and look for the right answer as you go through this module.

1. All are polynomials except _________.
   a. p
   b. $x^5 - 3$
   c. $a^3 + 2a^2 - 3a + 4$
   d. $\frac{1}{m^3} + \frac{1}{m^2} - \frac{1}{m} + 1$

2. The formula for changing temperature in degrees Fahrenheit (F) to degrees Celsius (C) is $C = \frac{5}{9}(F - 32)$. Find C if F = 68°.
   a. 20^0
   b. 25^0
   c. 32^0
   d. 68^0

3. What is $(x - 5)^2$?
   a. $x^2 + 25$
   b. $x^2 - 5x + 25$
   c. $x^2 + 10x + 25$
   d. $x^2 - 10x + 25$

4. Find the average of three consecutive integers whose sum is 99.
   a. 31
   b. 32
   c. 33
   d. 34

5. The area of rectangle is 100 in^2. If the base is 40 in, what is the height?
   a. 0.25 in
   b. 2.5 in
   c. 25 in
   d. 250 in

6. A number *n* increased by 5 is greater than 40. What numbers satisfy this condition?

a. $n < 35$
b. $n > 35$
c. $n \leq 35$
d. $n \geq 35$

7. If $m < 0$ and $n < 0$, which of the following is always true?
   a. $m + n > 0$
   b. $mn < 0$
   c. $m - n < 0$
   d. $m + n < 0$

8. What is the area of a square if one of the sides measures $(p - 7)$?
   a. $p - 7$
   b. $p^2 - 7$
   c. $p^2 - 7p + 7$
   d. $p^2 - 14p + 49$

9. RJCToda has released a new fare matrix as shown below.

From Terminal to	No. of kilometers	Fare
Matthew St.	4	P24
John St.	5	P 28
Luke St.	6	P 32
Mark St.	8	P 40

   If x is the number of kilometers, which expression represents the fare?
   a. $7x + 4$
   b. $20x + 4$
   c. $8 + 4x$
   d. $20 + 4 + x$

10. The cost of a blue shirt is PhP 300 while the cost of a white shirt is PhP 250. If *b* represents the number of blue shirts and *w* represents the number of white shirts, which expression represents the total cost of blue and white shirts?
   a. b + w
   b. 300 + 250
   c. 300b + 250w
   d. 300w + 250b

11. A megabyte is equal to 2^{20} bytes while a gigabyte is equal to 2^{30} bytes. How many mp3 songs, whose size is 8 megabytes each, can be saved on a 4-gigabyte disk?
   a. 128 songs
   b. 256 songs
   c. 512 songs

d. 1, 024 songs

12. A computer shop charges a special rate on Friday nights. The special rate includes a one-time charge of P50 plus P5 per hour to play computer games. What will be the formula to determine the rental cost C for y hours?
   a. $C = 5y$
   b. $C = 50 + y$
   c. $C = 50y$
   d. $C = 50 + 5y$

13. The grades of Angelo are 78, 74, 84 and 86. What should be his fifth grade to make his average at least 82?
   a. 85
   b. 86
   c. 87
   d. 88

14. Gary is younger than Jerry. The sum of their ages is 41. What is the age of Gary if Jerry is *m* years old?
   a. m + 41
   b. m – 41
   c. 41 – m
   d. 41m

15. Dr. Garcia prescribed to his patient Gloria 28 tablets for her sickness. She should take 8 tablets on the first day and then 4 tablets each day thereafter. If *x* represents the remaining days Gloria will take her medicine, which equation models this situation?
   a. 8 + 4x = 28
   b. 28 = 8 – 4x
   c. 8x + 4 = 28
   d. (8 + 4)x = 28

16. The cost of a car tire is 800 pesos plus 120 pesos per order regardless of the number of tires purchased. If Mr. Cruz placed an order worth 4920 pesos and *d* represents the numbers of tires, what equation models the situation?
   a. 800 + 120 = 4920
   b. 800d + 120 = 4920
   c. 800 + 120d = 4920
   d. 800 + 120 = 4920d

17. Jeremy is selling magazine subscriptions. He earns 300 pesos a day plus 50 pesos for each subscription he sold. If Jeremy earned 3200 pesos in 8 days, how many magazine subscriptions was he able to sell?
   a. 12 days
   b. 14 days
   c. 16 days

d. 18 days

18. At XYZ Sports Gym, the annual membership fee is 500 pesos. The rental fee for members is lower than the fee of non – members as shown in the table below:

Member rental fee per hour	Non – member rental fee per hour
60 pesos	150 pesos

Nestor while Jenny is not a member. If they spent 11 hours in the gym, who paid more?

a. Nestor paid 150 pesos more than Jenny.
b. Nestor paid 490 pesos more than Jenny.
c. Jenny paid 150 pesos more than Nestor.
d. Jenny paid 490 pesos more than Nestor.

19. You are a travel agent who is requested by a certain tourist group to provide a report regarding the cheaper tour package, between the two most recommended agencies. Which of the following standards will best assess your work?

a. clarity of report, manner of presentation, design
b. Fluency, subject-verb agreement, attractiveness
c. Accuracy, practicality, authenticity of data
d. neatness, cooperation, authenticity of data

20. You are a marketing associate of a certain network company, GLOBAL SMART making sales call to a group of executives who will be subscribing a regular cell phone plan. You need to convince these executives that GLOBAL SMART offers the best deals to meet their communication needs. Aside from an oral presentation, what do you need to present to convince the executives?

a. a gift to the executives
b. a brochure containing the comparison of the different plans
c. a colorful movie presentation
d. a camera to take photos with the executives

Lesson 1: Algebraic Expressions

Have you ever wondered why you have to study Algebra? In Algebra, you will encounter a lot of symbols like numbers and letters. What do these numbers and letters represent? You will be able to answer these questions as you go through this lesson. In the end of this lesson, you should be able to answer, ***how can real – life problems involving one variable be modeled and solved?***

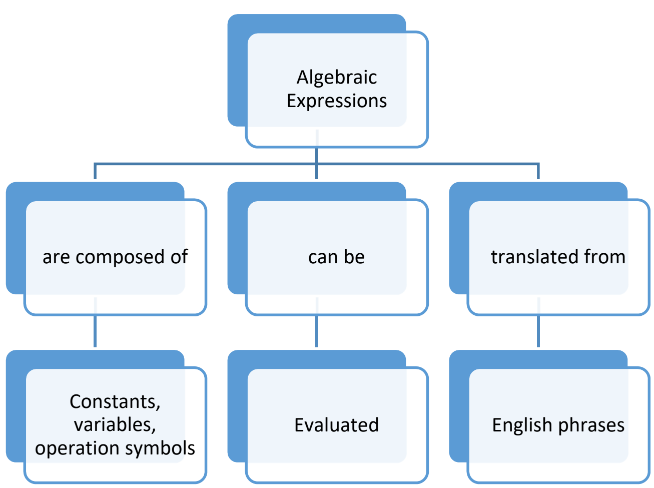
☑ LESSON COVERAGE:

This lesson has the following topics:

Topic	Title	You'll learn to…	Estimated Time
1	Algebraic Expressions	Differentiate between constants and variables in a given algebraic expression.	2 hours
2	Evaluating Algebraic Expressions	Evaluate algebraic expressions for given values of the variables.	1 hour
3	Mathematical Phrases	Translate English phrases to mathematical phrases and vice versa. Solve problems involving algebraic expressions.	2 hours

☑ Concept Map of the Module

Here is a simple illustration of the topics you will cover in this lesson:



EXPLORE

Let's begin by finding out what you know about algebraic expressions by doing the activity below.

ACTIVITY 1. The Magic Number

To determine the magic number, follow the steps below. Write the result for each step in the second column. In step 6, you will find out the magic number. Repeat the activity using other numbers and write the result for each step in the third and fourth column, respectively.

Steps			
1. Think of a positive integer.			
2. Multiply the number by 2.			
3. Add 6.			
4. Divide by 2.			
5. Subtract the original number in step 1.			
6. The magic number is			

ACTIVITY 2. IRF Worksheet

Complete the first part of the Initial Answer section of the worksheet below. You will revisit this worksheet as you progress in this lesson. Read aloud what you've written. Click "SAVE" when you're done.

Initial Answer
Algebraic expressions are used to ____ _______

Real – life problems can be modeled and solved by ______________________ ____ _____.
Revised Answer
Final Answer

PROCESS QUESTIONS:

1. What is the magic number?
2. Why is it that you arrive with the same number even if you changed your initial number?
3. Will the magic number be the same for all positive integers? Why?
4. Why are algebraic expressions useful?

End of Explore
The activity above is an example of a number puzzle. After experiencing the puzzle above, let us find out how you can use algebraic expressions to analyze and design other number puzzles.

FIRM-UP

Your goal in this section is to learn how to **model real – life situations** using varied strategies. You will learn how to use letters, numbers and symbols to represent given situations. As you move on, reflect on the following questions: ***Why are algebraic expressions useful? How can real – life problems involving one variable be modeled and solved?***

In Activity 1.1, you found out that the magic number is 3 and even if you keep on changing your initial number, you always arrive with 3 in step 6. Let's revisit the activity using 10 as the initial number.

Steps	result
1. Think of a positive integer.	10
2. Multiply the number by 2.	10(2) = 20
3. Add 6.	20 + 6 = 26
4. Divide by 2.	26 ÷ 2 = 13
5. Subtract the original number in step 1.	13 – 10
6. The magic number is	3

Since the number in the first step is any positive number, we cannot list all. After all, we have an infinite number of positive numbers. Since we cannot list all positive numbers, we can just represent the positive number using the letters in the English alphabet. Let say, ***n*** *is equal to a positive number*. In this case, the value of ***n*** can be 1,2,3,4 and so on. All the positive numbers that ***n*** represents is what we call the ***replacement set***. In other words, the replacement set is the set of values for ***n***. Zero or negative numbers are not included in our replacement set because the replacement only includes positive numbers. The letter or symbol, in this case ***n***, that represents any number in the replacement set is called ***variable***.

In contrast, in step 2, we can only add 2 and not any other number. Likewise, in step 3, we only add 6 and not any other number. In these cases, 2 and 6 are called ***constants*** because they have fixed values and we cannot use any other number. The constants, variables or a combination of constants and/or variable with operation symbols are called ***algebraic expressions***.

Now, let's revisit the table and represent the result of each step using algebraic expressions.

Steps		Let ***n*** be a positive integer
1. Think of a positive integer.	10	n
2. Multiply the number by 2.	10(2) = 20	n (2) = 2n
3. Add 6.	20 + 6 = 26	2n + 6
4. Divide by 2.	26 ÷ 2 = 13	$\frac{2n+6}{2} = \frac{2n}{2} + \frac{6}{2} = n + 3$
5. Subtract the original number in step 1.	13 – 10 = 3	n + 3 – n = 3
6. The magic number is	3	3

The result is also 3. Since ***n*** represents all positive integers, we are certain that the answer in step 6 is always 3 as long as ***n*** is a positive integer.

Let's try another puzzle.

ACTIVITY 3. You are X years old

Follow the steps below. Write the result for each step in the second column. The number in step 6 reveals your age and the number of siblings you have. Example, if you arrive with 155, it means you're 15 years old and you have 5 siblings.

Steps	result	Let ***x*** be your age and ***n*** be the number of your siblings
1. Write your age in years.		
2. Multiply your age by 2.		
3. Add 10.		
4. Multiply by 5.		
5. Add the number of siblings you have (must be less than 10).		
6. Subtract 50.		

You can repeat this activity with your friends. Let them do steps 1 – 6, then let them tell you the answer in step 6. They will be amazed that you're able to tell how old are they and how many siblings they have.

Next, try to repeat steps 1 – 6, but this time, represent your age with ***x*** and ***n*** represents the number of your siblings. Write your answer in the third column. Show it to your teacher.

PROCESS QUESTIONS:

1. What are the variables you used?
2. What are the constants you used?
3. What is the algebraic expression in step 3? _______________
   In words, it means "twice your age plus 10".

4. The algebraic expression in step 6 is ____________. In words, it means that
   ____
   _____.

Since age and the number of siblings can vary from one person to another, it is easier to describe the situation using algebraic expressions. The representation of a situation using algebraic expressions is called ***algebraic model***.

In the next activities, you will be asked to model various situations using algebraic expressions.

ACTIVITY 4. Model Me

Read the following situations then answer the follow up questions.

1. Richard's mobile plan charges a monthly fee of PhP 300 for unlimited text to all networks and PhP 7 per minute for every phone call to all networks.
   a. If Richard made 30 minutes of phone call for the month of January, how much is his bill? __________________
   b. If Richard made 75 minutes of phone call for the month of February, how much is his bill? _______________
   c. If ***m*** represents the total minutes of Richard's phone call made for a month, what algebraic expression models the monthly bill of Richard? _____________________
   d. Using your answer in ***c***, the constant is ___________ because ________________ while the variable is ______________ because ____________________________.
   e. Using your answer in ***c***, the algebraic model ______________ means that "____ _____".

2. Alex opened a new checking account. He deposits PhP10,000 each week. Justin and Gabriel made tables to represent the relationship between the number of deposits Alex made a deposit and the total amount of money he has in his checking account.

   Justin's Table

Number of deposits	Total Amount of Money (in Pesos)
1	10,000
2	20,000

3	30,000
10	100,000
w	100,000 + w

Gabriel's Table

Number of deposits	Total Amount of Money (in Pesos)
1	1 (10,000) = 10,000
2	2 (10,000)= 20,000
3	3 (10,000)= 30,000
10	10 (10,000)= 100,000
w	w (10,000) = 10,000w

PROCESS QUESTIONS:

a. What is wrong with Justin's table? Why?
b. Is Gabriel's table correct? Why?
c. What did Gabriel do in his table that made the situation easy to understand?
d. When modeling a situation using algebraic expressions, you can use ______________________ to illustrate the relationship of the related quantities.

3. Marilyn rented a baby car at a mall to help her stroll her toddler. The total cost includes a one-time rental fee of PhP100 plus PhP 50 for each hour. Complete the table that shows the relationship between the number of hours Marilyn rented the baby car and the total cost of the rental.

Number of Hours	Total Cost (in Pesos)
1	?
2	?
3	?
5	?
h	?

a. How much will Marilyn pay if she rented the baby car for 1 hour? 5 hours?
b. The total cost for ***h*** hours is represented by ___________________. The model can be interpreted as __________________________________

____
_____.

PROCESS QUESTIONS:

1. Are you able to correctly represent the three situations using algebraic expressions? If not, what makes it difficult for you to represent?
2. In problem # 1, no table was presented. In number 2 and 3, a table shows the relationship of the two quantities. Given another chance, will you use a table to represent situation? Why?
3. How can real – life problems involving one variable be modeled and solved?

Now that you're able to represent certain situations using algebraic expressions, move on to the next activity to find out some of the struggles of students involving algebraic expressions.

ACTIVITY 5. Misconception Check A

Analyze the pictures below. What is being shown in each picture? Write your reflection in the space provided. Explain how the errors can be corrected.

image adapted from from https://www.pinterest.com/ariascec/common-math-misconceptions/

____
____
____
____

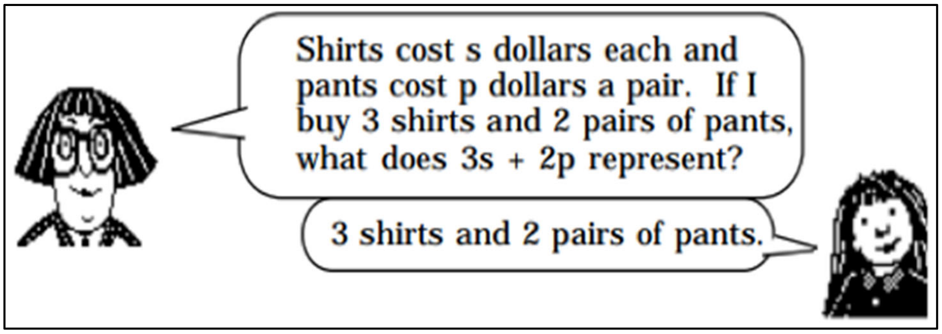
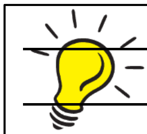


image adapted from http://www.learnquebec.ca/export/sites/learn/en/content/curriculum/mst/documents/algemisc.pdf



When you represent problems using mathematics just like what you have done in the previous activities, you are doing what is called **mathematical modeling**. Mathematical models can come in different forms such as:

- **oral and written form**
- **table/list of values**
- **graph, drawing or diagram; and**
- **algebraic models**

Consider the situation below.

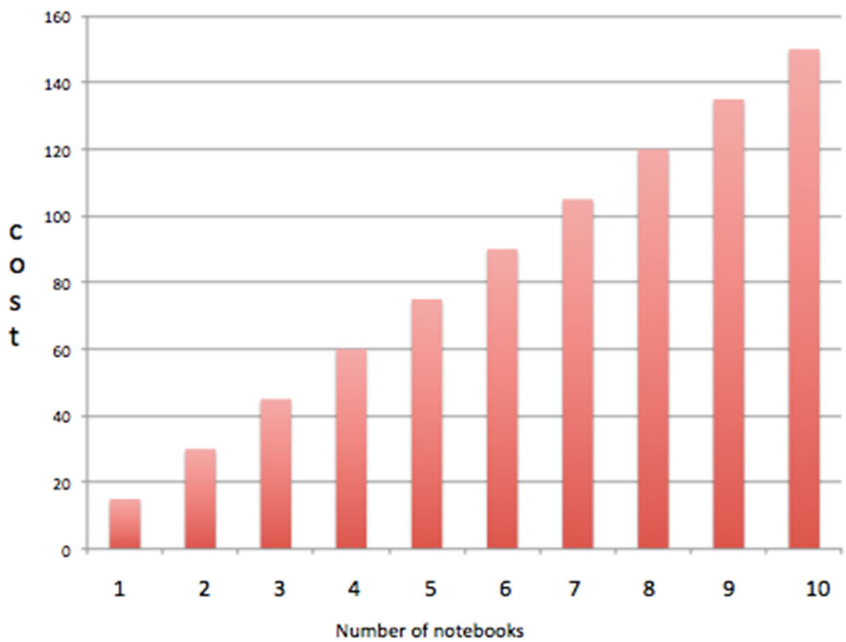
A notebook cost PhP 15. How much is the cost of 5 notebooks? 20 notebooks? *p* notebooks?

Using a table, the situation can be modeled as:

Number of notebooks	1	2	5	20	p
Cost (in pesos)	1(15)=15	2(15)=30	5(15)=75	2(15)=300	p(15)=15p

Tables and/or list of values are very helpful in looking for patterns. After all, you are familiar with repetitive computations. However, they take time to construct and maybe impractical sometimes. You should not be rely only to this model all the time.

A graphical model will also show the relationship of the number of notebooks sold and the total cost. However, graphs generally take more time and space to construct. Just look at the graph below, the greatest number for the number of books is 10 while in the problem, it asks for 20. This can be remedied if you use 5, 10, 15 and 20 as values in the x-axis but it might not help find the pattern easily.



Since the first activity in this module, you were tasked to represent the situation using algebraic models. Algebraic models describe the relationship among quantities using constants, variables and operation symbols. From the situation above, if we let *p* represent the number of notebooks and *c* represent the total cost of the notebooks, then the algebraic model is c = 15 p. In words, it means *"the total cost c is 15 times the number of notebooks p"*.

Tables, graphs and algebraic models are powerful tools to model word problems. These models will greatly help you analyze and understand the problems. Throughout the module in algebra, you will be asked to model situations using algebraic models. However, you can always use algebraic models in combination with the tables and graphs. In the end, you should be able to figure out the most appropriate strategy in modeling a certain situations.

ACTIVITY 6. My Models

Model the situation below using the varied strategies. Write your answer in the space provided.

The parking fee is 50 pesos for the first hour plus 10 pesos for every hour thereafter.

Use a table/list

Use a graph/chart

The best model is

because

Use algebraic expression

Use written words

Certain real – life situations can be modeled by ______________________________
____.
Modeling helps me ____.

ACTIVITY 7. Algebraic Models

Formulas are algebraic models. You might have encountered the following formulas before. In each formula, identify the variables and constants.

	Constant	Variable

1) $A = \frac{1}{2}bh$ formula for finding the area of a triangle		
2) $V = lwh$ formula for finding the volume of a rectangular prism		
3) $P = 2l + 2w$ formula for finding the perimeter of a rectangle		
4) $F = \frac{9}{5}C + 32$ formula for converting temperature from °C to °F		

Skills Readiness Check: Reflect on the level of your performance today. Check the first column if you **need more practice** or you are now **ready to move on to the next activity**.

I Need more practice (if some of your answers are incorrect)	I am ready to move on to the next activity (if you answered all items in Activity 1.7 correctly)
Ask for additional problems from your teacher. Visit the discussion forum and post your questions and clarifications to your classmates.	You may proceed to the next activity

Now that you have represented various situations using algebraic models through the use of constants, variables and operation symbols, its time for you to create your own algebraic expressions.

ACTIVITY 8. My Algebraic Models

Give 5 real – life situations and represent the situation using algebraic expressions. Two example as provided for you.

Real – life situation	Algebraic expression
The cost of a bottled water is PhP10. What is the cost of ***b*** bottles?	Let b = number of bottles **10b** is the cost of b bottles
The age of Rudy is twice the age of Mario. What is the age of Rudy?	Let a = age of Mario **2a** is the age of Rudy

1.	
2.	
3.	
4.	
5.	

PROCESS QUESTIONS:

1. What are variables you used? Did you use related letters to represent the quantities or you just used varied letters?
2. Where do algebraic expressions come from?
3. How can real – life problems involving one variable be modeled and solved?

After familiarizing yourself with algebraic expressions, proceed to the next page to learn on how to evaluate algebraic expressions.

You are already familiar in using algebraic expressions to model certain situations. In the previous activity, ***10000x*** models the total amount of savings Alex has in his account after making ***x*** deposits with 10,000 pesos each deposit.
If we are to calculate the total amount Alex has in his account after making 20 deposits, then, all we have to do is substitute ***x*** with ***20*** , thus 10000x = 10000(20)=200,000. This process of solving the value of an algebraic expression using assigned values is called **evaluation of algebraic expressions**.

Find out more about this process by doing the next two activities.

ACTIVITY 9. I Trick You

Just like the first activity in this module, follow the steps below. Try at least two different positive integers. In the second column are the equivalent algebraic expressions in each step. Substitute the value provided in each step.

Steps:	Let n = number	n = 3	n = 7
1.Think of a positive integer.	n		
2. Add 8 to it.	n + 8		

3. Multiply the result by 2.	2(n + 8)		
4. Subtract 6.	2(n + 8) – 6		
5. Divide by 2.	$\frac{2(n + 8) - 6}{2}$		
6. Subtract the number you first thought of.	$\frac{2(n + 8) - 6}{2} - n$		

PROCESS QUESTIONS:

1. Did you arrive with the same number after trying several times?
2. Do you think the result will be the same regardless of the number you start with? Why do you think so?
3. In what ways can we use algebraic expressions?

ACTIVITY 10. My Trick

Develop your own trick that has at most five steps. It should always lead to a constant number. Test it with three different numbers to see if the answers are the same. After that, write the algebraic equivalent of each step in the last column. You can try it with a friend. Then, explain to your teacher how come the result is always the same.

Steps	1st #	2nd #	3rd #	Let n = number
Step 1.				
Step 2.				
Step 3.				
Step 4.				
Step 5.				

PROCESS QUESTIONS:

1. Did you arrive with the same number after trying several times?
2. Do you think the result will be the same regardless of the number you start with? Why do you think so?
3. Why are algebraic expressions useful?

To further improve you understanding on how to evaluate algebraic expressions, do the next activity.

ACTIVITY 11. FAMILIAR OBJECTS

Continue to develop your understanding of evaluating algebraic expressions by doing this activity.

Basket A

Basket B

Basket C

a. Basket A has 7 oranges, basket B has 9 apples and basket C has 4 bananas.
How many baskets A, B or C, can you bring to have 43 pieces of fruits? Give 2 combinations and write them in the table below. An example is provided for you.

How many basket A? x	How many basket B? y	How many basket C? z	To have 43 pieces of fruits	Algebraic Expression
3	2	1	43	7x+9y+4z=7(3)+9(2)+4(1)=43
			43	
			43	

b. If the total number of fruits is 100, how many of each basket will you bring? What algebraic model represents this new situation?

In the last three activities, you tried **evaluating algebraic expressions**. This is the process of finding the value of the expression by replacing the variables with assigned values.

For example, evaluate $(x + y)z$ if x = 10, y=4 and z = 2.

$(x + y)z = (10 + 4)2$ Substitute the given values for x, y and z
$(14)2 = 28$ Simplify by following the order of operations.

Thus, the value of the expression $(x + y)z$ is 28 if x = 10, y=4 and z = 2.
Read more about evaluating algebraic expressions by using the online resources below.

ACTIVITY 12. Evaluating algebraic expressions online

Access the websites about evaluating algebraic expressions.
https://www.youtube.com/watch?v=i-RUMZT7FWg
A video showing how to evaluate algebraic expressions.

http://www.mathplanet.com/education/pre-algebra/introducing-algebra/evaluate-expressions
An article about evaluating algebraic expressions.

http://www.math.com/school/subject2/lessons/S2U2L3GL.html#sm1
The article shows the step by step process of evaluating algebraic expressions.

After watching the video and reading the articles, summarize the steps and write it on the space provided.

To evaluate algebraic expressions:
Step 1:
Step 2:
Step 3:
Step 4:
Step 5:

One of the important things to remember when evaluating algebraic expressions is ____.

Another one is ____

____

ACTIVITY 13. Interactive Online Games 1

Access the websites below and play the interactive games. As you play, you will test your skill of evaluating algebraic expressions.
http://www.math-play.com/Evaluating-Expressions-Basketball-Game/Evaluating-Expressions-Basketball-Game.html
Online game involving evaluating algebraic expressions

https://www.quia.com/rr/195129.html?AP_rand=712770679
Online game involving evaluating algebraic expressions
Now that you practiced evaluating algebraic expressions, it's time for you to test yourself by doing the next activity.

ACTIVITY 14. Misconception Check B

Analyze the picture below. Write your observations and realizations in the space provided

____

____

____

____

____

____

____

____

____

____

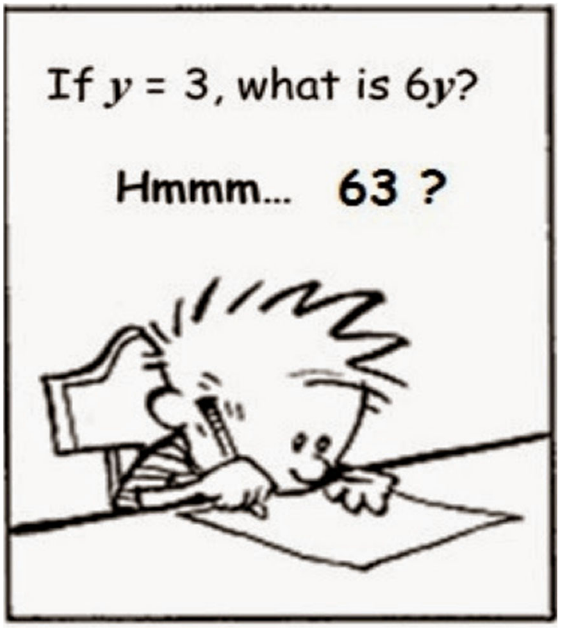


image adapted from http://www.resourceaholic.com/2014/08/algebra.htm

Skills Readiness Check: Reflect on the level of your performance today. Check the first column if you **need more practice** or you are now **ready to move on to the next activity**.

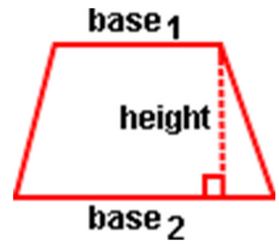
<u>I Need more practice</u> (if some of your answers are incorrect)	<u>I am ready to move on to the next activity</u> (if you answered all items in Activity 1.14 correctly)
Ask for additional problems from your teacher. Visit the discussion forum and post your questions and clarifications to your classmates.	You may proceed to the next activity

ACTIVITY 15. Evaluating Algebraic Expressions

Evaluate each algebraic expression by using the value/s provided. Submit to your teacher when finished.

1. Evaluate $2x^3 - 1$ when $x = 1$.
2. Let $x = -3$, $y = 4$, $z = -1$, evaluate the $2x(y + z)$.

3. The area *A* of trapezoid is given by this formula, $A = \frac{1}{2}(b_1 + b_2)h$



Find *A*

a. when b_1 = 2 cm, b_2 = 5 cm, and *h* = 4 cm.

b. when b_1= 3 cm, b_2 = 6 cm, and *h* = 5 cm.

PROCESS QUESTIONS:

1. Are you able to correctly evaluate all the algebraic expressions? If not, why?
2. If your friend has the same misconception as the one above, how will you correct the error?
3. Why are algebraic expressions useful?

You know already how to evaluate algebraic expressions. But do you know how to form algebraic expressions from a given situation? Yes, you tried it in the previous activities. You represented the total amount of money Alex has in the bank and the amount Marilyn will have to pay after renting the baby car for a certain number of hours. This time, you will have more activities in translating English phrases into mathematical phrases.

