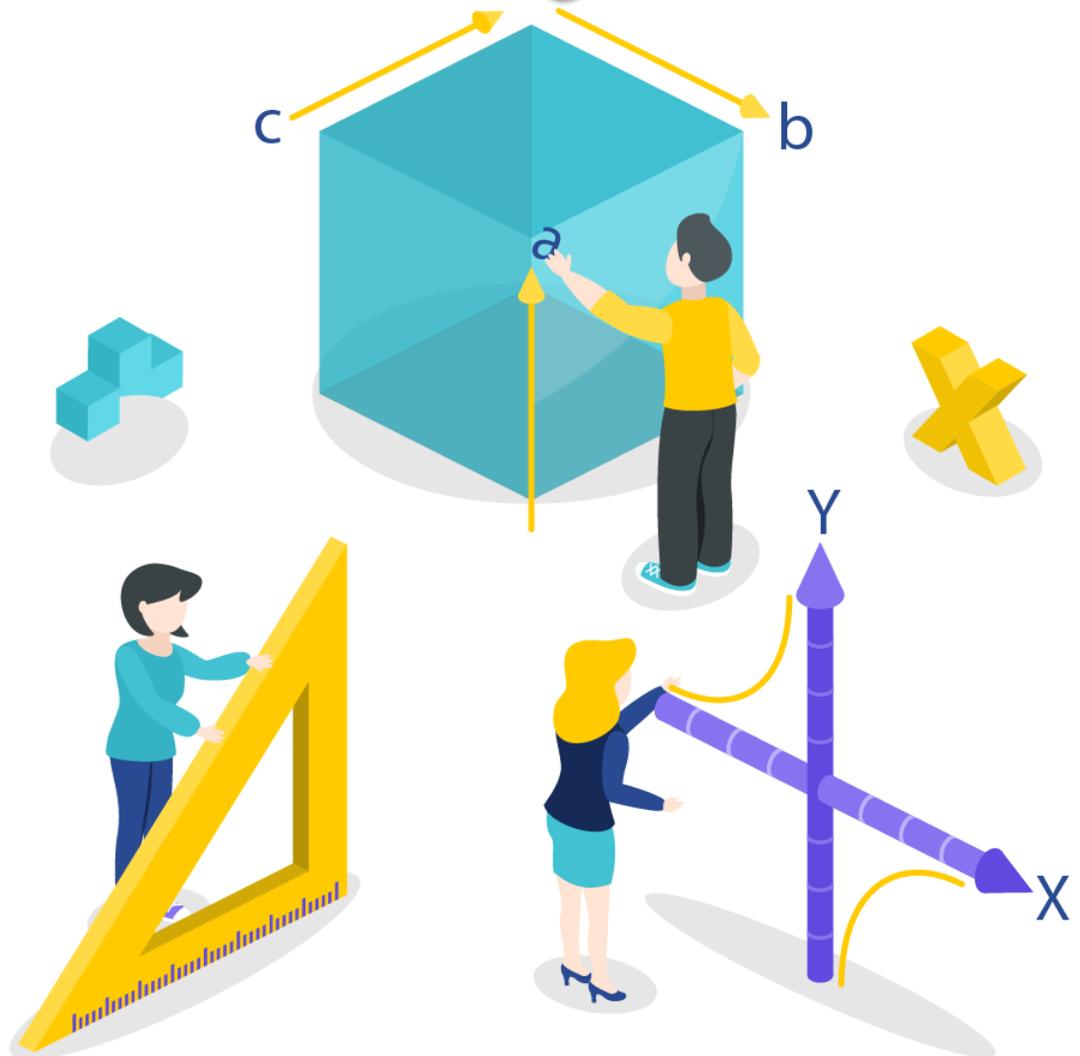# LEARNING MODULE

Mathematics G10 | Q2

Geometry

NOTICE TO THE SCHOOLS

This learning module (LM) was developed by the Private Education Assistance Committee under the GASTPE Program of the Department of Education. The learning modules were written by the PEAC Junior High School (JHS) Trainers and were used as exemplars either as a sample for presentation or for workshop purposes in the JHS In-Service Training (INSET) program for teachers in private schools.

The LM is designed for online learning and can also be used for blended learning and remote learning modalities. The year indicated on the cover of this LM refers to the year when the LM was used as an exemplar in the JHS INSET and the year it was written or revised. For instance, 2017 means the LM was written in SY 2016-2017 and was used in the 2017 Summer JHS INSET. The quarter indicated on the cover refers to the quarter of the current curriculum guide at the time the LM was written. The most recently revised LMs were in 2018 and 2019.

The LM is also designed such that it encourages independent and self-regulated learning among the students and develops their 21st century skills. It is written in such a way that the teacher is communicating directly to the learner. Participants in the JHS INSET are trained how to unpack the standards and competencies from the K-12 curriculum guides to identify desired results and design standards-based assessment and instruction. Hence, the teachers are trained how to write their own standards-based learning plan.

The parts or stages of this LM include Explore, Firm Up, Deepen and Transfer. It is possible that some links or online resources in some parts of this LM may no longer be available, thus, teachers are urged to provide alternative learning resources or reading materials they deem fit for their students which are aligned with the standards and competencies. Teachers are encouraged to write their own standards-based learning plan or learning module with respect to attainment of their school's vision and mission.

The learning modules developed by PEAC are aligned with the K to 12 Basic Education Curriculum of the Department of Education. Public school teachers may also download and use the learning modules.

Schools, teachers and students may reproduce the LM so long as such reproduction is limited to (i) non-commercial, non-profit educational purposes; and to (ii) personal use or a limited audience under the doctrine of fair use (Section 185, IP Code). They may also share copies of the LM and customize the learning activities as they see fit so long as these are done for non-commercial, non-profit educational purposes and limited to personal use or to a limited audience and fall within the limits of fair use. This document is password-protected to prevent unauthorized processing such as copying and pasting.

MATHEMATICS 10

Module 2: Geometry

☑ **INTRODUCTION AND FOCUS QUESTION(S):**

Coordinate Geometry: Get to the point!
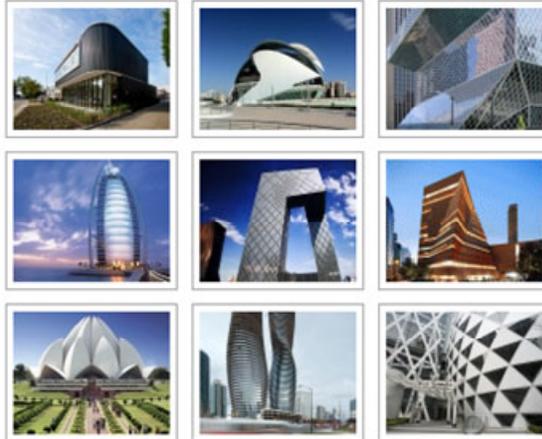
| FROM DREAM | TO REALITY |

Have you thought of why Geometry is difficult to understand and often perceived to have confusing set figures like circles, squares, rectangles and etc. Have you at a certain time asked yourself how can challenging problems involving geometric figures be analyzed and solved?

A very famous mathematician called Rene Descartes lay in bed one night. As he lay there, he looked up at the ceiling in his bedroom. He noticed a fly was asleep on the ceiling. Descartes, being a mathematician wondered if he could figure out a way of stating where exactly the fly was on the ceiling. Obviously it has to be a precise description he thought.

The discovery of Rene Descartes serves now as one of the foundations of geometry which was used as a tool for more discoveries which lead to modernization.

In this module, you will pay attention to circle and distance formula where the knowledge and skills learned in the previous lessons can be applied. You will find out the different concepts involved in distance formula. As you go over the exercises, you will be able to answer the questions: **How can problems where two quantities bounded by conditions are solved? How do related quantities affect each other? For example, how do you determine the strategic position of a camera/ video recorder in a specific location and finally you need to be able to answer the question,** how can challenging problems involving geometric figures be analyzed and solved?

☑ **LESSONS AND COVERAGE:**

In this module, you will examine this question when you take the following lessons:

Lesson 1 – Chords, Arcs, and Angles, Secants, Tangents, Segments and Sectors
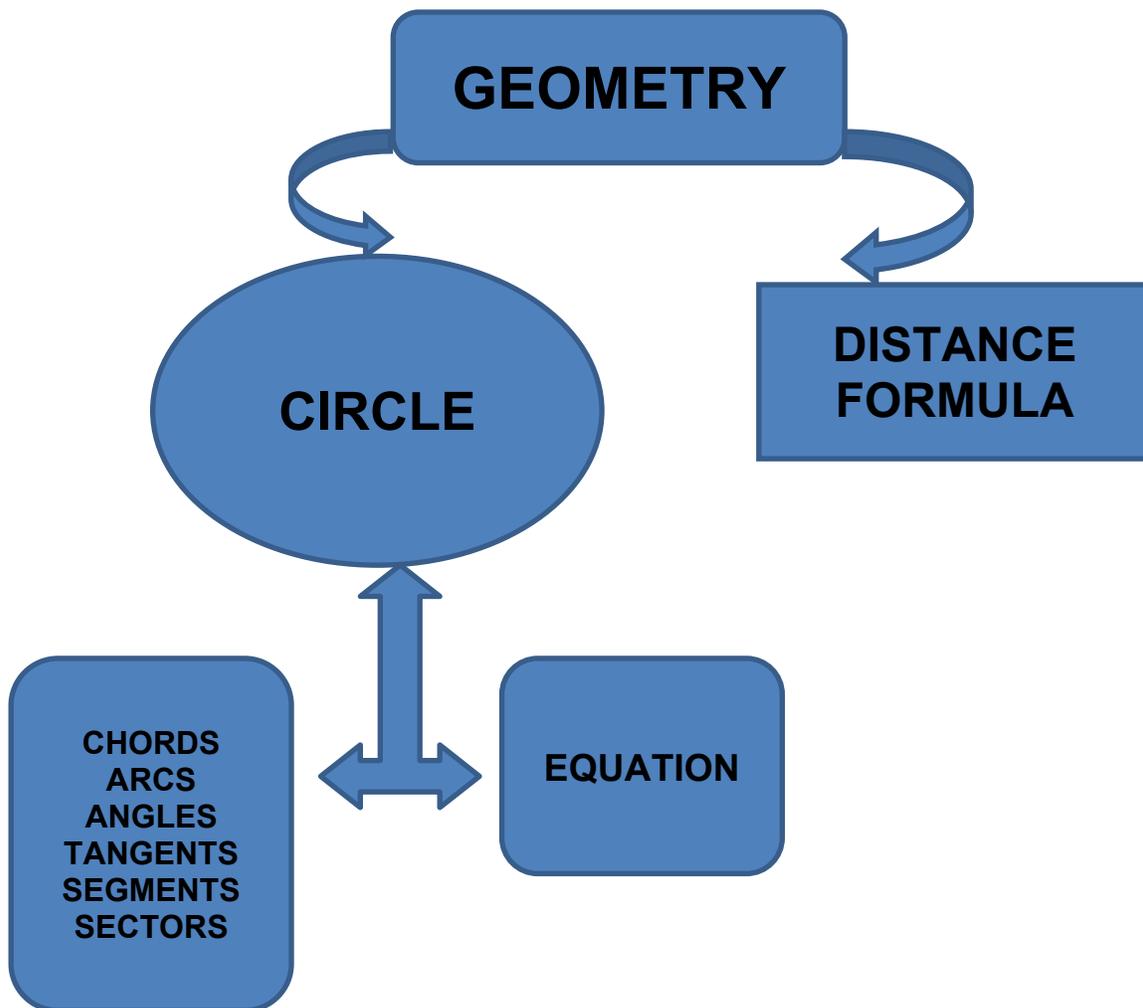*Lesson 2 – Distance Formula*
Lesson 3 – Equation of a Circle

In these lessons, you will learn the following:

Lesson 1	• Derives inductively the relations among chords, arcs, central angles, and inscribed angles. • Proves theorems related to chords, arcs, central angles, and inscribed angles. • Illustrates secants, tangents, segments, and sectors of a circle. • Proves theorems on secants, tangents, and segments. • Solves problems on circles.
Lesson 2	• Derives the distance formula • Applies the distance formula to prove some geometric properties • Graphs other geometric figures on the coordinate plane • Solves problems involving geometric figures on the coordinate plane
Lesson 3	• Illustrates the center-radius form of the equation of a circle. • Determines the center and radius of a circle given its equation and vice versa. • Graphs a circle and other geometric figures on the coordinate plane.

☑ **MODULE MAP:**

Here is a simple map of the above lessons you will cover:

```
                        GEOMETRY


         CIRCLE                        DISTANCE
                                       FORMULA



   CHORDS
   ARCS
   ANGLES            EQUATION
   TANGENTS
   SEGMENTS
   SECTORS
```

☑ **EXPECTED SKILLS:**

To do well in this module, you need to remember and do the following:
1. Look up the meaning of words you do not know.
2. Complete all activities and exercises.
3. Take note on the proper modeling of situations using oral, written, graphical and algebraic methods to solve problems.
4. Use the checklist and rubric provided to evaluate your work before submission.
5. Be mindful of the meaning of unfamiliar words you encounter in this module. A glossary of terms is provided in the last part of this module.
6. Maximize the use of online resources in each lesson. Online resources can be accessed multiple times. The summary of online resources is provided in the end of the module.
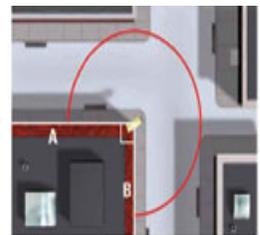
PRE-ASSESSMENT:

Let's find out how much you already know about this module. Click on the letter that you think best answers the question. Please answer all items. After taking this short test, you will see your score. Take note of the items that you were not able to correctly answer and look for the right answer as you go through this module.

1. Which of the following defines a chord?
   - A. A line that passes through the circle at exactly one point.
   - B. A line that passes through the circle at exactly two points.
   - C. A segment that passes through the circle at exactly one point.
   - D. A segment whose endpoints are on the circle.

2. If the measure of the central angle is 45^0, what is the measurement of the intercepted arc?
   - A. 22.5^0
   - B. 45^0
   - C. 90^0
   - D. 135^0

3. You are a professional photographer. Your camera has a 90 field of vision and you want to photograph the front of a statue. You move to a spot where the statue is the only thing captured in your picture, as shown. You want to change your position. Where else can you stand so that the statue is perfectly framed.

   - A. You may stand to any point on the semi- circle in front of the statue.
   - B. You may stand to any point equidistant from your distance to the statue.
   - C. You may move to the other side and stand to any point equidistant from your distance to the statue.
   - D. There is no other place where you can capture the whole picture of the statue but your original position.

4. A surveillance camera is mounted on a corner of a building. It rotates clockwise and counterclockwise continuously between Wall A and Wall B at a rate of 10^0 per minute as illustrated in the figure. How long does it take the camera to survey the entire area once? Justify your answer.
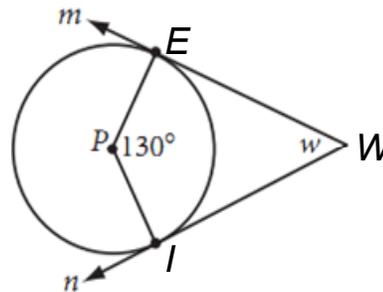

   - A. 4 minutes and 30 seconds, since the measure of the angle is half of the intercepted arc.
   - B. 9 minutes, since the angle and arc have equal measures.
   - C. 18 minutes, since the measure of the angle is twice the measure of the intercepted arc.

   D. 27 minutes, the camera needs to survey about 135^0 of the circle.
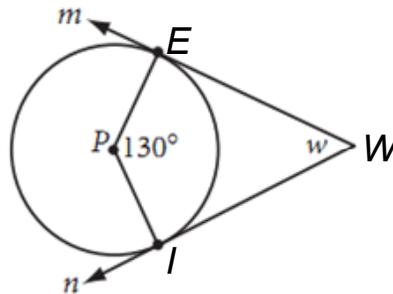
5. You are a professional photographer, a company requested you to submit a set of pictures of beautiful sceneries. Which among the following should you consider most to in selecting your pictures?
    A. Pictures are very clear but some important parts are not captured.
    B. All parts are captured but no highlighted subject.
    C. You have a focus subject and important parts are visible.
    D. All parts are captured but some parts are not that clear.

6. If rays WE and WI are both tangents to the circle P below, then which of the following is NOT necessarily true?

    A. EW = IW
    B. m∠PEW =m∠PIW
    C. ∠EWP ≅∠IWP
    D. ∠PEW and ∠EWI are supplementary.

7. If ray m and n are tangents to circle P below, then which of the following is the value of w?

    A. 30º   B. 40º   C. 50º   D. 60º

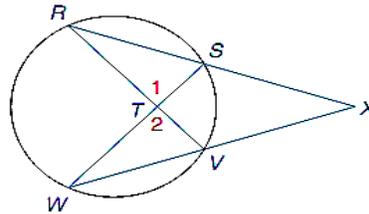8. In the figure below, if segment MT is tangent to the circle, segments NT and PT are secants, PS is a semicircle and the measure of minor arc MP is 140º, then what is the measure of ∠MTP?

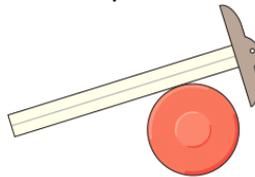   A. 50º   B. 60º   C. 90º   D. 100º

9. If RS = VW in the figure below, then which of the following will justify that △RVX is congruent to △WSX?
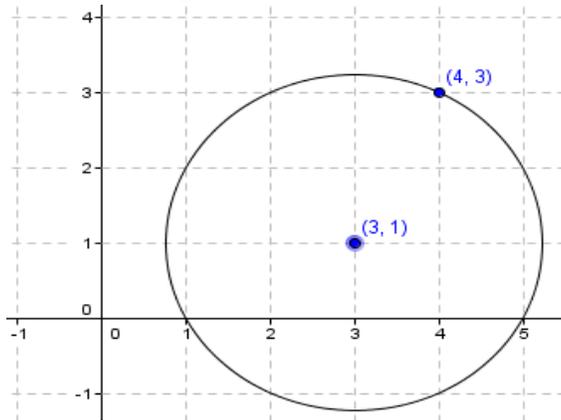

    A. SAA Congruence Theorem   B. SAS Congruence Postulate
    C. SSS Congruence Postulate   D. Hypotenuse-Leg Theorem

10. As an Archeologist, you volunteer to help a group of students to determine the center of a circular object that they have found. Which of the following will help you explain how the T-square will be an effective instrument to do so?

    A. Tangents are perpendicular to the radii at the points of tangency.
    B. The tangent segments intersecting at a point outside a circle are congruent.
    C. The measure of an angle formed by two secants intersecting outside a circle is half the measures of the difference of the intercepted arcs of the circle.
    D. The measure of an angle formed by a secant and a tangent to a circle, intersecting outside a circle is half the measures of the difference of the intercepted arcs of the circle.

11. What is the distance between the points (3,3) and (6,7)?
    A. 3
    B. 5
    C. 7
    D. 9

12. The point (5,4) lies on a circle. What is the length of the radius of this circle if the center is located at (3,2)?
    A. $\sqrt{8}$
    B. $\sqrt{3}$
    C. $\sqrt{18}$
    D. 8

13. If plotted in the coordinate plane, which of the following geometric figures best describes the coordinates A(-6,4), B(-6, -4), C(0,4) and D(0, -4)?
    A. Square
    B. Rectangle
    C. Rhombus
    D. Kite

14. On a map's coordinate grid, Brgy. Malaya is located at (2, 4) and Brgy. Pasong Tamo is located at (2, 2). Brgy. Katapan is the in the middle of the two barangay. What is the distance from Brgy. Malaya to Brgy. Katapatan?
    A. 2km
    B. 4km
    C. 6km
    D. 8 km

15. You are a city planner who is requested by the city mayor to provide a vicinity map to be used in the municipal strategic planning. Which of the following standards will be assessed your work?
    A. Accuracy, Clarity, application of the concept of distance formula
    B. Accuracy, Clarity, practicality
    C. Practicality, authenticity, use of grid lines
    D. Practicality, Accuracy, presentation

16. Low – Earth orbit satellites occupies a region of space from 180 – 2000 kilometers above Earth. If the satellite is moving in a circular path with Earth as the point of origin, which circular equation represents the lowest path of the satellite?
    A. $x^2 + y^2 = 180$
    B. $x^2 + y^2 = 2000$
    C. $x^2 + y^2 = 32,400$
    D. $x^2 + y^2 = 4,000,000$

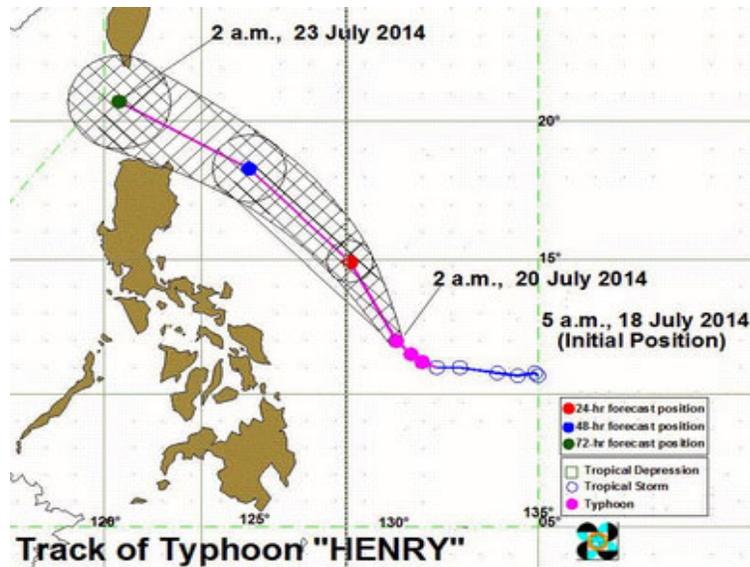17. Based on the figure below, what equation that represents the circle?