Mathematics is also a language. English phrases and sentences that deal with quantities can be changed into language of mathematics in a short and concise way. To translate English phrases to mathematical phrases or algebraic expressions, you need to associate keywords to their corresponding symbols. Try the next activity.

ACTIVITY 16. MATHching A

Can you match the different phrases to their corresponding symbols? Drag each phrase inside the basket with the correct symbols.

of

sum

less

over

more

times

minus

less than

divided by

the ratio of

multiplied by

Increased by

decreased by

subtracted to

the product of

the quotient of

the difference of

The symbols, + − × ÷ , are symbols of ***operations****.*

The symbols of ***relations*** *are*

$>$ read as 'is greater than' or ' is more than'
$\geq$ read as 'is greater than' or 'equal to', or ' is at least'
$<$ read as 'is less than'
$\leq$ read as 'is less than or equal to' or ' is at most'
$=$ read as 'is equal to', 'equals', or 'is'
$\neq$ read as 'is not equal to'

By knowing the symbols of operations and relations, you can translate English phrases to mathematical phrases/algebraic expression. Check how the following are translated.

English phrases	***algebraic expression***
1. $\underbrace{\text{a number}}_{y}\ \underbrace{\text{increased by}}_{+}\ \underset{\downarrow\ 4}{4}$	$y + 4$
2. $\underbrace{\text{twice a number}}_{2x}\ \underbrace{\text{decreased by four}}_{-4}$	$2x - 4$
3. $\underbrace{\text{five}}_{5}\ \underbrace{\text{less than}}_{-}\ \underbrace{\text{the quotient of } p \text{ and } q}_{\frac{p}{q}}$	$\frac{p}{q} - 5$
4. $\underbrace{\text{t}}_{\text{t}}\ \underbrace{\text{is less than}}_{<}\ \underset{\downarrow\ 3}{\text{thrice}}\ \text{the}\ \underbrace{\text{difference of m and n}}_{(\text{m} - \text{n})}$	$\text{t} < 3(m - n)$
5. $\underbrace{\text{twice}}_{2}\ \underbrace{\text{the sum of } p \text{ and } q}_{(p+q)}$.	$2(p + q)$

Notice the distinction between "less than" and "is less than".

To learn more about translating English phrases to algebraic expressions, do the next activity.

ACTIVITY 17. Translating online

Access the websites to learn more about translating English phrases to algebraic expressions.
https://www.youtube.com/watch?v=eIWsWDwoUoA
A video showing how to translate English phrases to algebraic expressions.

http://www.mathgoodies.com/lessons/vol7/expressions.html
An article about translating English phrases to algebraic expressions.

http://www.wwu.edu/tutoring/docs/englishtomathandvocab.pdf
An article about common phrases used in translating English phrases to algebraic expressions.

Now that you have watched the video and read the articles, it's time to test your skill through these online games.

ACTIVITY 18. Interactive Online Games 2

Access the websites below to play the interactive games. As you play the game, you will test you skill in translating English phrases to algebraic expressions and vice versa.
http://www.math-play.com/Algebraic-Expressions-Millionaire/algebraic-expressions-game.html
An online game about translating English phrases to algebraic expressions

https://www.quia.com/rr/520475.html
translating English phrases to algebraic expressions and vice versa.

After doing all the online tasks, it is time to test your skills in translating algebraic expressions.

ACTIVITY 19. MATHching B!

Match the English phrase in column A with their equivalent algebraic expressions in column B. Write the letter of your answer in the space provided before each number.

COLUMN A	COLUMN B
_____1.five times a number decreased by 4	A. $5 - x$
_____2.five decreased by four times a number	B. $\dfrac{5}{x-4}$
_____3.the square of a number decreased by four	C. $2x + 4$
_____4.four more than the square root of a number	D. $\sqrt{x} + 4$
_____5.five less than four times a number	E. $4 + \sqrt{x}$
_____6.twice a number increased by four	F. $5 - 4x$
_____7.the sum of the number and four	G. $5x - 4$
_____8.five divided by the difference of the number and four	H. $x^2 - 4$
_____9.the product of four and the number is increased by 5	I. $4x - 5$
_____10.a number subtracted from five	J. $x + 4$
	K. $4x + 5$

Skills Readiness Check: Reflect on the level of your performance today. Check the first column if you **need more practice** or you are now **ready to move on to the next activity**.

I Need more practice (if some of your answers are incorrect)	I am ready to move on to the next activity (if you answered all items in Activity 1.19 correctly)
Ask for additional problems from your teacher. Visit the discussion forum and post your questions and clarifications to your classmates.	You may proceed to the next activity

ACTIVITY 20. Translating

Write an algebraic expression for each of the following English phrases. Let n = number.

1. one-fourth of a number 1.______________
2. six less than the sum of the number and seven 2.______________
3. a number divided by 12 3.______________
4. the difference between the number and its squared 4.______________
5. fifteen less than a number is five 5.______________
6. the sum of eight and a square of a number 6.______________
7. four more than three times a number 7.______________
8. nine more than twice a number is twelve 8.______________
9. seven less than a number is 20 9.______________
10. one less than three times the difference of seven and a number is four 10.______________

Now that you acquire some important knowledge and skills about this lesson, revisit your IRF worksheet before you move to the deepen section.

ACTIVITY 21. IRF Worksheet Revisited

You completed the Initial Answer section of the worksheet below. This time, revise your ideas under the "Revised answer". Read aloud what you've written and when satisfied, click "SAVE".

Initial Answer
You completed this part in Activity 1.2
Revised Answer Algebraic expressions are used to ____. Real – life problems can be modeled and solved by ____.
Final Answer

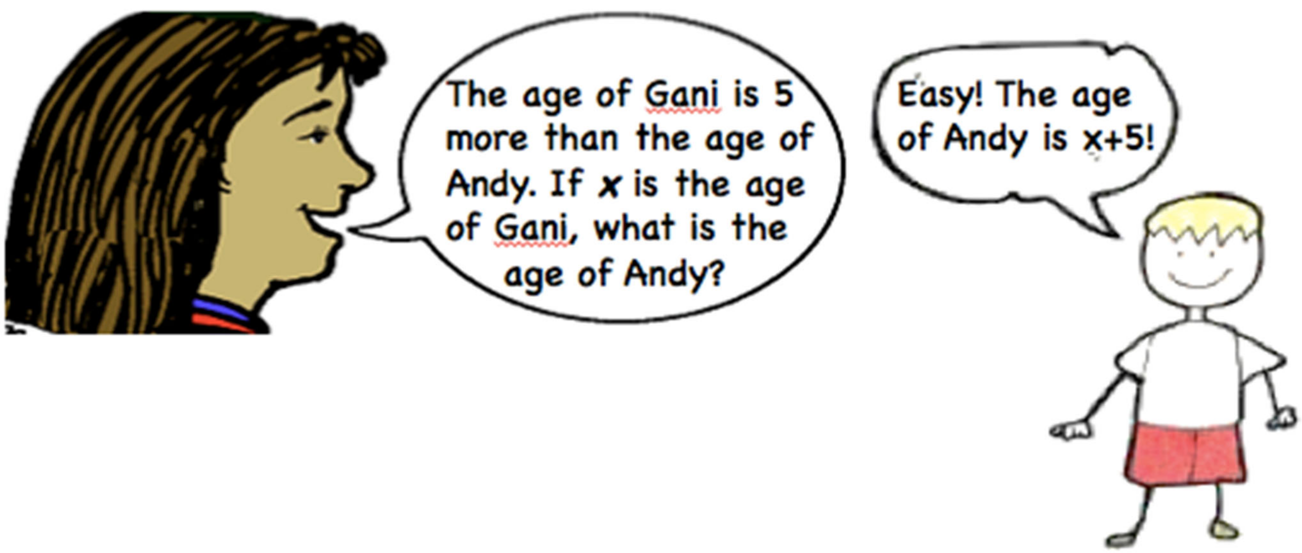
End of Firm Up
You learned in this section how algebraic expressions can be used to model certain situations. From a situation written in English phrase, you translated it into a mathematical phrase. You also evaluated various algebraic expressions. If there are some concepts that are not yet clear to you, you can go back to the previous activities or access the online videos, articles and games to improve your understanding. If you're ready, you can proceed to the next part of this module.

DEEPEN

our goal in this section is to deepen your understanding of the **sefulness of algebraic expressions in modeling real life ituations.** Having the skill of translating real life situations ıathematically will help you solve problems in an easier manner.

The next activities and exercises will help you deepen your understanding of algebraic expressions.

Using key words help in translating English phrases to mathematical phrases. However, in some cases, when keywords are translated directly, it results to incorrect representation. Examine the situation below.



Did you figure out what is wrong? Are you confused? Let's make the situation clearer by assigning values to the age of Gani.

If Gani is 30 years old, then Andy should be 30 – 5 = 25 years old.
If Gani is 23 years old, then Andy should be 23 – 5 = 18 years.

However, using **x+5** would have resulted to:

If Gani is 30 years old, then Andy is 30 + 5= 35 years old.
If Gani is 23 years old, then Andy is 23 + 5 = 28 years.

The ages 35 and 28 make Andy older than Gani, which is absolutely incorrect. Do you know why many commit this error? The error is the result of associating ***"more than"*** to ***addition only*** without trying to understand the situation. It is always important to understand the situation first before coming up with a representation. Do not rely only in the keywords. In the next activity, analyze and understand the situation first before you translate into algebraic expressions.

ACTIVITY 22. Revisiting Translating

Write an algebraic model for each of the following situations in the second column. Be careful not to rely on the keywords only. Write the explanation in the third column. An example is provided.

Situation	Algebraic expression	Explanation
The ***total cost*** of a meal with dessert is PhP110. What is the cost of the dessert if the meal only costs ***x*** pesos?	110 – x	If 110 is the total cost and the meal only cost ***x*** pesos, then the cost of the dessert is the total cost minus the cost of the meal only
1. Joy is older than Jane. The ***difference*** of their ages is 6. What is the age of Joy if Jane is ***b*** years old?		
2. The ***total*** income of Mr. and Mrs. Buyawe is PhP54,000. What is the income of Mrs. Buyawe if Mr. Buyawe's income is ***t*** pesos?		
3. The age of Robert is ***twice*** the age of Randy. What is the age of Randy if ***k*** is the age of Robert?		
4. The cost ***of*** 7 pencils is ***d*** pesos. What is the cost of each pencil?		
5. The ***total*** salary of the ***m*** employees is PhP112000. What is the salary of each employee?		
6. Jerry and John ***divided*** the cost of the food equally and each paid w pesos. How much did they pay in all?		

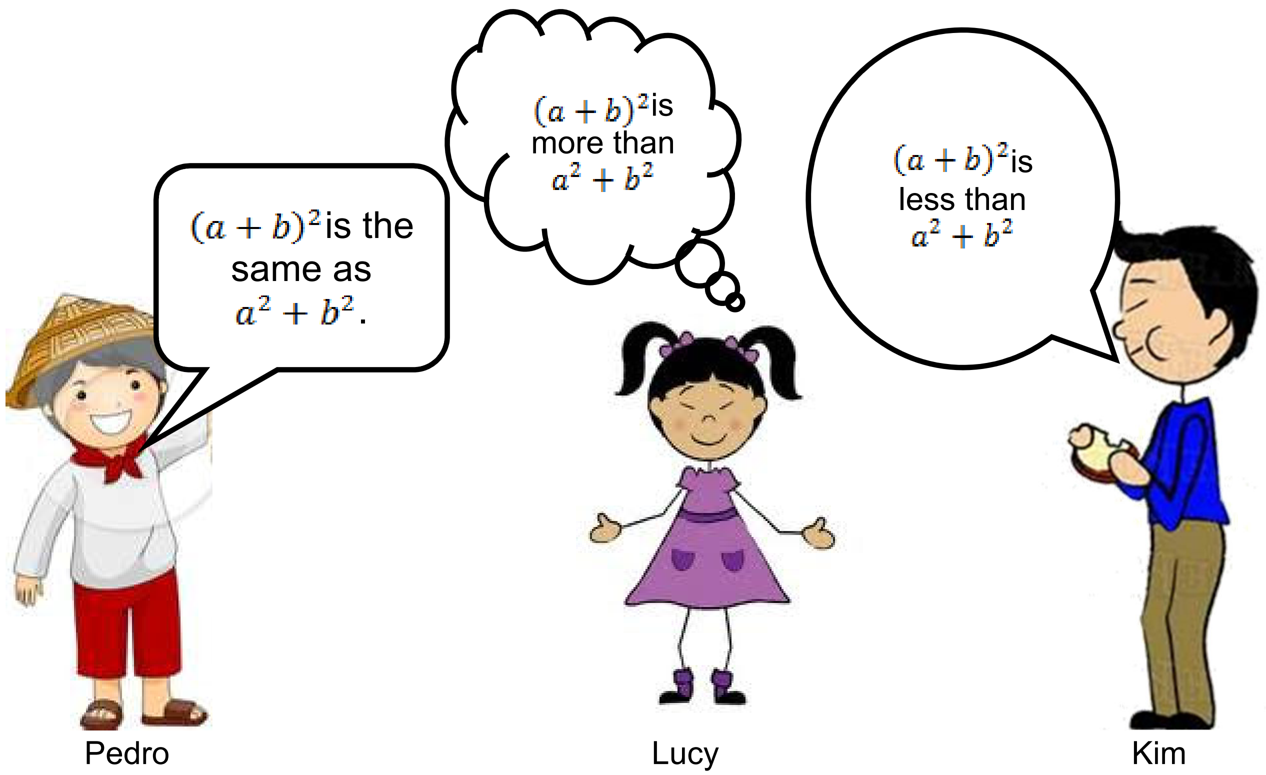
PROCESS QUESTIONS:

1. Are you able to represent all the given situations into algebraic expressions? If not, what makes it difficult for you?
2. What do you need to remember when translating English phrases into mathematical phrases/algebraic expressions?
3. Is it wise to rely only in keywords? Were you asked before not to take the "literal meaning" of word? Explain your answer.
4. How can real – life problems involving one variable be modeled and solved?

By now you should have realized that modeling a situation relying only in keywords is not wise. Modeling situations makes the situation easier to understand, reveals patterns and leads to possible solutions. It is a skill you need to acquire and develop as you can use it not only in math but also in varied real – life situations. Modeling helps you become a better critical thinker. Let's test your critical thinking skills in the next activity.

ACTIVITY 23. Who Is Telling The Truth?

Decide who is telling the truth. Put a check mark below the name of the person you think is telling the truth. After that, test your answer by completing the table below.



Assign values for ***a*** and ***b***. Then complete the table. An example is provided for you.

a	b	$a+b$	$(a+b)^2$	a^2	b^2	a^2+b^2	Who is correct?
1	2	1 + 2 = 3	3^2 = **9**	1^2 = 1	2^2 = 4	1 + 4 = **5**	Lucy

So, who is telling the truth? ____________________

PROCESS QUESTIONS:

1. If the values of ***a*** and ***b*** are both positive, is Lucy always correct?
2. If the values of ***a*** and ***b*** are both negative, is Kim always correct?
3. Who is correct if the values of ***a*** and ***b***, respectively, are 1,1; 0,0; -3,3; 4,-4?
4. Is there a change in your initial answer after question number 3? Did you consider all possible values for ***a*** and ***b*** before you made your decision?
5. When you examine all possible results before you decide, are you doing critical thinking? Why?
6. How does algebra enhance your critical thinking skills?

After improving your skills in modeling certain situations and making decisions, it is time to revisit your IRF Worksheet.

ACTIVITY 24. IRF Worksheet Finalized

You completed the Initial and Revised Answer sections of the worksheet below. This time, finalize your ideas under the "Final Answer". **Read aloud** what you've written and you're satisfied, click "SAVE".

Initial Answer
Revisit your answer in Activity 1.2
Revised Answer Revisit your answer in Activity 1.21
Final Answer Algebraic expressions are used to ____ _____. Real – life problems can be modeled and solved by ____ _____.

PROCESS QUESTIONS:

1. What did you change in your previous answers? Why?

Now that you have finalized your thoughts in the IRF worksheet, are you confident to answer problems related to algebraic expressions? Click **this link** to revisit the required competencies for the lesson. When ready, proceed to the next activity.

End of Deepen
You learned in this section how algebraic expressions can be useful in solving problems involving real life situations.

Now that you have a deeper understanding of the topic, you are ready to do the task in the next section

Your goal in this section is apply your learning to real – life situations. You will be given a practical task which will demonstrate your understanding.

TRANSFER

ACTIVITY 25. Happy Birthday Brother

Read and analyze the situation below. Apply the different skills you learned throughout the lesson. Refer to the rubric provided to see how your work will be evaluated.

TASK

Your 2-year old brother will be celebrating his birthday one month from now and your parents have decided to hold it at McBeenasal. Your mother showed you a brochure and asked your help to come up with possible set meals with one main meal, one side dish, one dessert and one beverage. There are 40 expected guests. She told you that the budget of PhP10,000 must be maximized. You are asked to come up with a model that shows the different set meals and a formula to compute the total cost. These will help your mother decide which is the best set meal. Below is the price list of McBeenasal.

MAIN MEAL	
FRIED CHICKEN	₱73
SPAGHETTI	₱44
CHEESE BURGER	₱43
SIDE DISH	
FRENCH FRIES (REGULAR)	₱23
FRENCH FRIES (SPECIAL)	₱35
MACARONI SOUP	₱41
DESSERT	
ICE CREAM (REGULAR)	₱15
ICE CREAM (SPECIAL)	₱25
MANGO PIE	₱23
BEVERAGE	
SOFTDRINK	₱43

ICED TEA (LARGE)	₱43
PINEAPPLE JUICE (LARGE)	₱43

When you are done with the task, use the rubric below to evaluate your work. Your work should be under “Proficient” or “Expert” for each criteria. If your work has these traits, you are ready to submit your work. Otherwise, revise your work before submitting it.

RUBRIC

CRITERIA	Expert 4	Proficient 3	Developing 2	Beginning 1
CONTENT	The needed data are comprehensive, authentic and very relevant.	The needed data are complete, authentic and relevant.	The needed data are incomplete, authentic and relevant.	The needed data are incomplete, unauthentic and some are not relevant.
MATHEMATICAL CONCEPTS	Demonstrates thorough understanding of algebraic expression.	Demonstrates understanding of algebraic expression.	Demonstrates some understanding of algebraic expression.	Demonstrates little understanding of algebraic expression.
ACCURACY	The solutions are logical and the computations are accurate and precise.	The solutions are orderly and the computations are correct.	The solutions are incomplete and some computations are incorrect.	The solutions are illogical and computations are inaccurate.

PROCESS QUESTIONS:

1. Among the combinations, which set meal is the cheapest?
2. Among the combinations, which set meal is the most expensive?
3. Among the combinations, which set meal do you think is the best? Why?
4. If five more guests are added, how much is the budget per person?
5. Which model did you use to analyze the situation? Did you use only one or a combination of different models? Why?
6. What formula did you use to facilitate easy computation? Define your variables.
7. How can real – life problems involving one variable be modeled and solved?

Now that you have finished the task, it is time to reflect on what happened in this lesson. Complete the synthesis journal in the next page before you move to the next lesson.

ACTIVITY 26. Synthesis Journal

Reflect on what you have done and learned in this lesson. Record your reflections in the journal below.

SYNTHESIS JOURNAL		
WHAT I DID	WHAT I LEARNED	HOW CAN I USE IT

End of Transfer
In this section, you completed a task that made you apply the different concepts and skills you gained throughout the module. Before you proceed to the next lesson, summarize what you've learned in the generalization table.

CONGRATULATIONS! You can now proceed to the next lesson.

GLOSSARY OF TERMS USED IN THIS MODULE:

algebraic expression is a variables, constants and/or a combination of constants and variables together with operation symbols

constant is a symbol that has a fixed quantitative value

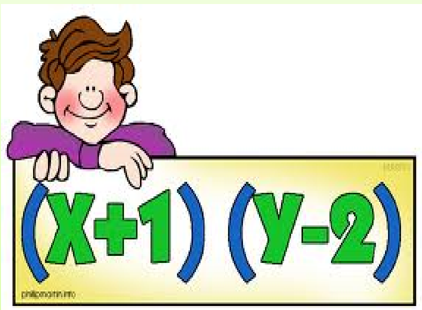
evaluating algebraic expression is the process of finding the value of the expression at a specified value of the variable

variable is a symbol or letter that does not have fixed quantitative value

WEBSITE RESOURCES AND LINKS IN THIS MODULE:

Lesson 2: Polynomials

Introduction and Focus Questions



In this lesson, you will focus on a special kind of algebraic expressions called polynomials. What makes polynomials different from other algebraic expressions? How can polynomials help in modeling real – life situations? As you learn more about polynomials, reflect on the answer of the question: ***How can real – life problems involving one variable be modeled and solved?***

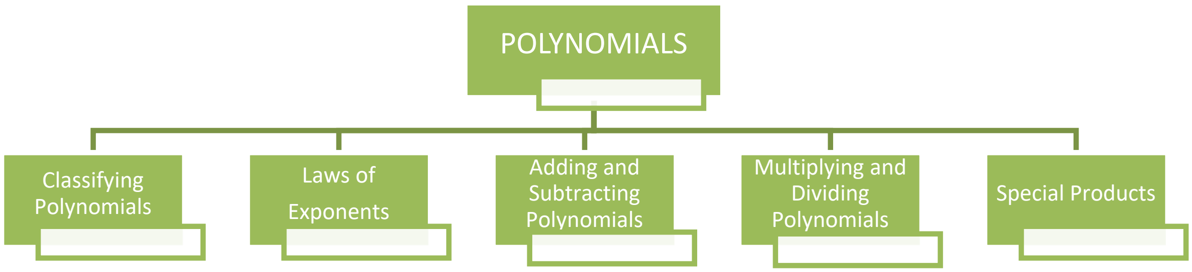
☑ LESSON COVERAGE

This lesson has the following topics:

Topic No.	Title	You'll learn to…	Estimated Time
Topic 1	Classifying Polynomials	Classify algebraic expressions which are polynomials according to degree and number of terms	2 hours
Topic 2	Laws of exponents	Interpret the meaning of a^n where n is a positive integer. Derive the laws of exponents.	3 hours
Topic 3	Adding and Subtracting Polynomials	Add and subtract polynomials.	3 hours
Topic 4	Multiplying and Dividing Polynomials	Multiply and divide polynomials	4 hours
Topic 5	Special Products	Use models and algebraic methods to find the: (i) product of two binomials; (ii) product of the sum and difference of two terms; (iii) square of a binomial; (iv) cube of a binomial; (v) product of a binomial and a trinomial. Solve problems involving special products	4 hours

☑ Concept Map of the Lesson

Here is a simple illustration of the topics you will cover in this lesson



☑ Expected Skills

To do well in this lesson, you need to remember and do the following:

1. Making outlines, concept maps and notes will be very helpful in understanding the module.
2. Follow the instructions provided for each activity.
3. Review and evaluate your work using the rubric provided before submission.
4. Complete all exercises.
5. Be mindful of the meaning of unfamiliar words you encounter in this module. A glossary of terms is provided in the last part of this module.
6. Maximize the use of online resources in each lesson. Online resources can be accessed multiple times. The summary of online resources is provided in the end of the module.

EXPLORE

In the previous lesson, you have learned about algebraic expressions. In this lesson, you will learn about polynomials. What makes polynomials different from algebraic expressions? You will find out the answer as you progress in this lesson. Also, in the end of the lesson, you should be able to answer: How can real – life problems involving one variable be modeled and solved?

ACTIVITY 1. Algebraic Expressions VS Polynomials

Classify the given as algebraic expression and/or polynomials by putting a check mark (√) under the appropriate column. As you do so, you are trying to answer the question: What makes polynomials different from algebraic expressions? Click on "Save" when you're finished.

Given	*Algebraic Expression*	*Polynomial*
1. 12		
2. – 7		
3. ¼ m		
4. $-6p^3$		
5. $\mathbf{5d^{-2}}$		
5. $61xy^4$		
6. $\frac{2x}{3}$		
7. $\frac{2}{3x}$		
8. $\sqrt{2}\,xy$		
9. $\sqrt{5xy}$		
10. $3b - 5c^2 + \frac{1}{2}w^4$		

Process Questions:

1. How did you differentiate algebraic expressions from polynomials?
2. Can we consider algebraic expressions as polynomials? Why? Why not?
3. Can we consider polynomials to be algebraic expressions? Why? Why not?

End of Explore
Are you sure of your answers or you have some doubts? Let's now find out more about polynomials and algebraic expressions by doing the next part.

FIRM-UP

Your goal in this section is to learn and understand key concepts about polynomials. You will learn how to classify and perform operations on polynomials. You will look at a number of examples and work on different activities. As you move on, reflect on the following questions: How can we use polynomials? How can real – life problems involving one variable be modeled and solved?

You learned in the previous lessons what algebraic expressions are. Some examples are of provided below. Are they familiar?

ALGEBRAIC EXPRESSIONS

1 $-8p^2$ $\frac{3m}{4}$ $\frac{\sqrt{6}}{m}$ $x^2 - \frac{2}{3}y + 4$ $x^{-2} - \frac{2}{3y} + 4$ $\frac{6}{5}$ p^{-2} 6^{-2} $p^{1/2}$ $\frac{3}{4m}$ $\frac{\sqrt{5m}}{6}$ $\sqrt{6}\ m$

Some algebraic expressions can be classified as polynomials. A polynomial is a special type of algebraic expressions.

Polynomial comes from "***poly***" *(meaning many)* and "***nomial***" *(meaning term)* so it means many terms. Term refers to each part of the polynomial together with its preceding sign. A polynomial is made up of terms that are only added, subtracted or multiplied.

Let us consider this polynomial: $4xy^2 + \frac{3}{5}x - 5$

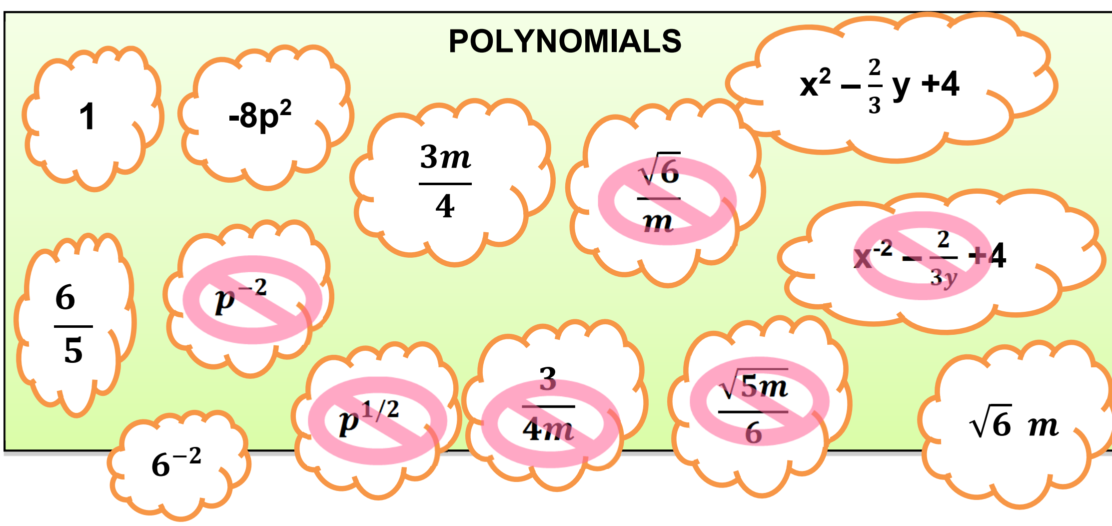
The three terms are $4xy^2$, $3x$ and – 5. TERM

A polynomial can have:

Constants like 3, - 20, ½ or $\sqrt{5}$
Variables like a, b, c, …, z
Exponents like 2 in y^2, 6 in p^6 but only 0,1,2,3,… as exponent for variables.

Polynomial or Not?

Earlier, you encountered examples of algebraic expressions. Let us cross out the non-polynomials.



These are **polynomials**: 1, $\frac{6}{5}$, - $8p^2$, 6^{-2}, $\frac{3m}{4}$, $\sqrt{6}$ m, $x^2 - \frac{2}{3}$ y +4.

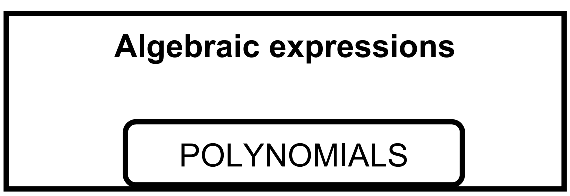
- **1** Yes, even "1" is a polynomial. It has one term which just happens to be a constant.
- $\frac{6}{5}$ Yes, even fractions are constants, and constants are polynomials.
- **$-8p^2$** It is a polynomial with a single term. The sign of the term is negative. The numerical coefficient is – 8 while the literal coefficient or variable is p raised to the exponent 2.
- **6^{-2}** It is a constant raised to a negative exponent. When simplified, $\frac{1}{6^2}$ is still a constant; therefore, it is a polynomial.

- $\frac{3m}{4}$ It is a polynomial. ¾ is the numerical coefficient and *m* is the variable.
- $\sqrt{6}$ **m** It is still a polynomial because the numerical coefficient is $\sqrt{6}$ while the variable is *m* which is not inside the square root symbol.
- $x^2 - \frac{2}{3}y + 4$ It is a polynomial with three terms namely x^2, $-\frac{2}{3}$ y, and 4.

And these are ***not*** polynomials:

- p^{-2} is not because the variable has a negative exponent. Variables in the numerator can only have 0,1,2,3 … as exponent.
- $p^{1/2}$ is not because the exponent is a fraction.
- $\frac{\sqrt{6}}{m}$ and $\frac{3}{4m}$ are not polynomials because division of variables is not allowed. Just remember, **there should not be a variable in the denominator**.
- $\frac{\sqrt{5m}}{6}$ is not a polynomial because the variable is inside the square root symbol.
- $x^{-2} - \frac{2}{3y} + 4$ is not a polynomial because the exponent of *x* is – 2 and *y* is in the denominator.

By now, you should have figured out that, **all polynomials are algebraic expressions but not all algebraic expressions are polynomials**. As shown in the Venn diagram, the set of polynomials is included in the set of algebraic expressions.



Now that you know what are polynomials, revisit Activity 1.

ACTIVITY 2. Algebraic Expressions VS Polynomials Revisited

Classify the given as algebraic expression and/or polynomials by putting a check mark (√) under the appropriate column. Write your reason in the last column. Take note of the changes in your thoughts. When finished, click "Submit".

Given	*Algebraic Expression*	*Polynomial*	*Reason*
1. 12			
2. – 7			
3. ¼ m			
4. $-6p^3$			
5. $\mathbf{5d^{-2}}$			
5. $61xy^4$			
6. $\frac{2x}{3}$			
7. $\frac{2}{3x}$			
8. $\sqrt{2}\,xy$			
9. $\sqrt{5xy}$			
10. $3b - 5c^2 + \frac{1}{2}w^4$			

Process Questions:

1. What makes polynomials different from algebraic expressions?
2. Can we consider algebraic expressions as polynomials? Why? Why not?
3. Can we consider polynomials to be algebraic expressions? Why? Why not?

Naming Polynomials

There are special names for polynomials with 1, 2, 3 or more terms. Let us look at the table below.

	3xy	**2ab – 4**	**2x – 3y + 7**	**x^2 – 4x – y – 6**	**a^4+3b^2+4c+5d+2**
Number of terms	One term (3xy)	Two terms (2ab and – 4)	Three terms (2x, - 3y and 7)	Four terms (x^2,– 4x, – y and – 6)	Five terms or more (a^4, 3b^2, 4c, 5d and 2)
Name	**Monomial**	**Binomial**	**Trinomial**	**Multinomial**	**Multinomial**
	POLYNOMIALS				

How can you remember the names? Think of cycles (monocycle, bicycle and tricycle).

The names monomials, binomials, trinomials and multinomials are just used to indicate the ***number of terms*** in a polynomial. As a whole, all monomials, binomials, trinomials and multinomials, are called polynomials.

Degree of Polynomials

Aside from the number of terms in a polynomial, another way to classify polynomials is through their degree. The degree of a polynomial with a single term is based on the exponent of the variable.

Examples:

5 The degree is of a constant is zero.

$6x$ The degree is one because the exponent of the variable is 1.

$7y^4$ The degree is one because the exponent of the variable is 4.

If there are multiple variables in the term, just add the exponents of the variable.

Examples:

$9m^2n^4$ The degree is 6. Add the exponents of the variables in the term.

$2^3x^2yz^5$ The degree is 8. Just add the exponents 2, 1 and 5. Do not add 3 because it is the exponent of the constant 2.

The degree of a polynomial with multiple terms is the highest degree among the terms.

Examples:

$\mathbf{2x^3 - 3x^2 - x}$ The degree of the first term $2x^3$ is 3. The degree of the second term is 2 while the third term has a degree of 1. Therefore, the degree of the polynomial $2x^3 - 3x^2 - x$ is 3.

$\mathbf{5b^7 - 6b^3 - 5x + 2}$ The degree of the first term is 7, the second term has a degree of 3, the third term has a degree of 1 and the last term has a degree of zero. Therefore, the polynomial $\mathbf{5b^7 - 6b^3 - 5x + 2}$ has a degree of 7.

$2x^3y^2 - 2x^4 - 3xy^2 + 5^6$ The degree of the first term is 5 (add the exponents of x and y which are 3 and 2), the degree of the second term is 4 (the exponent of x), the degree of the third term is 3 (add the exponent of x and y which are 1 and 2) and the degree of the last term is 0 because there is no variable involved. *Therefore, the degree of the whole polynomial is 5.*

$-5b^2d^3 + 3bd^3k^3 + 7k^{10}$ The degree of the first term is 5 (add the exponent of b and d which are 2 and 3), the degree of the second term is 7 (add the exponent of b,d and k which are 1, 3 and 3) and the degree of the last term is 10 (the exponent of k). *Therefore, the degree of the whole polynomial is 10.*

Standard Form of Polynomials

The standard form for writing a polynomial is to put the terms with the highest degree first especially those involving only one variable.

Examples: Rewrite these in standard form.

a) $3x^3 - 9 + 5x^6 + 2x^4$.
b) $19^7 - b^3 - 7b^4$

Solutions:

a) $5x^6 + 2x^4 + 3x^3 - 9$ The highest degree of x is 6, so that goes first, then 4, 3 and then the constant.
b) $-7b^4 - b^3 + 19^7$ The highest degree of b is 4, so that goes first, then 3 and then the constant. The constant is always the last no matter what is its exponent.

You don't have to use standard form, but it helps in organizing your answer especially when performing operations on polynomials.

Do the next activity which will summarize everything about classifying polynomials.

ACTIVITY 3. : CLASSIFIED Polynomials

Complete the table below. Provide examples of polynomials that have the degree specified.

For items 1 – 5, there should only be one variable in every term. An example is provided.

For items 6 – 10, there should be at least two variables in one term. An example is provided.

You can use constants for binomials, trinomials and multinomials. Your answers should be written in standard form. Click “SUBMIT” when you’re finished.

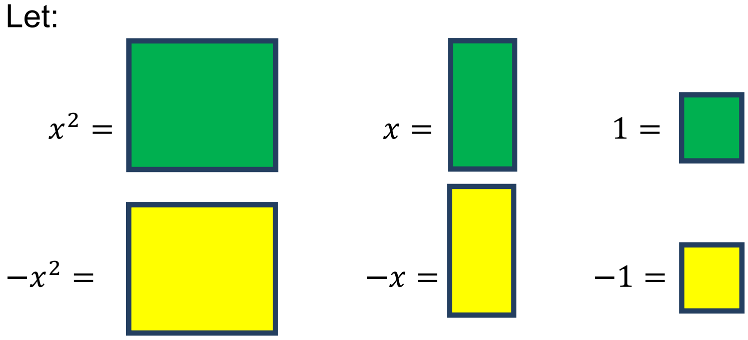
	Degree	Monomial	Binomial	Trinomial	Multinomial
	1	x	x + y	x + y + 1	x + y + z + 1
1	3				
2	5				
3	8				
4	10				
5	12				
	2	ab	ab + 1	$a^2 + ab + 1$	$b^2 + c^2 + 2d - 1$
6	3				
7	5				
8	8				
9	10				
10	12				

Since polynomials are algebraic expressions, then just like algebraic expressions, they represent certain values and are used to model certain situations. In the next activity, you will learn how to represent geometric figures using polynomials.

Exercise 1: Algebra Tiles

Read these online resources about Algebra Tiles and then work on the activity below.
http://staff.argyll.epsb.ca/jreed/math9/strand2/algetiles.htm
http://www.aplusalgebra.com/algebra-tiles.htm

Each algebra tile has its value. For items 1 – 3, the polynomial is provided, and you are to draw its equivalent using the algebra tiles. An example is provided. For items 4 – 5, you will provide a polynomial and draw its equivalent using algebra tiles. There are multiple ways of using the algebra tiles so don't hesitate to provide two or more answers if you want to. Click "SAVE" when you're finished.
Let:



$x^2 + 2x - 3$	
1. $2x^2 - 3x + 4$	
2. $-3x^2 + 5x - 6$	
3. $x^2 - 6x + 7$	
4.	
5.	

You have just created and represented polynomials using Algebra tiles. From the Algebra tiles, what makes x and x^2 different? What is the meaning of x^2? One key component of a polynomial is the exponent. Let's learn more about exponents in the next topic.

Googol turned Google

Reading 1

You surely know what Google is but do you know where it got its name? The founders of Google, Larry Page and Sergey Brin together with other Stanford graduate students including Sean Anderson were brainstorming of a good name that is related to the indexing of an enormous amount of data. Someone suggested "googolplex" and another suggested "googol" both of which are very large numbers. A search was made in the internet domain name registry database if the suggested names were available for registration and use. Anderson misspelled googol as "google" and found it available. Larry liked it and "google.com" was registered in September 15, 1997. Moreover, Google's corporate headquarters is called the Googleplex.

A GOOGOL is 1 followed by 100 zeros. It is larger than the number of elementary particles in the universe. The term was invented by Milton Sirotta in 1938, the 9 year old nephew of mathematician Edward Kasner, who had asked his nephew what he thought such a large number should be called. Such a number, Milton apparently replied after a short thought, could only be called something as silly as a "googol." He proposed further the term GOOGOLPLEX to be "one, followed by writing zeros until you get tired". Kasner decided to adopt a more formal definition "because different people get tired at different times". Thus, googolplex became standardized to be 10^{googol} *or* $10^{10^{100}}$. *How about you, what is the biggest number you ever imagined? In what way can you write it in a short yet accurate form?*

Process Questions:

1. What is the meaning of googol?
2. How will you write googol in a short yet accurate way?
3. How will you write googolplex in a short yet accurate way?
4. Why do we have to learn about exponents?

To learn more about exponents, read the next story.

ACTIVITY 4. Mathematical Investigation

Read the short story then complete the table that follows. When you're finished, click "SUBMIT".

The Squares of the Chessboard

Reading 2

There was a story about a court mathematician in India who, years ago, invented the game of chess for his king. The king was so pleased with the game that he offered to repay the mathematician whose request seemed modest enough. The mathematician requested a single grain of wheat on the first square of the chessboard, two grains on the second square, four on the third square, and so on, doubling the number of grains on each succeeding square until all squares had been used. Thinking that the mathematician's request was reasonable, the king readily approved it.

Did the king make a wise decision? Why or why not?

Chessboard Square	Number of Grains on the Square	Number of Grains on the square in exponential form
1st	1	$1 = 2^0$
2nd	2	$2 = 2^1$
3rd	4	$2 \times 2 = 2^2$
4th	________	$2 \times 2 \times 2 = 2^3$
5th	32	
6th	________	$2 \times 2 \times 2 \times 2 \times 2 =$ ________________
10th	1,024	______________________________
16th	65536	______________________________
24th	8388608	______________________________
50th	562949953421312	______________________________
64th	9223372036854775808	2^{63}

Questions:

1. How did you come up with your answers?
2. How will you determine the total numbers of grains on each square?
3. Do you agree with the king's estimation that the court mathematician's request is reasonable? Explain.
4. If you were the king, what will be your response to the request of the mathematician? Explain.
5. In what ways exponents present another strategy to solve problems?
6. How can real – life problems involving one variable be modeled and solved?

Now that you are able to answer the questions, let's have a discussion about exponents.

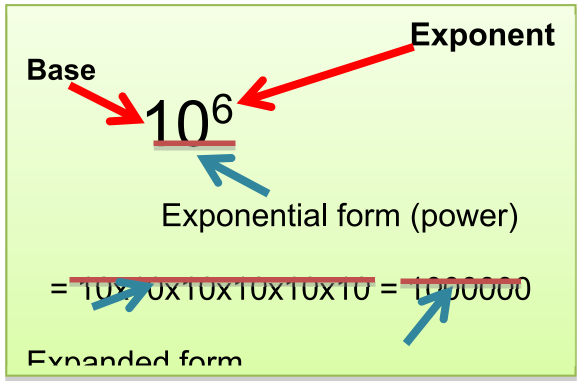
EXPONENTS

In Activity 4, the repeated multiplication under the third column is simplified when the expressions are written in exponential form. Consider the expressions 2 x 2 x 2 x 2 x 2 which is the number of grains on the sixth square of the chessboard in the chess story. This tedious and space consuming work will be eliminated by writing it in the exponential form 2^5. For 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 which is the number of grains in the tenth square, in exponential form is 2^9. Of course 2^5, $2^9$and 2^{63} can be written in standard form, as 32, 512 and 9223372036854775808 respectively.

However, writing very big numbers in standard notation is as tedious and space consuming like repeated multiplication. Consider the value of *googol* which when written in standard notation is
1000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000.

Writing and reading the number becomes extremely difficult and impractical. Through the use of exponents, very large numbers can be written in a concise yet accurate form. For googol, in exponential form it is written as 10^{100}. One million or 1,000,000 can be expressed as 10^6 since:

$$1{,}000{,}000 = \underbrace{10 \times 10 \times 10 \times 10 \times 10 \times 10}_{\text{6 factors}} = 10^6$$



Let's Do It. Do the sample items and compare your answers with the given solutions.

	Base	Exponent
a. 2^{10}		
b. -6^4		
c. $(9k)^m$		
d. $-3p^6$		

Solution:

	Base	Exponent
a. 2^{10}	2	10
b. -6^4	6	4
c. $(-9k)^m$	$-9k$	m
d. $-3p^6$	p	6

Let's Do It: Writing a number or expression in different forms.

a. d^3
b. 16^5
c. 2^7

Solution:

Exponential form	*Power*	*Expanded form*
a. d^3	d^3	d • d • d or (d) (d) (d)
b. 16^5	1048576	16 x 16 x 1 6 x 16 x 16
c. 2^7	128	2 x 2 x 2 x 2 x 2 x 2 x 2

Let's Do It: Write each of the following in exponential form.

a. 4 x 4 x 4
b. (h) (h) (h) (h) (h) (h) (h)
c. (– 5)(– 5)(– 5) (– 5)
d. – (5) (5) (5) (5)

Solution:

a. 4 x 4 x 4 = 4^3
b. (h) (h) (h) (h) (h) (h) (h) = h^7

c. Since (– 5) is used as a factor 4 times, enclose it in parentheses
$(-5)(-5)(-5)(-5) = (-5)^4$

d. Since 5 is used as a factor 4 times, then
$-(5)(5)(5)(5) = -5^4$

Note that $-5^4 \neq (-5)^4$ since $-5^4 = -625$ while $(-5)^4 = 625$. In -5^4, the base is 5 and not –5.

Let's Do It: Evaluate the following exponential expressions.

a. 3^4

b. -3^4

c. $(-3)^4$

d. $3(m)^4$

e. $\left\{\frac{1}{2}\right\}^2$

f. $\left\{\frac{-1}{3}\right\}^3$

g. $\left\{\frac{3}{2}\right\}^4$

h. $(-3)^2\left(\frac{1}{2}\right)^3$

Solution:

a. $3^4 = 3\cdot3\cdot3\cdot3 = 81$

b. $-3^4 = -3\cdot3\cdot3\cdot3 = -81$

c. $(-3)^4 = (-3)(-3)(-3)(-3) = 81$

d. $-3(m)^4 = -3(m)(m)(m)(m) = -3m^4$

e. $\left\{\frac{1}{2}\right\}^2 = \left\{\frac{1}{2}\right\}\left\{\frac{1}{2}\right\} = \frac{1}{4}$

f. $\left\{\frac{-1}{3}\right\}^3 = \left\{\frac{-1}{3}\right\}\left\{\frac{-1}{3}\right\}\left\{\frac{-1}{3}\right\} = \frac{-1}{27}$

g. $\left\{\frac{3}{2}\right\}^4 = \left\{\frac{3}{2}\right\}\left\{\frac{3}{2}\right\}\left\{\frac{3}{2}\right\}\left\{\frac{3}{2}\right\} = \frac{81}{16}$

h. $(-3)^2\left(\frac{1}{2}\right)^3 = (-3)(-3)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 9\left(\frac{1}{8}\right) = \frac{9}{8}$

Are you able to get the correct answer in all the items? If you committed an error, what do you think is the cause of that error? Rectify it before you move on to the next activity.

<u>*Exercise 2.*</u> Read Me Thrice.

An exponential expression can be read in varied ways. For example

$\mathbf{10^5}$ can be read as ***ten raised to five***

or ***ten to the fifth power***

or ***ten to the power of five*****.**

Read the following properly in multiple ways.

1. 12^7
2. p^4
3. -6^5
4. $(ab)^{11}$
5. $-7m^3$
6. $(-7m)^3$

As you read the expressions in different ways, do you think it is better to have a single standard way of reading exponential expressions? When solving problems, do you rely on your usual strategy or you try other possible strategies? What is the advantage of having multiple strategies? As you reflect on these questions, let's continue our discussion about exponents.

If there are n factors of d's multiplied together or used as factors, then

$$d^n = \underbrace{(d)(d)(d)(d) \ldots\ldots.(d)}_{n \text{ factors}}$$

If d is the base and n is the exponent, then

se is the number or symbol that is being multiplied. The exponent is a e number or symbol that tells how many times the base is used as a The power is the product of equal factors.

ACTIVITY 5. Complete and Compare.

To check how well you understand exponents, compare and contrast the exponential expressions by completing the statements below. Click "SUBMIT" when you're finished.

1. 5^3 and 3^5

   5^3 ________________________ while 3^5 is _______________________.

   The difference is because of ____________________________________

2. -4^3, $(-4)^3$ and $-(4)^3$

   The value of -4^3 is _______, $(-4)^3$ is _________ and $-(4)^3$ is ________.

   The difference between ____

3. $3x^2$, $(3x)^2$ and $3(x)^2$

   In expanded form, $3x^2$ is ________, $(3x)^2$ is ________ and $3(x)^2$ is ______.

   The difference between ______________ is because of ______________

   ____.

4. $(4^3)(4^2)$and 4^6

   In standard notation, $(4^3)(4^2)$ is _______ and 4^6 __________.

   The difference is caused by ____________________________________.

5. $(4^3)^2$ and 4^5

   In expanded form,$(4^3)^2$is ______________ while 4^5 ________________.

   They should have been equal if the exponent of _________ is _________.

6. $(2 + 3)^3$ and $2^3 + 3^3$

   In expanded form, $(2 + 3)^3$ is ___________________ while $2^3 + 3^3$is

   ___________.

   In standard notation, $(2 + 3)^3$ is ________________ while $2^3 + 3^3$is

   ____________.

   _______________ is greater than _____________

   because____

   _____.

Process Questions:

1. In comparing the exponential expressions for each item, did you use another strategy other than what was specified? Why?

2. Is it easier to spot differences when varied ways of comparing exponential expressions are used?

3. What is the advantage of using multiple representations?

After all the discussions, it is time to test your understanding about exponents by doing the next activity.

ACTIVITY 6. Raising to a Power

Direction: Answer the following. Do as directed. Click “SUBMIT” when you’re finished with all the items.

A. Given the following powers, identify the base and the exponent.

Base Exponent

1. 8^k ________ _________
2. s^3 ________ _________
3. $(d + m)^{10}$ ________ _________
4. $-9w^5$ ________ _________
5. $(-2y)^3$ ________ _________
6. $(7a - 9b)^t$ ________ _________

B. Express the following numbers in exponential form.

1. 27 = __________
2. –243 = __________
3. 100,000 = __________
4. 243 = __________
5. 2,048 = __________
6. –256 = __________
7. 256 = __________
8. 729 = __________

C. Evaluate the following.

1. $2^3 + 3^2$ = _________________
2. $2^5 + 5^2$ = _________________
3. $(-2)^4 + 2^4$ = _________________
4. $-3^3 - 3^3$ = _________________
5. $\left(\frac{-2}{5}\right)^3$ = _________________
6. $\left(\frac{3}{4}\right)^2 - \left(\frac{-1}{2}\right)^4$ = _________________

What is the size of the memory of your phone, tablet or computer? What is the total size occupied by your mp3 songs? What is the unit? As you do the next activity, you will make a connection between these and exponents.

ACTIVITY 7. The USB Flash Drive

Research about flash drives. A flash drive is a small, portable device that functions like a disk drive. On one end is a standard USB (Universal Serial Bus) connector that fits into USB ports. The capacity of flash drives is measured in bytes. Research about flash drives then complete the table below. Then, answer the questions that follow. Click “SUBMIT” when you’re finished.

A flash drive is It is used to				
Capacity (Unit)	Abbreviation	Actual size in bytes	Exponential Form	Usual value associated
1 Kilobyte	1 KB	1,024 bytes	2^{10}	1,000
1 Megabyte	1 MB			
1 Gigabyte	1 GB	1,073,741,824 bytes	2^{30}	
1 Terabyte	1 TB			
1 Petabytes				
1 Exabyte				
1 Zettabyte				
1 Yottabyte				

Process Questions:

1. Why are abbreviations like KB, MB and GB used instead of Kilobyte, Megabyte and Gigabyte, respectively? Relate it with the use of exponents.

2. Why is it that some numbers are purposely written in exponential form rather than standard notation?

3. When is the exponential form of a number more appropriate to be used than the standard form?

4. If you are selling and describing the capacity of a flash drive to a customer, which statement will you prefer to use. Explain.
   A. " A 1 Gigabyte flash drive can store up to 1,073,741,824 amount of data."
   B. " A 1 Gigabyte flash drive can store up to 1,000,000,000 amount of data."
   C. " A 1 Gigabyte flash drive can store up to 2^{30} amount of data."
   D. " A 1 Gigabyte flash drive can store up to 1,000 Megabytes amount of data."

Answer:

ACTIVITY 8. Term Frame

To summarize what you've learned about this topic, complete this term frame until "MY DEFINITIONS". When you're finished, click "SAVE".

<table>
<tr><td colspan="4">TERMS: EXPONENT, BASE, POWER, EXPONENTIAL NOTATION, STANDARD NOTATION</td></tr>
<tr><td>What I Already Know</td><td colspan="2">What I Am Learning</td><td>My Picture / Image</td></tr>
<tr><td></td><td colspan="2"></td><td></td></tr>
<tr><td colspan="2">EXAMPLES</td><td colspan="2">NON - EXAMPLES</td></tr>
<tr><td colspan="2"></td><td colspan="2"></td></tr>
<tr><td colspan="4">MY DEFINITIONS</td></tr>
<tr><td colspan="4"></td></tr>
<tr><td colspan="4">EXTENDING MY LEARNING</td></tr>
<tr><td colspan="4">Exponents can be</td></tr>
<tr><td colspan="4">Exponents are useful for</td></tr>
</table>

You gave your initial thoughts. At the end of this topic, you will revisit the same term frame. At this time, let us take a closer look at some aspects of the topic about exponents.

LAWS OF EXPONENTS

A. The Product Rule

Previously, you have illustrated that 64= 2 x 2 x 2 x 2 x 2 x 2 = 2^6. You know also that

64 = 2 x 32
64 = 4 x 16
64 = 8 x 8

So when 64 is written as a product of two numbers instead of the product of 2 multiplied to itself six times, how can it be equal to 2^6? Analyze the table below.

64	2 x 2 x 2 x 2 x 2 x 2			2^6
	2 x 32	2 x 2 x 2 x 2 x 2 x 2	2 x 2^5	
	4 x 16	2 x 2 x 2 x 2 x 2 x 2	2^2 x 2^4	
	8 x 8	2 x 2 x 2 x 2 x 2 x 2	2^3 x 2^3	

What made the product 2 x 2^5 equal to2^6? Does it apply to 2^2 x 2^4 and 2^3 x 2^3?

THE PRODUCT RULE

To multiply powers of the same base, just copy the base and add the exponents. In symbols, the product rule is illustrated by:

$a^m . a^n = a^{m+n}$ where ***a*** is the common base, ***m*** and ***n*** are positive integers.

Thus, 2 x 2^5 = 2^{1+5} = 2^6 ; 2^2 x 2^4 = 2^{2+4} = 2^6 ; and 2^3 x 2^3 = 2^{3+3} = 2^6.

Let's Do It. Multiply the following.

a. $(3^4)\,(3^5)$
b. $(m^{10})(m^6)$
c. $(-5y)^3(-5y)^{12}$
d. $4(2m)^2(2m)^{12}$

Solution:

a. $(3^2)\,(3^3) = 3^{2+3} = 3^5$
b. $(m^{10})(m^6) = m^{10+6} = m^{16}$
c. $(-5y)^3(-5y)^9 = (-5y)^{3+9} = (-5y)^{15}$
d. $4(2m)^2(2m)^{12} = 4(2m)^{2+12} = 4(2m)^{12}$

Exercise3. Apply the product rule to multiply the following. Click on "SUBMIT" to see your score. Take note of the items that you were not able to answer correctly and look for the right answer as you go through this module.

1. $10^6 \times 10^8$ = ____________
2. $(-5)^8(-5)^3$ = ____________
3. k^3k^2k = ____________
4. $6^3 \times 6^3 \times 6^3$ = ____________
5. $7^3 \times 7^4$ = ____________
6. $5^3 \times 3^5$ = ____________
7. $2^3 \times 3^2$ = ____________
8. $(9y^4)(9y^5)$ = ____________
9. $(3x^3)(x)(2x^2)(x^5)$ = ____________
10. $(x+7)^3(x+7)^{10}$ = ____________

Process Questions:

1. In answering Exercise 3, did you use the multiplication rule or did you use other methods like using the expanded form?

2. When the bases are not the same like in number 6 and 7, how did you multiply the exponential expressions?

To test and improve your skill of multiplying exponential expressions, play the online game.

Exercise 4. Rags to Riches (Online)

Play the game Rags to Riches by clicking on the hyperlink below. Answer questions using your knowledge about multiplying exponential expressions in a quest for fame and fortune.
http://www.quia.com/rr/180013.html

B. The Quotient Rule

Analyze how to perform division with exponential expressions. Try to derive the rule based from the following examples.

a. $\frac{m^6}{m^2} = \frac{m.m.m.m.m.m}{m.m} = \frac{\cancel{m}.\cancel{m}.m.m.m.m}{\cancel{m}.\cancel{m}} = m.m.m.m = m^4$

b. $\frac{2^5}{2^2} = \frac{2.2.2.2.2}{2.2} = \frac{\cancel{2}.\cancel{2}.2.2.2}{\cancel{2}.\cancel{2}} = 2.2.2 = 8$

c. $\frac{50\,b^4}{10b^3} = \frac{50.b.b.b.b}{10.b.b.b} = \frac{50.\cancel{b}.\cancel{b}.\cancel{b}.b}{10.\cancel{b}.\cancel{b}.\cancel{b}} = \frac{50b}{10} = 5b$

Based from the examples, what is the rule in dividing exponential expressions?

THE QUOTIENT RULE

To divide powers of the same base, just copy the base and subtract the exponents. In symbols, the quotient rule for exponential expressions states that: $\frac{a^m}{a^n} = a^{m-n}$ where ***a*** is the common base, ***m*** and ***n*** are positive integers.

Let's Do It. Divide the following. Evaluate if possible.

a. $\frac{9^8}{9^6}$
b. $\frac{w^{10}}{w^3}$
c. $\frac{(3x)^7}{(3x)^4}$
d. $\frac{-12r^{14}}{4r^9}$

Solution:

a. $\frac{9^8}{9^6} = 9^{8-6} = 9^2 = 81$
b. $\frac{w^{10}}{w^3} = w^{10-3} = w^7$
c. $\frac{(3x)^7}{(3x)^4} = (3x)^{7-4} = (3x)^3 = (3x)(3x)(3x) = 27x^3$
d. $\frac{-12r^{14}}{4r^9} = \frac{-12r^{14-9}}{4} = \frac{-12r^5}{4} = -3r^5$

Answer the next set of items. Remember that you can verify your answer by using another strategy, like expansion, aside from the quotient rule.

Exercise 5. Apply the quotient rule to simplify the following. Click on "SUBMIT" to see your score. Take note of the items that you were not able to answer correctly and review about the quotient rule to see what went wrong.

1. $\frac{10^{12}}{10^2}$ = ____________
2. $\frac{(-5)^8}{(-5)^3}$ = ____________
3. $\frac{k^3k}{k^2}$ = ____________
4. $\frac{-10k^{15}}{8k^9}$ = ____________
5. $\frac{(x+3)^5}{(x+3)^2}$ = ____________

The correct answers are:
1. 10^{10}
2. $(-5)^5$
3. k^2
4. $\frac{5k^6}{4}$
5. $(x + 3)^3$

C. Power of a Power Rule

Let's Do It. Try to simplify the following using definition of exponents and the product rule.

1. $(2^3)^2$

2. $(k^4)^5$
3. $(a^2b^3)^3$
4. $(-2k^2p^3)^5$

Solution:

1. $(2^2)^3 = (2^2)(2^2)(2^2) = 2^6 = 64$
2. $(k^4)^5 = (k^4)(k^4)(k^4)(k^4)(k^4) = k^{20}$
3. $(a^2b^3)^3 = (a^2b^3)(a^2b^3)(a^2b^3) = a^6b^9$
4. $(-2k^2p^3)^5 = (-2k^2p^3)(-2k^2p^3)(-2k^2p^3)(-2k^2p^3)(-2k^2p^3) = -32k^{10}p^{15}$

POWER OF A POWER RULE

To simplify a power raised to a power, just copy the base and multiply the exponents. In symbols, the power of a power rule for exponential expressions states that:

$(a^m)^n = a^{mn}$ where **a** is the base, **m** and **n** are positive integers.