I. $(x - 3)^2 + (y - 1)^2 = 5$
II. $x^2 + y^2 - 6x - 2y + 5 = 0$
III. $x^2 + y^2 - 6x - 2y - 5 = 0$

A. I only
B. II only
C. I and II only
D. I, II and III

18. To determine the radius and coordinates of the center of the circle $x^2 + y^2 - 6x + 2y + 8 = 0$, Nestor and Andy came up with the following solutions:

Nestor's Solution	Andy's Solution
$x^2 + y^2 - 6x + 2y + 8 = 0$	$x^2 + y^2 - 6x + 2y + 8 = 0$
$x^2 - 6x + \_\_ + y^2 + 2y + \_\_ = -8$	$x^2 - 6x + \_\_ + y^2 + 2y + \_\_ = -8$
$x^2 - 6x + (-3)^2 + y^2 + 2y + (1)^2 = -8$	$x^2 - 6x + (-3)^2 + y^2 + 2y + (1)^2 = -8 + 9$
$(x - 3)^2 + (y + 1)^2 = -8$	$+ 1$
Radius = $\sqrt{-8}$; center: (3, - 1)	$(x - 3)^2 + (y + 1)^2 = 2$
	Radius = $\sqrt{2}$; center: (3, - 1)

Which statement best describes the solutions of Nestor and Andy?
   A. The solution of Nestor is correct. Andy committed an error in when he added 9 and 1 on the right side of the equation.
   B. The solution of Andy is correct. Nestor committed an error when he failed to add 9 and 1 on the right side of the equation.
   C. The solution of Andy and Nestor are both correct.
   D. The solution of Andy and Nestor are both incorrect.

19. When delivering weather forecast during typhoons, weather forecasters uses diagrams and charts to illustrate the movement of the typhoon. The circles

indicate the affected area of the typhoon at a particular time. If you are the forecaster, which statement best describes typhoon Henry based on the illustration below?

Track of Typhoon "HENRY"

    A. Typhoon Henry grew stronger from July 20 – 23, 2014.
    B. Typhoon Henry move faster from July 20 – 23, 2014.
    C. Typhoon Henry's coverage increased from July 20 – 23, 2014.
    D. Typhoon Henry's coverage did not change from July 20 – 23, 2014.

20. The barangay officials recently reported alarming news that the number of crimes in your community is drastically increasing. The barangay council decided to put up a CCTV camera in the area with the most number of crime incidents. You are a marketing officer of a CCTV company that will participate in the bidding to provide the barangay with the CCTV unit. You need to identify the best location of the CCTV unit in the area that is identified in your barangay. You will present your recommendation and justification to the barangay officials. Which are the appropriate standards that your work should be evaluated?
    A. organization, neatness, practical recommendations, application of the concepts of the properties of circles
    B. neatness, presentation, practical recommendations, application of the concepts of the properties of circles
    C. creativity, accurate computation, practical recommendations, application of the concepts of the properties of circles
    D. authentic data, accurate computation, practical recommendations, application of the concepts of the properties of circles

Lesson 1. Chords, Arcs, Angles, Tangents and Secants

In this lesson you will learn the following:
1. Describes a chord, arc and angle.
2. Illustrates chords, arcs, and angles.
3. Describes the relationship between the central angle and the intercepted arc.
4. Describes the relationship between the inscribed angle and the intercepted arc.
5. Derives inductively the relations among chords, arcs, central angles, and inscribed angles.
6. Proves theorems related to chords, arcs, central angles, and inscribed angles.
7. Solves problems on circles.
8. Proves theorems on secants, tangents, and segments.
9. Solves problems on circles.

EXPLORE

You learned from the previous level some geometric figures involving circles. You also encountered some problems and terms about circle which are important in learning Geometry.

In this lesson, you will learn the concepts of chords, arcs, and angles which are used in many situations. You will also learn different relationships and theorems related to these lessons which are useful in solving real world problems. You will also gather ideas to answer the question **"How can challenging problems involving geometric figures be analyzed and solved?"** These concepts will also help you visualize situations and create solutions to the problems that you encounter. Answers to the question above will also help you do your performance task.

In this section you need to analyze pictures by answering different questions for you to discover important concepts. You will also do self-monitoring activity as you fill up the map of conceptual change.

Let's us start the lesson by analyzing the pictures and answering the questions that follow.

A. Circle

How can challenging problems involving geometric figures be analyzed and solved?"

Let's answer these questions by doing the activities below.

ACTIVITY 1.   Picture Analysis

(Eliciting of prior Knowledge, Motivation, Hook)
Observe the pictures below and answer the questions.

http://www.city101.net/
Featured are some of our installation of vandal-proof and weather resistant cameras. These domes, bullets, and PTZs are recommended for building entrances, parking lots, and streets to monitor activities around your property.

1. What geometric concepts can you associate with the pictures?

2. How are these concepts used in different situations?

3. Can you determine any purpose why these geometric concepts are present in the pictures? Please specify.

4. Can you cite any problem which can be answered through these geometric concepts? Describe at least one.

5. How can challenging problems involving geometric figures be analyzed and solved?

Now, try to write your initial answer about the essential question presented for you to monitor your understanding.

ACTIVITY 2.   CONCEPTUAL UNDERSTANDING CHECK

In the table below, write your answers on the initial part for the question how can challenging problems involving geometric figures be analyzed and solved?

INITIAL ANSWER
REVISED ANSWER
FINAL ANSWER

End of EXPLORE:

You just have tried to find out how mathematics can help you determine how can challenging problems involving geometric figures be analyzed and solved. Let us now strengthen that insight by doing the succeeding activities. What you will be learning in this section will help you perform well in your final performance task to formulate and solve challenging problems related to angles and distances by making recommendation and justification.

Now move to the next activity to learn the knowledge and skills you need to be a good problem solver and respond to different situations accurately.

FIRM-UP

Your goal in this section is to learn and understand key concepts of arcs, chords, angles, secant and tangent which are important in solving problems involving circles. In this section there are activities which will help you discover and understand the different theorems and terms which are useful tools in solving real life problem related to arcs, chords and angles of a circle.

To help us appreciate the important of the lessons, take a look at the situation in the next activity.

ACTIVITY 3.   GARDENING

In the previous activity, we looked at different pictures where the lessons can be used. Let us see what concepts can be used to answer the situation below.

You are a landscape designer. Three bushes are arranged in a garden as shown. Where should you place a sprinkler so that it is the same distance from each bush?

Process Questions:
1. How did you answer the problem above?
2. What concepts did you use to answer the problem?
3. What might happen if you can't be able to respond to the given situation?
4. How can challenging problems involving geometric figures be analyzed and solved?

Write your answers here

ACTIVITY 4.   Let's do an experiment

In the previous activity you are task to solve a real life problem concerning circle. Now, let's see if you can be able to develop the concepts that you learned by doing the next activity. For the online students you may accomplish the activity on your own or click the interactive website below before you answer the questions but for the non-online students you may do it with your classmates.

http://www.mathwarehouse.com/geometry/circle/inscribed-angle.php
This is an interactive website showing the relationships between the central angles and the intercepted arcs.

Directions: Follow the step by step procedure and then answer the questions.
Materials: compass, protractor, ruler, scissor

1. Draw a circle.
2. Divide the circle into equal parts, minimum of six and maximum of twelve.
3. Cut a part of the circle.
4. Measure the angle of that part together with the arc.
5. What part of the angle is cut from the whole?

6. What part of the arc is cut from the total circumference?

7. What relationship did you notice between the central angle and its intercepted arc?

In the previous activity you learned the relationships between the central angles and chords. Now, you will improve your knowledge in this lesson by doing the next activity.

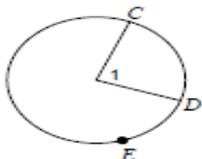**ACTIVITY 5.   Let's Consult the Expert**

Directions: Click the video below which explains further the concepts of central angle and the intercepted arc. After watching the video do the exercises below.
https://www.youtube.com/watch?v=xI2BZ5R_PrM
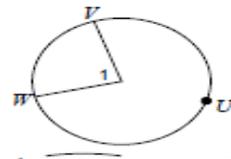This site contains video explaining the relationships between central angle and its intercepted arc.

Directions: Name the arc made by the given angle. Write your answers on the table provided below.
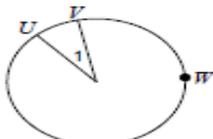
1) $\angle 1$

A) $\overarc{CED}$ B) $\overarc{CD}$

2) $\angle 1$

A) $\overarc{WUV}$ B) $\overarc{WV}$

3) $\angle 1$

A) $\overarc{UWV}$ B) $\overarc{UV}$

4) $\angle 1$

A) $\overarc{SUT}$ B) $\overarc{ST}$

5) $\angle 1$

A) $\overarc{JI}$ B) $\overarc{JHI}$

6) $\angle 1$

A) $\overarc{GH}$ B) $\overarc{GIH}$

7) $\angle 1$

A) $\overarc{DE}$ B) $\overarc{DFE}$

8) $\angle 1$
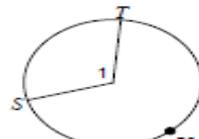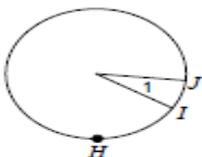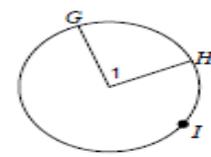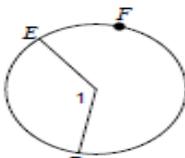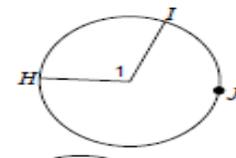
A) $\overarc{HJI}$ B) $\overarc{HI}$

1.		5.	
2.		6.	
3.		7.	
4.		8.	

Questions

1. How did you answer the questions above?

2. What relationship did you notice about the central angle and the intercepted arc?

3. Why is it important for you understand that relationship?

Now that you have enough exercises about the concepts of central angle and intercepted arc, you are ready for a short quiz.

ACTIVITY 6.   Let's fill this up (QUIZ)

Directions: Given the value of the central angle and the intercepted arc, find the value of x which will make each statement correct.

	Central Angle (degrees)	Intercepted Arc(degrees)	Value of (x)
1.	60	x + 20	
2.	2x + 20	100	
3.	3x + 20	10 – 2x	

| 4.	4x - 30	2x + 30	
5.	100 – 2x	3x – 30	

After learning the concepts of central angle and intercepted arc you are now ready to learn other concepts. To start your journey, take a look at the situation below and answer the questions.

ACTIVITY 7.   PHOTOGRAPH

In the previous section, we encountered problems involving the central angle and the intercepted arc. Let us take a look at the situation below to learn other concepts about circle.

You are a professional photographer. Your camera has a 90 field of vision and you want to photograph the front of a statue. You move to a spot where the statue is the only thing captured in your picture, as shown. You want to change your position. Where else can you stand so that the statue is perfectly framed**?**
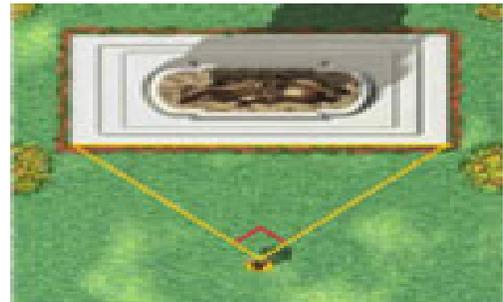
Process Questions:
1. How did you answer the problem above?
2. What concepts did you use to answer the problem?
3. What might happen if you can't be able to respond to the given situation?
4. How can challenging problems involving geometric figures be analyzed and solved?
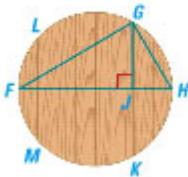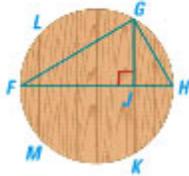
> **Write your answers here:**

To verify the concepts you used in the situation above, let us do the next activity to discover other ways we can use in solving the problem above.

ACTIVITY 8.   LET'S DISCOVER!

Directions: For the online students please do the left side while the non-online students do the activity on the right side. You may also click the interactive website below before answering the questions.
http://www.mathwarehouse.com/geometry/circle/interactive-central-angle-of-circle.php
This website contains interactive activities showing the relationships between arcs and angles of the circle. You may also discover the angles formed by three points using the diameter of the circle.

ONLINE LEARNER	NON-ONLINE LEARNER
1. Draw a circle similar to what is illustrated below. 2. Get a point on the circle and then connect it to the end points of the diameter. 3. Measure the angle formed. 4. Repeat the process until you got five different points.	1. Group yourselves with 8 or 10 members. 2. Form a circle similar to the points on the circle below. 3. Using your arms (one by one) try to point your arms to the persons which you consider as the endpoints of the diameter of a circle. 4. Try to estimate the angle formed by your arms.
Questions: 1. What did you discover from the activity? 2. What conjecture or conclusion can you give from what you learned? 3. How will you validate your answer? 4. Be ready to share what you discovered?	
Answers: 	

Now let us check if your generalization in the previous activity is correct by answering a worksheet related to your answers.

ACTIVITY 9.   PROVE IT!

Directions: Analyze the figures below and then answer the questions. However, if you are not yet ready to answer the problem below, you may open the video below to help you strengthen your understanding of the topic.
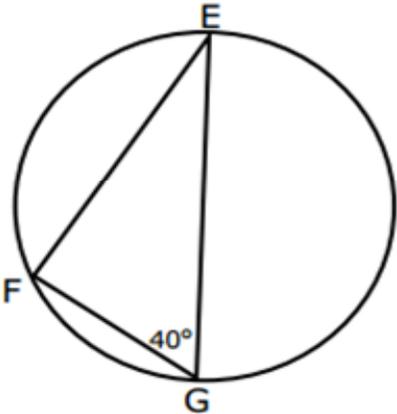https://www.youtube.com/watch?v=Ybzcm2-rU4M
This video contains discussion about the concept of inscribed right angle.
If you want practice exercises before answering the problem below you may open the interactive quiz below.
http://www.ixl.com/math/geometry/angles-in-inscribed-right-triangles
This contains interactive quiz related to the problem below.

Given:	Explanation and Justification
EG - diameter What is m∠E =? E F 40° G	

Now that you learned the concepts of inscribed right triangles it now time for you to learn other angles. The next activity will help you understand other relationships of arcs and angles which are very important for you to do your performance task.

ACTIVITY 10.   Watch and Learn

Directions: Click the website below and listen to the video then answer the worksheets below. Write your answer on the table provided.
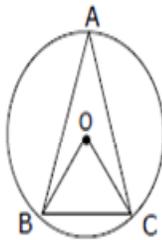
https://www.youtube.com/watch?v=-QBJtmEVg9s
This video contains detailed lecture and explanation how other angles are related to their intercepted arcs.

Find the value of x

1.

5.1
x 4

4.

x
14 12.1

2.

2.2
x
2

5.

x
4.2
8
11.3

3.

x 5
3.4
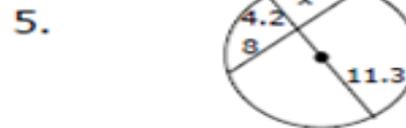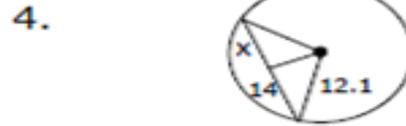
6. In the figure, O is the center of the circle. What is m∠ABC, if m∠ OBC = 67?

Write your answers here:

1.		3.		5.	
2.		4.		6.	

Questions:
1. How did you answer each question above? What concepts did you use?
2. What did you learn about the relationships between the inscribed angles and the intercepted arcs?
3. How can challenging problems involving geometric figures be analyzed and solved?

Answers:

You are now ready for more challenging activities. Since you have already acquired some knowledge and skills let's see if you can be able to use to respond to the challenge.

ACTIVITY 11. Let's us ask the expert

Directions: Click the videos below. These videos will help you answer the activity below. However, if you want to challenge yourself you may go directly to the activity but you need to complete each part.
https://www.youtube.com/watch?v=z-YxfG42P2M
This site contains video about tangent line.

https://www.youtube.com/watch?v=z-YxfG42P2M
This site contains video about common tangent to two circles.

https://www.youtube.com/watch?v=CJNAO6LQmAw
This site contains video about secants, tangents and angles measure.

Process Questions:
1. What did you notice about the videos above?
2. What relationships did you observe about the angles and the intercepted arcs?
3. How can you use those relationships in real life problems?
4. How can challenging problems involving geometric figures be analyzed and solved?
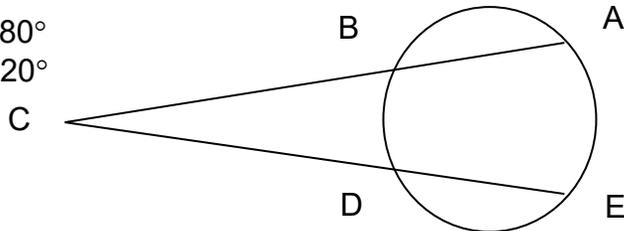
Write your answers here:

To check if you can be able to use the concepts you learned from the videos; let us solve together the problem below.

ACTIVITY 12.   Let's model

Directions: Complete the graphic organizer below to answer the problem below.

Find the measurement of arc CD in the figure.

Given: m AE = 80°
       M ∠C = 20°

FORMULATE:

The longer arc subtracted by the shorter are within the angle divided by two equals the measurement of the angle. Hence,

$$\frac{\text{arc AE} - \text{arc BD}}{2} = \underline{\hspace{2cm}}$$

COMPUTE:

$$\frac{\text{m AE} - \text{m BD}}{2} = \underline{\hspace{2cm}}$$

$$\frac{80° - \text{m BD}}{2} = 20°$$

Continue solve for BD.

VALIDATE:

If arc AE is 100° and arc BD is 35°, what will be the measure of ∠C?

_____

INTERPRET:

Therefore, the measure of arc BD if arc AE=80° and angle C = 20° is _____.

Now that you have used the modelling technique, let us see if you can be able to do it on your own.

ACTIVITY 13. Let's make it real

Directions: Read the problem carefully and then complete the boxes.

PROBLEM:

GLOBAL POSITIONING SYSTEM (GPS) GPS satellites orbit about 11,000 miles above Earth. The mean radius of Earth is about 3959 miles. Because GPS signals cannot travel through Earth, a satellite can transmit signals only as far as points A and C from point B, as shown. Find BA and BC to the nearest mile.
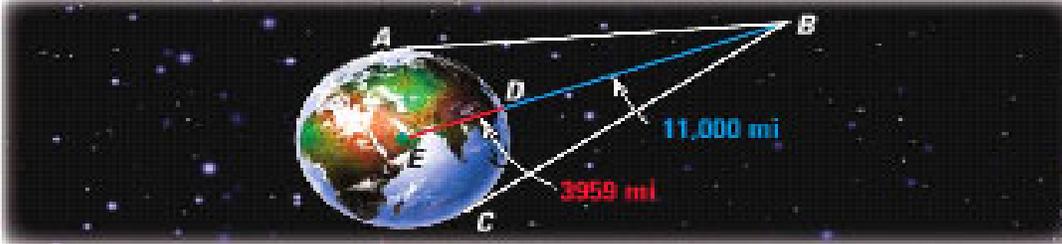
FORMULATE: (Write the concepts or formula you need to answer the problem)

COMPUTE: (Write your complete solution)

INTERPRET: (Write a paragraph about the interpretation of your answer)

VALIDATE: (Provide one more example to validate your answer)

REPORT:
NON-ONLINE: Be ready to report your work in front of the class.
ONLINE: Use voki.com to be able to report your work. You may also send your written report through email or using the student dash board. Just attached your file or send it the discussion forum.

You learned a lot of concepts and theorems from the previous activity. To test if you can use it correctly, do the next activity.

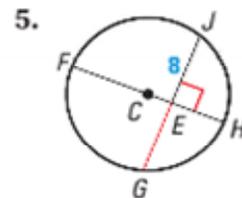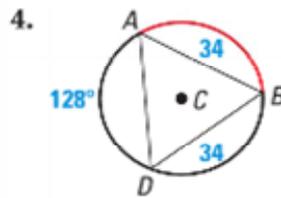## ACTIVITY 14.   Let's use it (Quiz)

Directions: Answer the problems below.

> 1. Describe what it means to bisect an arc.

> 2. Two chords of a circle are perpendicular and congruent. Does one of them have to be a diameter? Explain your reasoning.

> Find the value of the **red** arc or chord in the circle C.

3.

4.

5.
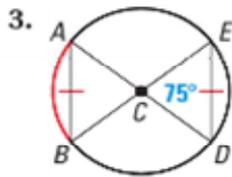
> Find the value of **x** in circle Q.

6.

7.

8.

9.

10.

11.
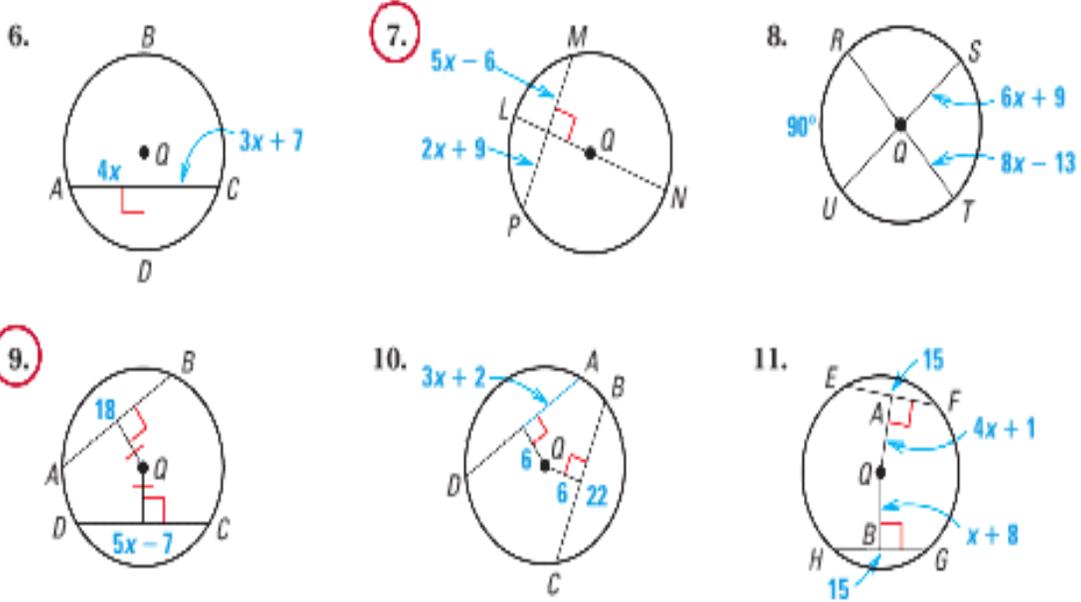
Write your answers here:

1.		
2.		
3.	4.	5.
6.	7.	8.
9.	10.	11.

Questions:

1. What concepts in geometry did you use to answer each problem?

2. What are the difficulties they you encounter?

3. How do geometric concepts related to one another?

4. How can challenging problems involving geometric figures be analyzed and solved?

ACTIVITY 15.   3 – 2 - 1

You encountered a lot of concepts related to circle. Now it's time to pause for a while and reflect to your learning process by doing the 3 – 2 – 1.

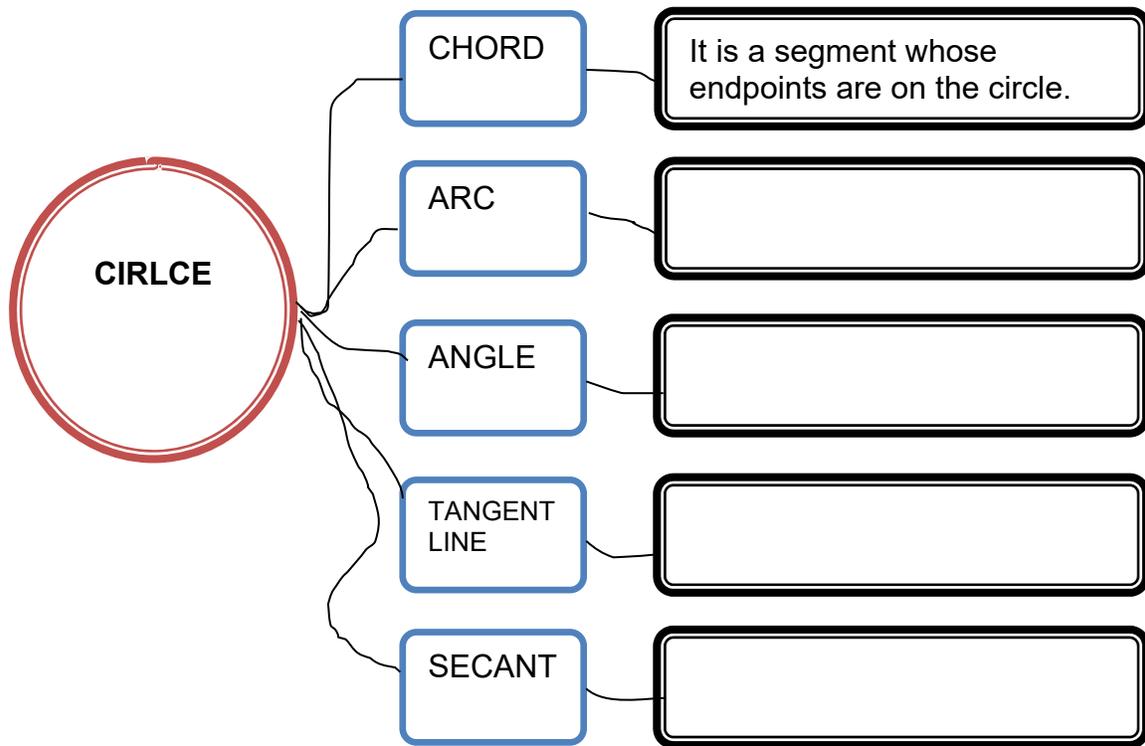What are the 3 most important things you learned?

What are the 2 things you are not sure about?

What is 1 thing that you want to clarify immediately?

To help you summarize and remember important concepts that you learned about triangle similarity, try to complete the graphic organizer below.

ACTIVITY 16.   Do the Map

Directions: Observe the diagram below and complete the missing parts.

CIRLCE		
	CHORD	It is a segment whose endpoints are on the circle.
	ARC	
	ANGLE	
	TANGENT LINE	
	SECANT	

End of FIRM UP:

In this section, the discussion was about circle and other related concepts. It also involves theorems relating the angles and the intercepted arcs.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision? What new learning goal should you now try to achieve?

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.
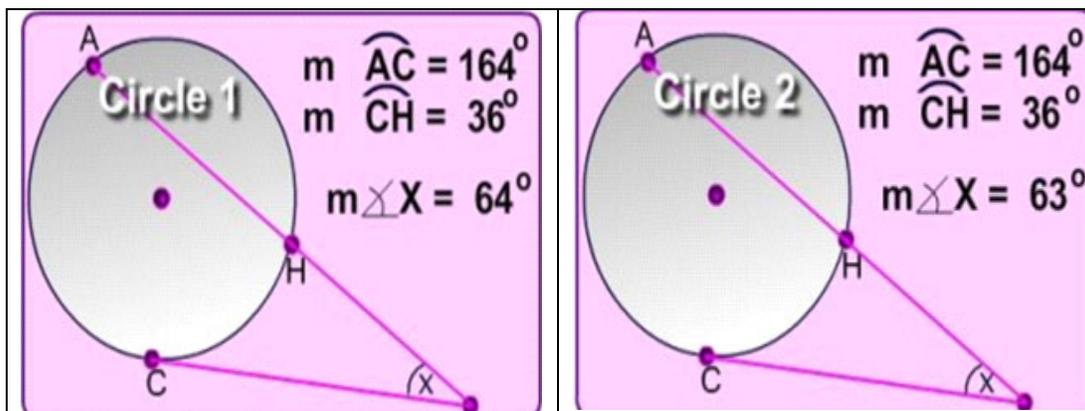
DEEPEN

Your goal in this section is to take a closer look at some aspects of the topic. With the activities that you have accomplished, do you think you are now ready for more challenging situations? Can you assess now how can challenging problems involving geometric figures be analyzed and solved?

To test your analysis and understanding of the concepts let's see if you can be able to identify mistakes and be able to correct it with justification. This activity will also help you check your understanding and possible misconceptions you absorb from the previous activities.

ACTIVITY 17.   Concept Analysis

Directions: Analyze each situation carefully and write your answer, solution and justification.

Note: Only one of the two circles BELOW includes the intersection of a tangent and a secant. Which one is it? Explain your answer.

Circle 1

m $\overgroup{AC}$ = 164°
m $\overgroup{CH}$ = 36°

m$\angle$X = 64°

Circle 2

m $\overgroup{AC}$ = 164°
m $\overgroup{CH}$ = 36°

m$\angle$X = 63°

Write your explanation here!

Before you continue the learning process, it will be better if you stop and reflect. After taking different activities, what happened to your initial answers? You may now answer the R part of you IRF worksheet.

ACTIVITY 18.   CONCEPTUAL UNDERSTANDING CHECK

In the table below, write your answers on the revised answer for the question how can challenging problems involving geometric figures be analyzed and solved?

INITIAL ANSWER
REVISED ANSWER
FINAL ANSWER

To validate your understanding about the concepts of circles, answer the next activity.

ACTIVITY 19.   TRUE OR FALSE AND WHY?

Analyze each statement carefully whether it is right or wrong and then explain your answer.

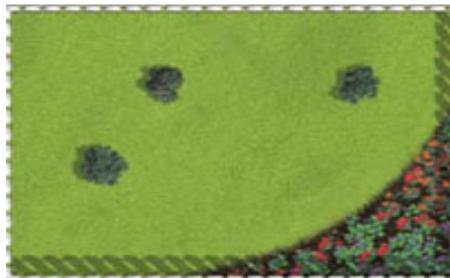| STATEMENT	ANSWER (TRUE/ FALSE)	EXPLANATION
1. If the measure of the central angle is 120° then its intercepted arc is 60°.		
2. If the measure of the inscribed angle is 78° then the measure of its intercepted arc is 39°.		
3. If two chords intersect on the circle then the sum of two intercepted arcs equals to the measure of the angle formed.		
4. Some tangent lines are perpendicular to the radius.		

5. If two secant lines intersect outside the circle, then the measure of the angle formed is the measure of the major arc minus the minor arc.		

Now that you validate your understanding, you are ready for more challenging activities. The next three activities will help you develop your understanding of the concepts and importance of the lesson.

ACTIVITY 20.   GARDENING

In the previous section, we looked at different problems about the application of circle and its properties. Let us put together in the table below our answers to the essential question that we asked for each problem.

You are a landscape designer. Three bushes are arranged in a garden as shown. Where should you place a sprinkler so that it is the same distance from each bush?

What property was used to solve the given problem? Why?

Show your solution/justification:

![PEAC logo] **PEAC** PRIVATE EDUCATION ASSISTANCE COMMITTEE · Trustee of the Fund for Assistance to Private Education   **DepED** DEPARTMENT OF EDUCATION

JHS INSET Learning Module Exemplar

Complete the statement:

Problems in real life involving gardening can be analyzed and solved by

_____.

The choice of properties to be used depends on

_____.

ACTIVITY 21.   PHOTOGRAPH

In the previous section, we looked at different problems about the application of circle and its properties. Let us put together in the table below our answers to the essential question that we asked for each problem.

You are a professional photographer. Your camera has a 90 field of vision and you want to photograph the front of a statue. You move to a spot where the statue is the only thing captured in your picture, as shown. You want to change your position. Where else can you stand so that the statue is perfectly framed?

What property was used to solve the given problem? Why?

Show your solution/justification:

Complete the statement:

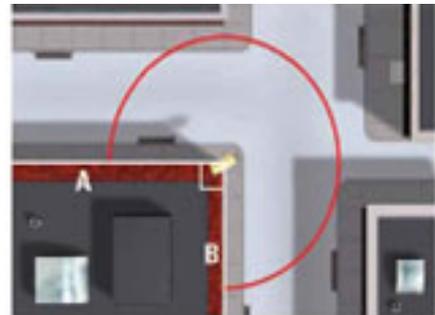**Problems in real life involving photography can be analyzed and solved by**

_____.

The choice of properties to be used depends on

_____.

ACTIVITY 22.   DO THE SURVEY

In the previous section, we looked at different problems about the application of circle and its properties. Let us put together in the table below our answers to the essential question that we asked for each problem.

A surveillance camera is mounted on a corner of a building. It rotates clockwise and counterclockwise continuously between Wall A and Wall B at a rate of   10 per minute as illustrated in the figure. How long does it take the camera to survey the entire area once? Justify your answer.

What property was used to solve the given problem? Why?

Show your solution/justification:

Complete the statement:

Problems in real life involving monitoring places can be analyzed and solved by

_____.

The choice of properties to be used depends on

_____.

ACTIVITY 23.   SUM IT UP

Based from your answer on the three previous activities, answer the following.

What are the common words or phrase among your answers?

Using the words/phrases above, write a one paragraph statement summarizing what you learned from this activity?

PROCESS QUESTIONS:
1. Look at your answers above; what do all the solutions have in common?
2. Were you able to answer all the problems?
3. How did you come up with correct property for each situation?
4. Have you encountered problems which can be solved with two or more ways? What made you choose the solution that you used?
5. How is each property useful in solving the problem?
6. If the situations will differ, how do you know which one to use? Explain.
7. Complete the statements below.

Answer to the process questions:

| In general, problems in real life can be analyzed and solved by….. |
|---|
| In general, the choice of properties to be used depends on …… |
| How can challenging problems involving geometric figures be analyzed and solved? |

Submit

After summing up everything, try solving the problem below. Try to see if the generalization that you have made can also be used in the new situation.

ACTIVITY 24.   TRY IT TO OTHERS

Directions: Read the problem below and answer each question.

You are an accident investigator, you learned from research that the speed of the car can be modelled by the equation $S = 3.86\sqrt{fr}$ where S = car's speed, f = coefficient of friction, and r = radius of the circle in feet. If a car goes around a turn too quickly, it can leave tracks that form an arc of a circle, what is the estimated speed of the car? You may refer to the given illustration.
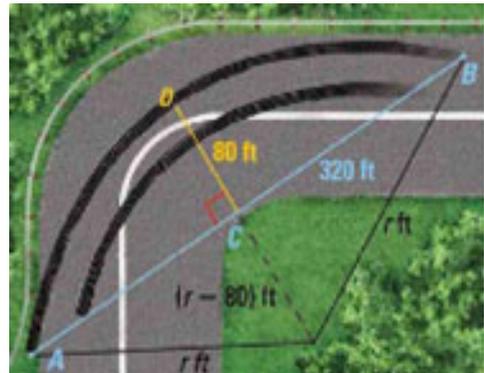
Process Questions:
1. What information would help you solve the given problem?
2. What property can be used to solve the problem below? Why?
3. Show your solution and justification.
4. How can challenging problems involving geometric figures be analyzed and solved?

Answers:

End of DEEPEN:

In this section, the discussion was about circle and other related concepts.

What new realizations do you have about the topic? What new connections have you made for yourself? What helped you make these connections?

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.

TRANSFER

Your goal in this section is apply your learning to real life situations. You will be given a practical task which will demonstrate your understanding.

To help you summarize everything that you learned in this lesson, try to make a problem posing and solving activities with the use of mathematical modelling framework.

ACTIVITY 25.   LET'S INVESTIGATE

Directions: Read the procedure carefully and follow. You may do your work using MS Word and send it to your teacher. For your explanation and justification you may do it face to face or you may try another web 2.0 **present.me.** Here you may record your explanation, justification and generalization of the lessons. In your presentation, do not forget to answer the questions, *how can challenging problems involving geometric figures be analyzed and solved?*
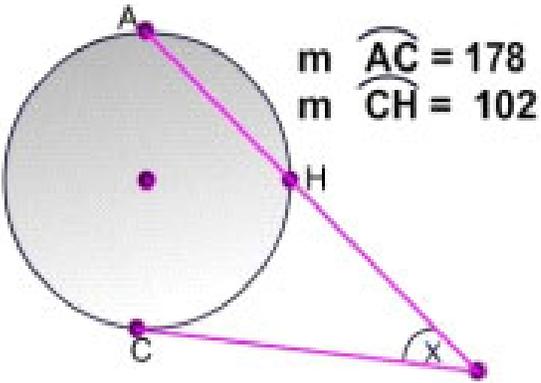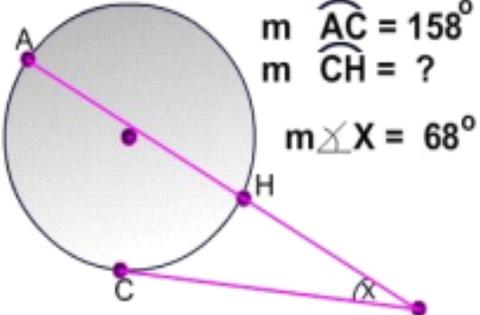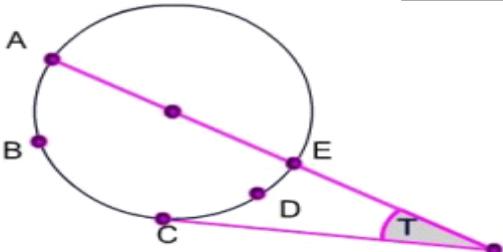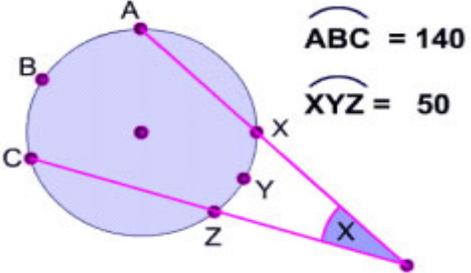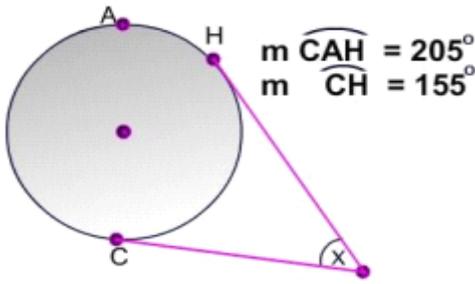
PROCEDURE:
1. **Problem:** Think of a problem or situation where the concepts of the lesson are very important. Create your own word problem.
2. **Formulate:** Write the concept, theorem or formula you need to be able to solve the problem.
3. **Compute:** Write your complete solution (step by step).
4. **Interpret:** Write a paragraph about the interpretation of your answer.
5. **Validate:** Provide more examples about your answer and concepts you used.
6. **Report:** Report the result of your investigation.

After doing your own investigation you make take a long quiz to assess more the knowledge and skills you need before doing your transfer task.

ACTIVITY 26.   LONG TEST

Directions: Read each problem carefully and answer each question.

1. What is the value of x in the problem on below?	2. What is the measure of mAC?
m $\widehat{AC}$ = 178 m $\widehat{CH}$ = 102	m $\widehat{AC}$ = 158° m $\widehat{CH}$ = ? m∠X = 68°
Answer:	Answer:
3. $\overline{AE}$ is a diameter , **What is m ∠T?**	4. What is the value of x?
m $\widehat{ABC}$: m $\widehat{CDE}$ = 3:1	$\widehat{ABC}$ = 140 $\widehat{XYZ}$ = 50
Answer:	Answer:
5. What is the value of x in the problem below?	6. What is the measure of m CH?

$m\ \overset{\frown}{CAH} = 205°$ $m\ \overset{\frown}{CH} = 155°$	$m\ \overset{\frown}{CAH} = 210°$ $m\ \overset{\frown}{CH} = ?$ $m\angle X = 62°$
Answer:	Answer:
7. What is the value of k in the problem below?	8) The diagram on the right is not to scale. $ABC : XYZ = 3:2$, arc $AX = 80$ and arc $CZ = 170$. What is k?
$K = ?$ $\overset{\frown}{ABC} = 90$ $\overset{\frown}{XYZ} = 30$	
Answer:	Answer:
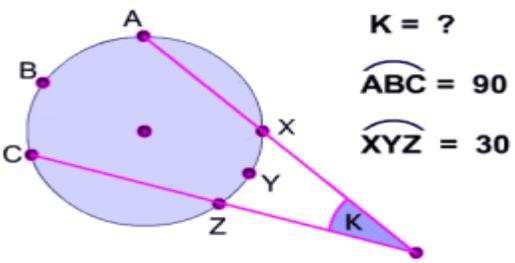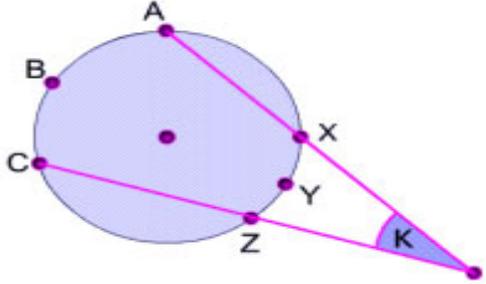
Now you may have enough knowledge, skills and understanding to do your transfer task. To assess what you learned and understand, you will apply the concepts of the lesson in an actual situation by doing the transfer task of this lesson Scaffold 1.

ACTIVITY 27.   SCAFFOLD 1

Directions: Read the procedure in the box and do what is asked.

> Students will be asked to conduct an experiment involving different situations. Students may choose among the three situations below.
>
> In a theater, what will be the best locations of the actors so that they will always be visible to their audience?
>
> For a cameraman, what will be his best locations so that he can take good shots to the actors performing on the stage?
>
> For a gardener, what should be the best position of the lawn sprinkler to maximize its use?
>
> Students will submit a written output containing explanation and justification. Students will be using the knowledge they learned about the concepts of circle and other related terms.