Let's Do It. Try to simplify the items using the power to a power rule.

1. $(3^3)^2$
2. $(m^3)^4$
3. $(bd^2)^4$
4. $(-3x^4y^5)^2$

Solution:

1. $(3^3)^2 = 3^{3(2)} = 3^6 = 729$
2. $(m^3)^4 = m^{3(4)} = m^{12}$
3. $(bd^2)^4 = b^4d^{2(4)} = b^4d^8$
4. $(-3x^4y^5)^2 = (-3)^2x^{4(2)}y^{5(2)} = 9x^8y^{10}$

Exercise 6. Simplify the following. Click on "SUBMIT" to see your score. Take note of the items that you were not able to answer correctly and review about the power of a power rule to see what went wrong.

1. $(10^4)^{11}$ = ___________
2. $(k^{12})^3$ = ___________
3. $(-k^{12})^3$ = ___________
4. $(5k^2)^3$ = ___________
5. $(-5k^2)^3$ = ___________
6. $(-4m^{30}n^{12})^4$ = ___________

The correct answers are:

1. 10^{44}
2. k^{36}
3. $-k^{36}$
4. $125k^6$
5. $-125k^6$
6. $256m^{120}n^{48}$

To test your skill in simplifying exponential expressions, play the online game Exponent Asteroids.

Exercise 7. Exponent Asteroids (Online)

Play the game Math Dork – Exponent Asteroids by clicking on the hyperlink below. You will be using your knowledge with simplifying exponential expressions with positive exponents to win the battle.
http://www.mathdork.com/games/asteroidsexp3/asteroidsexp3.html

If after the game, you still have some confusions about simplifying algebraic expressions, access the website below for additional explanation and activities.

Exercise 8. One More Time (Online)

The website is about the laws of exponents. You can access it for additional explanations and activities.
http://www.algebra-class.com/laws-of-exponents.html

ACTIVITY 9. Term Frame

Previously, you have completed this term frame until “MY DEFINITIONS”. This time, complete until “EXTENDING MY LEARNING” as you reflect on what you’ve learned about exponents. Click “SUBMIT” when you’re finished.

<table>
<tr><td colspan="4">TERMS: EXPONENT, BASE, POWER, EXPONENTIAL NOTATION, STANDARD NOTATION</td></tr>
<tr><td>What I Already Know</td><td colspan="2">What I Am Learning</td><td>My Picture / Image</td></tr>
<tr><td></td><td colspan="2"></td><td></td></tr>
<tr><td colspan="2">EXAMPLES</td><td colspan="2">NON - EXAMPLES</td></tr>
<tr><td colspan="2"></td><td colspan="2"></td></tr>
<tr><td colspan="4">MY DEFINITIONS</td></tr>
</table>

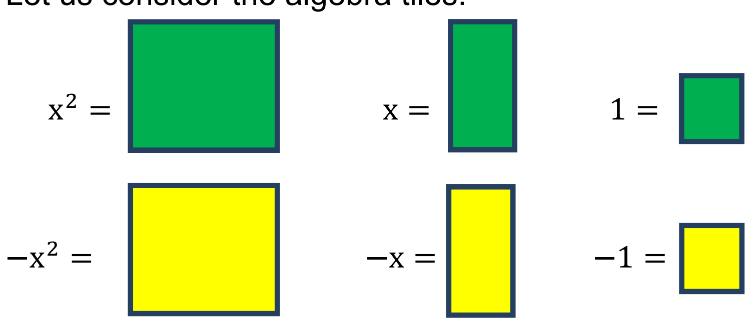
EXTENDING MY LEARNING
Exponents can be
Exponents are useful for

So far, you have learned about a special type of algebraic expression called polynomials. You tried classifying polynomials based on the number of its terms and arrange it based on the degree. The degree depends on the exponent of the variables. The use of exponents makes writing very big numbers and very long algebraic expressions short and accurate.

Let us learn more about performing operations on polynomials.

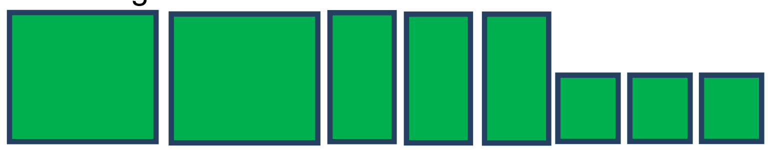
Previously, you made use of algebra tiles to represent polynomials. In the process, you were actually adding or subtracting polynomials. Can you figure out how to add or subtract polynomials? Let us look at some examples.
Example 1: Add $x^2 + x$ and $x^2 + 2x + 3$

Let us consider the algebra tiles:



Using the tiles, $x^2 + x$ and $x^2 + 2x + 3$ will be represented as

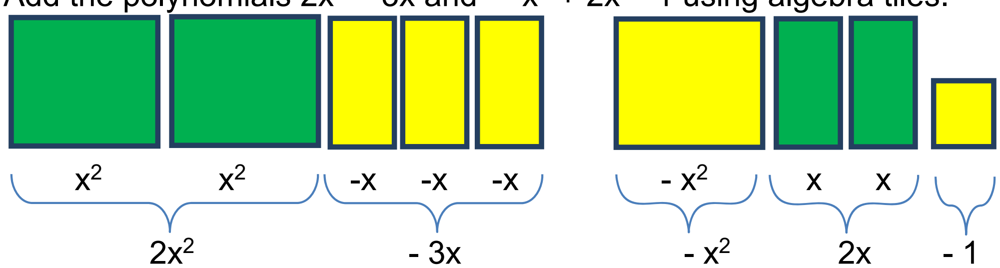
Group similar tiles together.



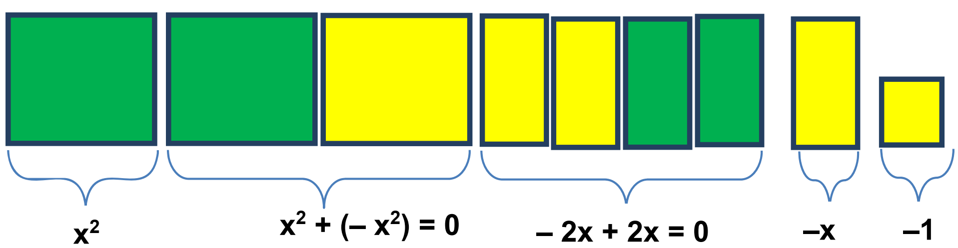
Therefore, the answers is $\mathbf{2x^2 + 3x + 3.}$

$2x^2 + 3x + 3$ can't be added furthermore because based from the tiles; they are of different sizes and can't be combined. Only those of the same sizes can be combined.

Example 2: Add the polynomials $2x^2 – 3x$ and $– x^2 + 2x – 1$ using algebra tiles.



Group similar tiles. Some tiles are opposite each other so when combined becomes zero.



So the answer is $\mathbf{x^2 – x – 1}$.

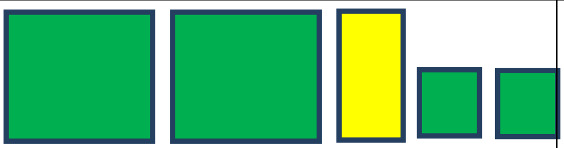
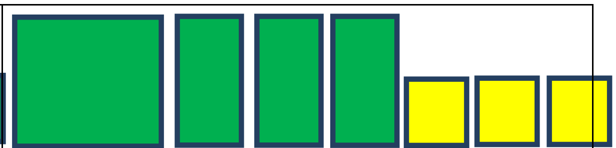
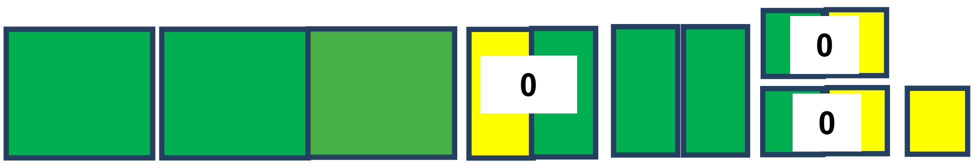
Based on the activity, how can you add polynomials? Complete the statement below.

To add polynomials,

____.

ACTIVITY 10. Adding Polynomials using Algebra Tiles.

Find the sum of the two polynomials using algebra tiles. An example is provided for your guidance. Click “Submit” when you’re finished.

	
$2x^2 - x + 2$	$x^2 + 3x - 3$
 $(2x^2 - x + 2) + (x^2 + 3x - 3) = \mathbf{3x^2 + 2x - 1}$	
$-x^2 + 4x - 5$	$3x^2 - 5x - 1$
$-2x^2 - 2x - 1$	$4x^2 + x + 5$

Process Questions:
1. How did you add the polynomials using Algebra tiles?
2. Without the Algebra tiles, how can you add polynomials?

Exercise 9. Adding and Subtracting Polynomials Using Algebra Tiles (Online)

http://staff.argyll.epsb.ca/jreed/math9/strand2/poly_sum.htm

The website provides animated examples of algebra tiles. Scroll down and you will encounter adding and subtracting polynomials using algebra tiles. Do the following:

1. Study the examples for **ADDING POLYNOMIALS** by clicking the “Examples” button.
2. Pay attention to the discussion of Like term and Zero Property on the right side.
3. Click on “Algebra Tile Practice” button.
4. Work on some practice activities by clicking on monomial, binomial or trinomial.
5. Click on Step, Answer, and Check/Verify and compare with your answer.
6. Repeat the process until you understood the topic.
7. Study the examples for **SUBTRACTING POLYNOMIALS** by clicking the “Examples” button.
8. Pay attention to the discussion of Like term and Zero Property on the right side.
9. Click on “Algebra Tile Practice button”.
10. Work on some practice activities by clicking on monomial, binomial or trinomial.
11. Click on Step, Answer, and Check/Verify and compare with your answer.
12. Repeat the process until you understood the topic.

Process Questions:

1. Can you now add/subtract polynomials?
2. What is the rule you need to remember to easily add/subtract polynomials?
3. Do you have to draw algebra tiles all the time to add/subtract polynomials?
4. Do you think using more strategies is better than sticking with a single strategy?

You can always use Algebra tiles but there are other ways to add and subtract polynomials. Study the other methods and decide which is best for you.

Exercise 10: Adding and Subtracting Polynomials Using Other Strategies (Online)

These websites show other ways of adding/subtracting polynomials.

http://www.mathsisfun.com/algebra/polynomials-adding-subtracting.html

This website focuses on how to add/subtract polynomials with LIKE TERMS. Click on to play the animations. Then, you can try to answer some questions by clicking on Question 1 Question 2 Question 3 Question 4 Question 5

http://www.regentsprep.org/regents/math/algebra/AV2/sp_subt.htm

This website shows several ways of subtracting polynomials. Choose among the methods which is most effective for you.

Can you still remember how to evaluate algebraic expression? Yes, to evaluate means to replace the variable by its assigned value. If x = 4, then 2x = 2(4) = 8; $x^2 = 4^2 = 16$. How can we use the concept of evaluating expressions to explain how polynomials are added/subtracted? Let us look at the table.

In the first column are the polynomials to be added and in the second column is the sum of the polynomial. Each will be evaluated using the values provided. Observe the resulting value when the polynomials are evaluated and when the sum of the polynomials are evaluated? Are they the same? What are like terms?

Evaluate if x = 1, y = 2	Sum
1. $x^2 + (x + 4)$	**$= x^2 + x + 4$**
$1^2 + (1 + 4) = 1 + 5 =$ **6**	$1^2 + 1 + 4 = 1+1+4 =$ **6**
2. $3x^2 + (2x^2 + 4)$	**$= 5x^2 + 4$**
$3(1)^2 + [2(1)^2 + 4] = 3 + (2 + 4) =$ **9**	$5(1)^2 + 4 =$ **9**
3. $(x + y) + (2x + 3y)$	**$= 3x + 4y$**
$(1 + 2) + [2(1)+3(2)] = 3+ [2+6] =$ **11**	$3(1) + 4(2) = 3 + 8 =$ **11**
4. $(3x + 4) + (x^2 + 2x)$	**$= x^2 + 5x + 4$**
$[3(1)+4] + [1^2 + 2(1)] = 7 + 3 =$ **10**	$1^2 + 5(1) + 4 =$ **10**
5. $(2x^2 + y) + (x^2 + y^2 + y)$	**$= 3x^2 + y^2 + 2y$**
$[2(1)^2 + 2] + (1^2 + 2^2 + 2) = 4 + 7 =$ **11**	$3(1)^2 + 2^2 + 2(2) = 3 + 4 + 4 =$ **11**

The table above shows that when polynomials are evaluated, it should have the same value when the polynomial sum is evaluated. This holds true for both addition and subtraction. It also shows that only like terms are added or subtracted. Can you relate it with the previous activities? Is it better to have a single strategy or use varied strategies to solve problems?

Exercise 11: Adding and Subtracting (Online)

Answer the activities in these websites using any method you prefer. Then, compare your answer with the correct answer provided by the website. Did you master how to add/subtract polynomials? Remember, practice makes perfect.

http://www.regentsprep.org/regents/math/algebra/AV2/sprac_a.htm

http://www.algebralab.org/practice/practice.aspx?file=Algebra1_10-1.xml

Process Questions:

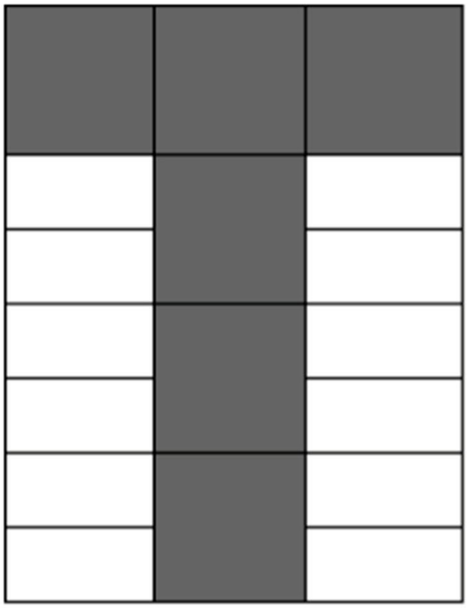
1. After doing different activities and trying different strategies to add and subtract polynomials, what do you have to remember when adding/subtracting polynomials?
2. Is the use of varied strategies better than using only one strategy?

Now that you know how to add/subtract polynomials, let's solve this problems.

ACTIVITY 11. Let's Tile It.

Answer the following. Show your solution in the space provided. You can use several methods to solve a single problem. Click "Submit" when you're finished.

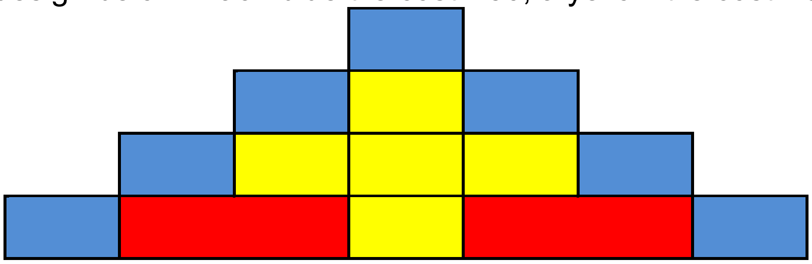
1. Mr. T will use 2 types of tiles for his floor. He came up with the design shown below. The area of each gray tile is 144 in^2 and the area of each white tile is $72in^2$.



a) What is the total floor area?

b) Mr. T said, *"if x = 144 and y = 72, then the total area of the tiles is 6x + 12y"*. Is he correct? Evaluate the polynomial.

2. Mrs. Pyramid wants to use blue, yellow and orange tiles in his floor following the design below. Each blue tile cost ₱50, a yellow tile cost ₱55 and a red tile costs ₱90.



a) If b = cost of blue tiles, y = cost of yellow tiles and r = cost of red tiles, which polynomial models the total cost of all tiles? Why?

a. 7b + 5y + 2r
b. 50b + 55y + 90r
c. 50(7b) + 55(5y) + 90(2r)

b) What is the total cost of all the tiles?

c) If Mr. Pyramid will replace the red tiles with blue tiles instead, what is the total cost of the tiles?

d) Finally, Mr. Pyramid decided to use only one color of tiles. He will choose the tile with the lowest price. Kindly help him decide by using a polynomial model to compute the cost if he will use blue tiles, yellow tiles or red tiles.

	a. blue tiles	b. yellow tiles	d. red tiles
Polynomial model:	____________	____________	____________
Total cost	____________	____________	____________

Your suggestion to Mr. Pyramid: ____

You have learned a lot about polynomials and possible ways of using it. But there is more about polynomials. For now, complete this journal to reflect on what you've learned so far about polynomials. Remember the important question: How can real – life problems be solved?

ACTIVITY 12. Journal writing

Complete the sentences. Click 'Submit" when you're finished.

At first, I thought that polynomials are ______________________________ ____. However, I learned that ______________________________ ____ ______________________. Moreover, ______________________ ____ ______________________. Polynomials can be ____________ ____ ____. Some applications of polynomials are ______________________ ____ ____ I think it can be used also in______________________________ ____ Hopefully, I will learn more about ______________________________ ____.

Aside from addition and subtraction, what are the two major operations? Yes, multiplication and division. The next topic is about multiplying and dividing polynomials.

MULTIPLYING AND DIVIDING POLYNOMIALS
In multiplying and dividing polynomials, your knowledge about the laws of exponents is very important. As such, apply the laws of exponents in the next activity.

ACTIVITY 13. You Complete Me A.

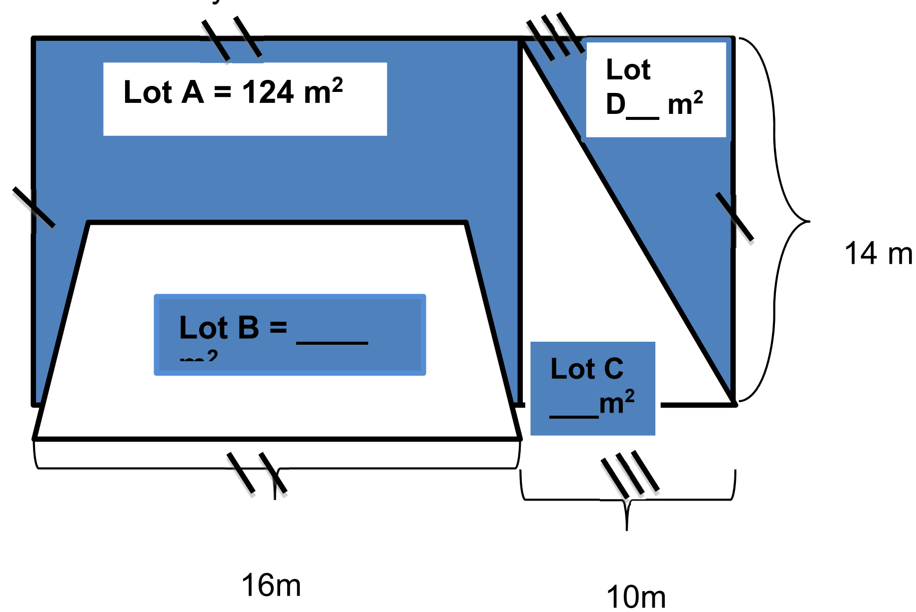
A. Search for pattern in the given examples. Following the same pattern, complete the table. Answer the questions that follow. Click "Submit" when you're finished.

	Factors	***Expanded Form***	***Writing the Product Using Exponents***
1	$3^3.3^2$	$3.3.3.3.3$	3^5
2	$m^2.\ m^4$	$m.m.m.m.m.m$	m^6
3	$5^2\ .\ 5^3$		
4	$p^7.p^2$		
When powers of the same bases are multiplied, what happens to the exponent? Answer:			
5	$(3^2)^4$	$3^2.\ 3^2.\ 3^2.\ 3^2$	3^8
6	$(k^3)^5$	$k^3.\ k^3.\ k^3.\ k^3.\ k^3.$	k^{15}
7	$(2^5)^2$		
8	$(m^5)^7$		
What pattern do you observe between the exponents in the second and the third columns? Answer:			
9	$(2x^4y)^2$	$(2x^4y)(2x^4y)$	$2^2x^8y^2$
10	$(b^2\,d^3)^4$	$(b^2\,d^3)\ (b^2\,d^3)\ (b^2\,d^3)\ (b^2\,d^3)$	$b^8\,d^{12}$
11	$(6m^2n^3)^3$		
12	$(p^{10}\,q^2)^5$		
What pattern do you observe between the exponents in the second and the third columns? Answer:			

Now that you refreshed your mind about exponents, let's have an activity to refresh your mind about areas where the main operation is multiplication.

ACTIVITY 14. You Complete Me B.

Situation: You are a surveyor and you were assigned to determine the dimensions and area of each lot in your neighborhood. Unfortunately, you were given an incomplete data and it's up to you to complete all dimensions. Click "Submit" when you're finished.



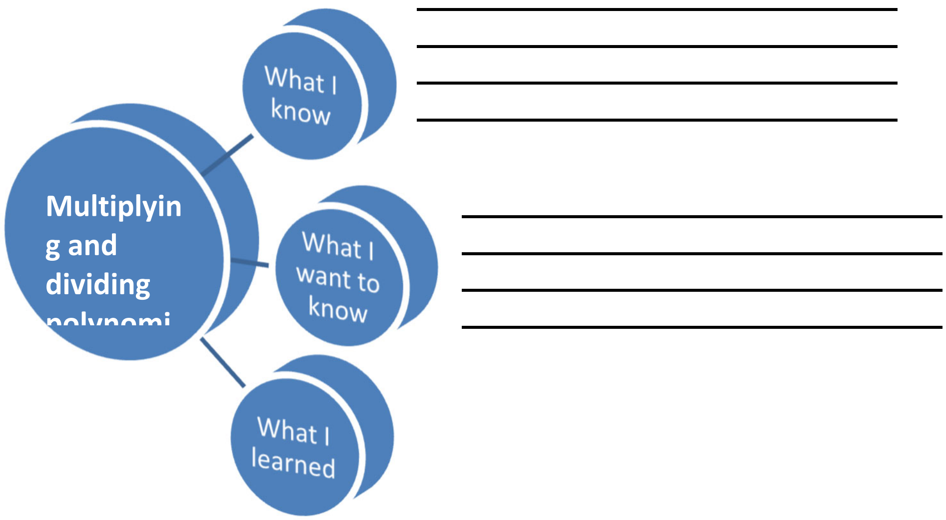
The correct answers are:
Lot B = 100 m^2
Lot C = 35 m^2
Lot D = 35 m^2

Process Questions:

1. How did you find the answers?
2. Have you considered representing the unknown by a variable?
3. Do you think multiplication or division of polynomials is just the application of the laws of exponents?
4. In what ways multiplying polynomials present another strategy to solve emerging problems?
5. How can real – life problems involving one variable be modeled and solved?

ACTIVITY 15. KWL Chart

Try to complete the first parts of the diagram. After the topic, you will be requested to complete all parts. Click “Save” when you’re finished.



You gave your initial answers, let’s see if it will change after learning more about polynomials.

A. Multiplication of Monomials

In Activity 13, you tried multiplying monomials. Based on the patterns, these rules can be derived. They are similar with the laws of exponents you encountered in the other module.

ying powers with like bases (Product Rule)

For any real number ***a*** and for any positive integers ***m*** and ***n***:

$a^m . a^n = a^{m+n}$

Raising a power to a power (Power to a Power Rule)

For any real number ***a*** and for any positive integers***m*** and ***n***:

$(a^m)^n = a^{mn}$

Raising a product to a power

For any real number ***a*** and***b*** and for any positive integer ***m***:

$(ab)^m = a^m b^m$

Let's do it. Multiply the following.

a. $b^4 . b^9$
b. $4x^2 (5x^{12})$
c. $(-2ab^2)(6ab^4)$
d. $(4kp^2)^3$
e. $(3a^2b)^2(6a^3b)(2)$

Solution:

a. $b^4 . b^9 = b^{4+9} = b^{13}$
b. $4x^2 (5x^{12}) = 20x^{2+12} = 20x^{14}$
c. $(-2ab^2)(6ab^4) = -12a^{1+1}b^{2+4} = -12a^2b^6$
d. $(4kp^2)^3 = 4^3k^3p^{2(3)} = 64k^3p^6$
e. $(3a^2b)^2(6a^3b) = (3^2a^{2(2)}b^2)(6a^3b) = (9a^4b^2)(6a^3b) = 54a^7b^3$

Exercise 12. Multiply the following. Click "Submit" to see the correct answers. Pay attention to the items where you got a wrong answer. You can review the previous discussion if you need to.

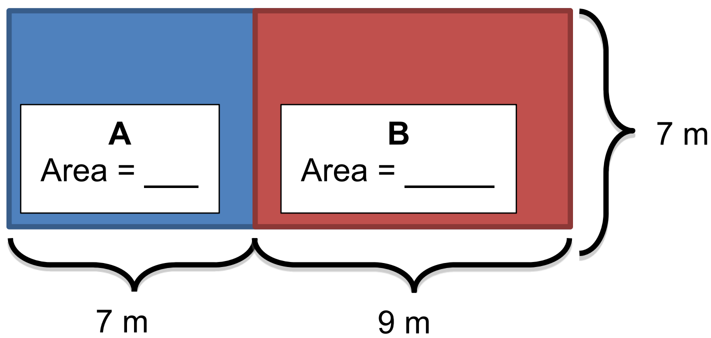
a. $a^4.a^5$ = ____________
b. $4x^3.5x^7$ = ____________
c. $(5y^2)(-y^5)$ = ____________
d. $(ab)(a^2b^3)$= ____________
e. $(6n)(m)(5)$= ____________
f. $b^y.b^{5y}$ = ____________
g. $x^{3+a}.x^{7-a}$ = ____________
h. $(3b^3d^2)^2(2bd^5)$ = ____________
i. $(-8m^2)^2 (4d^3)^2$ = ____________
j. $(-pq)^{11}(-pq)^{18}$ = ____________

The correct answers are:

a. a^9
b. $20x^{10}$
c. $-5y^7$
d. a^3b^4
e. $30mn$
f. b^{6y}
g. x^{10}
h. $18b^7d^9$
i. $1024d^6m^4$
j. $-p^{29}q^{29}$

B. Multiplying a Monomial by a Polynomial

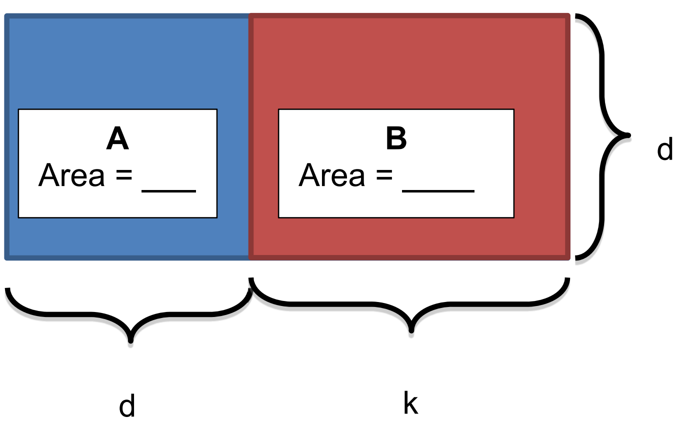
Consider the figure below. What is the area each shaded rectangle? What is the area of the whole rectangle? How will you get the area of each figure?



Since A is a rectangle, to get the area, multiply the length and width so that 7x7 to get $49m^2$. Since B is also a rectangle, the area will be $7x9=63m^2$. The area of the whole rectangle will be $7(7+9)=7(16)=112m^2$ which is the same as when the areas of A and B are added, $49m^2 + 63m^2 = 112m^2$.

Now, what if the measure of each side is represented by a variable instead? Try to get the area of each shaded figure and the total area of the figure.

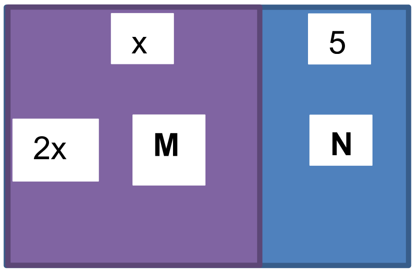
Remember what you have done to get the area of the figure above.



Since, A and B are rectangles, then the area of A is $d(d)=d^2$ and the area of B is dk. The total area will be $d^2 + dk$. It is the same result when d is multiplied to the whole length of the rectangle, d+k so that:

Area = (length) (width)
= (d + k) (d)
= $d^2 + dk$ $units^2$

Consider the next figure. What will be the area of M and N and the total area of the rectangle.