Note: To make your written output appealing and accurate you may use the **geogebra.** To do it, download **geogebra** then you may use it for free.

For your presentation and explanation, you may use **voki.com**. This will help you record your presentation in the most exciting way.

After doing your transfer task (Scaffold 1) it's time again to reflect on the learning process to check if there are ideas which you need to change, to revise or improve. You may now complete you IRF worksheet by writing your ideas on the F part.

ACTIVITY 28. CONCEPTUAL UNDERSTANDING CHECK

In the table below, write your answers on the final answer for the question how can challenging problems involving geometric figures be analyzed and solved?

INITIAL ANSWER
REVISED ANSWER
FINAL ANSWER

To complete the learning process, reflect again and complete the table below. This will also check if you have absorbed some misunderstandings which need to be corrected.

ACTIVITY 29. Let's Reflect!

CORNELL' S NOTES	
TOPICS	**THINGS I LEARNED**
Questions I want to be answered:	

To summarize what you learned, you may complete the synthesis journal below.

ACTIVITY 30.   Synthesis Journal

The lesson was on _____. One key idea was _____. This is important because _____ _____. Another key idea was _____ _____. This is also important because _____ _____. In summary, the lesson _____ _____.

End of TRANSFER:

In this section, your task was to make a written explanation and justification about the situation that you encountered.

How did you find the performance task? How did the task help you see the real world use of the topic?
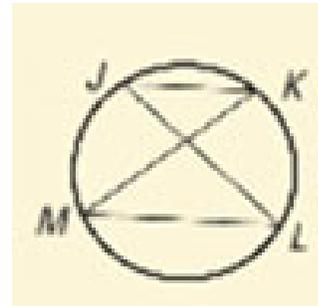
You have completed this lesson. But you have two more lessons before you finish this module. You need to learn more about circles and coordinate geometry to complete what you need in doing your performance task.

POST ASSESSMENT

1. Which of the following is true about the measures of central angles in relation to the intercepted arc?
    A. The measure of the intercepted arc is half of the measure of the central angle.
    B. The measure of the intercepted arc is twice the measure of the central angle.
    C. The measure of the central angle is equal to the measure of the intercepted arc.
    D. The measure of the central angle is half of the measure of the intercepted arc.

2. Name two pairs of congruent angles using the figure below.
    A. ∠JKM ≅∠KJL &∠JLM ≅∠KML
    B. ∠JLM ≅∠KJL &∠JKM ≅∠KML
    C. ∠JKM ≅∠JLM &∠KJL ≅∠KML
    D. ∠JLM ≅∠KJL &∠JLM ≅∠JKM

3. A professional photographer has a camera that has a 90^0 field of vision and he wants to photograph the front of a statue. He moves to a spot where the statue is the only thing captured in your picture, as shown. After seeing this, you are also encouraged to capture the photograph of the statue but your camera has less than 90^0 field of vision. Which of the following should be your strategy to capture the whole picture?

    A. You may move closer to the statue.
    B. You may move farther from the statue.
    C. You may move to the other side of the statue.
    D. Look for an elevated place to capture the statue.

4. You are a member of the board who will assess the different proposals who are applying for bidding for CCTV cameras to be installed in different places. Which among the following criteria should you consider most?
    A. Authenticity of data, accuracy, practicality of proposal
    B. Accuracy, organization, practicality of proposal
    C. Practicality, neatness, accuracy of proposal
    D. Organization, practicality, neatness of proposal

5. A surveillance camera is mounted on a corner of a building. It rotates clockwise and counterclockwise continuously between Wall A and Wall B at a rate of 15^0 per minute as illustrated in the figure. How long does it take the camera to survey the entire area once? Justify your answer.

   A. 3 minutes, since the measure of the angle is half of the intercepted arc.

   B. 6 minutes, since the angle and arc have equal measures.

   C. 12 minutes, since the measure of the angle is twice the measure of the intercepted arc.

   D. 18 minutes, the camera needs to survey about 135^0 of the circle.

6. If rays WE and WI are both tangents to the circle P below, then which of the following is NOT necessarily true?

   A. AO = OB

   B. m∠PAO =m∠PBO

   C. ∠AOB ≅∠APB

   D. AP = BP

7. If ray m and n are tangents to circle P below, then which of the following is the value of w?

   A. 30°   B. 40°   C. 50°   D. 60°

8. In the figure below, if segment MT is tangent to the circle, segments NT and PT are secants. If MP is 140° and the measure of ∠MTP= 50°, what is the measure of arc MS?

A. 30°   B. 40°   C. 50°   D. 60°
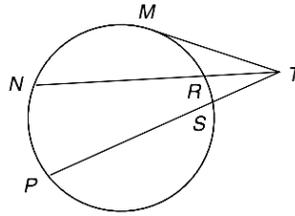
9. If RS = VW in the figure below, then which of the following will justify that ΔRTS is congruent to ΔWTV?
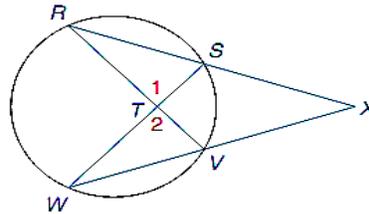
    A. SAA Congruence Theorem
    B. SAS Congruence Postulate
    C. SSS Congruence Postulate
    D. Hypotenuse-Leg Theorem

10. What is the distance between the points (2,2) and (5,6)?
    A. 3
    B. 5
    C. 7
    D. 9

11. The point (5,4) lies on a circle. What is the length of the radius of this circle if the center is located at the origin?
    A. $\sqrt{8}$
    B. $\sqrt{3}$
    C. $\sqrt{18}$
    D. $\sqrt{41}$

12. Which of the following coordinates must added to A(2,3), B(4,4), D(-1,-1) to prove that the given figure is a kite?
    A. C(3,2)
    B. C(3,5)
    C. C(3,7)
    D. C(2,5)

13. On a map's coordinate grid, Brgy. Malaya is located at (2, 4) and Brgy. Pasong Tamo islocated at (2, 2). Brgy. Katapan is the in the middle of the two barangay. What is the distance from Brgy. Malaya to Brgy. Katapatan?
    A. 2km
    B. 4km
    C. 6km
    D. 8 km

14. The JDC Cargo ship is sailing from Zhanjiang China to Honolulu and then to Long Beach. The amount of diesel used of the cargo sheep in travelling is 100 gallons for every 10 miles. Using the information in the figure below, determine the number of gallons used by the cargo from Zhanjiang to Long Beach.

    A .616.56 gallons
    B. 600 gallons
    C. 700 gallons
    D. 716.65 gallons

15. In the municipal vicinity map the coordinate of your barangay hall is (2, -1) and the school is (-2,2). How long will it take you to go to school from the barangay hall if the tricycle that you are riding is at 10km/hr?
    A. 30minutes
    B. 45 minutes
    C. 1 hour
    D. 1 hr and 10 minutes

16. Low – Earth orbit satellites occupies a region of space from 180 – 2000 kilometers above Earth. If the satellite is moving in a circular path with Earth as the point of origin, which circular equation represents the maximum path of the satellite?
    A. $x^2 + y^2 = 180$
    B. $x^2 + y^2 = 2000$
    C. $x^2 + y^2 = 32,400$
    D. $x^2 + y^2 = 4,000,000$

17. Based on the figure below, what equation that represents the circle?

I. $(x + 1)^2 + (y - 3)^2 = 9$
II. II. $(x - 1)^2 + (y + 3)^2 = 9$
III. $x^2 + y^2 + 2x - 6y + 1 = 0$
    A. I only
    B. II and III only
    C. I and III only
    D. I, II and III

18. To determine the radius and coordinates of the center of the circle $x^2 + y^2 + 4x - 8y + 4 = 0$, Jerry and Akzie came up with the following solutions:

Jerry's Solution	Akzie's Solution
$x^2 + y^2 + 4x - 8y + 4 = 0$ $x^2 + 4x + \_\_ + y^2 - 8y + \_\_ = -4$ $x^2 + 4x + 2^2 + y^2 - 8y + (-4)^2 = -4$ $(x + 2)^2 + (y - 4)^2 = -4$ Radius $= \sqrt{-4}$; center: $(-2, 4)$	$x^2 + y^2 + 4x - 8y + 4 = 0$ $x^2 + 6x + \_\_ + y^2 - 8y + \_\_ = -4$ $x^2 + 4x + 2^2 + y^2 - 8y + (-4)^2 = -4 + 4 + 16$ $(x + 2)^2 + (y - 4)^2 = 16$ Radius $= 4$; center: $(-2, 4)$

Which statement best describes the solutions of Jerry and Akzie?
    A. The solution of Jerry is correct.
    B. The solution of Akzie is correct.
    C. The solution of Jerry and Akzie are both correct.
    D. The solution of Jerry and Akzie are both incorrect.

19. When delivering weather forecast during typhoons, weather forecasters uses diagrams and charts to illustrate the movement of the typhoon. The multiple colored circles indicate the effect of the typhoon in the area at a given time at. If you are the weather forecaster, which statement best describes typhoon Haiyan based on the illustration below?

A. The area affected by 128 kph winds of Haiyan decreased from Nov. 7 – 10.

B. The area affected by 95 kph winds of Haiyan decreased from Nov. 7 – 10.

C. The area affected by 95 – 128 kph winds of Haiyan was maintained from Nov. 7 – 10.

D. The area affected by Haiyan decreased from Nov. 7 – 10.

20. The barangay officials recently reported alarming news that the number of crimes in your community is drastically increasing. The barangay council decided to put up a CCTV camera in the area with the most number of crime incidents. You are a marketing officer of a CCTV company that will participate in the bidding to provide the barangay with the CCTV unit. You need to identify the best location of the CCTV unit in the area that is identified in your barangay. Your work should have authentic data, accurate computation, should have practical recommendations, and with correct application of the concepts of the properties of circles. What should you do to convince the barangay officials?

A. You will present the CCTV to the barangay officials.

B. You will present the CCTV and layout to the barangay officials.

C. You will present your recommendation to the barangay officials.

D. You will present your recommendation and justification to the barangay officials.

GLOSSARY OF TERMS USED IN THIS LESSON:

Circle. A circle is a plane figure bounded by one line, and such that all right lines drawn from a certain point within it to the bounding line, are equal.

Arc. Any connected part of the circle.

Center. It is a point equidistant from the points on the circle.

Radius. It is a line segment joining the center of the circle to any point on the circle itself; or the length of such a segment, which is half a diameter.

Sector. It is a region bounded by two radii and an arc lying between the radii.

Secant. It is an extended chord, a coplanar straight line cutting the circle at two points.

Tangent. It is a coplanar straight line that touches the circle at a single point.

Central Angle. It is the angle subtended by the arc to the center of the circle of which it is a part

Inscribed Angle. It is the angle subtended by the arc to any point on the circumference of the circle of which it is a part.

REFERENCES AND WEBSITE LINKS USED IN THIS LESSON:

https://www.youtube.com/watch?v=xI2BZ5R_PrM
This site contains video explaining the relationships between central angle and its intercepted arc.
https://www.youtube.com/watch?v=Ybzcm2-rU4M
This video contains discussion about the concept of inscribed right angle.

http://www.ixl.com/math/geometry/angles-in-inscribed-right-triangles
This contains interactive quiz related to the problem below.

https://www.youtube.com/watch?v=-QBJtmEVg9s
This video contains detailed lecture and explanation how other angles are related to their intercepted arcs.

https://www.youtube.com/watch?v=z-YxfG42P2M
This site contains video about tangent line.

https://www.youtube.com/watch?v=z-YxfG42P2M
This site contains video about common tangent to two circles.

https://www.youtube.com/watch?v=CJNAO6LQmAw
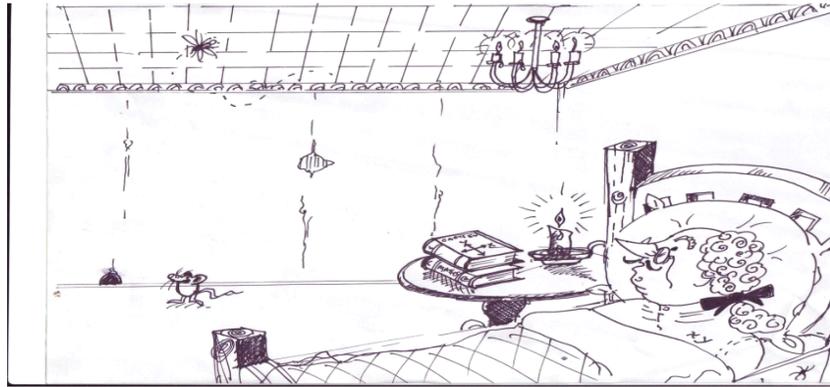This site contains video about secants, tangents and angles measure.

http://www.mathwarehouse.com/classroom/worksheets/circles/secants-tangents-arcs-angle.pdf
This site contains worksheets about secants, tangents and angles measure.

Lesson 2: Distance Formula

☑ **INTRODUCTION AND FOCUS QUESTION(S):**

Coordinate Geometry: Get to the point!

Have you thought of why Geometry is difficult to understand and often perceived to have confusing set figures like circles, squares, rectangles and etc. Have you at a certain time asked yourself how can challenging problems involving geometric figures be analyzed and solved?

A very famous mathematician called Rene Descartes lay in bed one night. As he lay there, he looked up at the ceiling in his bedroom. He noticed a fly was asleep on the ceiling. Descartes, being a mathematician wondered if he could figure out a way of stating where exactly the fly was on the ceiling. Obviously it has to be a precise description he thought.

In this module, you will pay attention to distance formula where the knowledge and skills learned in the previous lessons can be applied. You will find out the different concepts involved in distance formula. As you go over the exercises, you will be able to answer the questions: **How can problems where two quantities bounded by conditions are solved? How can challenging problems involving geometric figures be analyzed and solved? For example, how do you determine the strategic position of a camera/ video recorder in a specific location?**

☑ **LESSONS AND COVERAGE:**

In this module, you will examine this question when you take the following lessons:
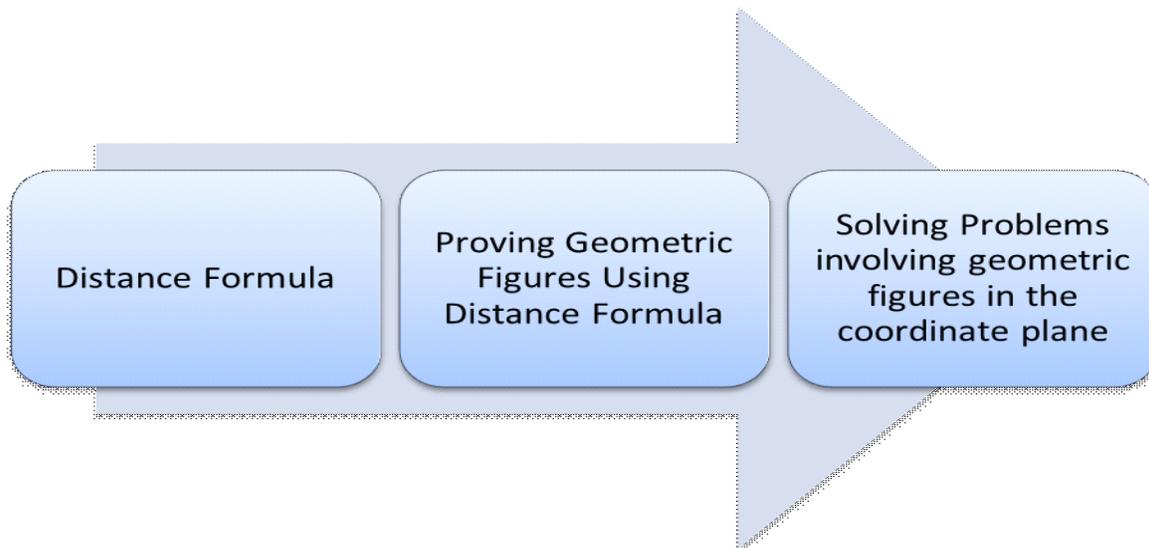
Lesson 1 –
Lesson 2 –   Distance Formula
Lesson 3 –

In this lesson, you will learn the following:

Lesson 1	
Lesson 2	• Derives the distance formula • Applies the distance formula to prove some geometric properties • Graphs other geometric figures on the coordinate plane • Solves problems involving geometric figures on the coordinate plane
Lesson 3	

☑ **MODULE MAP:**

Here is a simple map of the above lessons you will cover:

Distance Formula → Proving Geometric Figures Using Distance Formula → Solving Problems involving geometric figures in the coordinate plane

☑ **EXPECTED SKILLS:**

To do well in this module, you need to remember and do the following:

1. Look up the meaning of words you do not know.
2. Complete all activities and exercises.
3. Take note on the proper modeling of situations using oral, written, graphical and algebraic methods to solve problems.
4. Use the checklist and rubric provided to evaluate your work before submission.
5. Be mindful of the meaning of unfamiliar words you encounter in this module. A glossary of terms is provided in the last part of this module.
6. Maximize the use of online resources in each lesson. Online resources can be accessed multiple times. The summary of online resources is provided in the end of the module.

PRE-ASSESSMENT:

Let's find out how much you already know about this module. Click on the letter that you think best answers the question. Please answer all items. After taking this short test, you will see your score. Take note of the items that you were not able to correctly answer and look for the right answer as you go through this module.

1. Which of the following defines a chord?
    A. A line that passes through the circle at exactly one point.
    B. A line that passes through the circle at exactly two points.
    C. A segment that passes through the circle at exactly one point.
    D. A segment whose endpoints are on the circle.

2. If the measure of the central angle is 45^0, what is the measurement of the intercepted arc?
    A. 22.5^0
    B. 45^0
    C. 90^0
    D. 135^0

3. You are a professional photographer. Your camera has a 90 field of vision and you want to photograph the front of a statue. You move to a spot where the statue is the only thing captured in your picture, as shown. You want to change your position. Where else can you stand so that the statue is perfectly framed?

    A. You may stand at any point on the semi- circle in front of the statue.
    B. You may stand at any point equidistant from your distance to the statue.
    C. You may move to the other side and stand to any point equidistant from your distance to the statue.
    D. There is no other place where you can capture the whole picture of the statue but your original position.

4. A surveillance camera is mounted on a corner of a building. It rotates clockwise and counterclockwise continuously between Wall A and Wall B at a rate of 10^0 per minute as illustrated in the figure. How long does it take the camera to survey the entire area once? Justify your answer.
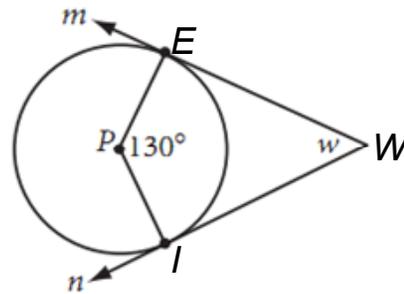

    A. 4 minutes and 30 seconds, since the measure of the angle is half of the intercepted arc.
    B. 9 minutes, since the angle and arc have equal measures.
    C. 18 minutes, since the measure of the angle is twice the measure of the intercepted arc.
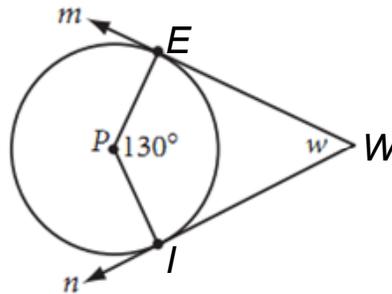
D. 27 minutes, the camera needs to survey about 135^0 of the circle.

5. You are a professional photographer, a company requested you to submit a set of pictures of beautiful sceneries. Which among the following should you consider most to in selecting your pictures?
    A. Pictures are very clear but some important parts are not captured.
    B. All parts are captured but no highlighted subject.
    C. You have a focus subject and important parts are visible.
    D. All parts are captured but some parts are not that clear.

6. If rays WE and WI are both tangents to the circle P below, then which of the following is NOT necessarily true?

    A. EW = IW
    B. $m\angle PEW = m\angle PIW$
    C. $\angle EWP \cong \angle IWP$
    D. $\angle PEW$ and $\angle EWI$ are supplementary.

7. If ray m and n are tangents to circle P below, then which of the following is the value of w?
    B.   30°   B. 40°   C. 50°   D. 60°

8. In the figure below, if segment MT is tangent to the circle, segments NT and PT are secants, PS is a semicircle and the measure of minor arc MP is 140°, then what is the measure of $\angle MTP$?

        A. 50°   B. 60°   C. 90°   D. 100°

9. If RS = VW in the figure below, then which of the following will justify that △RVX is congruent to △WSX?

A. SAA Congruence Theorem Postulate

B. SAS Congruence

C. SSS Congruence Postulate

D. Hypotenuse-Leg Theorem

10. As an Archeologist, you volunteer to help a group of students to determine the center of a circular object that they have found. Which of the following will help you explain how the T-square will be an effective instrument to do so?

A. Tangents are perpendicular to the radii at the points of tangency.
B. The tangent segments intersecting at a point outside a circle are congruent.
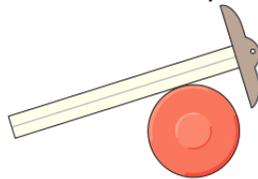C. The measure of an angle formed by two secants intersecting outside a circle is half the measures of the difference of the intercepted arcs of the circle.
D. The measure of an angle formed by a secant and a tangent to a circle, intersecting outside a circle is half the measures of the difference of the intercepted arcs of the circle.

11. What is the distance between the points (3,3) and (6,7)?
    A. 3
    B. 5
    C. 7
    D. 9

12. The point (5,4) lies on a circle. What is the length of the radius of this circle if the center is located at (3,2)?
    A. $\sqrt{8}$
    B. $\sqrt{3}$
    C. $\sqrt{18}$
    D. 8

13. If plotted in the coordinate plane, which of the following geometric figures best describes the coordinates A(-6,4), B(-6, -4), C(0,4) and D(0, -4)?
    A. Square
    B. Rectangle
    C. Rhombus
    D. Kite

14. On a map's coordinate grid, Brgy. Malaya is located at (2, 4) and Brgy. Pasong Tamo is located at (2, 2). Brgy. Katapan is the in the middle of the two barangay. What is the distance from Brgy. Malaya to Brgy. Katapatan?
    A. 2km
    B. 4km
    C. 6km
    D. 8 km

15. You are a city planner who is requested by the city mayor to provide a vicinity map to be used in the municipal strategic planning. Which of the following standards will be assessed your work?
    A. Accuracy, Clarity, application of the concept of distance formula
    B. Accuracy, Clarity, practicality
    C. Practicality, authenticity, use of grid lines
    D. Practicality, Accuracy, presentation

16. Low – Earth orbit satellites occupies a region of space from 180 – 2000 kilometers above Earth. If the satellite is moving in a circular path with Earth as the point of origin, which circular equation represents the lowest path of the satellite?
    A. $x^2 + y^2 = 180$
    B. $x^2 + y^2 = 2000$
    C. $x^2 + y^2 = 32,400$
    D. $x^2 + y^2 = 4,000,000$

17. Based on the figure below, what equation that represents the circle?

III. $(x - 3)^2 + (y - 1)^2 = 5$

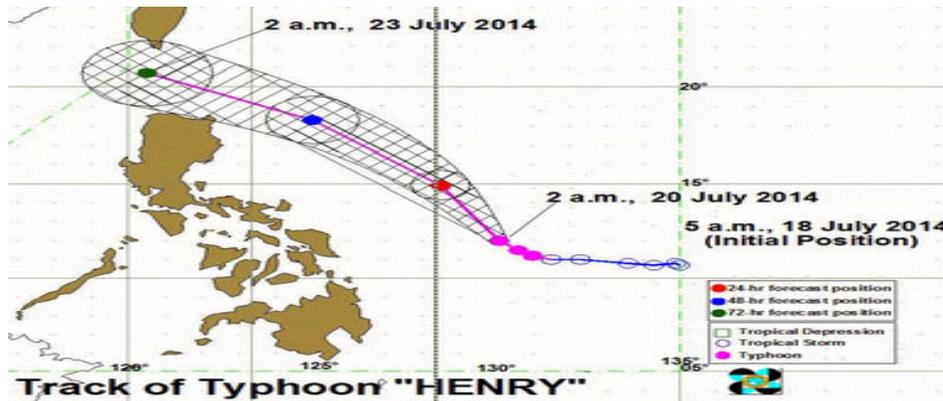IV. $x^2 + y^2 - 6x - 2y + 5 = 0$

V. $x^2 + y^2 - 6x - 2y - 5 = 0$

A. I only

B. II only

C. I and II only

D. I, II and III

18. To determine the radius and coordinates of the center of the circle $x^2 + y^2 - 6x + 2y + 8 = 0$, Nestor and Andy came up with the following solutions:

Nestor's Solution	Andy's Solution
$x^2 + y^2 - 6x + 2y + 8 = 0$	$x^2 + y^2 - 6x + 2y + 8 = 0$
$x^2 - 6x + \underline{\ \ } + y^2 + 2y + \underline{\ \ } = -8$	$x^2 - 6x + \underline{\ \ } + y^2 + 2y + \underline{\ \ } = -8$
$x^2 - 6x + (-3)^2 + y^2 + 2y + (1)^2 = -8$	$x^2 - 6x + (-3)^2 + y^2 + 2y + (1)^2 = -8 + 9 + 1$
$(x - 3)^2 + (y + 1)^2 = -8$	$(x - 3)^2 + (y + 1)^2 = 2$
Radius = $\sqrt{-8}$; center: (3, -1)	Radius = $\sqrt{2}$; center: (3, -1)

Which statement best describes the solutions of Nestor and Andy?

     E. The solution of Nestor is correct. Andy committed an error in when he added 9 and 1 on the right side of the equation.

     F. The solution of Andy is correct. Nestor committed an error when he failed to add 9 and 1 on the right side of the equation.

     G. The solution of Andy and Nestor are both correct.

     H. The solution of Andy and Nestor are both incorrect.

19. When delivering weather forecast during typhoons, weather forecasters uses diagrams and charts to illustrate the movement of the typhoon. The circles indicate the affected area of the typhoon at a particular time. If you are the forecaster, which statement best describes typhoon Henry based on the illustration below?

Track of Typhoon "HENRY"

A. Typhoon Henry grew stronger from July 20 – 23, 2014.
B. Typhoon Henry move faster from July 20 – 23, 2014.
C. Typhoon Henry's coverage increased from July 20 – 23, 2014.
D. Typhoon Henry's coverage did not change from July 20 – 23, 2014.

20. The barangay officials recently reported alarming news that the number of crimes in your community is drastically increasing. The barangay council decided to put up a CCTV camera in the area with the most number of crime incidents. You are a marketing officer of a CCTV company that will participate in the bidding to provide the barangay with the CCTV unit. You need to identify the best location of the CCTV unit in the area that is identified in your barangay. You will present your recommendation and justification to the barangay officials. Which are the appropriate standards that your work should be evaluated?

    A. organization, neatness, practical recommendations, application of the concepts of the properties of circles
    B. neatness, presentation, practical recommendations, application of the concepts of the properties of circles
    C. creativity, accurate computation, practical recommendations, application of the concepts of the properties of circles
    D. authentic data, accurate computation, practical recommendations, application of the concepts of the properties of circles

.

EXPLORE

Have you ever wondered or asked how did Rene Descartes discovered or formulated the distance formula, or the midpoint formula? Or wonder how was the fly became part of the history of mathematics in the creation of the Cartesian Plane. In the first activity you will learn how Rene Descartes formulated the distance formula.

ACTIVITY 1. Learn Your History.

Read the article below. After reading the article answer the process questions.

History now shows that the two Frenchmen Rene Descartes and Pierre de Fermat seem to have earned at the idea of analytical geometry at about the same time. Descartes' work "*La Geometrie*", however, was published first (in 1637) and Fermat's "*Introduction to Loci*" was not published until after his death. Today, they are considered the co-founders of this important branch of mathematics, which links algebra and geometry.

Their initial approaches were quite different. Basically, Descartes began with a line or curve and then found the equation which described it. Fermat, to a large extent, started with an equation and investigated the shape of the curve it described. The interaction between algebra and geometry shows the power of analytical geometry as a branch of mathematics. Analytical geometry and its use of coordinates, provided the mathematical tools which enabled Isaac Newton to later develop another important branch of mathematics called calculus.

Newton humbly said: *"if I have seen further than Descartes, it is because*

Process Questions:
1. What do you think is the article all about?

2. Do you think the discovery of Fermat and Descartes helped us dealing with geometrical problems?

3. What do you think Newton means when he said, *"if I have seen further than Descartes, it is because I have stood on the shoulders of giants?"*

ACTIVITY 2.   What Descartes Saw

When Descartes looked up at his ceiling, this is what he saw. A fly resting there. He began to think about how he might be able to describe the exact position of the fly.
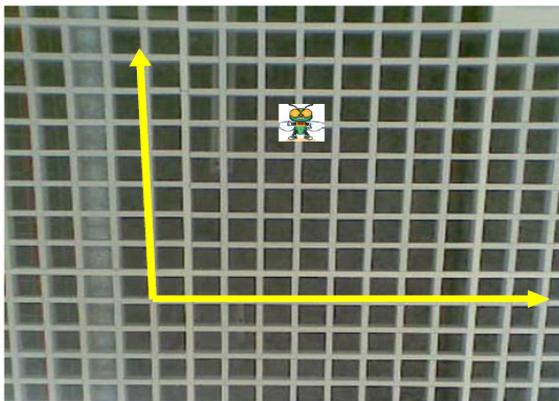
Descartes decided that if he drew two lines at right angles to each other, then he might be able to come up with a way of describing the exact position of the fly.

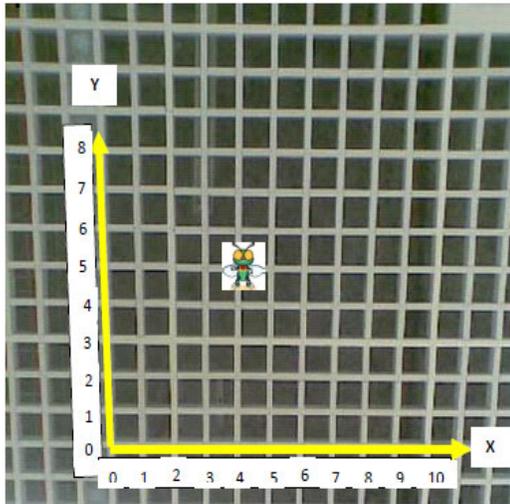How do you think this would have helped him?

_____
_____
_____

Descartes decided to place numbers on the bottom (horizontal) row and on the side (vertical) row. He could now state accurately where exactly the fly was on the ceiling.

But there was a problem: should he give the vertical number of tiles followed by horizontal, i.e. go up 5 squares and move across 4 squares? Or, should he give the horizontal number first, then the vertical, i.e. go across 4 squares then move up 5?

He decided to give the HORIZONTAL NUMBER **FIRST** and THE VERTICAL NUMBER **SECOND.** To help people remember this, he called the horizontal line X and the vertical line Y (Because X comes before Y in the alphabet)
So, in this diagram, the position of the fly can be found by moving 4 units across, then 5 units up. These are known as X, Y values and are written like this:

Position of fly = (4, 5)
                X value Y value (First) (Second)
In honour of Rene Descartes, the graph showing the coordinates of the fly is known as the **Cartesian Plane** (or X Y Plane).

View from the Past

> If you still have questions about the coordinate plane, you may click the link below. The Youtube link below is about The Cartesian coordinate system, formed from the Cartesian product of the real number line with itself, allows algebraic equations to be visualized as geometric shapes in two or three dimensions.
>
> https://www.youtube.com/watch?v=RrrYInyIEGo

A rectangular coordinate system is formed by two lines intersecting at a right angle. The horizontal line is usually called the **x – axis** and the vertical line is usually called the **y – axis**. The point of intersection of the two axes is called the **origin** and the axes separate the plane into four regions called the **quadrants**.

Each point in the rectangular coordinate system corresponds to an ordered pair (x, y) called the **coordinates**. The first number, called **x – coordinate** or **abscissa**, tells how far to the left or right the point is from the vertical axis and the second number called the **y – axis** or the **ordinate**, tells how far up and down the point is from the horizontal axis.

ACTIVITY 3.   Generalization Table

Fill in the first column of the generalization table below by stating your initial thoughts on the question, then save your answer.

How can challenging problems involving geometric figures be analyzed and solved?				
My Initial Thoughts	*My Findings and Corrections*	*Supporting Evidence*	*Qualifying Conditions*	*My Generali-zations*

End of Explore

You just tried finding out how the distance formulas was discovered and formulated by Rene Descartes. Through this formula, Descartes showed that the distance formula can be used to model and solve real-life problems. It is now time to learn more about the distance formula. What you will learn in the next sections will also enable you to do the final project which involves looking at real-life situations that involve decision making and coming up with sound recommendations.

We will start by doing the next activity.

FIRM-UP

Recall that on a number line, the distance between two points is the absolute value of the difference of their coordinates. In a rectangular coordinate plane, distance between two points can be determined by the use of the Pythagorean Theorem.

In this section you will learn more on distance formula. You will be given different sets of activities on how to determine the distance between two locations in real life context which will be very useful in doing the final project.

ACTIVITY 4.   Read and Think Aloud

Read the article below about the process of deriving the distance formula. Using the box below, you may take important notes or concepts that may be useful in the next activities.

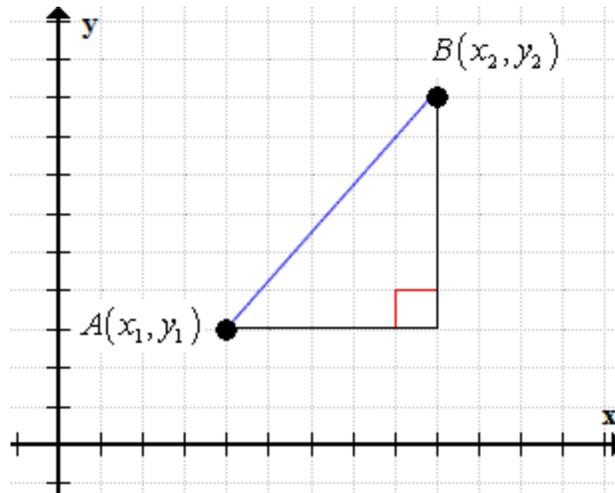Suppose you're given two arbitrary points **A** and **B** in the Cartesian plane and you want to find the distance between them.
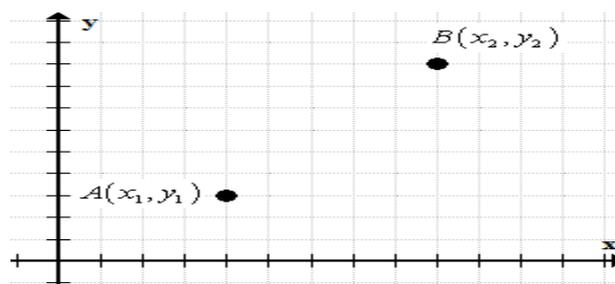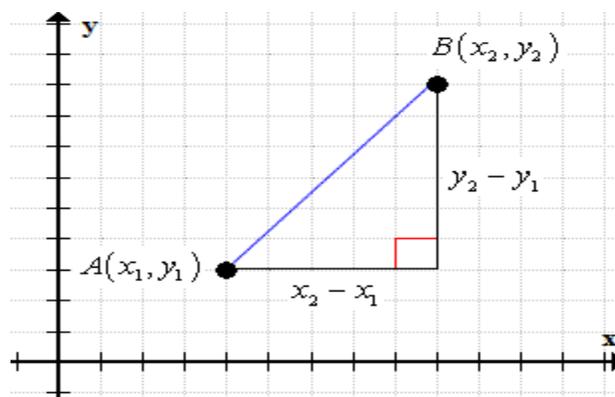

First, construct the vertical and horizontal line segments passing through each of the given points such that they meet at 90 degree angle.
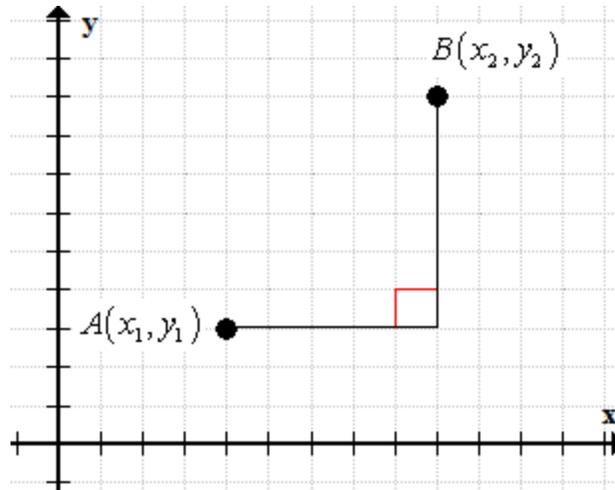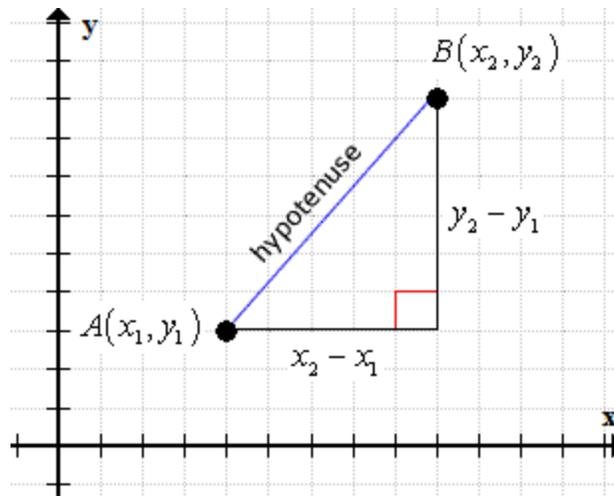
Next, connect points **A** and **B** to reveal a right triangle.

Find the legs of the right triangle by subtracting the x-values and the y-values accordingly.

The distance between the points **A** and **B** is just the hypotenuse of the right triangle. Remember that the Hypotenuse is always the side opposite the 90 degree angle.

Finally, applying the concept of the Pythagorean Theorem, the distance formula is calculated as follows:

$$\text{hypotenuse} = \text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Examples:
Given the coordinates of A and B, Find AB.

1. $A(1, -1); B(-2,2)$
    Solution:

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$AB = \sqrt{(-2 - 1)^2 + (2 - (-1))^2}$$
$$AB = \sqrt{(-3)^2 + (3)^2}$$
$$AB = \sqrt{9 + 9}$$
$$AB = \sqrt{18} \ or \ 3\sqrt{2}$$

2. $A(3,4); B(-3, -4)$

    Solution:

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$AB = \sqrt{(-3 - 3)^2 + (-4 - 4)^2}$$
$$AB = \sqrt{(-6)^2 + (-8)^2}$$
$$AB = \sqrt{36 + 64}$$

$$AB = \sqrt{100} \ or \ 10$$

My Note Pad
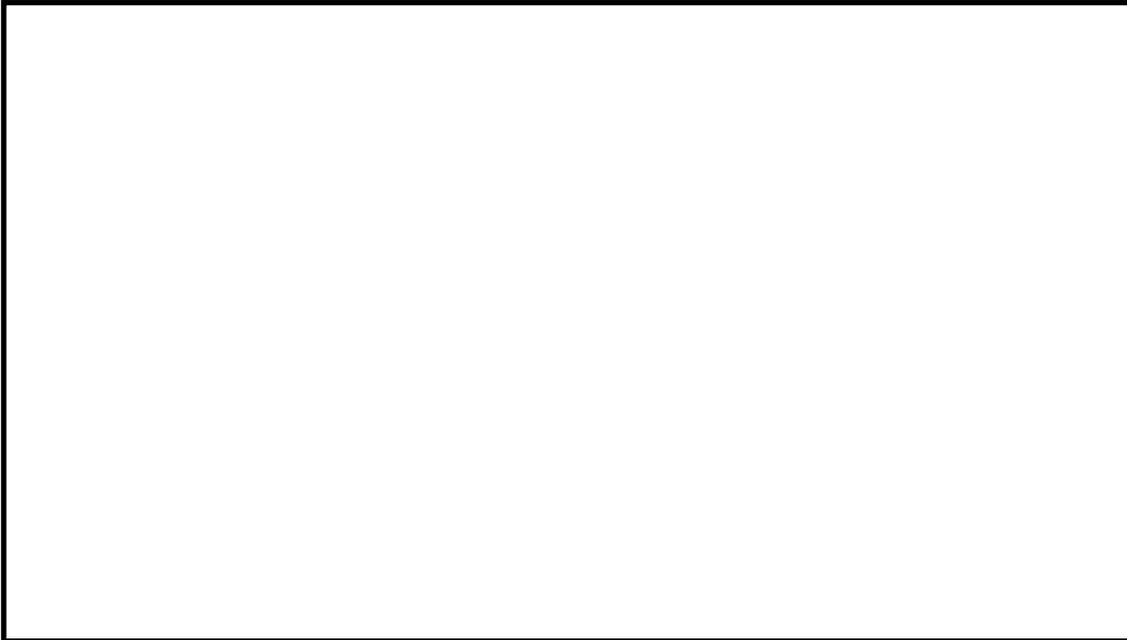
ACTIVITY 5.   Find the Distance

A. Using the distance formula presented earlier. Find the distance between each pair of points on the coordinate plane.

1. (2, 5) and (4, 7)
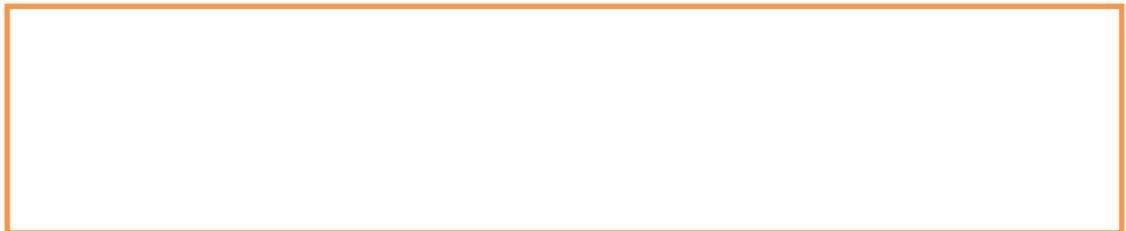
2. (-1, 4) and (7, 10)

3. (3, 1) and (-2, 8)

4. (2, -6) and (5, 3)

5. (-1, -4) and (1, 8)

B. Find the midpoint of each segment with the given endpoints.

6. (4, 1) and (8, 2)

7. (3, 6) and (5, 11)

8. (-2, -5) and (4, 9)

9. (-1, 7) and (4, -2)

10. (5, 0) and (0, 10)

ACTIVITY 6.   Map it Out

Use a graphing paper in plotting the points that will be given in the activity. You may use geogebra in doing the activity below.