Area of M	Total Area
Area = (length)(width)	Solution 1
= x(2x)	Area = M + N
= $2x^2$ units^2	= $2x^2 + 10x$
Area of N	Solution2
Area = (length)(width)	Area = (length)(width)
= 5 (2x)	= (x+5)(2x)
= 10x units^2	= $2x^2 + 10x$ units^2

As you can observe, in the three examples, two solutions were presented to get the total area. First, add the areas. Second, multiply the width with the total length of the rectangle. The second method is similar with the Distributive Property which shows how to multiply a monomial by a polynomial. In short,

To multiply a Monomial by a Polynomial:
a (b + d) = ab + ad

Let's Do It. Multiply.

a. 4 (x+2)
b. –5(3d+4)
c. – 2m (3m – 1)
d. 2n ($6n^2$ + 5n + 2)

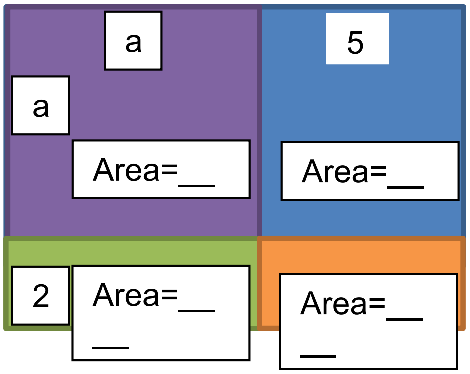
Solutions:

a. 4 (x+2) = 4x + 8
b. –5(3d+4) = – 15d – 20
c. – 2m (3m – 1) = $-6m^2 + 2m$
d. 2n ($6n^2$ + 5n + 2) = $12n^3 + 10n^2 + 4n$

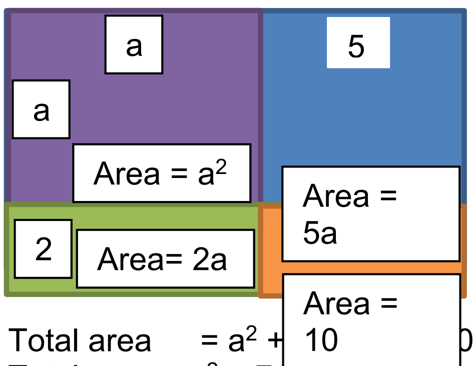
When multiplying a monomial by a polynomial, what is the rule/pattern you need to remember?

C. Multiplying a Binomial by a Binomial

Try to multiply a binomial by another binomial by getting the total area of the rectangle. Take note the total length and width are not given.



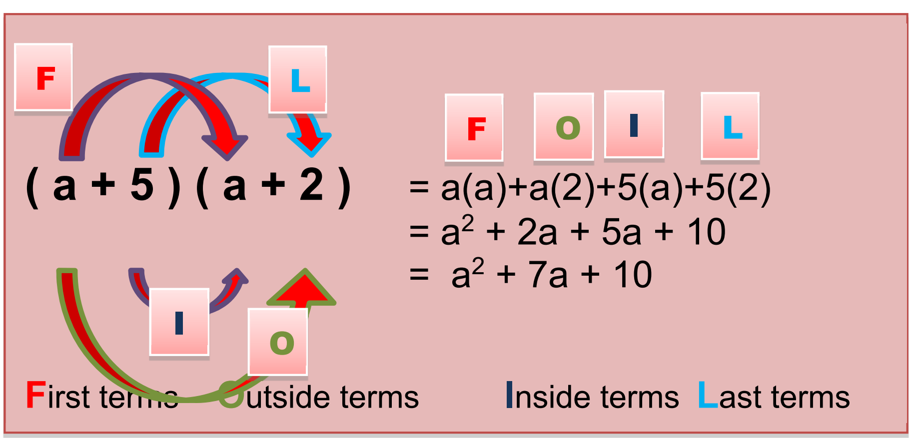
Solution1:



Total area = $a^2 +$ … 0
Total area = $a^2 + 7a + 10$ units^2

Solution2:
using the distributive property
Area of rectangle
Area = (length)(width)
= (a + 5) (a + 2)
= $a^2 + 5a + 2a + 10$
= $a^2 + 7a + 10$ units^2

In multiplying two binomials, the distributive property can easily be remembered by using the FOIL method.



Let's Do It. Multiply the following using the FOIL method.

a. (b+3) (b+2)
b. (m+3) (m – 7)
c. (4x – 5)(x + 2)
d. $(2k^2 – 10) (5k^2 – 1)$
e. (3d + 5) (2b + m)

Solution:

F O I L

a. (b+3) (b+2) =b(b) + b(2) + 3(b) + 3(2) = $b^2+2b+3b+6 = b^2+5b+6$

b. (m+3) (m – 7) =m(m)+m(–7)+3(m)+3(–7) = $m^2–7m+3m–21 = m^2–4m–21$

c. (4x – 5)(x + 2) =4x(x)+4x(2)+(–5)(x)+(–5)(2) = $4x^2+8x–5x–10 = 4x^2+3x–10$

d. $(2k^2–10) (5k^2–1) =2k^2(5k^2)+2k^2(–1)+(–10)(5k^2)+(–10)(–1)$
$=10k^4 – 2k^2 – 50k^2 + 10 = 10k^4 – 52k^2 + 10$

e. (3b + 5) (2d + m) =3b(2d)+3b(m)+5(2d)+5(m) = 6bd+3bm+10d+5m

Remember to apply the laws of exponents as you multiply the terms and add only the common terms. In letter e, there is no common term.

Process Questions:

1. When multiplying two binomials, what rule/pattern do you need to remember?

2. What is the meaning of FOIL in relation to multiplying two binomials?

D. Multiplying a Polynomial by a Polynomial

So far, you tried to multiply a monomial and a polynomial, and a binomial with another binomial. What if two polynomials are to be multiplied? To multiply two polynomials, just apply the distributive property. Check the example below.

$$(a+2)(a^2+3a+5) = a(a^2)+a(3a)+a(5)+2(a^2)+2(3a)+2(5)$$
$$= a^3 + 3a^2 + 5a + 2a^2+ 6a + 10$$
$$=a^3 + 5a^2 + 11a + 10$$

Let's Do It. Multiply.

a. $(x + 2)(x^2 + 4x + 1)$
b. $-3n(n^3 + n^2 - n + 8)$
c. $(2n - 3)(5n^2 + n - 4)$

Solution:

a. $(x + 2)(x^2 + 4x + 1)$
$= x(x^2) + x(4x) + x(1) + 2(x^2) + 2(4x) + 2(1)$ *Use the distributive property*
$= x^3 + 4x^2 + x + 2x^2 + 8x + 2$ *Multiply each term*
$= x^3 + 6x^2 + 9x + 2$ *Combine similar terms*

b. $-3n(n^3 + n^2 - n + 8)$
$= (-3n)(n^3) + (-3n)(n^2) + (-3n)(-n) + (-3n)(8)$ *Use the distributive property*
$= -3n^4 - 3n^3 + 3n^2 - 24n$ *Multiply each term*

c. $(2n - 3)(5n^2 + n - 4)$
$= (2n)(5n^2) + (2n)(n) + (2n)(-4) + (-3)(5n^2) + (-3)(n) + (-3)(-4)$ *Use the distributive property*
$= 10n^3 + 2n^2 - 8n - 15n^2 - 3n + 12$ *Multiply each term*
$= 10n^3 - 13n^2 - 11n + 12$ *Combine similar terms*

Exercise 13. Multiply the following. Click "Submit" to see the correct answers. Pay attention to the items where you got a wrong answer. You can review the previous discussion if you need to.

a. $(x + 9)(x + 6)$
b. $(3m - 4)(3m - 4)$
c. $(b + 7)(b + 1)$
d. $(a^2 + 3)(a^2 + 6)$
e. $(4k + 3)(k - 2)$
f. $(2+m)(4m^2 - m + 10)$
g. $-6p^2(p^3 + 5p^2 - p + 2)$
h. $(t + 2)(2t^2 - t + 1)$
i. $(5d - 1)^2$
j. $(x^2 + 2x + 1)(x^2 - x - 3)$

The correct answers are:

a. $x^2 + 15x + 54$
b. $9m^2 – 24m + 16$
c. $b^2 + 8b + 7$
d. $a^4 + 9a^2 + 18$
e. $4k^2 – 5k – 6$
f. $4m^3 + 7m^2 + 8m + 20$
g. $– 6p^5 – 30p^4 + 6p^3 – 12p^2$
h. $2t^3 + t^2 – t + 2$
i. $25d^2 – 10d + 1$
j. $x^4 + x^3 – 4x^2 – 7x – 3$

Process Questions:

1. While answering the exercise, what did you observe?

2. What is the general rule when you multiply polynomials?

3. In multiplying polynomials, is there another strategy aside from the distributive property?

If the steps in multiplying polynomials are not clear yet, you can access the following websites which will show you more examples and activities about multiplying polynomials.

Activity 14. Multiplying Polynomials (Online)
These online resources will be helpful in understanding the concepts. Feel free to access and learn from it. You can access it multiple times.

Multiplying Polynomials
http://www.klvx.org/DocumentView.asp?DID=161
This powerpoint presentation is about multiplying a polynomial by a monomial and multiplying a polynomial by a polynomial. It also includes a review of the Distributive Property. At the end of the presentation are some items you can work on.

Multiplying binomials - FOIL
http://www.coolmath.com/crunchers/algebra-problems-multiplying-polynomials-FOIL-1.html
This website, Algebra Cruncher, generates an endless number of practice problems for multiplying binomials with FOIL with solutions. Follow the instruction provided.

After learning how to multiply polynomials, it is time to solve some problems. Remember that at the end of the lesson, you should answer the question, "how can real – life problems be solved?"

ACTIVITY 16. Appreciating Polynomials

Answer the following problems. Show your complete solution. Item 1 is done for your guidance.

1. Two friends Alex and Ben challenges Charles and Dominic for individual badminton games. How many games will be played if they play each other once?

Solution:

a. First, assign a variable for each: Alex can be represented by the expression A, Ben by B, Charles by C and Dominic by D.
b. Next, represent or model the situation using a polynomial. Since they are going to play each other once, then it can be represented by (A + B) (C + D).
c. Solve the polynomial. (A + B) (C + D) = AC + AD + BC + BD
d. Finally, interpret the result. An answer which is not interpreted is meaningless. In this case, Alex will be playing against Charles as represented by AC and Dominic as represented by AD; and Ben will play against Charles as represented by BC and Dominic as represented by BD. There will be **four** games in all.

The diagram below summarizes the steps you can follow when solving word problems.

STEP 1 Assign variable/s to known and unknown values. → **STEP 2** Model the situation using polynomials. → **STEP 3** Solve. Perform the operations. → **STEP 4** Interpret results and write final answer.

2. Three groups of students: L&M, O&P and Q&R decided to play scrabble. In each game, there will be three players, one from each group. A different set of players will be playing for every game. All players will have an equal number of games. If there are 8 games, show all the possible combinations of players?
   [Hint: binomial x binomial x binomial]

3. The amount and ingredients needed to prepare a pancake good for three (3) servings are:
   1 cup all-purpose flour
   2 tablespoons sugar
   2 teaspoons baking powder

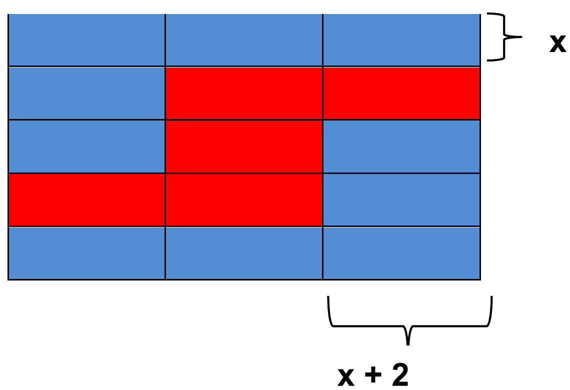
1/2 teaspoon salt
1 large egg, slightly beaten
2 tablespoons vegetable oil

If you wish to prepare 12 servings, what is the amount for each ingredient? Provide a monomial times a polynomial that models the amount of each ingredient for any desired number of servings.

4. Watch the video in the website then answer the problem that follows.

http://www.phschool.com/atschool/academy123/english/academy123_content/wl-book-demo/ph-269s.html

For each rectangle in the figure, the width is x and the length is x + 2. a) What is the area of the whole rectangle? What is the area of the part colored with blue? If x = 3cm, what is the actual area of the rectangle?



5. http://www.regentsprep.org/regents/math/algebra/AV3/Papply.htm

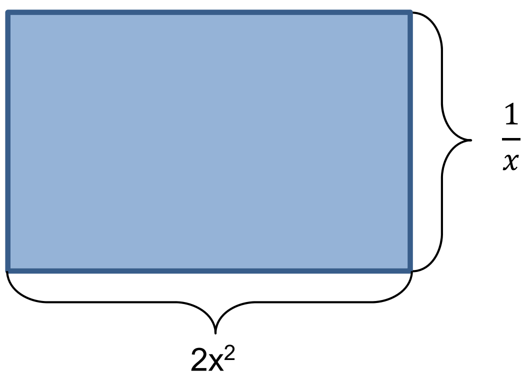
This website presents 6 problems that can be solved using the concept of multiplying polynomials. Answer the items. Compare your solution with the solution of the website. Do you have the same solution or different approaches but the same answers?

In the introduction of polynomials, you have learned that algebraic expressions with variables in the denominator are not polynomials. Does it mean that polynomials can only be added, subtracted or multiplied but can't be divided?

The answer is *NO. Well,* $\frac{2}{x}$ is not a polynomial per se but if we treat the numerator and denominator independently, then 2 is a polynomial and x is another polynomial. Therefore, a polynomial can be divided by another polynomial.

DIVISION OF POLYNOMIALS

Applying your knowledge about multiplying polynomials, what is the area of the rectangular figure?



To solve the area, multiply the length and the width so that $(2x^2)(\frac{1}{x}) = \frac{2x^2}{x}$. However, the product is not fully simplified. Using the definition of exponents,

$\frac{2x^2}{x} = \frac{2\,.\,x\,.\,x}{x} = 2x.$

In certain situations, division of polynomials will be performed. Some rules can be derived based on the examples using the definition of exponents.

QUOTIENT RULE

1. $\frac{a^m}{a^n} = a^{m-n}$ if m > n
2. $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$ if m < n
3. $\frac{a^m}{a^n} = a^0 = 1$ if m = n

Power of a Quotient

4. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Examples**.** First, the polynomials are simplified using the definition of exponents. Then, the rules are applied. Observe how the rules are derived. Take note of the conditions where each rule is applicable.

1. $\frac{a^m}{a^n} = a^{m-n}$ if m > n

   a. $\frac{6^5}{6^3} = \frac{6.6.6.6.6}{6.6.6} = 6.6 = 36$; $6^{5-3} = 6^2 = 36$

   b. $\frac{d^4}{d^3} = \frac{d.d.d.d}{d.d.d} = d$; $d^{4-3} = d$

2. $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$ if m < n

   a. $\frac{4^2}{4^3} = \frac{4.4}{4.4.4} = \frac{1}{4}$; $\frac{1}{4^{3-2}} = \frac{1}{4}$

   b. $\frac{m}{m^5} = \frac{m}{m.m.m.m.m} = \frac{1}{m^4}$; $\frac{1}{m^{5-1}} = \frac{1}{m^4}$

3. $\frac{a^m}{a^n} = a^0 = 1$ if m = n

   a. $\frac{7^3}{7^3} = \frac{7.7.7}{7.7.7} = 1$; $7^{3-3} = 7^0 = 1$

   b. $\frac{x^5}{x^5} = \frac{x.x.x.x.x}{x.x.x.x.x} = 1$; $x^{5-5} = x^0 = 1$

4. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

   a. $\left(\frac{4}{3}\right)^2 = \frac{4}{3} \times \frac{4}{3} = \frac{16}{9}$; $\frac{4^2}{3^2} = \frac{16}{9}$

   b. $\left(\frac{b}{d^2}\right)^3 = \frac{b}{d^2}\frac{b}{d^2}\frac{b}{d^2} = \frac{b^3}{d^6}$; $\frac{b^3}{d^{2(3)}} = \frac{b^3}{d^6}$

Let's Do It. Divide.

a. $\frac{x^5y^3}{x^3y^2}$

b. $\frac{m^6n^3}{m^9n^5}$

c. $\frac{10x^4y^8}{x^7y^2}$

d. $\frac{-7a^4b^2d^2}{14\,ab^9d^2}$

e. $\left(\frac{-8x^2y^6z^2}{2x^5y^2z^9}\right)^3$

Solution:

a. $\frac{x^5y^3}{x^3y^2} = x^{5-3}y^{3-2} = x^2y^1 = x^2y$

b. $\frac{m^6n^3}{m^9n^5} = \frac{1}{m^{9-6}n^{5-3}} = \frac{1}{m^3n^2}$

c. $\frac{10x^4y^8}{x^7y^2} = \frac{10y^{8-2}}{x^{7-4}} = \frac{10y^6}{x^3}$

d. $\frac{-7a^4b^2d^2}{14\,ab^9d^2} = \frac{-7a^{4-1}d^{2-2}}{14\,b^{9-2}} = \frac{-a^3(1)}{2b^7} = \frac{-a^3}{2b^7}$

e. $\left(\frac{-8x^2y^6z^2}{2x^5y^2z^9}\right)^3 = \left(\frac{-4y^4}{x^3z^7}\right)^3 = \frac{-64y^{12}}{x^9z^{21}}$ Simplify the numerator and denominator before raising to a power.

A. Dividing a Polynomial by a Monomial

To divide a polynomial by a monomial, apply the property of fractions.

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

To divide a polynomial by a monomial:

1. Arrange the powers of the dividend in descending order.
2. Divide each term of the polynomial by the monomial
3. Add the resulting quotient.

Exercise 15. Division of Polynomial by Monomial (Online)
Access the website below for more discussion about dividing a polynomial by a monomial. Next, do the activities provided in the website.
http://www.shmoop.com/basic-algebra/dividing-polynomials.html

B. Dividing a Polynomial by a Binomial

To divide a polynomial by a polynomial, apply the long division of whole numbers.

To divide a polynomial by a monomial:

1. Arrange the terms of the binomial and the polynomial in descending order. Insert a 0 for any missing term.
2. Divide the first term of the polynomial by the first term of the binomial. The result is the first term of the quotient.
3. Multiply the first term of the quotient by the binomial and subtract the results from the first two terms of the polynomial. Bring down the next term.
4. If the degree of the first term of the remainder is less than the degree of the first term of the binomial divisor, STOP. If not, repeat step 2 and continue the process.

Exercise 16. Division of Polynomial by Binomial (Online)

Access the website below for more discussion about dividing a polynomial by a binomial. Next, do the activities provided in the website.
http://www.mathsisfun.com/algebra/polynomials-division-long.html
http://www.ltcconline.net/greenl/courses/152a/polyexp/polydiv.htm

Exercise 17. Dividing Polynomials (Online)
This online resource will be helpful in understanding the concepts. Feel free to access and learn from it. You can access it multiple times.

Dividing Polynomials
This website, Algebra Cruncher, generates an endless number of practice problems for dividing polynomials. Follow the instruction provided.
http://www.coolmath.com/crunchers/algebra-problems-exponent-rules-2.htm

Now that you've learned how to divide polynomials, let's try to solve some word problems.

1. The area of a rectangle is ($108x^6 + 135x^3 - 162x^2$) square meters. What is the length if the width is $9x^2$.

Solution:

   a. Represent. Let A = area, l = length and w = width.
   b. Model the situation using polynomials.
      Area = (length)(width)
      A = LW
      $108x^6 + 135x^3 - 162x^2 = (L)(9x^2)$
   c. Solve the length (L).
      To solve the length, divide $108x^6 + 135x^3 - 162x^2$ by $9x^2$.

      $$\frac{108x^6+135x^3-162x^2}{9x^2} = 12x^4 + 15x - 18$$

   d. Interpret the answer.
      The length of the rectangle is $12x^4 + 15x - 18$ meters.

2. The volume of rectangular box is $2x^3 + 8x^2 + 2x - 12$ and its height is $2x + 4$. What is the area of its base?
   a. Represent Volume = V, height = H and Area of base = A.

   b. Model situation in a formula.
      V = AH

   c. Solve the height by dividing the Volume by the Height. Apply the concepts of dividing a polynomial by a polynomial.
      V = AH
      $2x^3 + 8x^2 + 2x - 12 = (A)(2x + 4)$

$A = \frac{2x^3+8x^2+2x-12}{2x+4} = x^2 + 2x - 3.$

d. Interpret the answer. The area of the base is $x^2 + 2x - 3$. To check if the area of the base is correct, multiply $x^2 + 2x - 3$ and $2x + 4$ and the result should be the volume.

Process Questions:

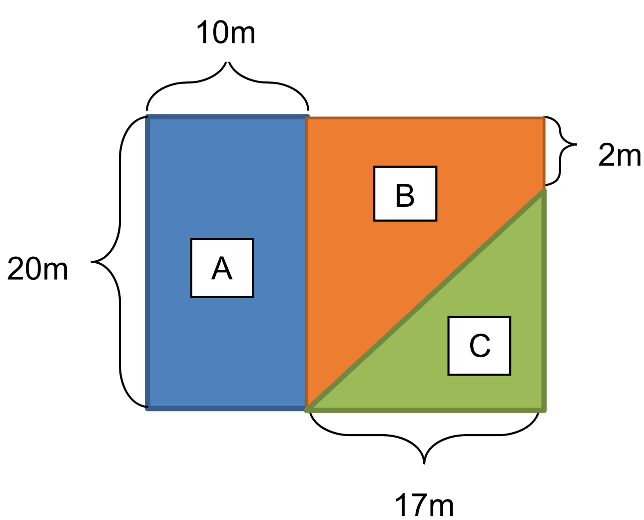
1. What new realizations do you have about the topic?
2. To divide expressions, how many operations need to be performed?

ACTIVITY 17. Appreciating Polynomials 2

Answer the following. Click "Submit" when you're finished.

1. A father is to divide a piece of land for his three children such that the Pedro will have the largest area, Dennis the smallest area and the remaining goes to Jordan.

   a) Based on the figure below, what is the total area of the lot?
   b) Which lot (A, B , C) is for each sibling? Why?
   c) If x = 10, what algebraic expression represent the area of each lot?



The correct answers are:
a)Total area = 340 m^2
b)
Lot A is for Pedro (200 m^2)
Lot B is for Jordan (187 m^2)
Lot C is for Dennis (153 m^2)

c) Land Area
Lot A: $2x(x) = 2x^2\ m^2$
Lot B: $2x(x+7) - \frac{(x+7)(2x-2)}{2} = x^2+8x + 7m^2$
Lot C: $\frac{(x+7)(2x-2)}{2} = x^2 + 6x - 7m^2$
When you evaluate each expression (x = 10), the value is the same with those of b).

2. Find the width of a rectangle if the area is $2x^2 + 19x + 24$ and the length is $2x + 3$.

Solution:

Correct Answer: The width is x+ 8.

3. Find The length of a rectangle whose area is $28x^2 + 3x – 1$ and whose width is $4x + 1$.
Solution:

Correct Answer: The length is $7x – 1$.

4. The distance covered by a car in 2x hours is $4x^3 – 2x^2 + 6x$. Find the rate of the car.
Solution:

Correct Answer: The rate is $2x^2 – x + 3$.

5. The volume of a rectangular solid is $x^3 – x^2 – x + 1$. If the height is $x – 1$, find the area of its base. If x is 3cm, what is the actual height and area of the base of the rectangular solid?
Solution:

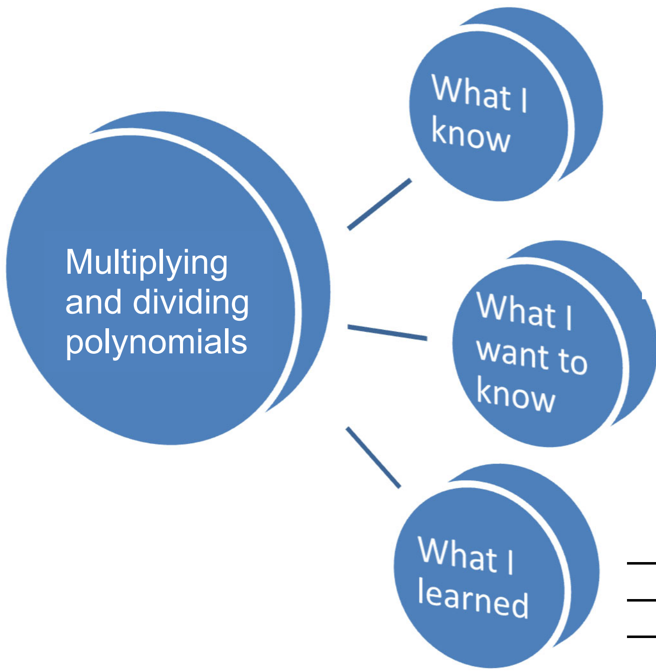
Correct Answer: The area of the base is $x^2 – 1$.
The actual height is 2cm and the area is $8cm^2$

Process Questions:

1. How did you find the activities task?
2. Did it help for you to see the real world use of polynomials?
3. How can real – life problems involving one variable be modeled and solved?

ACTIVITY 18. KWL Chart

Complete the KWL chart you started to accomplish previously. Click “Submit” when you’re finished.



Now that you have learned about classifying po
exponents, added, subtracted, multiplied and divided polynomials, can you now answer the question - how will polynomials help in your daily life? Well, surely you can do that.

End of Firm Up
In this section, the discussion was all about classifying polynomials, exponents, and operations with polynomials. Now that you know some of the important ideas about polynomials, in what ways can it help solving real – life problems?

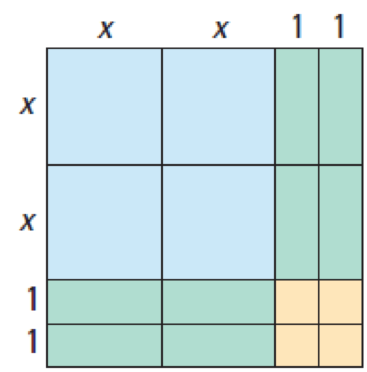
Let’s go deeper by moving on to the next section.

DEEPEN

Your goal in this section is to take a deeper look about polynomials.

SPECIAL PRODUCTS

In the previous lesson, you dealt with addition/subtraction and multiplication of polynomials. How are they related? Well, after multiplying the polynomials, you have to check for similar terms to be added/subtracted. For multiplying polynomials, is there a shorter way of doing it aside from FOIL and the use of algebra tiles? You will learn about these in this section about special products. In the end, you must relate it with the previous topics and be able to answer: how can polynomials help in your daily life?



ACTIVITY 19. Mathematical Model

Basing on the figure on the right, answer the following. Click "Submit" when you're finished.

1. Write expressions that represent the length and width of the figure.
   Length ____________________

   Width _____________________

   Correct answer: length = $2x + 2$; width = $2x + 2$

2. What is the area of the whole figure? What is your basis in saying so?

   Solution:

   Correct answer: Area = $4x^2 + 8x + 4$; $A = lw$

3. Tell whether the following statement is true or false.
   The product of $(2x+2)$ and $(2x+2)$ is $4x^2 + 2^2$. Explain.

   ____

   ____

   ____

   Correct answer: False, it should be $4x^2 + 8x + 4$.

4. Find the missing term: $(a - b)^2 = a^2 - \underline{?} + b^2$.

The missing term is ___________.

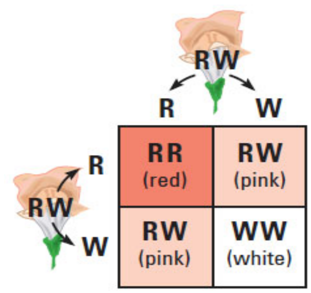
Correct answer: The missing term is – 2ab.

Process Questions:
1. How did you find the area? Is the topic about algebra tiles helpful in this activity?

2. What do you need to remember when multiplying a binomial by another binomial?

3. How can real – life problems involving one variable be modeled and solved?

ACTIVITY 20. Mathematical Investigation: Punnet Squares



Punnet Squares are used in genetics to model the mixing of parents' genes in the resulting offspring. The Punnet square on the right is a model that shows the possible results of crossing two pink snapdragons (flower), each one with red gene R and one white gene W. Each parent snapdragon passes along only one gene color to its offspring.

1. What will be the possible color of the offspring?
2. How many snapdragon of each color will there be?

Solution:

Each parent snapdragon has half red genes and half white genes. You can model the genetic makeup of each parent as 0.5R + 0.5W. The genetic makeup of the offspring can be modeled by $(0.5R + 0.5W)^2$.

$$(0.5R + 0.5W)^2 = (0.5R)^2 + 2(0.5R)(0.5W) + (0.5W)^2$$
$$= \underbrace{0.25R^2}_{\text{Red}} + \underbrace{0.5RW}_{\text{Pink}} + \underbrace{0.25W^2}_{\text{White}}$$

Based on the Punnet square and the mathematical model, 25% of the offspring will be red, 50% will be pink and 25% will be white.

Process Questions:
1. What are the common physical features you share with your family members?
2. How can you predict the Blood type of your children in the future?

3. Given the same situation, which do you prefer to use, a mathematical model or a Punnet square? Explain what you think is the advantage and disadvantage of each method.
4. When is it appropriate to use polynomial models? Explain.
5. How can real – life problems involving one variable be modeled and solved?

After working on the two activities, have you observed anything special about? Let's begin with special products by learning about multiplying binomials.

A. Multiplying Binomials

ACTIVITY 21. Let's Do the Pattern

Use the FOIL method to find the product. Observe any pattern for each group. Click "Submit" when you're finished.