1. On a graphing paper, draw a coordinate plane. Plot the points P(-2, 3) and Q(8, 11). You may use the space below for your graph.

[ ]
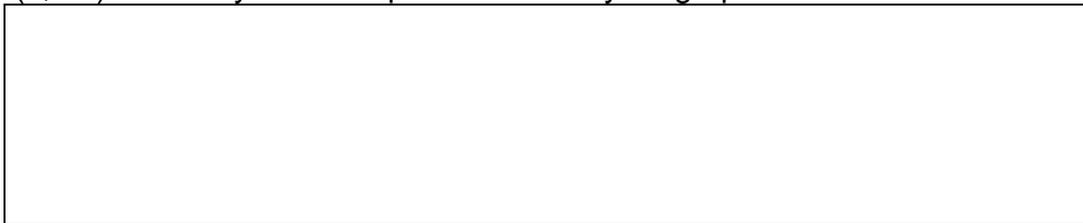
2. Get the average of the x-coordinates of the endpoints of $\overline{PQ}$

[ ]

3. Get the average of the y-coordinates of the endpoints of $\overline{PQ}$

[ ]

4. Compare the values you obtain in #2 to the coordinates of the midpoint (x, y) of $\overline{PQ}$

[ ]

5. What statements could you make?

[ ]

6. If the coordinates of the endpoints of $\overline{PQ}$ are $P(x_1, y_1)$ and $Q(x_2, y_2)$, how would you get the average of the x-coordinates?

[ ]

7. If the coordinates of the endpoints of $\overline{PQ}$ are $P(x_1, y_1)$ and $Q(x_2, y_2)$, how would you get the average of the y-coordinates?

8. Write the formula for the midpoint M of line segment PQ

Exercise 1. Find the distance between each pair of points. Round your answer to the nearest tenth, if necessary.

1)

Answer:

2)

Answer:

3)

Answer:

4)

Answer:

5)
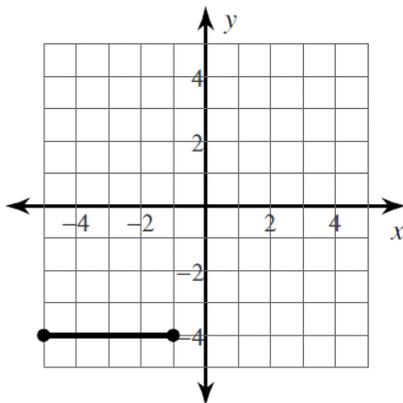
Answer:

ACTIVITY 7.   Watch Me

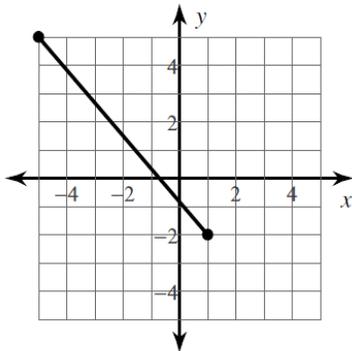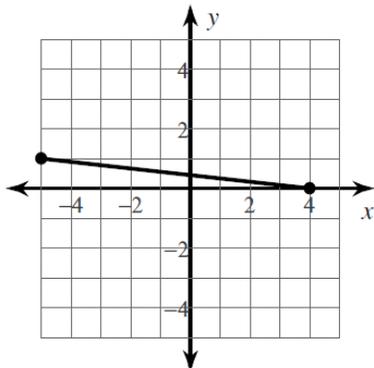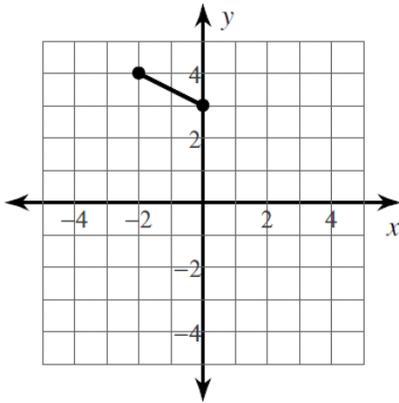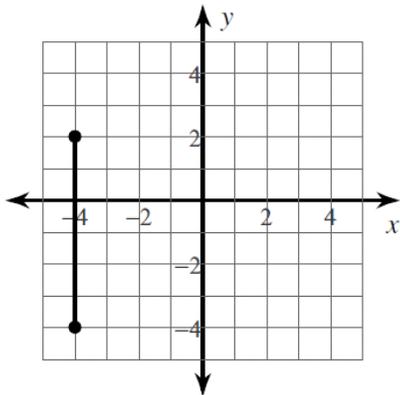Click on the links given below. You may use the notepad for taking important notes that may be useful in doing the next activities.

https://www.youtube.com/watch?v=RrrYInyIEGo
- the website is about the Cartesian coordinate system, formed from the Cartesian product of the real number line with itself, allows algebraic equations to be visualized as geometric shapes in two or three dimensions.

https://www.youtube.com/watch?v=QPIWrQyeuYw
- the website is about finding the "distance formula for two points" "Derive The Equation" "Distance Formula For Two Points" "Math Help" "Math Problem" "Distance Formula" "Pythagorean Theorem" "equation" "midpoint Formula" "coordinate Geometry"

https://www.youtube.com/watch?v=w5ZhGjTnbSA
- The website is about How to Memorize Math Formulas | Math Formula Memorization

ACTIVITY 8.   Throwback!

Recall Activity 2, when a very famous mathematician called Rene Descartes lay in bed one night. As he lay there, he looked up at the ceiling in his bedroom. He noticed a fly was asleep on the ceiling. Descartes, being a mathematician wondered if he could figure out a way of stating where exactly the fly was on the ceiling. Obviously it has to be a precise description he thought.

Descartes decided to place numbers on the bottom (horizontal) row and on the side (vertical) row. He could now state accurately where exactly the fly was on the ceiling.
But there was a problem: should he give the vertical number of tiles followed by horizontal, i.e. go up 5 squares and move across 4 squares? Or, should he give the horizontal number first, then the vertical, i.e. go across 4 squares then move up 5?

Example 1. Let's see what you have learned from the previous Youtube videos. What if, for example, the fly, which Rene Descartes saw, flew from one location to another location? Let's determine the distance.

Given:

Location 1 of the fly: (4, 5)
Location 2 of the fly: (9, 3)

Determine the distance using the formula: (Reference: https://www.youtube.com/watch?v=QPIWrQyeuYw)

$$Distance\ Formula = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(9 - 4)^2 + (3 - 5)^2}$$
$$= \sqrt{(5)^2 + (-2)^2}$$
$$= \sqrt{25 + 4}$$
$$= \sqrt{29}\ or\ approximately\ 5.385$$

Example 2. In this example, the fly, which Rene Descartes saw, flew again to a third location (9,7). Let's compare the distances of the three locations.
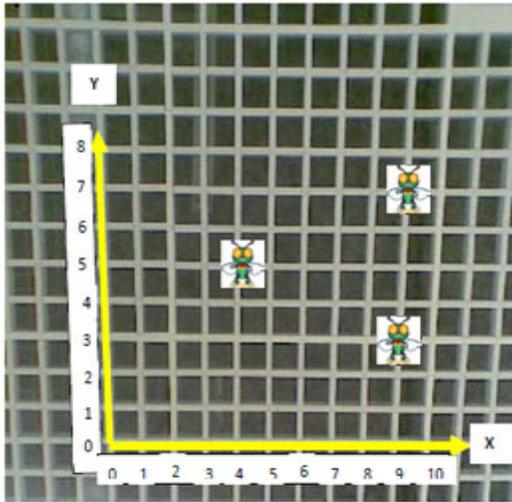
Location A(4, 5)
Location B(9, 3)
Location C(9, 7)

Solutions:
Distance Location A to Location B

$$Distance\ A\ to\ B = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(9 - 4)^2 + (3 - 5)^2}$$
$$= \sqrt{(5)^2 + (-2)^2}$$

$$= \sqrt{25+4}$$
$$= \sqrt{29} \ or \ approximately \ 5.385$$

Distance Location B to Location C

$$Distance \ B \ to \ C = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(9-9)^2+(7-3)^2}$$
$$= \sqrt{(0)^2+(4)^2}$$
$$= \sqrt{16}$$
$$= 4$$

Distance Location A to C

$$Distance \ A \ to \ C = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(9-4)^2+(7-5)^2}$$
$$= \sqrt{(5)^2+(4)^2}$$
$$= \sqrt{25+16}$$
$$= \sqrt{41} \ or \ approximately \ 6.401$$

ACTIVITY 9.   What I Learned

A. Find the distance between each pair of points on the coordinate plane.
1.

1. (12, 5) and (6, 7)

2. (2, 14) and (9, 1)

3. (-4, 11) and (-12, -19)

4. (-2, 6) and (5, 10)

5. (-1, 4) and (1, -8)

B. Find the midpoint of each segment with the given endpoints.

1. (-4, -1) and (-8, -2)

2. (-1, 6) and (2, 10)

3. (2, -5) and (-4, -9)

4. (-11, 6) and (14, 2)

5. (5, 10) and (7, 0)

C. Find the unknown coordinates that would satisfy the given condition.

The distance of a line segment is 5 units with the following endpoints: (2, 1) and (x, 4). Find the missing x coordinate.

The length of $\overline{PQ}$ = 15 units. If the coordinates of **P** are (9, -6) and the coordinates of **Q** are (-3, y), find y.

1. The midpoint of segment CD has coordinates (4, 2). If the coordinates of point **C** are (5, -8), find the coordinates of point **D**.

2. Find the coordinates of the midpoint of a line segment whose endpoints are S(4, 8) and T(-8, 2). Label the midpoint M. Verify if SM = TM.

ACTIVITY 10. Generalization Table

Fill in the unshaded columns of the generalization table below by stating your findings and corrections, supporting evidence and qualifying conditions on the question, then save your answer.

How can challenging problems involving geometric figures be analyzed and solved?				
My Initial Thoughts	My Findings and Corrections	Supporting Evidence	Qualifying Conditions	My Generalizations

ACTIVITY 11.   CHECKPOINT

Choose between the color of the traffic light to describe your present knowledge about the lesson.
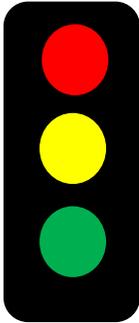
RED – I still need more activities to understand all the concepts

YELLOW – More than 50 % of the concepts I fully understand

GREEN – I understand and will be able to apply

End of Firm Up

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.

DEEPEN

In the previous activities you were able to derive the distance formula, determine the distance formula between two locations, and solve for the midpoint. The next activity will test you on how well you can apply the previous concepts of distance formula in coordinate proof. You will see how the knowledge of distance formula may be used to prove some geometric properties, as well as graph other geometric figures on the coordinate plane and solve problems involving geometric figures on the coordinate plane.

As you move on, think of these questions asked in the previous activity: How can challenging problems involving geometric figures be analyzed and solved?

Coordinate proof is a proof that uses figures in a coordinate plane so that geometric relationships and properties can be proven by means of algebra. Generally uses formula such as distance formula, slope formula, midpoint formula and starts with the coordinates of the points (which can be variable or numbers). A Coordinate geometry proof involves placing of a geometric figure in the coordinate plane. Also Coordinate geometry proofs require that we are well familiar with the geometric coordinate formulas.

> Guidelines for Placing Figures on a Coordinate Plane:
> 1. Use the origin as vertex or center of a figure.
> 2. Place at least one side of the polygon on either the x-axis or y-axis.
> 3. If possible, the figure should be placed on the first quadrant.
> 4. Assign coordinates that make computation simple and easy. Sometimes using coordinates that are multiples of two will make the computation easier.

In coordinate proofs, you use algebraic tools like variables, equations, and formulas. You may use the following steps in proving involving coordinates.

1. Assign appropriate coordinates to the points of the figure, making sure that the figure is as general as necessary. For example, if we are proving the characteristics of a triangle, we cannot choose points to form special triangles, such as isosceles triangles or right triangles. The simplification we may use is to assign the x – or y – axis as one of the sides of our figure or assigning the origin as one of our points.

2. Use the coordinates to arrive at the conclusion. Some of the tools we have for proving are:
   a. The slope formula, which we can use to show that the segments are parallel or perpendicular.
   b. midpoint formula, which may be used to show if a segment is a bisector.
   c. distance formula, and recall that the length of the segment is defined as the distance between the endpoints, which means we may use the distance formula to show that two segments are congruent.

ACTIVITY 12.   Find Out How.

Study the properties of different geometric figures given below. Learn how the distance formula is used to prove these properties. Use the notepad below to write down important concepts or questions that you still have.

Geometric Figure No. 1 Parallelograms
- Parallelograms are also said to be a kind of quadrilaterals which has two sides as parallel sides.
- Parallelograms contain some of the types in it. The diagonals of the parallelograms also bisect each other at the centre point.
- The parallelograms and types are discussed in detail with their basic problem solving methods as below.

> **Steps to follow when working on Coordinate Geometry Proofs (Parallelograms)**
>
> **Step1**: First we draw and label the graph.
> **Step2**: Write down the geometric conditions that are required.
> **Step3**: State the formulas that are used
> **Step4:** Show all work
> **Step 5**: Have a concluding statement stating what you have proven and why it is true.

Example 1 The coordinates of the vertices of a quadrilateral are A(2,2), B(3, 5), C(6, 7) and D(5,4). Show that ABCD is a parallelogram.

Solutions:
Let's compare the distances like what we did in determining the location of the fly in the previous pages.

Use the steps to follow when working on coordinate geometry proofs involving parallelograms.

STEP 1: Draw and label the Figure or Graph.
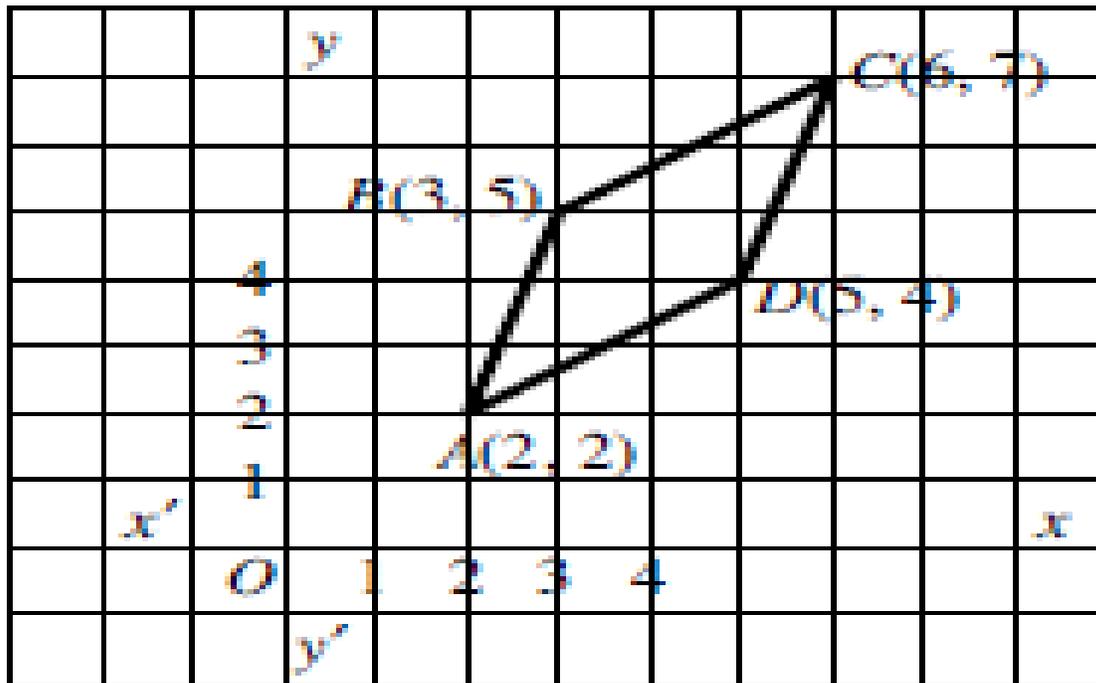
STEP 2: Write down the geometric conditions that are required.

Condition: Opposite sides are congruent.

STEP 3: State the formulas that are used

$$Distance\ A\ to\ B = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

STEP 4: Show all work

Solution 1: Distance from A to B

$$Distance\ A\ to\ B = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$Distance\ A\ to\ B = \sqrt{(3 - 2)^2 + (5 - 2)^2}$$
$$Distance\ A\ to\ B = \sqrt{(1)^2 + (3)^2}$$
$$Distance\ A\ to\ B = \sqrt{10}$$

Solution 2: Distance from C to D

$$Distance\ C\ to\ D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$Distance\ C\ to\ D = \sqrt{(6 - 5)^2 + (7 - 4)^2}$$

$$\text{Distance } C \text{ to } D = \sqrt{(1)^2 + (3)^2}$$
$$\text{Distance } C \text{ to } D = \sqrt{10}$$

Solution 3: Distance from B to C

$$\text{Distance } B \text{ to } C = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$\text{Distance } B \text{ to } C = \sqrt{(6-3)^2 + (7-5)^2}$$
$$\text{Distance } B \text{ to } C = \sqrt{(3)^2 + (2)^2}$$
$$\text{Distance } B \text{ to } C = \sqrt{13}$$

Solution 4: Distance from A to D

$$\text{Distance } C \text{ to } D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$\text{Distance } C \text{ to } D = \sqrt{(5-2)^2 + (4-2)^2}$$
$$\text{Distance } C \text{ to } D = \sqrt{(3)^2 + (2)^2}$$
$$\text{Distance } C \text{ to } D = \sqrt{13}$$

STEP 5: Have a concluding statement stating what you have proven and why it is true.

Conclusion: Thus $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$. Since opposite sides are congruent then ABCD is a parallelogram.

Example 2:
Show that the quadrilateral MNOP is a parallelogram where the coordinates A (-2, 0), B (2, 4), C (4, 1) and D (0, -3) defines their vertices.
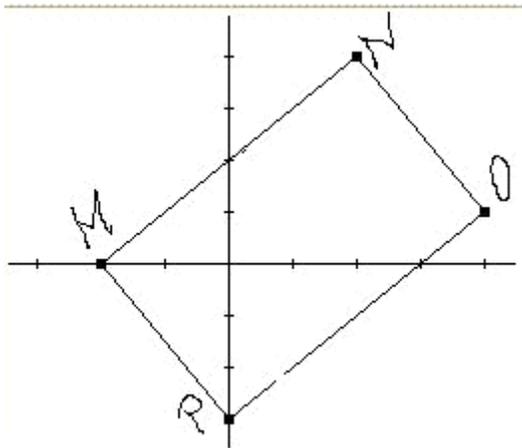
Solution:

By finding the slopes and distance of the given quadrilateral we can prove that this given quadrilateral is a parallelogram.

Step 1:
First, Determine the Slope of AB

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{4 - 0}{2 - (-2)}$$

$$m = \frac{4}{2 + 2}$$

$$m = \frac{4}{4}$$

$$m = 1$$

Second, Determine the Slope of BC

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{1 - 4}{4 - 2}$$

$$m = \frac{-3}{4}$$

Third, Determine the slope of CD

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-3 - 1}{0 - 4}$$

$$m = \frac{-4}{-4}$$

$$m = 1$$

Fourth, Determine the slope of AD

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-3 - 0}{0 - (-2)}$$

$$m = \frac{-3}{2}$$

Step 2: Determine the distance

First, Distance of the side AB

$$Distance\ AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(2 - (-2)^2 + (4 - 0)^2}$$
$$= \sqrt{(4)^2 + (4)^2}$$
$$= \sqrt{16 + 16}$$
$$= \sqrt{32}\ or\ 4\sqrt{2}$$

Second, Distance of the side BC

$$Distance\ BC = \sqrt{(2)^2 + (-3)^2}$$
$$= \sqrt{4 + 9}$$
$$= \sqrt{13}$$

Third, Distance of the side CD

$$Distance\ CD = \sqrt{(4)^2 + (4)^2}$$
$$= \sqrt{16 + 16}$$
$$= \sqrt{32}\ or\ 4\sqrt{2}$$

Fourth, Distance of side AD

$$Distance\ AD = \sqrt{(-2)^2 + (3)^2}$$
$$= \sqrt{4 + 9}$$
$$= \sqrt{13}$$

Conclusions:

- Segments AB and CD are parallel since they have equal slopes and also their lengths are seemed to be equal.
- Segments BC and DA are parallel since they have equal slopes and also their lengths are seemed to be equal.

Therefore, the properties of a parallelogram are proved in the given quadrilateral.

Geometric Figure No. 2 Trapezoid

Example: Given Coordinates of A (1,3) B (-1,1) C (-1,-2) D (4,3) . To prove ABCD is a isosceles trapezoid
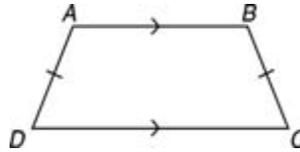
Solution:

First we will prove that it is Trapezoid. A trapezoid is a quadrilateral that has one pair of parallel sides and other pair as nonparallel sides. If the 2 non parallel sides are equal then we say the trapezoid is isosceles. We know that two lines are parallel when they have equal slopes

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope of AB $= \dfrac{1-3}{-1-1} = \dfrac{-2}{-2} = -1$

Slope of BC $= \dfrac{-3}{0} = 0$

Slope of CD $= \dfrac{5}{5} = 1$

Slope of AD $= \dfrac{0}{3} = 0$

Hence BC is not parallel to AD

Since slope of AB is equal to CD they are parallel

Conclusion: Since only 1 pair of sides have equal slopes, then we get only 1 pair of parallel sides. Hence, ABCD is a trapezoid. Now, we use the distance formula to prove that it is isosceles.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Se we will see distance of non parallel sides

Distance of BC$= \sqrt{(-1+1)^2 + (-2-1)^2}$

$= \sqrt{(0)^2 + (-3)^2}$

$= \sqrt{9}$

Distance of BC= 3

Distance of AD= $\sqrt{(4-1)^2+(3-3)^2}$

$= \sqrt{(3)^2+(0)^2}$

Distance of AD = 3

Since the non parallel sides have equal distance then they are congruent and trapezoid ABCD is isosceles.

ACTIVITY 13.   What am I?

The following coordinates represent the vertices of a polygon. Plot the points, and then classify the polygon. If the polygon is a triangle, classify it as equilateral, isosceles or scalene. If it is a quadrilateral, classify it as a trapezium, trapezoid or parallelogram.

1. (-2, 5), (0, -1), and (12, 3)

2. (0, 0), (-4, 3), (-1, 7) and (10, 5)

3. (-10, 5), (-6, 15), (14, 11) and (10, -9)

4. (2,2), (2, 4) and (4, 2)

5. (12, 3), (0, 12), (9, -1) and (-3, 8)

ACTIVITY 14. Test for Understanding

In the previous section, we looked at different problems about distance formula. In the table below, let's put together our answers to the essential question that we asked for each problem.

Essential Question	Problem 1	Problem 2	Problem 3
How can challenging problems involving geometric figures be analyzed and solved?	A new court which is rectangular in nature will be built in your barangay. The sports committee would like to paint the floor of the court. The paint to be used is .5 gallons per square meter.	The Sangguniang Kabataan would like to fence a vacant lot to be used as an herbal garden for the barangay. Pole 1 (4,7) (8,6) Pole 2 (9,2) Pole 3 (1,1) Pole 4	Your barangay is posted by 4 barangay outpost for any barangay concerns and to show its boundaries. How much do you think is covered by the barangay? (1:5square meters)
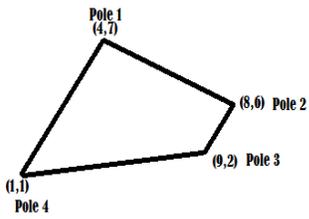
	How much wood will you be using for the fence. (1 meter = 5 pcs of wood)	
(1,6) (8,6) (1,1) (8,1) How many gallons of paint would you use if the plan of the court is given below? The problem was used using….	The problem was used using….	Outpost 1 (2,8) Outpost 3 (7,8) Outpost 2 (2,3) Outpost 4 (7,3) The problem was used using….
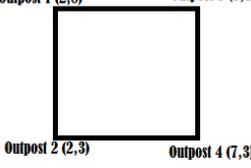
Process Questions:

1. Look at your answers to the essential question in the above table. What do all the answers have in common?
2. When will you know if a problem calls for the application of distance formula? What general statement can we make to guide us? Complete the following: We use the distance formula…

ACTIVITY 15.   Generalization Table

Fill in the unshaded columns of the generalization table below by stating your findings and corrections, supporting evidence and qualifying conditions for the question, then save your answer.

How can challenging problems involving geometric figures be analyzed and solved?				
My Initial Thoughts	*My Findings and Corrections*	*Supporting Evidence*	*Qualifying Conditions*	*My Generali-zations*

ACTIVITY 16.   Check This Out

Check the appropriate column based on your knowledge on the skills mentioned.

Skills	Crystal Clear	Somewhat Clear	Not Fully Understood
1. derive the distance formula			
2. solve using the distance formula			
3. determine the midpoint			
4. apply the distance formula to prove some geometric properties			
5. Solve problems involving geometric figures on the coordinate plane			

END OF DEEPEN

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.

TRANSFER

All throughout the lesson, the question about how can radicals help in your daily life was asked repeatedly. The Transfer section of the lesson will guide you in determining the best answer to the question. **How can problems where two quantities bounded by conditions are solved? How do related quantities affect each other?** How is the knowledge of radical expressions and equations used to solve real-life problems?

Your goal in this section is apply your learning to real life situations. You will be given a practical task which will demonstrate your understanding.
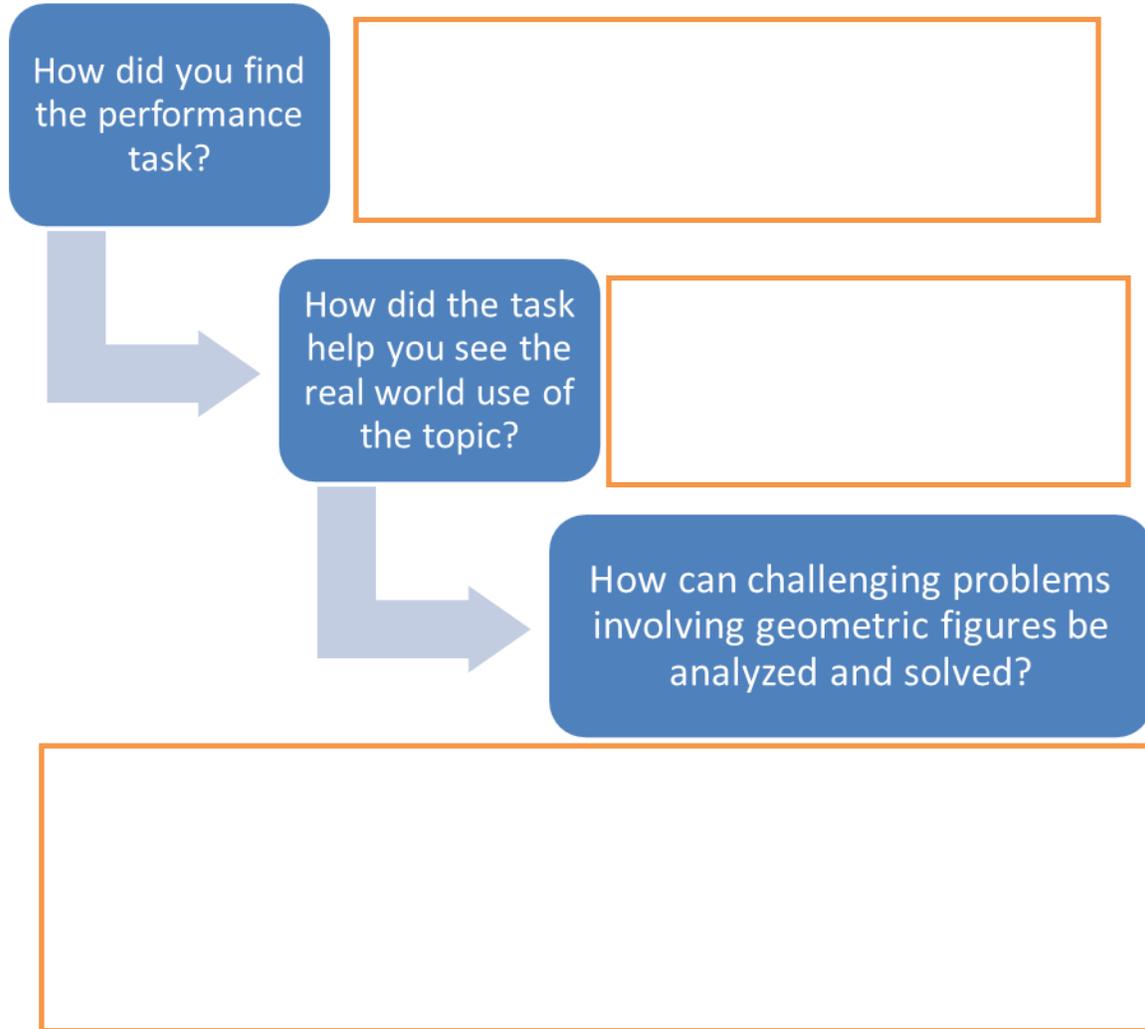
ACTIVITY 17.   The Performance Task

Suppose you are the head of the barangay security in your barangay that is planning to purchase CCTV cameras to be distributed in the barangay. Your group wants to make sure that the CCTV are evenly distributed, so, the barangay captain asks you to prepare a vicinity map of your barangay in which the CCTV cameras are to be distributed. Your product will be graded according to authenticity of data, accuracy, and applications of the mathematical concept

Criteria	4 Excellent	3 Proficient	2 Progressing	1 Beginning
Authenticity of Data	The data used are authentic and updated. Data are taken from reliable resources.	The data used are authentic. Data are taken from reliable resources.	Some of the data used are not authentic.	None of the data are useful and authentic.
Accuracy of the Computations	Computations are accurate and supported with correct and clear interpretation.	Computations are accurate and supported with correct interpretation	Computations are correct but interpretation is incorrect.	Most of the computations ad interpretations are erroneous.
Application of the Concepts of Distance Formula	The Distance Formula created is correct with additional model to support it.	The Distance Formula created is correct.	The Distance Formula created does not use the defined variables.	The Distance Formula used is incorrect.

ACTIVITY 18.   Connect Me

Answer the following by reflecting on the following questions.

How did you find the performance task?	

How did the task help you see the real world use of the topic?	

How can challenging problems involving geometric figures be analyzed and solved?

Now that you have completed your project, you may do the next activity.

End of TRANSFER:

In this section, your task was to create a Powerpoint presentation and a written proposal containing the possible or the best location for the CCTV to be installed that will help the barangay in terms of security. In this section it is expected that you are able to learn how distance between coordinates using the distance formula and be able to prove the properties of different polygons given coordinates in the coordinate plane.

How did you find the performance task? How did the task help you see the real world use of the topic?

You have completed this lesson. Before you go to the next lesson, you have to answer the following post-assessment.

POST-ASSESSMENT:

It's now time to evaluate your learning. Click on the letter of the answer that you think best answers the question. Your score will only appear after you answer all items. If you do well, you may move on to the next module. If your score is not at the expected level, you have to go back and take the module again.
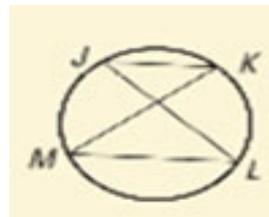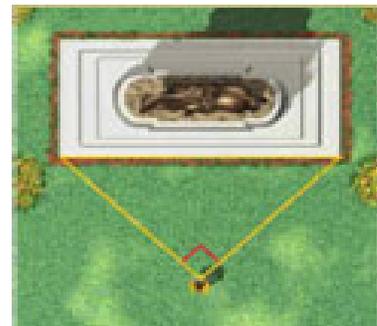
1. Which of the following is true about the measures of central angles in relation to the intercepted arc?
    A. The measure of the intercepted arc is half of the measure of the central angle.
    B. The measure of the intercepted arc is twice the measure of the central angle.
    C. The measure of the central angle is equal to the measure of the intercepted arc.
    D. The measure of the central angle is half of the measure of the intercepted arc.

2. Name two pairs of congruent angles using the figure below.

    A. $\angle JKM \cong \angle KJL \ \& \ \angle JLM \cong \angle KML$
    B. $\angle JLM \cong \angle KJL \ \& \ \angle JKM \cong \angle KML$
    C. $\angle JKM \cong \angle JLM \ \& \ \angle KJL \cong \angle KML$
    D. $\angle JLM \cong \angle KJL \ \& \ \angle JLM \cong \angle JKM$

3. A professional photographer has a camera that has a 90^0 field of vision and he wants to photograph the front of a statue. He moves to a spot where the statue is the only thing captured in your picture, as shown. After seeing this, you are also encouraged to capture the photograph of the statue but your camera has less than 90^0 field of vision. Which of the following should be your strategy to capture the whole picture?
    A. You may move closer to the statue.
    B. You may move farther from the statue.
    C. You may move to the other side of the statue.
    D. Look for an elevated place to capture the statue.

5. You are a member of the board who will assess the different proposals who are applying for bidding for CCTV cameras to be installed in different places. Which among the following criteria should you consider most?
    A. Authenticity of data, accuracy, practicality of proposal
    B. Accuracy, organization, practicality of proposal
    C. Practicality, neatness, accuracy of proposal
    D. Organization, practicality, neatness of proposal

5. A surveillance camera is mounted on a corner of a building. It rotates clockwise and counterclockwise continuously between Wall A and Wall B at a rate of 15^0 per minute as illustrated in the figure. How long does it take the camera to survey the entire area once? Justify your answer.
   A. 3 minutes, since the measure of the angle is half of the intercepted arc.
   B. 6 minutes, since the angle and arc have equal measures.
   C. 12 minutes, since the measure of the angle is twice the measure of the intercepted arc.
   D. 18 minutes, the camera needs to survey about 135^0 of the circle.

6. If rays WE and WI are both tangents to the circle P below, then which of the following is NOT necessarily true?
   A. AO = OB
   B. m∠PAO =m∠PBO
   C. ∠AOB ≅∠APB
   D. AP = BP

7. If ray m and n are tangents to circle P below, then which of the following is the value of w?

   A. 30°   B. 40°   C. 50°   D. 60°

8. Low – Earth orbit satellites occupies a region of space from 180 – 2000 kilometers above Earth. If the satellite is moving in a circular path with Earth as the point of origin, which circular equation represents the maximum path of the satellite?
   A. $x^2 + y^2 = 180$
   B. $x^2 + y^2 = 2000$
   C. $x^2 + y^2 = 32,400$
   D. $x^2 + y^2 = 4,000,000$

11. Which of the following points give a distance of 4 units?
    A. (3,3) and (8,6)
    B. (11,3) and (8,6)
    C. (3,9) and (8,6)
    D. (7,3) and (8,6)

12. What is the perimeter of the square if the endpoints of one side are (6,5) and (9,5)
    A. 18
    B. 16
    C. 12
    D. 8

13. On a map's coordinate grid, how far is Brgy. Bangal to Brgy. Canda if their location in the coordinated grid is (2, 4) and (2, 2) respectively.
    A. 2km
    B. 4km
    C. 6km
    D. 8 km

14. Which of the following coordinates must added to A(2,3), B(4,4), D(-1,-1) to prove that the given figure is a kite?
    A. C(3,2)
    B. C(3,5)
    C. C(3,7)
    D. C(2,5)

The JDC Cargo ship is sailing from Zhanjiang China to Honolulu, and then to Long Beach. The amount of diesel used of the cargo sheep in travelling is 100 gallons for every 10 miles. Using the information in the figure below, determine the number of gallons used by the cargo from Zhanjiang
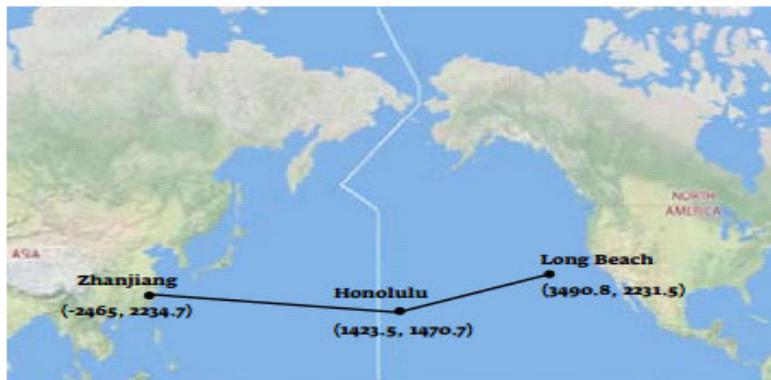
to Long Beach.
    a. 616.56 gallons
    b. 600 gallons
    c. 700 gallons
    d. 716.65 gallons

15. In the municipal vicinity map the coordinate of your barangay hall is (2, -1) and the school is (-2,2). How long will it take you to go to school from the barangay hall if the tricycle that you are riding is at 10km/hr?
    a. 30minutes
    b. 45 minutes
    c. 1 hour
    d. 1 hr and 10 minutes

16. Low – Earth orbit satellites occupy a region of space from 180 – 2000 kilometers above Earth. If the satellite is moving in a circular path with Earth as the point of origin, which circular equation represents the maximum path of the satellite?
    A. $x^2 + y^2 = 180$
    B. $x^2 + y^2 = 2000$
    C. $x^2 + y^2 = 32,400$
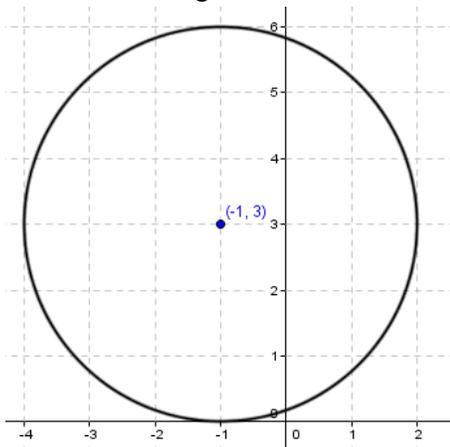   D. $x^2 + y^2 = 4000000$

17. Based on the figure below, what equation that represents the circle?

III. $(x + 1)^2 + (y - 3)^2 = 9$
IV. $(x - 1)^2 + (y + 3)^2 = 9$
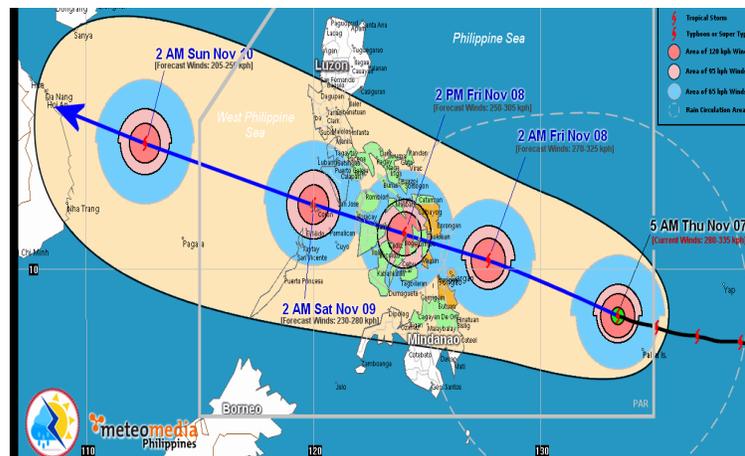III. $x^2 + y^2 +2x - 6y + 1= 0$

A. I only
B. II and III only
C. I and III only
D. I, II and III

18. To determine the radius and coordinates of the center of the circle $x^2 + y^2 + 4x - 8y + 4 = 0$, Jerry and Akzie came up with the following solutions:

Jerry's Solution	Akzie's Solution
$x^2 + y^2 + 4x - 8y + 4 = 0$	$x^2 + y^2 + 4x - 8y + 4 = 0$
$x^2 + 4x +$ ___ $+ y^2 - 8y +$ ___ $= -4$	$x^2 + 6x +$ ___ $+ y^2 - 8y +$ ___ $= -4$
$x^2 + 4x + 2^2 + y^2 - 8y + (-4)^2 = -4$	$x^2 + 4x + 2^2 + y^2 - 8y + (-4)^2 = -4 + 4 + 16$
$(x + 2)^2 + (y - 4)^2 = -4$	$(x + 2)^2 + (y - 4)^2 = 16$
Radius = $\sqrt{-4}$; center: ($-2, 4$)	Radius = 4; center: ($-2, 4$)

Which statement best describes the solutions of Jerry and Akzie?
   A. The solution of Jerry is correct.
   B. The solution of Akzie is correct.
   C. The solution of Jerry and Akzie are both correct.
   D. The solution of Jerry and Akzie are both incorrect.

19. When delivering weather forecast during typhoons, weather forecasters use diagrams and charts to illustrate the movement of the typhoon. The multiple colored circles indicate the effect of the typhoon in the area at a given time. If you are the weather forecaster, which statement best describes typhoon Haiyan based on the illustration below?

   A. The area affected by 128 kph winds of Haiyan decreased from Nov. 7 – 10.
   B. The area affected by 95 kph winds of Haiyan decreased from Nov. 7 – 10.

C. The area affected by 95 – 128 kph winds of Haiyan was maintained from Nov. 7 – 10.
D. The area affected by Haiyan decreased from Nov. 7 – 10.