1. (x + 2) (x + 3)	**=**
2. (m + 6) (m + 4)	=
3. (3n + 1) (3n + 9)	=
4. (m + n) (m + p)	=
Pattern:	
5. (x + 2) (x + 2)	
6. (m + 6)^2	=
7. (3n + 1) (3n + 1)	=
8. (m + n)^2	=
Pattern:	
9. (x – 2) (x – 2)	=
10. (m – 6)^2	=
11. (3n – 1) (3n – 1)	=
12. (m – n)^2	=
Pattern:	
13. (x + 2) (x – 2)	=
14. (m – 6) (m + 6)	=
15. (3n + 1) (3n – 1)	=
16. (m – n) (m + n)	=
Pattern:	

Based on the previous activity, the product of binomials are predictable if they are in these special cases:

Miscellaneous Binomials:
$(x + a) (x + b) = x^2 + (a+b)x + ab$
$(ax + b) (cx + d) = acx^2 + (ad+bc)x + bd$

Square of a Binomial
$(a+b)^2 = (a+b)(a+b)= a^2 + 2ab +b^2$
$(a–b)^2 = (a–b)(a–b)= a^2–2ab +b^2$
The square of the sum or difference of two terms is the square of the first term plus or minus twice the product of the two terms plus the square of the last term.

Sum and difference of two terms
$(a+b) (a–b) = a^2 – b^2$
The product of the sum and difference of two terms is the square of the first term minus the square of the second term.

Let's Do It. Multiply.

a. $(x + 5) (x + 3)$
b. $(2x + 4) (2x + 4)$
c. $(2x – 4)(2x – 4)$
d. $(2x – 4) (2x + 4)$

Solution:

a. $(x + 5) (x + 3) = x^2 + (5 + 3)x + 5(3) = x^2 +8x + 15$
b. $(2x + 4) (2x + 4) = (2x)^2 + 2 (2x)(4) + 4^2 = 4x^2 + 16x + 16$
c. $(2x – 4)(2x – 4) = 2x)^2 – 2 (2x)(4) + 4^2 = 4x^2 – 16x + 16$
d. $(2x – 4) (2x + 4) = (2x)^2 – (4)^2 = 4x^2 – 16$

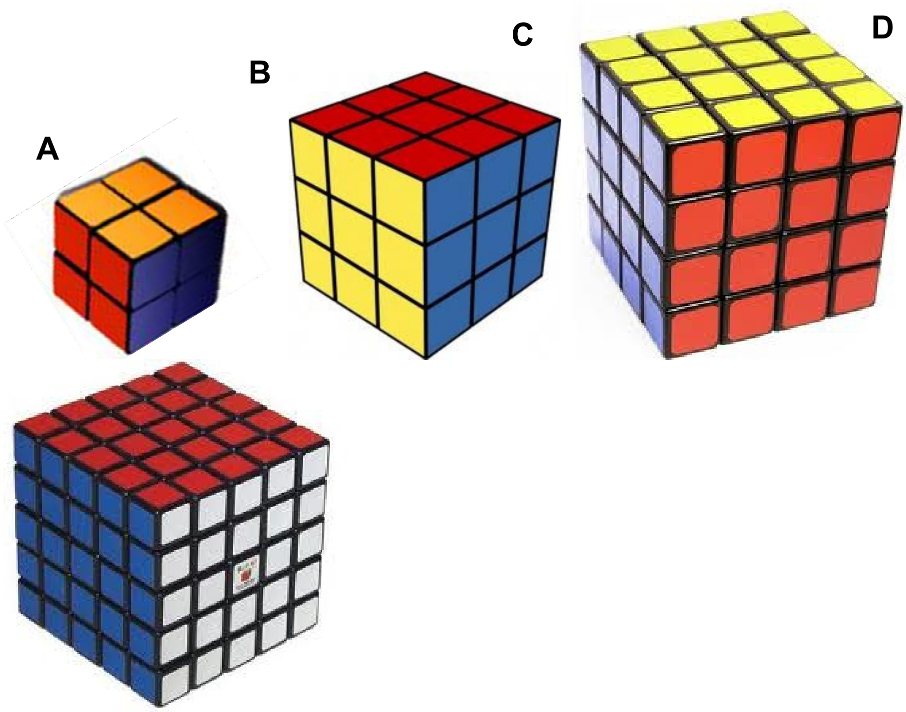
Did you notice how the patterns were applied in the examples? Did the patterns make multiplying special types of binomials easier? Is $(x + y)^2 = x^2 + y^2$. Why or why not?

B. Cube of a Binomial

ACTIVITY 22. Rubik's Cube.

Find the area and volume of each Rubik's cube. The length of each square in each cube is 1 unit. Click "Submit" when you're finished.

Rubik's Cube	Area of one side	Volume
A (2x2)		
B (3x3)		
C (4x4)		
D (5x5)		



ACTIVITY 23. Cubed.

Find the area and volume of each Rubik's cube with the given dimension of each side. Click "Submit" when you're finished.

Rubik's Cube	Length of a side	Area of one side	Volume
A(2x2)	x + y		
A(2x2)	a + b		
A(2x2)	x – y		

A(2x2)	a – b	
B(3x3)	x + y + z	[redacted]
B(3x3)	a + b + c	

Process Questions:
1. Is there any pattern you observed?
2. Did you apply the concept about special products in solving the area of the cubes?
3. What did you do when solving the volume? Is there a pattern too?

There is a pattern for getting the cube of a binomial. The pattern is:

CUBE OF A BINOMIAL
$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
$(a – b)^3 = a^3 – 3a^2b + 3ab^2 – b^3$

Let's Do It. Multiply or expand.
a. $(x + 4)^3$
b. $(x – 4)^3$
c. $(2m + n)^3$

Solution:

a. $(x + 4)^3$ $= (x)^3 + 3(x)^2(4) + 3(x)(4)^2 + (4)^3$
$= x^3 + 3x^2(4) +3x(16) + 64$
$= x^3 + 12x^2 + 48x + 64$

b. $(x – 4)^3$ $= (x)^3 – 3(x)^2(4) + 3(x)(4)^2 – (4)^3$
$= x^3 – 3x^2(4) + 3x(16) – 64$
$= x^3 – 12x^2 + 48x – 64$

c. $(2m + n)^3$ $= (2m)^3 + 3(2m)^2(n) + 3(2m)(n)^2 + (n)^3$
$= 8m^3 + 3(4m^2)(n) + 6m(n^2) + n^3$
$= 8m^3 + 12m^2n + 6mn^2 + n^3$

C. Square of a Trinomial

In Activity 22, when you were solving the area of Rubik's Cube B, you multiplied a trinomial with itself. What is the pattern you observed?

SQUARE OF A TRINOMIAL
$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$

Let's Do It. Multiply or expand.
a. $(x + y + 4)^2$
b. $(x + 2y – 5)^2$

Solution:

a. $(x + y + 4)^2 = (x)^2 + (y)^2 + (z)^2 + 2(x)(y) + 2(x)(4) + 2(y)(4)$
$= x^2 + y^2 + z^2 + 2xy + 8x + 8y$

b. $(3x + 2y – 5)^2 = (3x)^2 + (2y)^2 + (–5)^2 + 2(3x)(2y) + 2(3x)(–5) + 2 (2y)(–5)$
$= 9x^2 + 4y^2 + 25 + 12xy – 30x – 20y$

Process Questions:

1. From the different special products you performed, is it helpful to observe patterns on certain phenomenon and make predictions?

2. In your opinion, what is the advantage of using polynomial models?

Exercise 18. One More Time (Online)
This website is about multiplying special cases of binomials. You can play the animations for each case. Try to answer some questions in the end.
http://www.mathsisfun.com/algebra/special-binomial-products.html

Now that you've learned some of the skills about special product, let's try to apply it in solving some problems. Try to observe the steps suggested when solving word problems.

1. The width of a rectangle is y – 2. Its length is 4 more than its width. What is the area of the rectangle?

Solution:

a. First, represent the length and width using polynomials.
   Let: width = y – 2; length = (y – 2) + 4 = y + 2
b. Represent the situation using a mathematical model (polynomials).
   Area = length x width
   Area = (y + 2) (y – 2)
c. Solve. Apply concepts of multiplying polynomials and special products.
   Area = (y + 1) (y + 2)
   Area = $y^2 + 7y + 10$
d. Interpret the result.
   The area of the rectangle is $y^2 + 7y + 10$ square units.

2. Juan dela Cruz is negotiating for a cost – of – living allowance (COLA) increase for his union's members and asked for an 6% annual increase. In the proposal, someone earning ₱120,000 this year would earn ₱120,000 + (₱120,000)(6%) next year. What is the actual salary in the first year, second year and third year of increase?

Solution:

a. Represent the salary for 1st year as S1, 2nd year as S2 and 3rd year as S3.

b. Represent the situations using mathematical models.
S1 = ₱120,000 + (₱120,000) (6%)
S2 = S1 + S1 (6%)
S3 = S2 + S2 (6%)

c. Solve.
S1 = ₱120,000 + (₱120,000) (0.06) = 120,000 + 7,200 = ₱127,200.00
S2 = ₱127,200 + (₱127,200) (0.06) = 127,200 + 7,632 = ₱134,832.00
S3 = ₱134,832 + (₱134,832) (0.06) = 134,832 + 8089.92 = ₱142,921.92

d. Interpret the results.
The annual salary increases by six percent every year.

3. In the salary problem in number 2, let the annual percentage increase be represented by the variable ***c***. What polynomial represents the salary for the 1st year? 2nd year? 3rd year?

Solution:

a. Represent the salary for 1st year as S1, 2nd year as S2 and 3rd year as S3. Represent annual percentage increase as ***c.***

b. Represent the situations using mathematical models.
S1 = ₱120,000 + (₱120,000) (c%)
S2 = S1 + S1 (c%)
S3 = S2 + S2 (c%)
There is no problem with this representation but to know the annual salary for any year, we have to know the salary of the previous year. So, if we want to know the salary for the tenth year after the increase, we have to know the salary from the first to the ninth year. That's so tedious! Let's try to look at another way.
In number 2,
S1 = ₱120,000 + (₱120,000) (0.06) = 120,000 + 7,200 = ₱127,200.00
Observe the situation can also be represented by
₱120,000 (1 + 0.06) = ₱120,000 + (₱120,000) (0.06)
It follows that: S1 = ₱120,000 (1 + c).
Since S2 = S1 + S1 (c%), then S2 = ₱120,000 (1 + c) (1+c).
Also, S3 = S2 + S2 (c%), then S3 = ₱120,000 (1 + c) (1+c) (1 + c).
Try to substitute 6% for c and you will arrive at the same answers.

c. Solve. Multiply the polynomials. Special products are useful in solving S2 and S3.
S1 = ₱120,000 (1 + c) = 120,000 + 120,000c

S2 = ₱120,000 (1 + c) (1+c) = 120,000 $(1 + 2c + c^2)$ = 120,000 + 240,000c + $120{,}000c^2$

S3 = ₱120,000 (1 + c) (1+c) (1 + c) = 120,000 (1 + 3c + $3c^2$ + c^3) = 120,000 + 360,000c + 360,000c^2 + 120,000c^3

d. Interpret the results.
The salary can be represented by a polynomial 120,000 $(1+c)^n$ where n is the number of years.

Process Questions:
1. In the earlier activities, you dealt with multiplying binomials together or multiplying trinomials together. What if a binomial or a trinomial are multiplied, will there be a pattern too?
2. After the examples, what do you think is the answer to the question "how can real – life problems be solved?"

D. Multiplying a Binomial and a Trinomial of the Form $(a \pm b)(a^2 \pm ab + b^2)$

Let's Do It. Multiply using the Distributive property.

a. $(a+b)(a^2 - ab + b^2)$
b. $(a - b)(a^2 + ab + b^2)$

Solution: Using

a. $(a+b)(a^2 - ab + b^2) = (a)(a^2) + (a)(-ab) + (a)(b^2) + (b)(a^2) + (b)(-ab) + (b)(b^2)$
$= a^3 - a^2b + ab^2 + a^2b - ab^2 + b^3$
$= a^3 + b^3$

b. $(a - b)(a^2 + ab + b^2) = (a)(a^2) + (a)(ab) + (a)(b^2) + (-b)(a^2) + (-b)(ab) + (-b)(b^2)$
$= a^3 + a^2b + ab^2 - a^2b - ab^2 + b^3$
$= a^3 - b^3$

The factors $(a+b)(a^2 - ab + b^2)$ have the following characteristics:

1. The first term of the trinomial is the square of the first term of the binomial.
2. The second term of the trinomial is the additive inverse of the product of the first and second terms of the binomial.
3. The third term of the trinomial is the square of the second term of the binomial.

PRODUCT OF A BINOMIAL AND A TRINOMIAL OF A SPECIAL FORM

$(a+b)(a^2 - ab + b^2) = a^3 + b^3$
$(a - b)(a^2 + ab + b^2) = a^3 - b^3$

Let's Do It. Verify if the factors follow the special form. If it does, find the product.

a. $(x+2)(x^2 + 2x + 4)$
b. $(2x - 3)(4x^2 + 12x + 9)$
c. $(x+5)(x^2 - 5x + 25)$
d. $(3x - 2)(9x^2 + 6x + 4)$

Solution:

a. $(x+2)(x^2 + 2x + 4)$

No. The middle term of the trinomial should be negative.

b. $(2x - 3)(4x^2 + 6x + 9) = (2x)^3 - (3)^3 = 8x^3 - 27$
Yes

c. $(x+5)(x^2 - 5x + 25) = (x)^3 + (5)^3 = x^3 + 125$

Yes

d. $(3x - 2)(9x^2 - 6x + 4)$

No. The middle term of the trinomial should be negative.

Exercise 19. One More Time
These online videos are about special products. Feel free to view it by clicking to the hyperlinks below.
http://www.brightstorm.com/math/algebra/polynomials-2/multiplying-polynomials-special-cases/
http://www.brightstorm.com/math/algebra/polynomials-2/multiplying-polynomials-using-area-models/

ACTIVITY 24. How Special.

Apply special products to find the indicated product. Click "Submit" when you're finished.

1. $(x+3)(x - 5)$ = ______________________
2. $(y - 4)(y - 1)$ = ______________________
3. $(m+8)(m - 2)$ = ______________________
4. $(x + 3y)^2$ = ______________________
5. $(6x - 5)^2$ = ______________________
6. $(1 - 2y^2)^2$ = ______________________
7. $(b - 8d)^2$ = ______________________
8. $(7x + 4y)^2$ = ______________________
9. $(n - 8)(n + 8)$ = ______________________
10. $(m^2 - 9)(m^2 + 9)$ = ______________________

11. $(4n^3 - 6m)(4n^3 + 6m)$ = ________________________
12. $(y+3)^3$ = ________________________
13. $(m^2 - 5)^3$ = ________________________
14. $(5x^2 - 3y^3)^3$ = ________________________
15. $(x + y + 2)^2$ = ________________________
16. $(3a - 2b - c)^2$ = ________________________
17. $(x+4)(x^2 - 4x + 16)$ = ________________________
18. $(x+5)(x^2 - 5x + 25)$ = ________________________
19. $(y - 3)(y^2 + 3y + 9)$ = ________________________
20. $(5x - 2y)(25x^2 + 10xy + 4y^2)$ = ________________________

The correct answers are:

1. $x^2 - 2x - 15$
2. $y^2 - 5y + 4$
3. $m^2 + 6m - 16$
4. $x^2 + 6xy + 9y^2$
5. $36x^2 - 60x + 25$
6. $1 - 4y^2 + 4y^4$
7. $b^2 - 16bd + 64d^2$
8. $49x^2 + 56xy + 16y^2$
9. $n^2 - 64$
10. $m^4 - 81$
11. $16n^6 - 36m^2$
12. $y^3 + 9y^2 + 27y + 27$
13. $m^6 - 15m^4 + 75m^2 - 125$
14. $125x^6 - 150x^4y^3 + 135x^2y^6 - 27y^9$
15. $x^2 + y^2 + 2xy + 4x + 4y + 4$
16. $9a^2 + 4b^2 + c^2 - 12ab - 6ac + 4bc$
17. $x^3 + 64$
18. $x^3 + 125$
19. $y^3 - 27$
20. $125x^3 - 8y^2$

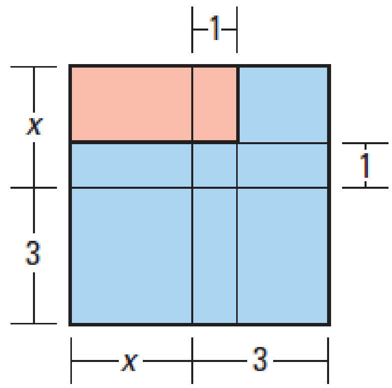
Process Questions:

1. Are you able to apply special products in answering the items?
2. Is it easier to multiply polynomials if you know about special products? Explain.

ACTIVITY 25. Appreciating Polynomials 3

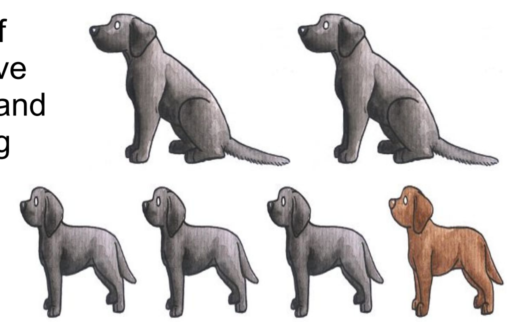
Answer the following. Click "Submit" when you're finished.

1. With the given dimensions of the figure, what is the expression that represents the area of the blue region? Show your complete solution.



Solution:

2. Your brother is breeding Labrador dogs. Both of his dogs are black but when the mother dog gave birth with four puppies, three are colored black and one is colored liver (chocolate). He is requesting your help in explaining what happened so that he knows which dog he is going to breed to get a colored black or colored liver in the future.



   a. First, research about dominant and recessive genes. In this case, black (B) is the dominant and liver (b) is the recessive gene.
   b. Draw a Punnet square that explains the colors of the four puppies.
   c. Which dog is he going to breed if he prefers to have all liver Labrador? Support it with a Punnet square.
   d. Which type of dog is he going to breed so that there is a 50% percent chance of having a black or liver dog? Support it with a Punnet square.

e. How van real – life problems involving one variable be solved?
____
____
____

3. The area of a rectangle is given by the expression $6x^3 - 13x - 5$.
   a. Find the length if the width is $2x - 5$.

   b. If $x = 8cm$, find the length, width and area of the rectangle.

4. The volume of a rectangular box is $x^3 + 27$.
   a. Find the area of its base in terms of x.

   b. What is the height in terms of x?

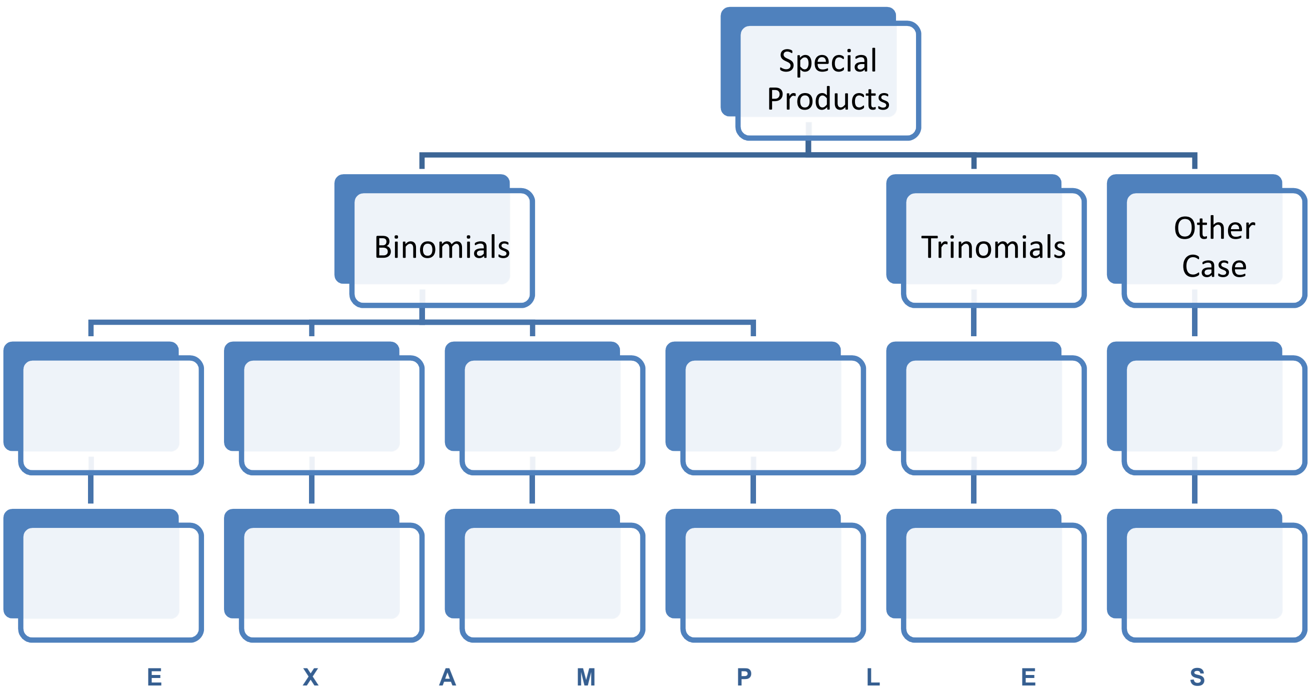
   c. If $x = 10cm$, find the dimensions and volume of the box.

End of DEEPEN:

In this section, the discussion was about other forms of special products. What new realizations do you have about the topic? What new connections have you made for yourself?

ACTIVITY 26. So Special

Complete the concept map to summarize what you've learned in this topic then click "Submit" when you're finished. Take a picture of your output and upload in your facebook/instagaram account. Solicit comments from your friends.



Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section

TRANSFER

At the start and throughout the lesson, this question was asked repeatedly: ***How can real – life problems be solved?*** The Transfer section of the lesson will guide you in determining the best answer to the question.

Your goal in this section is apply your learning to real life situations. You will

Performance Task. Flash Drive

You are working in DX2 Computers. One day, a representative of an organization is inquiring about flash drives, which they plan to give as giveaway for their upcoming activity. He said that mp3 and mp4 files will be saved on each drive and he is not sure what is the suppose size of the flash drive he will buy. When given the proper explanation, he will be buying 150 units of flash drives. His main priorities are the capacity of the flash drive and the brand. The cost is the last priority. You are to convince him through oral and written presentation using graphical and algebraic models. When convinced, your boss will see that you are a very important person in the staff.

1. What flash drive, considering capacity (memory) are you going to suggest? Give two suggestions.
2. How many mp3 and mp4 files will fit in each drive? Give detailed explanation.
3. Which flash drive are you going to suggest? Why?

Now that you are done with your work, use the rubric below to evaluate your work. Your work should show the traits listed as Competent or 3. If your work has these traits, you are ready to submit your work. Click on "Submit".

If you want to do more, your work should show the traits listed as Accomplished or 4. If your work does not have the traits for 3 or 4, revise your work before submitting it.

Rubric of the Performance Task

Scoring Level	Accuracy	Analysis & Evaluation	Presentation	Organization
4 - Accomplished	Solutions are accurate and demonstrate understanding of the structure of the problem. Demonstrates a clear knowledge and **application of math skills**.	Examines conclusions. Uses reasonable judgment Synthesizes data of the **mathematical model**. Views information critically	Discusses issues thoroughly by using **multiple strategies**. Justifies decisions. **Mathematical terminology** is prevalent and used correctly.	Presents information in logical, interesting sequence which audience can follow.
3 - Competent	Solutions to problems are accurate. Demonstrates a general knowledge and application of math skills.	Formulates conclusions Recognizes arguments Notices differences. Analyzes data based on mathematical model. Seeks out information	Identifies issues by using two acceptable strategies Suggests solutions to identified problems Mathematical terminology correctly used.	Presents information in logical sequence which audience can follow.
2 - Developing	Solutions are only slightly inaccurate, resulting from errors in calculations. Demonstrates a limited knowledge and application of math skills.	Identifies some conclusions but these are erroneous. Sees some arguments but some are faulty. Identifies some differences Paraphrases data Assumes information valid	Misconstructs arguments Presents a single option Overlooks some information Some mathematical terminology is presented, but not correctly used.	Audience has difficulty following presentation because student jumps around.
1 - Beginning	Solutions contain many inaccuracies. Demonstrates little or no knowledge or application of math skills.	Unable to draw conclusions Overlooks differences Repeats data Omits research	Omits argument Misrepresents issues Excludes data No mathematical terminology is used or attempted.	Audience cannot understand presentation because there is no sequence of information.

End of TRANSFER:

In this section, your task was about making explanations and appropriate decisions based on graphical and polynomial models using the concepts about polynomials.

How did you find the performance task? How did the task help you see the real world use of the topics about polynomials? involving one variable be modeled and solved? You have completed this lesson. Before you go to the next lesson, you have to complete the term frame to summarize what you've learned in this lesson.

<table>
<tr><td colspan="6">TERMS: POLYNOMIALS, EXPONENTS, OPERATIONS ON POLYNOMIALS, SPECIAL PRODUCTS</td></tr>
<tr><td colspan="2">What I Already Know</td><td colspan="2">What I Am Learning</td><td colspan="2">My Picture / Image</td></tr>
<tr><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td></tr>
<tr><td colspan="3">EXAMPLES</td><td colspan="3">NON - EXAMPLES</td></tr>
<tr><td colspan="3"></td><td colspan="3"></td></tr>
<tr><td colspan="6">MY DEFINITIONS</td></tr>
<tr><td colspan="6"></td></tr>
<tr><td colspan="6">EXTENDING MY LEARNING</td></tr>
<tr><td colspan="6">Polynomials can be</td></tr>
<tr><td colspan="6">Polynomials are useful for</td></tr>
</table>

You have completed this lesson. You can move on to the next lesson.

GLOSSARY OF TERMS USED IN THIS MODULE:

Base is the number or symbol that is being multiplied

binomial is a polynomial with two terms

constant is a number on its own

exponent is a positive number or symbol that tells how many times the base is used as a factor

monomial is a polynomial with only one term

multinomial is a polynomial with four or more terms

polynomial in x is an algebraic expression that contain only terms of the form ax^n, where a is any real number and n is any whole number. Other than x, other letters may also be used

power is the product of equal factors.

term in a polynomial is separated by plus or minus sign. It is a collection of numerical and literal factors

trinomial is a polynomial with three terms

WEBSITE RESOURCES AND LINKS IN THIS LESSON
(Arranged according to order of appearance in the lesson)

http://staff.argyll.epsb.ca/jreed/math9/strand2/algetiles.htm
The website is about Algebra Tiles

http://www.aplusalgebra.com/algebra-tiles.htm
The website is about Algebra Tiles

http://www.quia.com/rr/180013.html
The website is about the game Rags to Riches

http://www.mathdork.com/games/asteroidsexp3/asteroidsexp3.html
The website is about the game Math Dork – Exponent Asteroids

http://www.algebra-class.com/laws-of-exponents.html
The website is about the laws of exponents.

http://staff.argyll.epsb.ca/jreed/math9/strand2/poly_sum.htm
The website provides animated examples of algebra tiles

http://www.mathsisfun.com/algebra/polynomials-adding-subtracting.html
This website focuses on how to add/subtract polynomials with LIKE TERMS.

http://www.regentsprep.org/regents/math/algebra/AV2/sp_subt.htm
This website shows several ways of subtracting polynomials.

http://www.regentsprep.org/regents/math/algebra/AV2/sprac_a.htm
The website provides activities for adding and subtracting.

http://www.algebralab.org/practice/practice.aspx?file=Algebra1_10-1.xml
The website provides activities for adding and subtracting.

http://www.klvx.org/DocumentView.asp?DID=161
The website is about multiplying a polynomial by a monomial and multiplying a polynomial by a polynomial. It also includes a review of the Distributive Property.

http://www.coolmath.com/crunchers/algebra-problems-multiplying-polynomials-FOIL-1.html
This website, Algebra Cruncher, generates an endless number of practice problems for multiplying binomials with FOIL with solutions.

http://www.phschool.com/atschool/academy123/english/academy123_content/wl-book-demo/ph-269s.html
The website is about solving word problems involving multiplying polynomials.

http://www.regentsprep.org/regents/math/algebra/AV3/Papply.htm
The website is about solving word problems involving multiplying polynomials.

http://www.shmoop.com/basic-algebra/dividing-polynomials.html
The website is about dividing a polynomial by a monomial.

http://www.mathsisfun.com/algebra/polynomials-division-long.html
http://www.ltcconline.net/greenl/courses/152a/polyexp/polydiv.htm
The websites are about dividing a polynomial by a binomial.

http://www.coolmath.com/crunchers/algebra-problems-exponent-rules-2.htm
This website, Algebra Cruncher, generates an endless number of practice problems for dividing polynomials.

http://www.mathsisfun.com/algebra/special-binomial-products.html
This website is about multiplying special cases of binomials.

http://www.brightstorm.com/math/algebra/polynomials-2/multiplying-polynomials-special-cases/
The online video is about special products.

http://www.brightstorm.com/math/algebra/polynomials-2/multiplying-polynomials-using-area-models/
The online video is about special products.

Lesson 3: Linear Equations

☑ INTRODUCTION AND FOCUS QUESTION(S):

Almost all situations in the real world where certain relationships exist are represented by algebraic expressions. In this context, the knowledge and skills in dealing with algebraic expressions particularly on linear equations and inequalities in one variable are of utmost importance, which will pave the way to solve real-life problems in a variety of ways. This module aims to address these questions:

Why are algebraic expressions useful?

How can real – life problems involving one variable be modeled and solved?

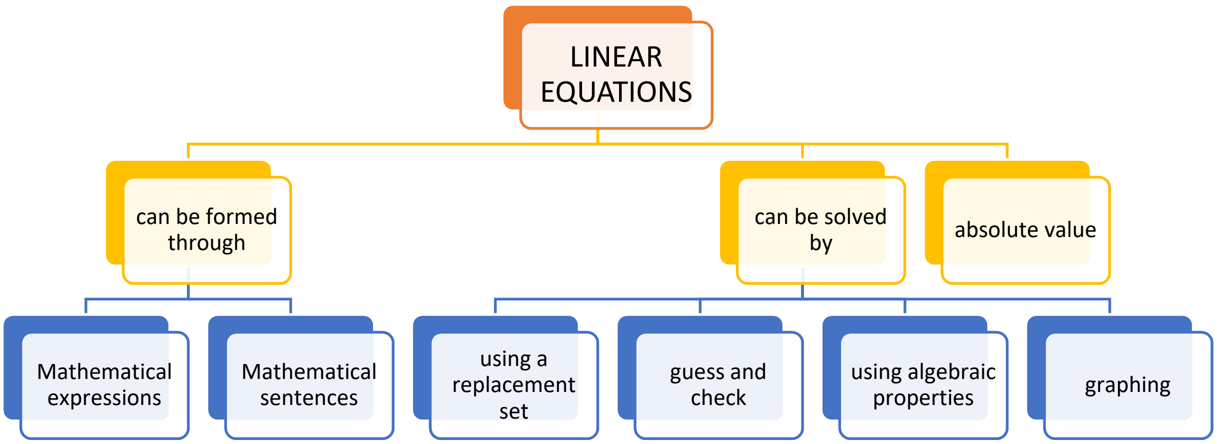
☑ LESSON COVERAGE

This lesson will cover the following

Title	You'll learn to…	Estimated Time
Linear Equations	a. Differentiate between mathematical expressions and mathematical equations b. Translate English sentences to mathematical sentences and vice versa. c. Find the solution of linear equation in one variable. d. solves linear equations in one variable involving absolute value by: (a) graphing; and (b) algebraic methods e. solves problems involving equations in one variable	10 hours

☑ Concept Map of the Lesson

Here is a simple illustration of the topics you will cover in this lesson



☑ Expected Skills

To do well in this lesson, you need to remember and do the following:

7. Making outlines, concept maps and notes will be very helpful in understanding the module.
8. Follow the instructions provided for each activity.
9. Review and evaluate your work using the rubric provided before submission.
10. Complete all exercises and activities.
11. Be mindful of the meaning of unfamiliar words you encounter in this module. A glossary of terms is provided in the last part of this module.
12. Maximize the use of online resources in each lesson. Online resources can be accessed multiple times. The summary of online resources is provided in the end of the module.

EXPLORE

Let's start this module by eliciting your prior knowledge about linear equation and its applications.

ACTIVITY 1. KWL Chart

Fill-up the first 2 columns of the KWL chart.

K W L CHART

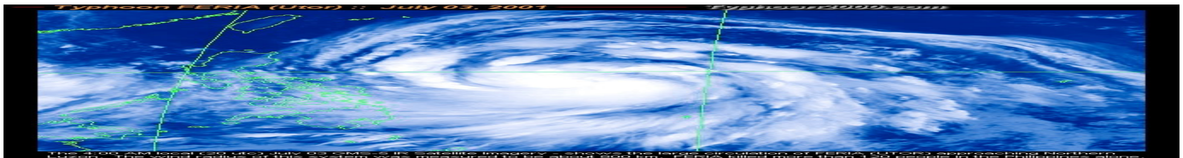
What I know about algebraic expressions, polynomials and linear equations and inequalities	What I want to know	What I learn

When you are done, proceed to the next activity.

ACTIVITY 2. Problem/Situational Analysis

You work together with a partner. Observe the data in the table below and answer the questions that follow. Reporting of your answers to the class will be done after 10 minutes

The number of typhoons reported in the Philippines:



Year	Number	Year	Number
2001	17	2006	20
2002	13	2007	
2003	25	2008	21
2004	25	2009	22
2005	18	2010	11

http://www.typhoon2000.ph/stormarchives/2001/images_full/feria_large.jpg

A certain NGO is designing a plan of actions for environmental protection program and it needs a complete data of the number of typhoons in the Philippines for the past 10 years for the purpose of creating trends and making predictions. The records in 2007 were missing due to a fire that happen in that particular year but the clerk can still remember that the number of typhoons was 7 less than the number of typhoons reported during the preceding year.

Questions to reflect and answer:

1. How many typhoons took place in 2007?
2. How did you determine such number?
3. What mathematical expression models this situation?
4. What kind of an expression is it?
5. Are algebraic expressions useful?
6. ***Why are algebraic expressions useful?***
7. ***How can real – life problems* involving one variable be modeled and solved?**

After you have given your answers, proceed by answering the ff. process questions:

1. What have you noticed of the trend from 2007 up to 2009?
2. What is the average rate of increase/decrease?
3. Based from that rate, how many possible typhoons will take place in 2012?

4. To what factors do you attribute the increase/decrease?
5. How can you help minimize the number of typhoons?

End of Explore
You have already given your initial ideas on algebraic expressions/ linear equations in one variable, now you will find out the correct answer by doing the next activity.

FIRM-UP

Your goal in this section is to learn and understand key concepts of linear equations in one variable and the skills in solving their

http://images.search.yahoo.com/search/images;_ylt=A2KJkK1u.95O5yEAREiLuLkF?p=playing+with+dice&ei=utf-8&iscqry=&fr=sfp

ACTIVITY 3. Playing Together

Instructions:
1. You and your partner will need 2 dice or an improvised cube with numbers1-6 on each side.
2. Trial: One of you will roll the dice and 2 numbers will show up.
3. Add, subtract, multiply or divide these numbers to form mystery numbers.
4. Game: Player #1 rolls the 2 dice, covers one number, announces the mystery number or gives the clue (Example: the sum of the 2 numbers is 11 or three times my number subtracted by the other number is 5).
5. Player #2 makes a guess.
6. Play for 5 rounds and announce the score.
7. Player# 2 takes his or her turn in playing.

Discuss with your partner how you find the number. Write on the board sample solutions. Reflect and answer the ff. process questions;
1. Which of the processes on the board give the correct answer?
2. Which procedure is easy to follow?

3. How did you come up with that solution?
4. What did you do to the given clue?
5. What have you noticed about the algebraic expressions?
6. Are algebraic expressions useful in playing mind games? Explain why.

Remember?

Algebraic expressions are combinations of numbers, operations and variables. A **variable** usually represents a number. The value of an algebraic expression depends on the value of the variables in it. To evaluate an expression, substitute a value for the variable.

EXAMPLE 1

Evaluate each expression for x = 2

a.) $4x - 3 = 4 \cdot 2 - 3$
$= 8 - 3$
$= 5$

b.) $7x^2 = 7 \cdot 2^2$
$= 7 \cdot 4$
$= 28$

The set of numbers that can be substituted for the variable is the **replacement set**.

EXAMPLE 2

Find the values of the expression 5m + 1 if the replacement set of m is { -2, 3, 6 }

m	5m + 1
-2	$5 \cdot -2 + 1 = -10 + 1 = -9$
3	$5 \cdot 3 + 1 = 15 + 1 = 16$
6	$5 \cdot 6 + 1 = 30 + 1 = 31$

TRY THESE:

Exercise 1: For the replacement set of y { -1, 2, 4, 5 }, find the values of

a.) 3y – 4 b.) 2y + 7

ACTIVITY 4. Concept Building:

In the previous activity on playing together, you were able to solve for the unknown number by translating the clues into mathematical statements. Now you need to know these basic terminologies

A statement which expresses that 2 numbers or algebraic expressions are equal is called an **equation.** An equation that contains a variable is an **open sentence** which can be true or false depending on the value of the variable. The value of the variable that makes the equation true is a **solution** of the equation.

Some equations can be solved by **mental math** just like what you did in the game.

TRY THESE:

Practice: Use mental math to solve each equation. Check your answer.

1.) x + 10 = 17 2.) p – 3 = 9 3.) 24 – y = 11 4.) 4t = 20

But if the solution is not obvious, you can try guessing it and test by substituting it into the equation. If the guessed value makes the equation true, then your guess was the correct solution, otherwise you make another guess. This is referred to as the **guess and check method**.

EXAMPLE:

Find the value of c in this equation 8c – 3 = 29

Solution:

First guess: c = 6	Second guess: c = 2	Third guess: c = 4
Substitute c with 6	Substitute c with 2	Substitute c with 4
8c – 3 = 29	8c -3 = 29	8c – 3 = 29
8(6) – 3 = 29	8(2) -3 = 29	8(4) – 3 = 29
48 – 3 = 29	16 – 3 = 29	32 – 3 = 29
45 $\neq$ 29	13 $\neq$ 29	29 = 29 Hence,
So, 6 is not a solution.	So, 2 is not a solution	4 is a solution of 8c -3=29

EXERCISE 2:

Use guess and check to solve each equation. Check your answer.

1.) 3s + 5 = 11 2.) 6a – 13 = 17 3.) 4x – 9 = - 1 4.) y/5 = 3

Answer exercise nos. 1-4 for practice.

ACTIVITY 5. Modeling solving linear equations in one variable using Manipulative (Algeblocks)

After learning mental math and guess and check methods of solving equations in one variable, you will explore another method. This time you will use algeblocks which include the basic mat which is divided into 2 parts, 1 positive and 1 negative, unit blocks consist of green squares and yellow rectangles which represent integers and the variables respectively.

ACTIVITY 6. A. Work in triads. Your group will prepare two basic mats.

In your Math Club, you and 9 other members will form 2 committees with the same number of members. Use the Algeblocks to model the problem and answer the following questions;

1. How many members should there be in each committee?
2. How did you solve the problem?
3. What blocks are shown on each basic mat?
4. How did you arrange the blocks to solve the problem?
5. What expressions represent each mat?
6. What equation is modeled by the algeblocks?
7. What equation represents the problem?
8. What have you observed about the equations in numbers 6 & 7?
9. How will this activity help you solve other linear equations in one variable?

For your exit ticket, submit in ¼ sheet of paper your solution verifying your answer using the other methods that you have already learned.

You already know the importance of using algeblocks in solving linear equations in one variable, you may now engage on other hands-on activity using equation mat, cups and caps (red for positive integers & blue for negative)

Think Back:
When you add a number and it's opposite, the sum is zero. The two numbers are called **zero pairs.**

ACTIVITY 6. B. Hands-on Activity: Group of 4 students

Activity materials: basic mat, cups, blue and red caps
Task: Model and solve the equation $3x - 7 = 2x - 3$

Instructions:
1. Place 3 cups and 7 blue caps on one side of the equation mat. On the other side of the mat place 2 cups and 3 blue caps.
2. Since 2 sides of the mat represent equal quantities, you can remove an equal number of cups and caps from each side without changing the value of the equation.
3. If there is nothing left on one side, it means it is equal to zero, so add zero pairs equivalent to the number of caps left on the other side.
4. Remove again the same number of caps of the same color from each side.
5. Whatever is left, this answers the question how many caps are contained in each cup.

Seatwork exercise by pair:
Model each equation and solve using the same materials listed above.

1.) $3x + 4 = 10$ 2.) $2y - 3 = 5$ 3.) $4m + 7 = 3m + 8$ 4.) $5a - 3 = 2a - 9$

ACTIVITY 6. C. Perform activity 6c in a triad (see attached worksheets at the end)

Exercise 3: Solve the following equations:

1.) $x + 5 = 6$ 2.) $3y = 12$ 3.) $2m - 7 = 5$ 4.) $5a/6 = 10$

The correct answers are:
1. $x = 1$ 2. $y = 4$ 3. $m = 6$ 4. $a = 12$

Linear equations can also be solved using a number line. The distance of a point from zero is called an **absolute value**. A very good model of this is the elevator. If you go down 4 floors below the ground, in Algebra it is represented by -4 but your distance from the ground is 4 not -4. Hence, the **absolute value of -4** which in symbol is ***/-4/*** is 4. If you go up to the tenth floor, your distance from the ground is 10 not +10, so /+10/= 10.

Example of an absolute value of a linear equation:

a. How many floors will the elevator rise, if it started two floors below the ground (basement) to the sixth floor?

If you count from basement 2 up to the 6th floor, the elevator will climb 8 floors. This situation is modeled by the equation -2 + x = 6 and solving for x it will give you a value equal to 8.

To solve for the absolute value of a linear equation, there should be 2 solutions; /x/ = ±c. First /x/ = +c and the second is /x/ = - c

/-2 + x/ = 6	/-2 + x/ = -6
X = 6 + 2	x = - 6 + 2
X = 8	x = - 4

In this example 8 is the accepted solution, because distance cannot be a negative number.

After accomplishing activity 6C, complete the flow chart that follows. Click "SUBMIT" when you're done. (see attached flow chart)

FLOW CHART
ON
SOLVING LINEAR EQUATIONS IN ONE VARIABLE

A. With a variable on one side

1. Add or subtract on both sides of the equation a number that forms a zero pair with the number on the side of the equation with the variable.

↓

2. Equate the variable with the number on the other side, and such number is the solution of the equation.
(If the numerical coefficient of the variable is one) →

Solving linear equations with variables on both sides

1. Transfer all terms with variables on one side of the equation and the constants on the other side.

↓

2.

↓

3.

THE CORRECT ANSWERS IS:

FLOW CHART
ON
SOLVING LINEAR EQUATIONS IN ONE VARIABLE

B. With a variable on one side

1. Add or subtract on both sides of the equation a number that forms a zero pair with the number on the side of the equation with the variable.

↓

2. Equate the variable with the number on the other side, and such number is the solution of the equation.
(If the numerical coefficient of the variable is one)

→ OTHERWISE, divide both sides of the equation by the numerical coefficient of the variable.

Solving linear equations with variables on both sides

1. Transfer all terms with variables on one side of the equation and the constants on the other side.

↓

2. **Combine all the similar terms.**

↓

3. **Divide both sides of the equation by the numerical coefficient of the variable. The quotient is the solution of the equation.**

Process Questions:

1. What is used to represent the given figure?
2. ***Are algebraic expressions useful?***
3. ***How can real-life problems* involving one variable be modeled and solved?**

The first 3 websites contain interactive exercises on solving linear equations in one variable with increasing degree of difficulty. Do not proceed to the next if you cannot answer at least 80% of the items.

http://www.freeed.net/sweethaven/Math/algebra/linearEq/LinEqOne01_EE.asp?iNum=3

This website contains interactive activities on solving linear equations in one variable which can be solved by applying addition/subtraction property of equality.

http://www.freeed.net/sweethaven/Math/algebra/linearEq/LinEqOne01_EE.asp?iNum=6

This contains interactive activities on solving linear equations in one variable which can be solved by applying the multiplication/division property of equality.

http://www.freeed.net/sweethaven/Math/algebra/linearEq/LinEqOne01_EE.asp?iNum=9

This contains interactive activities on solving linear equations in one variable which can be solved by applying a combination of at least 2 different properties of equalities.

http://www.wtamu.edu/academic/anns/mps/math/mathlab/video.htm?video=algebra/college/14/01

This website contains a video on the explanation of solving linear equations in one variable.

End of FIRM UP:
In this section, the discussion was about finding solutions of linear equations in one variable using different strategies.

Exercise 4: Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision? What have you realized?

Let's go back to the KWL chart. (Click on the link.) Now make some revision and click "Save."
Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.

DEEPEN

Your goal in this section is to take a closer look at some aspects of the topic. After learning how to solve linear equations in one variable, your next task is to find out where to apply these knowledge and skills in real-life so that you will find meaning in learning all of these things.

Consider this for instance;

EXAMPLE 1: At the start of the experiment there was 17 grams of salt but when it was time to return the remaining material, there was only 8 grams left. How many grams of salt were used in the experiment?

In solving this problem you can use any strategy that you know, but the more logical method is the algebraic solution.

These are the suggested steps in problem solving:

a. Read the problem.
b. Identify what is asked and represent it with a variable.
c. Translate the entire statement into mathematical symbols.
d. Solve for the unknown.

Let x = be the number of grams of salt used in the experiment
Since there are 17 grams at the start and 8 grams left, so the equation would be

$17 - x = 8$
$(17 - x) - 17 = 8 - 17$
$(-x)(-1) = (-9)(-1)$
$x = 9$

Therefore 9 grams of salt were used in the experiment.

EXAMPLE 2:
How many teams of 8 members can be formed if there are 136 first year students?

Let x = be the number of teams
Since there are 8 members in each team and 136 first year students, then the equation is

$8x = 136$
$\frac{8x}{8} = \frac{136}{8}$
$x = 17$; Therefore 17 teams of 8 members can be formed out of 136 students.

EXAMPLE 5:
The taxi fare charges Php 40 for the first 2 km and Php 3.50 for every succeeding km. How far is the destination if the total fare is Php 89.00?

Let x = be the distance of the destination
x- 2 = the succeeding distance after the first 2 km.

$40 + 3.5(x-2) = 89$
$40 + 3.5x - 7 = 89$
$33 + 3.5x = 89$
$-33 + 33 + 3.5x = 89 - 33$
$3.5x/3.5 = 56/3.5$
$X = 16$; Therefore the destination is 16 km. away.

Q & A:

a. What did you do to solve the given problems?
b. What do you used to represent the unknown?
c. How did you determine the equation?
d. What kind of an equation is formed?
e. How many variables are involved?

f. ***Are algebraic expressions useful?***
g. ***How can problems* involving one variable be modeled and solved?**

ACTIVITY 7. Problem Solving:

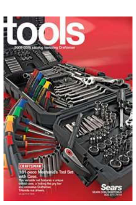
Solve the given problems with a partner. You explain your solutions and answers to the class. You and your partner will have to submit you work.

1. How much money was given by her mother if after paying Php 75 for the contribution of the field trip, Liza has Php 50 left for her allowance?
   - Given:
   - Asked:
   - Representation:
   - Equation:
   - Solution:
   - Answer in sentence form:

2. If the average of the student's score in the three tests is 9 and his scores in the first 2 tests were 6 and 13, what is his score in the third test?
   - Given:
   - Asked:
   - Representation:
   - Equation:
   - Solution:
   - Answer in sentence form:

3. If the labor charge of encoding data is Php 65 per hour. How many hours will he be encoding to earn Php 780?
   - Given:
   - Asked:
   - Representation:
   - Equation:
   - Solution:
   - Answer in sentence form:

Once reporting is done, you will proceed to work on another task with your learning pair.



Exercise 6: TASK: You are given different options of labor packages of a certain repair job at home; you have to decide which is a better option. Applying the skills in solving linear equations, first translate each option to

mathematical symbols; then solve the equation to determine the total cost incurred in each package.

First: a carpentry package of Php 20,000.

Second: Php 800 daily for 24 days.

Third: Php 750 daily for 28 days.

You need to justify your choice by showing a comparison of the computations using tables and formula. Which is the cheapest package? You may give the disadvantages of the options which are not chosen?

ACTIVITY 8. : Individual Practice.

Work on the exercise on problem solving.

1. During the last hour of the school fair, the prices of all items in a certain booth were cut down to Php 6. How many items were sold if they were able to come up with a sales amounting to Php 252?

2. An appliance store offers two payment options of a LED television set;
Option A: Php 10,000 down payment & a monthly installment of Php 1,500.
Option B: Php 5,000 down payment & a monthly installment of Php 2,000.
How long can the customers fully pay the same amount?

Process questions:
a. Were you able to solve all the problems?
b. How did you manage to solve them?
c. What knowledge and skills are proven useful to you?
d. What important insights did you gain from the problem solving activities?
e. Would you prefer to work alone? Why?/ Why not?

ACTIVITY 9. COOPERATIVE LEARNING

Together with your 4 group mates, perform another task which will require you to apply your new found knowledge and skills.

TASK 2: You are to gather data of different savings accounts from banks and other savings institutions. Use your knowledge about solving linear equation to come up with a proposal of the best savings plan. Make your proposal in written and oral form.

How did you come up with the proposal?
Describe the process of determining the best savings plan.
What linear equations model each savings plan?

What makes a good savings plan?
Is interest the only consideration for choosing the savings plan?
What other factors are considered in choosing the best savings plan?

Q & A

1. How do you find working in groups?
2. How do rate your group work? Justify it.
3. What rating do you give to the group's cooperation? Explain.

ACTIVITY 10. PROBLEM POSING

In each situation given, you are to make 2-3 problems and solve them. You will do these in your group composed of 5 members. Since there are10 groups, the even groups will work on situation 1 and the odd groups will do situation 2. You will post your work around the walls of your room to prepare for the **Gallery Walk.**

Situation 1: **The reading of the water meter is 2356 cubic meters. The daily water consumption is 2 cubic meters.**

Situation 2: **A car rental company charges Php 500 a day and Php 4.00 per km of travel.**

Process Questions:
1. How did you find the problem posing activity?
2. ***Did you find algebraic expressions useful?*** Explain.
3. ***How can real – life problems involving one variable be modeled and solved?***

End of Deepen
In this section, the discussion was about solving problems which are represented by linear equations in one variable.

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.

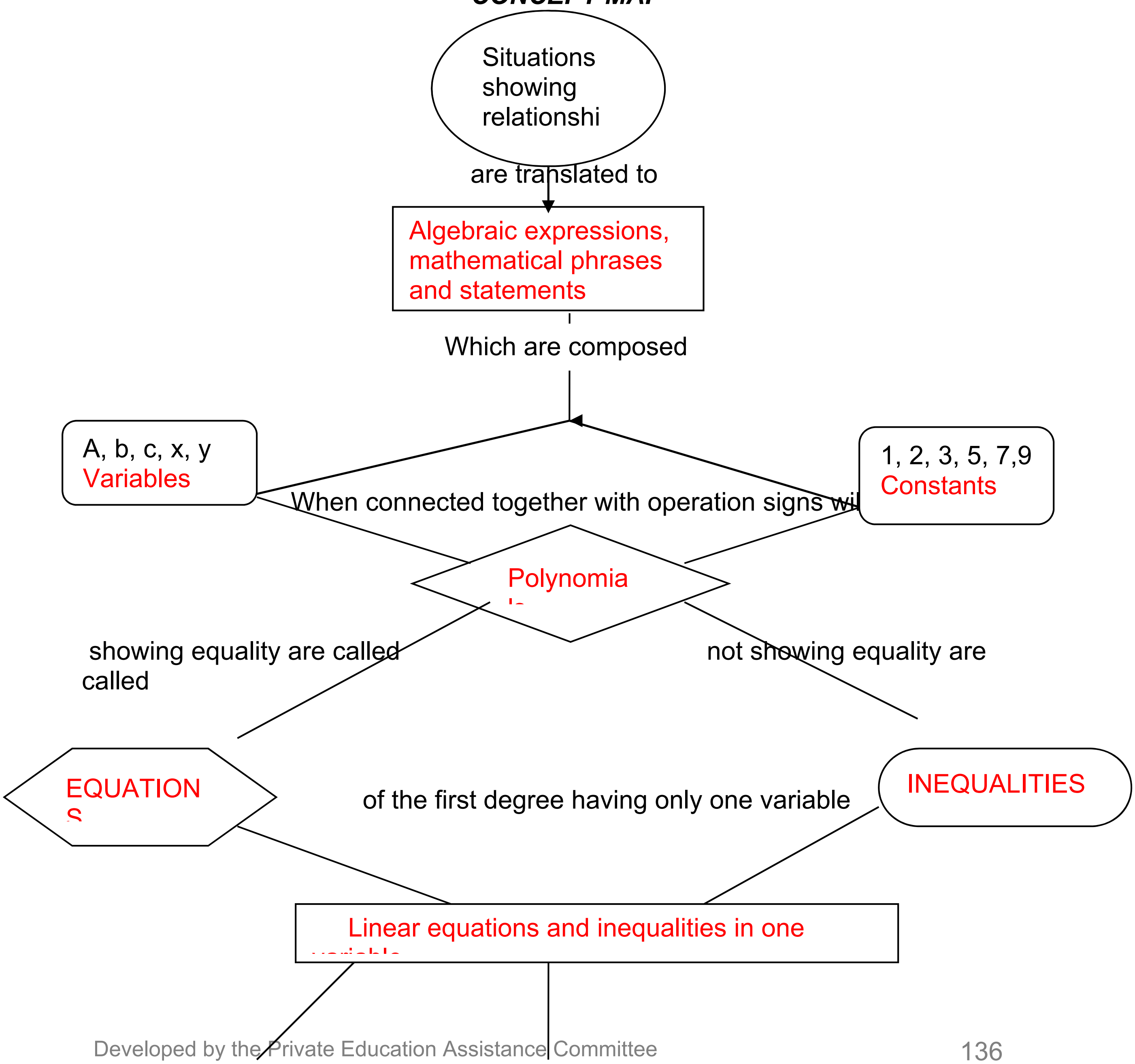
TRANSFER

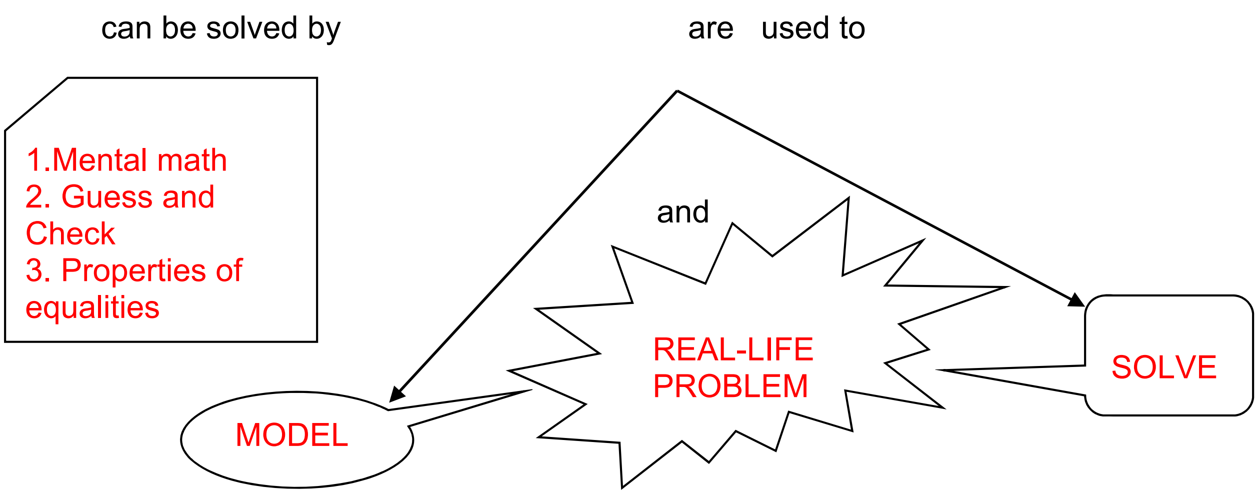
Your goal in this section is apply your learning to real life situations. You will be given a practical task which will demonstrate your understanding.

Exercise 6: What new realizations do you have about the topic? What new connections have you made for yourself? Click on the KWL chart again and complete it. Click on “Save” once you are done.

To make a summary of the lessons, complete the concept map.

CONCEPT MAP





To check your understanding of the concepts and skills on algebraic expressions, polynomial and solving linear equations and inequalities in one variable, answer the mastery test that follows. (This is optional)

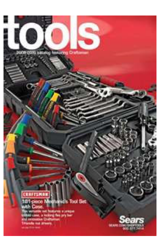
REFLECTIONS:

In your journal notebook, write the answers of the following questions;

a. What insights did you gain from the lessons on solving equations and inequalities in one variable?
b. Do you find the concepts and skills learned useful? If yes, in what ways? If no, why do you think they are not useful?

At the start of the unit you were given a pre-test, now you will answer the post test to see if there is learning taking place and check whether understanding is evident.

To show evidence of understanding the lesson, perform the transfer task given below. You may work in a group of 4 members. (This is to be submitted after 2 meetings)



PERFORMANCE TASK

Suppose you are an officer of the fundraising committee of the Student Council. Your committee is planning to sell T-shirts with the school emblem to raise money for an upcoming activity. Your committee received bids from two companies, *Plain T* and *Tee Company. Plain T* charges P 20,000 plus P115 per T-shirt while *Tee Company* charges P195 per T-shirt. Your committee must prepare a presentation to deliver to the Student Council moderator, recommending which company the school should use. You need to convince the moderator which company should be granted the bid at different possible number of orders and justify it using mathematical formulas or models. Your work will be evaluated according to clarity of presentation, conciseness, use of mathematical concepts, and accuracy.

PERFORMANCE TASK RUBRIC

CRITERIA	Expert 4	Proficient 3	Developing 2	Beginning 1
CLARITY	Very clear, convincing and confident explanation all throughout the entire presentation.	Generally clear oral presentation	Important ideas are not clear.	Ambiguous and rambling oral presentation.
CONCISENESS	The needed data are comprehensive, authentic and very relevant.	The needed data are complete and relevant.	The needed data are incomplete, partially relevant.	The needed data are missing.
USE OF MATHEMATICAL CONCEPTS	Demonstrates thorough understanding of the concepts and properties of equations.	Demonstrates understanding of the concepts and properties of equations.	Demonstrates partial understanding of the concepts and properties of equations.	Demonstrates erroneous understanding of the concepts and properties of equations.
ACCURACY	The solutions are logical and the computations are accurate and precise.	The solutions are orderly and the computations are correct.	The solutions are incomplete and some computations are incorrect.	The solutions are illogical and computations are inaccurate.

After the submission, answer the ff. process questions;

1. How did you find the performance task?
2. How did the task help you see the real world use of the topic?
3. If you are to rate your work from 1- 10, what is your rating?
4. How do you rate your group mates?
5. Where you able to make use of your learning?
6. What facilitates ease in solving certain daily-life problems?

7. ***Are algebraic expressions really useful?* Why do you think so?**
8. ***How can real – life problems involving one variable be modeled and solved?***

End of TRANSFER:
In this section, your task was to accomplish the performance task which showed evidence of your learning and understanding of the concepts and skills of linear equations in one variable.

CLOSURE:
In life many things relate to one another, and in some instances one affects the other. It is for this reason that you need to quantify the effects so informed/sound decisions can be achieved.

You have completed this lesson. Before you move on to the next lesson check that done all activities.

ACTIVITIES
Materials (worksheet, readings, references, Website links, equipment)
Procedures
Questions to Answer
Questions for Processing Information and Reflecting on Answers
Discussion
Performance task

GLOSSARY OF TERMS USED IN THIS MODULE:

1. Absolute value – the exact value of a number regardless of sign. It is the distance of a point from zero or point of origin.

2. Algebraic expression – is a combination of numbers, variables and operation signs.

3. Linear equation – a mathematical statement which shows equality between number and algebraic expression of the first degree.

4. Replacement set – a set whose elements are values that can be substituted for the variable

5. Solution – is the value of the variable which will make the equation true.

6. Solution set – is a collection of all solutions.

7. Zero pairs – are numbers or expressions which when added gives a sum zero.

WEBSITE RESOURCES AND LINKS IN THIS MODULE:

http://www.typhoon2000.ph/stormarchives/2001/images_full/feria_large.jpg
This website contains the records of all typhoons which visited the Philippines since 2000.
http://images.search.yahoo.com/search/images;_ylt=A2KJkK1u.95O5yEAREiLuLkF?p=playing+with+dice&ei=utf-8&iscqry=&fr=sfp
This site contains different images of playing with dice.

http://www.wtamu.edu/academic/anns/mps/math/mathlab/video.htm?video=algebra/college/14/01
This website contains a video on the explanation of solving linear equations in one variable.

http://www.studygs.net/equations.htm
This page contains explanations and examples of solving linear equations.

http://www.drdelmath.com/intermediate_algebra/chapter_summary/intermediate_algebra_chapter2_summary.htm
This contains important information about linear equations, properties of equalities and problem solving tips. This also provides links to other web pages for supplemental exercises.

http://www.freeed.net/sweethaven/Math/algebra/linearEq/LinEqOne01_EE.asp?iNum=3
This website contains interactive activities on solving linear equations in one variable which can be solved by applying addition/subtraction property of equality.

http://www.freeed.net/sweethaven/Math/algebra/linearEq/LinEqOne01_EE.asp?iNum=6
This contains interactive activities on solving linear equations in one variable which can be solved by applying the multiplication/division property of equality.

http://www.freeed.net/sweethaven/Math/algebra/linearEq/LinEqOne01_EE.asp?iNum=9
This contains interactive activities on solving linear equations in one variable which can be solved by applying a combination of at least 2 different properties of equalities.

Printed Resources:

Gerver, Robert, et al. Algebra 1 An Integrated Approach. Lincolnwood, Illinois, USA: National Textbook Company/Contemporary Publishing Group, Inc., 1998.

Leschensky, William, et al. Pre-Algebra. Columbus, Ohio: Glencoe/McGraw Hill, 2002.

Stewart, James, et al. Algebra and Trigonometry 2nd edition. Singapore: Brooks/Cole a division of Thomson Learning Asia, 2007.

LESSON 4: LINEAR INEQUALITIES

INTRODUCTION AND FOCUS QUESTION(S):

In this lesson, you will pay attention to linear inequalities where the knowledge and skills learned in algebraic expression can be applied. What makes linear inequality different from linear equations? ***How can linear inequality help us solve real life problems?***

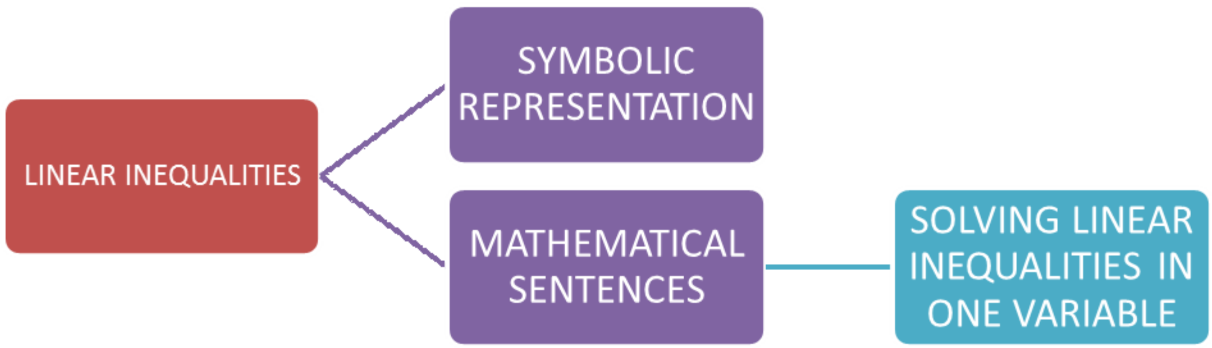
☑ LESSON COVERAGE

This lesson has the following topics:

Topic No.	Title	You'll learn to…	Estimated Time
Topic 1	Linear Inequalities	a. Find the solution of an inequality involving one variable (i) from a given replacement; (ii) intuitively by guess and check; (iii) by algebraic procedures (applying the properties of inequalities; (iv) graphing. b. Solves problems that use inequalities.	5 hours

☑ Concept Map of the Lesson

Here is a simple illustration of the topics you will cover in this lesson



☑ Expected Skills

To do well in this lesson, you need to remember and do the following:

13. Making outlines, concept maps and notes will be very helpful in understanding the module.
14. Follow the instructions provided for each activity.
15. Review and evaluate your work using the rubric provided before submission.
16. Complete all exercises and activities.
17. Be mindful of the meaning of unfamiliar words you encounter in this module. A glossary of terms is provided in the last part of this module.
18. Maximize the use of online resources in each lesson. Online resources can be accessed multiple times. The summary of online resources is provided in the end of the module.

EXPLORE

Let's begin by finding out the meaning of Linear Inequality.

ACTIVITY 1. READ AND THINK ALOUD

1. Read the problem and answer the following questions below.

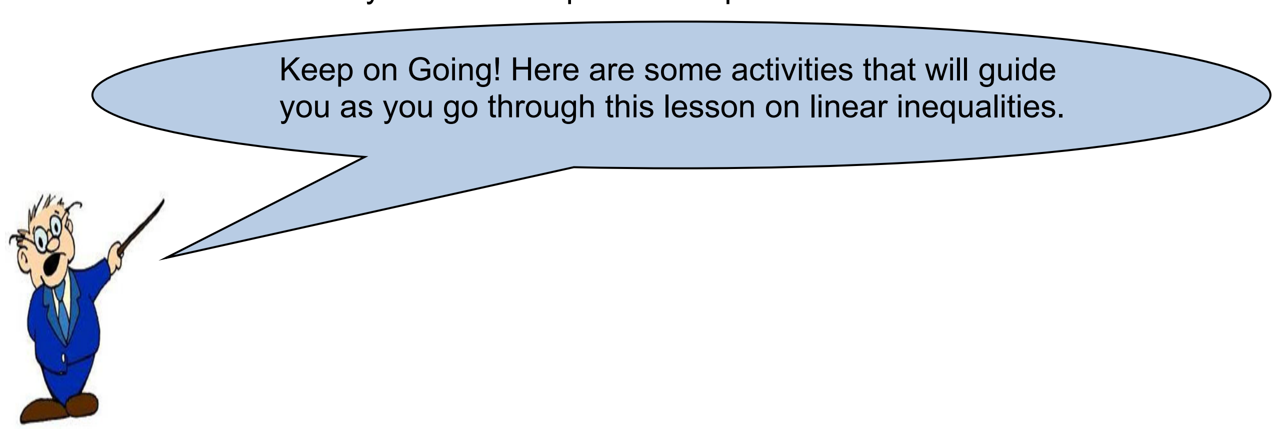
Suppose you were given a gift certificate by your friend worth P1,000.00 for CD-R Queens and want to spend on DVDs or LDs. You want to buy a special edition of Laser Disk that costs P350.00 and the DVDs you want to buy are on promo for P10.50 each. Assuming that you will use only the gift certificate to purchase the LD, what is the maximum DVDs you can buy?

You know you can buy at least one DVD. What about two DVDs? It can be time consuming to keep checking. To solve this problem, you can express it as an inequality. Let D represent the number of DVDs you can buy.

Cost of LD	Cost of each DVD	Number of DVDs	≤	Amount of Gift Certificate
350.00	10.50	D	≤	1000.00

Questions:

5. If you were the owner of the gift certificate, how many DVDs are you going to purchase?
6. How will you determine the total numbers of DVDs are you going to buy?
7. What question can you formulate to help you find the answer?
8. How can the study of linear inequalities help us?



ACTIVITY 2. KWL SHEET

1. In activity 2, a problem is given on the number of DVD that can be bought given a gift certificate. Based on activity 2, accomplish this chart and use it to record what you already know and what you would like to learn about linear inequalities. Leave the "Learned" section blank to fill in after you have finished this module.

Topic: Linear Inequalities		
What do you already know (or think you know) about this topic	**What do you wonder about this topic**	**What new things have you learned about this topic**

End of Explore
You gave your initial ideas on Algebraic Expressions. Let's now find out what the answer is by doing the next part.

FIRM-UP

Your goal in this section is to learn and understand key concepts about Linear Inequalities and its properties. You will look at a number of examples and illustrations about Linear Inequalities.

ACTIVITY 3. Buzz Session

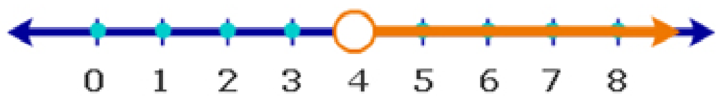
First, read the text and discuss it with the group. After the discussion, you may answer the practice exercise and then may proceed to the next activity.

In the first activity, you were asked to determine the maximum number of DVDs that can be bought. Before we solve this inequality for D, let's try to review what you might know about inequalities.

An inequality is really just any mathematical statement relating two quantities using an inequality symbol such as <, >, ≤, or ≥. For inequalities that contain variable expressions, you may be asked to solve the inequality for that variable.
This just means that you need to find the values of the variable that make the inequality true. Remember that when you solve a linear equation there is usually one value that makes the equation true. But when you solve an inequality, there can be many values that make the statement true!

Look at this inequality: $x > 4$.

The solution to this inequality includes every number that is greater than 4. What numbers are greater than 4? Well, 5 is greater than 4. And so are 6, 7, 8, 9, and so on. What about 4.5? What about 4.99? And 4.000001? All of these numbers are greater than 4! We couldn't possibly list all of them—there are infinitely many solutions! Instead, we can use a graph to show all the solutions. To graph the solution, place an open circle on the number 4. An open circle indicates that the number is not apart of the solution set. Then, to show all numbers to the right of 4 are included in the solution, draw a ray to the right of 4.



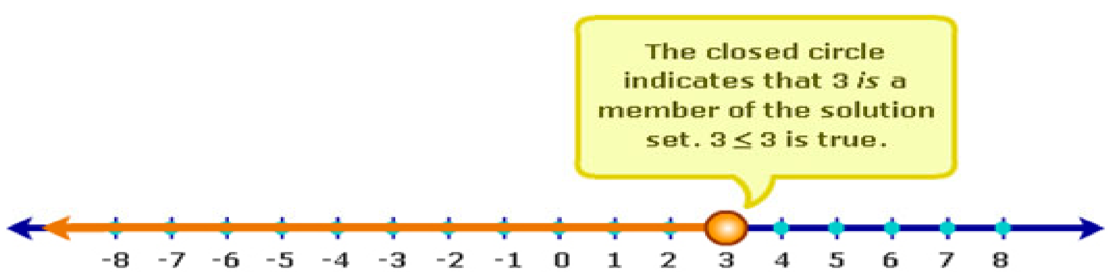
Example:

Jessan was asked to find the values of x that will make this inequality true: $3x - 2 \leq 9$. How could you solve this inequality? You can use what you know about solving equations to help you. You can use those skills to help you solve an inequality and graph the solution set.

SOLUTION STEPS:

$3x - 9 \leq 9$	Original Inequality
$3x - 2 + 2 \leq 7 + 2$	Add two from each side to undo the subtraction
$3x \leq 9$	Simplify
$\frac{3x}{3} \leq \frac{9}{3}$	Divide 3 from each side to undo the multiplication
$x \leq 3$	Answer

To graph the solution $x \leq 3$ on a number line, put a closed circle on 3, and draw a ray to the left.



Exercise 1: Solve each inequality. Sketch a graph of the solution.

1. x – 6 ≤ 15
2. –2x > 8
3. –8x – 4 ≤ 4
4. 3x – 2 > 4
5. –10x + 5 ≥ 25

Question for Reflection: As you solved the given inequalities, did you stick to one given strategy or did try to explore other possible ways of solving the inequalities? Is there an advantage of having multiple strategies? Justify your answer.

ACTIVITY 4. AMAZING GENIE!!

Play the amazing genie by clicking on the hyperlink below. Answer the questions using your knowledge about linear inequalities.
http://www.math-play.com/Inequality-Game.html

Previously, you just saw the process in determining the solution set of an inequality 3x - 2 ≤ 9, where the given solution the application of the addition and multiplication property of inequality.

Given another example solve the inequality.
Example 1:
SOLUTION STEPS:

$5x - 15 > 8x + 6$	Original inequality
$5x - 15 + 15 > 8x + 6 + 15$	Add 5 to each side to undo subtraction
$5x > 8x + 21$	simplify
$5x - 8x > 21$	Subtract -8x to each side to undo addition
$-3x > 21$	simplify
$\frac{-3x}{-3} > \frac{21}{-3}$	Divide -3 to each side to undo multiplication
$x < -7$	Answer

What do you notice in the solution?

Now that you know how to solve and graph Linear Inequalities, use the compare and contrast chart to differentiate Linear Inequality and Linear Equality

ACTIVITY 5. COMPARE AND CONTRAST

To see how well you understand the process of determining the solution set of an inequality, accomplish Compare and Contrast Chart based on what you saw in the solution.

Compare and Contrast Chart Graphic Organizer

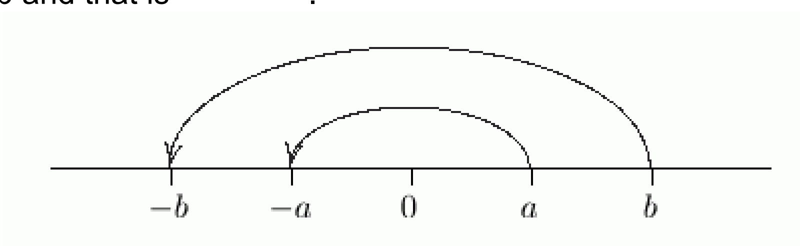
How are they alike?

How are they different?

Based on the comparison and contrast activity, what do you think are the rules to follow in terms of inequality? One of those rules which can be noticed in the given examples is the division property of inequality. This property basically says that you can divide both sides of the inequality by the same number. However, you should be careful in terms of the direction of the inequality. You should find that the direction of the inequality symbol is reversed only when you multiply or divide by a negative number.

This property also says that for all real numbers a, b and c, where c is less than zero, $a < b$ which is also equivalent to $ac > bc$. Have you noticed the symbol ' < 'for $a < b$ and ' > ' for $ac > bc$? This shows that when you divide each side of an inequality by the same negative number, the direction of the symbol of the inequality must be changed to preserve the truthfulness of the inequality.

Take a look at the diagram to better understand the situation. The diagram shows the truthfulness in $a < b$ because a lies to the left of b. Multiplying each side by -1 sends a and b to its opposite. Now the opposite of a is to the right of opposite of b and that is $-a > -b$.

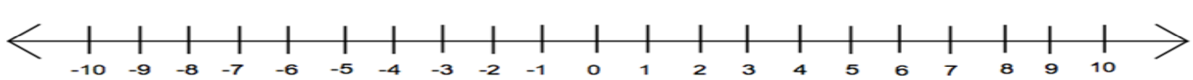


ACTIVITY 6. DO IT YOURSELF

1. Solve the given inequalities then represent each solution set on the number line.

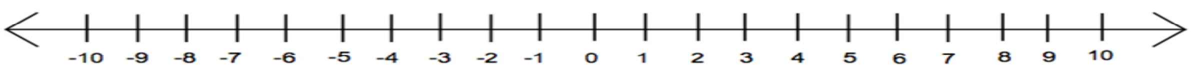
a. $3x + 2(x + 4) \geq 8x + 29$

Solution:



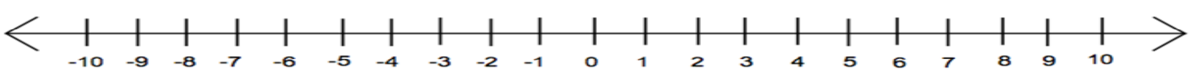
b. $7x + 20 < 19x + 8$

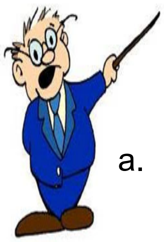
Solution:



c. $3(2x + 4) + x > 6(3x - 5) - 11$

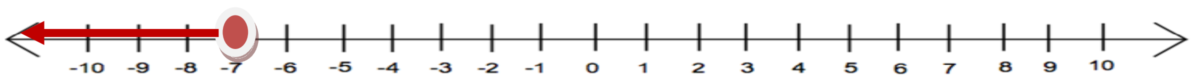
Solution:



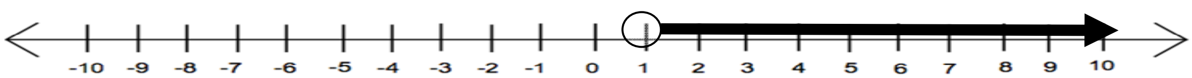


Answer Key:

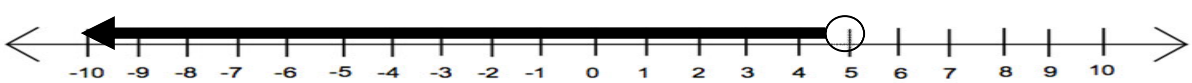
a. $3x + 2(x + 4) \geq 8x + 29$
$3x + 2x + 8 \geq 8x + 29$
$5x + 8 \geq 8x + 29$
$5x - 8x \geq 29 - 8$
$-3x \geq 21$
$\frac{-3x}{-3} \geq \frac{21}{-3}$
$x \leq -7$



b. $\mathbf{7x + 20 < 19x + 8}$
$7x - 19x < 8 - 20$
$-12x < -12$
$\frac{-12x}{-12} < \frac{-12}{-12}$
$x > 1$



c. $\mathbf{3(2x + 4) + x > 6(3x - 5) - 13}$
$6x + 12 + x > 18x - 30 - 13$
$\mathbf{7x - 12 > 18x - 43}$
$\mathbf{7x - 18x > -43 - 12}$
$\mathbf{-11x > -55}$
$\mathbf{\frac{-11x}{-11} > \frac{-55}{-11}}$
$\mathbf{x < 5}$



Let us now summarize the process of involved in solving linear inequalities.

a. *Remove the grouping symbols using distributive property.*
b. *Combine like terms.*
c. *Use the addition property in collecting all the variable terms opposite to the constant terms.*
d. *Reverse the sense of inequality, if there is a negative coefficient.*
e. *Graph the solution set in the number line.*

Previously, you just have learned the processes in determining the solution set and graphing of a linear inequality. Now, given the following inequality symbols and their meaning, try to work on the next exercise.

Inequality Symbols	
Symbols	Meaning
>	" is greater than"
<	"is less than"
≥	" is greater than or equal to" "is at least" "is no less than"
≤	"is less than or equal to" "is at most" "do not exceed"

EXERCISE 1 Translate the following mathematical statements into symbolic representations.

a. Your age 5 years ago	
b. The temperature outside is less than 20ºF	
c. The cost of the resort's entrance is less than P350	
d. The number of recitation days for December is at most 19	
e. The number of participants of the last convention is no less than 500	
f. The number of passers of the previous exam does not exceed 80.	
g. Your allowance everyday is at most P75	
h. My gifts last Christmas was not less than 17	

As you read and translate each statement, do you think there should be a standard way of translating them into symbolic representations? Explain
In your daily life were you able to translate situations mathematically? Do you use different strategy or only one step by step rule? What is the advantage of having multiple strategies? In what ways do you think algebra helps us? Explain

Online Activity: Solving Linear Activities
Access the website below for more examples of solving linear activities. Next, do the activities provided in the website.
http://www.intmath.com/inequalities/2-solving-linear-inequalities.php

End of FIRM UP:

In this section, the discussion was about solving and graphing linear Inequalities, and translating mathematical sentences into its symbolic representations.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.

DEEPEN

Your goal in this section is to take a closer look on Linear Inequalities.

Previously you have translated different statements into its equivalent symbolic representations using inequality translation guide. Let us now apply the previous exercise in solving different worded problems.

EXAMPLE 1: Jessan went to Enchanted Land with a pocket money of P700. He decided to use the money to ride the Extreme Rides that costs P350 and play arcade games at the Arcade Centrum. What amount is the most he can spend on for arcade games?

Solution:

We need to determine the maximum amount of money that Jessan can spend on arcade games.

Let ‘ a ’ be the amount of money that Jessan can spend on games.

We need to write first an inequality that will represent Jessan’s problem. Take note of “most that he can spend” that means less than or equal to.

Cost of riding Extreme	Plus	Cost of arcade games	must be less than or Equal to	Total Amount
P350	**+**	**x**	**≤**	**P700**

After translating to its symbolic representation, we may now start solving for the solution set.

SOLUTION STEPS:

$350 + x \leq 700$	Write the inequality
$350 + x - 350 \leq 700 - 350$	Subtract 350 on both sides
$x \leq 350$	Simplify

EXAMPLE 2: Karen has P4,650.00 in a savings bank in the beginning of the school year. She wants to have at least P2,250.00 in the account by the end of the year. She withdraws P600.00 per week for food, clothes, and coffee breaks. Write an inequality that will represent Karen's situation. How many weeks can Karen withdraw money from her account? Justify your answer.

Solution:

Started with P4,650 at the beginning of the school year	withdraws	P600 per week	must be less than o Equal to	End of the school year amount
P4650	**-**	**P600x**	**≥**	**P2250**

After translating to its symbolic representation, we may now start solving for the solution set.

SOLUTION STEPS:

$4650 - 600w \geq 2250$	Write the inequality
$4650 - 600w - 2250 \geq 2250 - 2250$	Subtract 2250 on both sides
$2400 - 600w \geq 0$	
$2400 - 600w + 600w \geq 0$ +600w	Add 600w on both sides
$2400 \geq 600w$	
$\frac{2400}{600} \geq \frac{600w}{600}$	Divide each side by 600
$4 \geq w$	Answer

For Karen to have P2, 250 before the end of the school year, she should have at least 4 withdrawals.

For checking, if Karen will have 4 withdrawals by the end of the school year she will have on her account P2,250. However, if Karen will have 5 withdrawals or more she will get less than P2,250.

ACTIVITY 7. PROBLEM SOLVING

Previously, you have seen different examples on how to solve linear inequality. Study the given situations below and do the tasks or answer the questions that follow.

1. Sun Smart Company offers a basic monthly plan that charges P300 for the first 60 minutes of calls plus P4.50 on the succeeding minutes. If you have spent P1,050 to spend on your monthly bill, how many minutes can you see? Let m represent the number of additional minutes.

Solution:

2. Jessan was just given a new job in shoe selling. He was given two salary options. He can receive a salary of P2,300 per week with no commission or you can receive a salary of P1450 per week plus 8% of your monthly sales. What amount of product must you see each week in order for the commission option to be the better deal?

Solution:

3. To get a passing grade in your course, you must have an average of at least 86% which is equivalent to B on 7 examinations. You have taken the first 6 examinations and got the scores of 84%, 77%, 78%, 84%, 88% and 75%. What score would you need on the last score to get a grade of B or better?

Solution:

End of Deepen
In this section, the discussion was about solving worded problems which are represented by linear inequalities in one variable.

What new realizations do you have about the topic? What new connections have you made for yourself? What is the important of using different strategies in solving a given problem? In what ways do you think algebra helps us?

ACTIVITY 8. Gimme More!!

Here are some online videos about linear inequalities that will help you all throughout this lesson. Just click the hyperlink below.

http://www.youtube.com/watch?v=MJ4dCBmYwvU
This video contains explanation of solving and graphing linear inequalities in one variable.

http://www.youtube.com/watch?v=mCdqgwWuNBk
This video contains a demonstration on how to set up and solve a word problem that involves the use of linear inequalities.

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.

TRANSFER

Your goal in this section is apply your learning to real life situations. You will be given a practical task which will demonstrate your understanding.

You are a marketing associate of a certain network company, GLOBAL SMART making sales call to a group of executives who will be subscribing a regular cell phone plan. You need to convince these executives that GLOBAL SMART offers the best deals to meet their communication needs which are easy on their budget, so that you can also meet your quota. You have to come up with an oral presentation and a brochure containing a comparison of the different plans and further proposal for a large group plan. The presentation must be clear and concise and the brochure must be catchy and have an accurate and satisfactory explanation of the solution.

PERFORMANCE TASK RUBRIC

CRITERIA	Expert 4	Proficient 3	Developing 2	Beginning 1
CLARITY	Very clear and convincing explanation all throughout the entire presentation.	Clear oral presentation.	Important ideas are not clear.	Ambiguous and unsure oral presentation.
CONCISENESS	The needed data are complete, comprehensive, authentic and relevant. Explanations in the brochure are thoroughly done and insightful.	The needed data are complete, comprehensive and relevant. Explanations in the brochure include the necessary information and direct to the point.	The needed data are incomplete and not relevant. Explanations in the brochure lack some pertinent information and include the unnecessary.	The needed data are incomplete and not relevant. Explanations in the brochure are difficult to understand.
APPEAL	The design and layout of the brochure are attractive and readily capture the reader's attention.	The design and layout of the brochure are attractive.	The design and layout of the brochure are presentable.	The design and layout of the brochure are not presentable.
Mathematical concepts	Demonstrates thorough understanding of the concepts and properties of linear equations and inequalities.	Demonstrates understanding of the concepts and properties of linear equations and inequalities.	Demonstrates partial understanding of the concepts and properties of linear equations and inequalities.	Demonstrates a lack of understanding of the concepts and properties of linear equations and inequalities.
ACCURACY	The solutions are logical and the computations are accurate and precise. Explanations are provided for each solution.	The solutions are orderly and the computations are correct.	The solutions are incomplete and some computations are incorrect.	The solutions are illogical and computations are inaccurate.

In this section, your task was to come up with a brochure containing a comparison of the different cell phone plans.

How did you find the performance task? How did the task help you see the real world use of the topic?

You have completed this lesson. Before you go to the next lesson, you have to answer the following post-assessment.

Topic: Linear Inequalities		
What do you already know (or think you know) about this topic	**What do you wonder about this topic**	**What new things have you learned about this topic**

End of Transfer

How did you find the performance task? How did the task help you see the real world use of the topic? What is the importance of using different strategies in solving a given problem? In what ways do you think algebra helps us?

Now, you are ready to take the post- assessment test.

POST-ASSESSMENT:
It's now time to evaluate your learning. Click on the letter of the answer that you think best answers the question. Your score will only appear after you answer all items. If you do well, you may move on to the next module. If your score is not at the expected level, you have to go back and take the module again.

GLOSSARY OF TERMS USED IN THIS MODULE:

Algebraic Expression - is an expression that contains one or more numbers, one or more variables, and one or more arithmetic operations.

Constant - is a number on its own

Distributive Property - states that the product of a number and a sum is equal to the sum of the individual products of the addends and the number.
That is, a(b + c) = ab + ac.

Linear Inequality - is a mathematical sentence that uses symbols such as <, ≤, >, or ≥ to compare two quantities.

Module 3: Algebra > Lesson 4: Linear Inequalities

WEBSITE RESOURCES AND LINKS IN THIS MODULE:

http://www.math-play.com/Inequality-Game.html
The website is about the game Amazing Genie.

http://www.intmath.com/inequalities/2-solving-linear-inequalities.php
This website is about solving linear inequalities.

www.purplemath.com/modules/ineqlin.htm
This website is about linear inequalities, this site demonstrates how to solve **linear inequalities** step-by-step; shows four different solution formats.