20. The barangay officials recently reported alarming news that the number of crimes in your community is drastically increasing. The barangay council decided to put up a CCTV camera in the area with the most number of crime incidents. You are a marketing officer of a CCTV company that will participate in the bidding to provide the barangay with the CCTV unit. You need to identify the best location of the CCTV unit in the area that is identified in your barangay. Your work should have authentic data, accurate computation, should have practical recommendations, and with correct application of the concepts of the properties of circles. What should you do to convince the barangay officials?
   A. You will present the CCTV to the barangay officials.
   B. You will present the CCTV and layout to the barangay officials.
   C. You will present your recommendation to the barangay officials.
   D. You will present your recommendation and justification to the barangay officials.

GLOSSARY OF TERMS USED IN THIS LESSON:

Distance is an amount of space between two things or people.

Distance Formula is used to determine the **distance**, d, between two points. If the coordinates of the two points are (x_1, y_1) and (x_2, y_2), the **distance** **e**quals the square root of $x_2 - x_1$ squared $+ y_2 - y_1$ squared.

REFERENCES AND WEBSITE LINKS USED IN THIS LESSON:

https://www.youtube.com/watch?v=RrrYInyIEGo
The Cartesian coordinate system, formed from the Cartesian product of the real number line with itself, allows algebraic equations to be visualized as geometric shapes in two or three dimensions.

Lesson 3: Circles

☑ **INTRODUCTION AND FOCUS QUESTION(S):**

Is the figure above familiar? When do you usually see it?

When describing the size of a typhoon, weather forecasters usually use geometric figures to describe the situation. What is the significance of the circles?

In this module, you will learn more about circles and how they are used. Remember to search for the answer to the following question: **How challenging situations are to be analysed and solved?**

☑ **LESSONS AND COVERAGE:**

In this module, you will examine this question when you take the following lessons:
Lesson 1 – Graphing circles
Lesson 2 – Equations of circles

In these lessons, you will learn the following:

Lesson 1	• Graphs a circle on the coordinate plane
Lesson 2	• Illustrates the center-radius form of the equation of a circle
	• Determines the center and radius of a circle given its equation and vice versa

☑ **MODULE MAP:**

Here is a simple map of the above lesson you will cover:
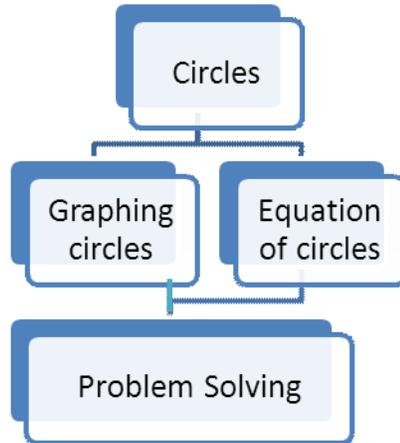
☑ **EXPECTED SKILLS:**

To do well in this module, you need to remember and do the following:
1. Follow the instructions provided for each activity.
2. Complete all activities and exercises.
3. Review and evaluate your work using the rubric provided before submission.
4. Be mindful of the meaning of unfamiliar words you encounter in this module. A glossary of terms is provided in the last part of this module.
5. Maximize the use of online resources in each lesson. Online resources can be accessed multiple times. The summary of online resources is provided in the end of the module.

PEAC PRIVATE EDUCATION ASSISTANCE COMMITTEE *Trustee of the Fund for Assistance to Private Education*   **DepED** DEPARTMENT OF EDUCATION

JHS INSET Learning Module Exemplar

PRE-ASSESSMENT:

> 💬 Let's find out how much you already know about this module. Answer the pre-test below.

Click on the letter that you think best answers the question. Please answer all items. After taking this short test, you will see your score. Take note of the items that you were not able to correctly answer and look for the right answer as you go through this module.

1. Which of the following defines a chord?
   A. A line that passes through the circle at exactly one point.
   B. A line that passes through the circle at exactly two points.
   C. A segment that passes through the circle at exactly one point.
   D. A segment whose endpoints are on the circle.

2. If the measure of the central angle is 45^0, what is the measurement of the intercepted arc?
   A. 22.5^0
   B. 45^0
   C. 90^0
   D. 135^0

3. You are a professional photographer. Your camera has a 90 field of vision and you want to photograph the front of a statue. You move to a spot where the statue is the only thing captured in your picture, as shown. You want to change your position. Where else can you stand so that the statue is perfectly framed?

   A. You may stand to any point on the semi- circle in front of the statue.
   B. You may stand to any point equidistant from your distance to the statue.
   C. You may move to the other side and stand to any point equidistant from your distance to the statue.
   D. There is no other place where you can capture the whole picture of the statue but your original position.

4. A surveillance camera is mounted on a corner of a building. It rotates clockwise and counterclockwise continuously between Wall A and Wall B at a rate of 10^0 per minute as illustrated in the figure. How long does it take the camera to survey the entire area once? Justify your answer.

A. 4 minutes and 30 seconds, since the measure of the angle is half of the intercepted arc.
B. 9 minutes, since the angle and arc have equal measures.
C. 18 minutes, since the measure of the angle is twice the measure of the intercepted arc.
D. 27 minutes, the camera needs to survey about 135^0 of the circle.

5. You are a professional photographer, a company requested you to submit a set of pictures of beautiful sceneries. Which among the following should you consider most to in selecting your pictures?
A. Pictures are very clear but some important parts are not captured.
B. All parts are captured but no highlighted subject.
C. You have a focus subject and important parts are visible.
D. All parts are captured but some parts are not that clear.

6. If rays WE and WI are both tangents to the circle P below, then which of the following is NOT necessarily true?

A. EW = IW
B. m∠PEW =m∠PIW
C. ∠EWP ≅∠IWP
D. ∠PEW and ∠EWI are supplementary.

7. If ray m and n are tangents to circle P below, then which of the following is the value of w?

    C. 30°   B. 40°   C. 50°   D. 60°

If ray m and n are tangents to circle P below, then which of the following is the value of w?

      A. 30°   B. 40°   C. 50°   D. 60°

8.   In the figure below, if segment MT is tangent to the circle, segments NT and PT are secants, PS is a semicircle and the measure of minor arc MP is 140°, then what is the measure of ∠MTP?

      A. 50°   B. 60°   C. 90°   D. 100°

9.   If RS = VW in the figure below, then which of the following will justify that △RVX is congruent to △WSX?

      A. SAA Congruence Theorem   B. SAS Congruence Postulate
      C. SSS Congruence Postulate   D. Hypotenuse-Leg Theorem

10. As an Archeologist, you volunteer to help a group of students to determine the center of a circular object that they have found. Which of the following will help you explain how the T-square will be an effective instrument to do so?

      A. Tangents are perpendicular to the radii at the points of tangency.
      B. The tangent segments intersecting at a point outside a circle are congruent.
      C. The measure of an angle formed by two secants intersecting outside a circle is half the measures of the difference of the intercepted arcs of the circle.

Developed by the Private Education Assistance Committee
under the GASTPE Program of the Department of Education
    102

D. The measure of an angle formed by a secant and a tangent to a circle, intersecting outside a circle is half the measures of the difference of the intercepted arcs of the circle.

11. What is the distance between the points (3,3) and (6,7)?
    A. 3   C. 7
    B. 5   D. 9

12. The point (5,4) lies on a circle. What is the length of the radius of this circle if the center is located at (3,2)?
    A. $\sqrt{8}$ C. $\sqrt{18}$
    B. $\sqrt{3}$ D. 8

13. If plotted in the coordinate plane, which of the following geometric figures best describes the coordinates A(-6,4), B(-6, -4), C(0,4) and D(0, -4)?
    A. Square   C. Rhombus
    B. Rectangle   D. Kite

14. On a map's coordinate grid, Brgy. Malaya is located at (2, 4) and Brgy. Pasong Tamo is located at (2, 2). Brgy. Katapan is the in the middle of the two barangay. What is the distance from Brgy. Malaya to Brgy. Katapatan?
    A. 2km   C. 6km
    B. 4km   D. 8km

15. You are a city planner who is requested by the city mayor to provide a vicinity map to be used in the municipal strategic planning. Which of the following standards will be assessed your work?
    A. Accuracy, Clarity, application of the concept of distance formula
    B. Accuracy, Clarity, practicality
    C. Practicality, authenticity, use of grid lines
    D. Practicality, Accuracy, presentation

16. Low – Earth orbit satellites occupies a region of space from 180 – 2000 kilometers above Earth. If the satellite is moving in a circular path with Earth as the point of origin, which circular equation represents the lowest path of the satellite?
    A. $x^2 + y^2 = 180$
    B. $x^2 + y^2 = 2000$
    C. $x^2 + y^2 = 32{,}400$
    D. $x^2 + y^2 = 4{,}000{,}000$

Developed by the Private Education Assistance Committee
under the GASTPE Program of the Department of Education
103

17. Based on the figure below, what equation represents the circle?

    I. $(x - 3)^2 + (y - 1)^2 = 5$
    II. $x^2 + y^2 - 6x - 2y + 5 = 0$
    III. $x^2 + y^2 - 6x - 2y - 5 = 0$

A. I only
B. II only
C. I and II only
D. I, II and III

18. To determine the radius and coordinates of the center of the circle $x^2 + y^2 - 6x + 2y + 8 = 0$, Nestor and Andy came up with the following solutions:

Nestor's Solution	Andy's Solution
$x^2 + y^2 - 6x + 2y + 8 = 0$	$x^2 + y^2 - 6x + 2y + 8 = 0$
$x^2 - 6x + \_\_ + y^2 + 2y + \_\_$ $= -8$	$x^2 - 6x + \_\_ + y^2 + 2y + \_\_ =$ -8
$x^2 - 6x + (-3)^2 + y^2 + 2y +$ $(1)^2 = -8$	$x^2 - 6x + (-3)^2 + y^2 + 2y + (1)^2$ $= -8 + 9 + 1$
$(x - 3)^2 + (y + 1)^2 = -8$	$(x - 3)^2 + (y + 1)^2 = 2$
Radius = $\sqrt{-8}$; center: (3, - 1)	Radius = $\sqrt{2}$; center: (3, - 1)

Which statement best describes the solutions of Nestor and Andy?
    A. The solution of Nestor is correct. Andy committed an error in when he added 9 and 1 on the right side of the equation.
    B. The solution of Andy is correct. Nestor committed an error when he failed to add 9 and 1 on the right side of the equation.
    C. The solution of Andy and Nestor are both correct.
    D. The solution of Andy and Nestor are both incorrect.

19. When delivering weather forecast during typhoons, weather forecasters uses diagrams and charts to illustrate the movement of the typhoon. The circles indicate the affected area of the typhoon at a particular time. If you are the forecaster, which statement best describes typhoon Henry based on the illustration below?

Track of Typhoon "HENRY"

    A. Typhoon Henry grew stronger from July 20 – 23, 2014.
    B. Typhoon Henry move faster from July 20 – 23, 2014.
    C. Typhoon Henry's affected area increased from July 20 – 23, 2014.
    D. Typhoon Henry's affected area did not change from July 20 – 23, 2014.

20. The barangay officials recently reported alarming news that the number of crimes in your community is drastically increasing. The barangay council decided to put up a CCTV camera in the area with the most number of crime incidents. You are a marketing officer of a CCTV company that will participate in the bidding to provide the barangay with the CCTV unit. You need to identify the best location of the CCTV unit in the area that is identified in your barangay. You will present your recommendation and justification to the barangay officials. Which are the appropriate standards that your work should be evaluated?

    A. organization, neatness, practical recommendations, application of the concepts of the properties of circles
    B. neatness, presentation, practical recommendations, application of the concepts of the properties of circles
    C. creativity, accurate computation, practical recommendations, application of the concepts of the properties of circles
    D. authentic data, accurate computation, practical recommendations, application of the concepts of the properties of circles

EXPLORE

Let's begin by gathering your thoughts about circles.

Can you draw a perfect circle? How? How are circles used in describing areas and distances?

ACTIVITY 1. Drawing a Circle

DESCRIPTION: Visit the website below and observe how the circle is drawn. Try to draw a circle in a piece of paper.

https://www.youtube.com/watch?v=4jLN56GlNHQ
A video showing how to draw a perfect circle

PROCESS QUESTIONS:

1. What are the materials used to draw the circle?

2. What is the secret in drawing a perfect circle?

3. Drawing circles can be challenging to others. How will you help your friend draw a perfect circle?

ACTIVITY 2.   Picture Analysis A

DESCRIPTION: Below is an image you need to identify.
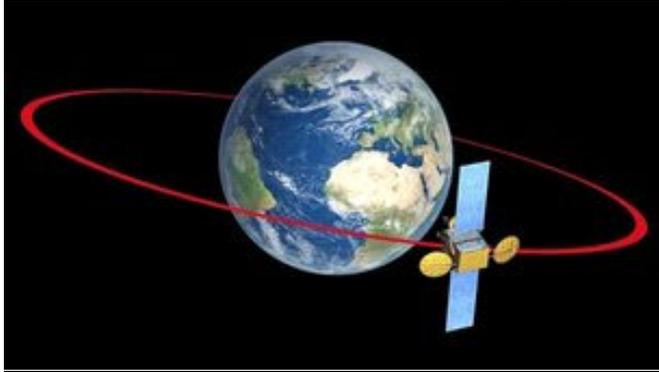
http://www.esa.int/Our_Activities/Telecommunications_Integrated_Applications/Orbits

PROCESS QUESTIONS:

1. What is the image all about?

2. Describe the path.

3. What is the location of the Earth with respect to the path of the object?

ACTIVITY 3.   Picture Analysis B

DESCRIPTION: Below is a graphical representation of the forecast of typhoon Santi given last October 10, 2013. Analyze the picture then answer the process questions. Click "SUBMIT" when you're finished.

http://weather.com.ph/announcements/tropical-storm-nari-santi-update-number-004
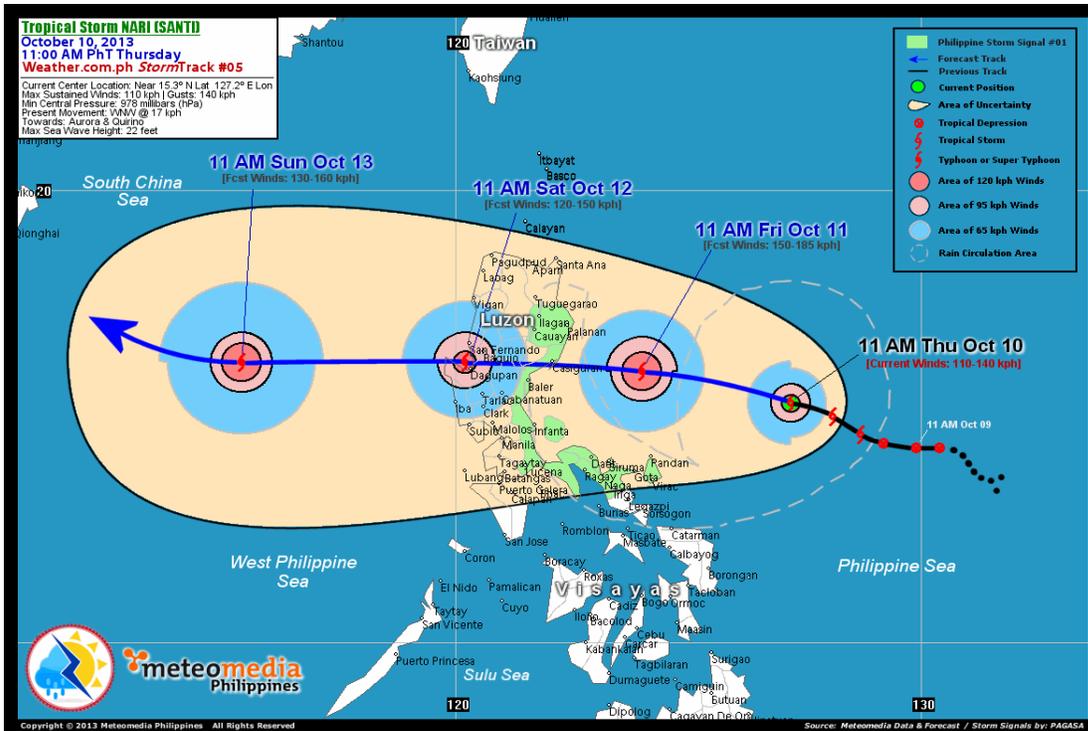
PROCESS QUESTIONS:

1. What is your observation based on the image?

2. How wide is the area affected by the typhoon?

3. What is the significance of the circles used in the forecast?

4. When forecasters provide image like this, does it help you understand the situation better? Why?

After drawing circles and analyzing pictures related to circles, try to complete the table below. You will be asked in the next section to revisit and possibly revise you answer.

ACTIVITY 4.   GENERALIZATION TABLE

DESCRIPTION: Fill in the first column of the generalization table below by stating your initial thoughts on the question, then save your answer. As you progress through this module, you will be asked to revisit your initial ideas. Click "SAVE" when you're finished.

How can challenging problems involving geometric figures be analyzed and solved?

My Initial Thoughts	My Findings and Corrections	Supporting Evidence	Qualifying Conditions	My Generalizations

End of EXPLORE:

The activities above show how circles can be drawn and how circles can be used in a real – life scenario. You'll get to learn more about circles and its properties and applications. You will also find out the answer to the question "How challenging problems involving geometric figures be analysed and solved?"

FIRM-UP

Your goal in this section is to learn and understand key concepts about circles.

As the concepts of circles become clear to you through the succeeding activities, do not forget to think about how to apply these concepts in solving real – life problems. Throughout the module, you will be asked "How challenging problems involving geometric figures be analysed and solved?"

In Activity 2, you have seen the path of Typhoon _____.

In Activity 3, you have seen the path of Typhoon Santi. The circle indicates the predicted area affected by the typhoon at a particular time. The highlighted center of the circle is the center of the typhoon. You can read more about Typhoon Santi in http://weather.com.ph/announcements/tropical-storm-nari-santi-update-number-004. In Activity 2, the satellite revolves around Earth, Earth being the center of the circle.

As you have seen also in the video of Activity 1, the center of the circle and the radius determines the size of the circle.

Also in Lesson 2, you tried to plot points and draw figures in the rectangular coordinate plane. Circles can be drawn also in the coordinate plane if the coordinates of the center and length of the radius are provided. Example, the circle whose center is at (3,2) and the radius is 2 units is drawn below.
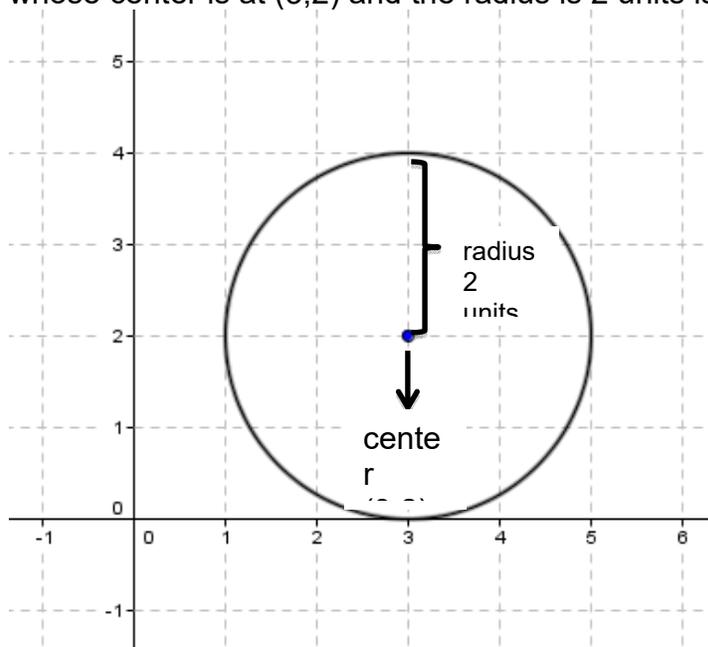
Earlier in this module, a circle is described as a plane figure. However, with the illustration above, we have to be more explicit in defining a circle.
A **circle** is the set of all points on a plane that are equidistant from a fixed point on the plane. The fixed point is called the **center**, and the distance from the center to any point on the circle is called the **radius**.

Try to draw circles with the help of Geogebra software by doing the next activity.

ACTIVITY 5.   Practice with Geogebra

DESCRIPTION: Access the Geogebra website then download and install the software. Follow the instructions below.

1. Downloading and installing
   a. Download the software in http://www.geogebra.org/cms/en/download/
   b. Install in your computer by clicking the installation file. Follow the instructions to install Geogebra.

2. Using Geogebra
   a. Open Geogebra. To explore tutorials, click "Help" then choose "tutorials". You will be directed to a website for the tutorials.
   b. To graph a circle, open Geogebra. Click "View" then choose "Algebra".
   c. Close "Algebra" by clicking "x".

   d. Click ⊥ ▦ to show or hide the axes and grid. Make the grid appear.

   e. Clik ⊙ to draw a circle.
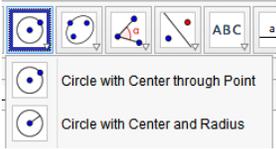
   f. Choose "Circle with center and radius".

   g. Click the cursor at (3,2) in the coordinate plane. You should see a dot. That is the center of the circle. Then, you will asked to input the length of the radius.
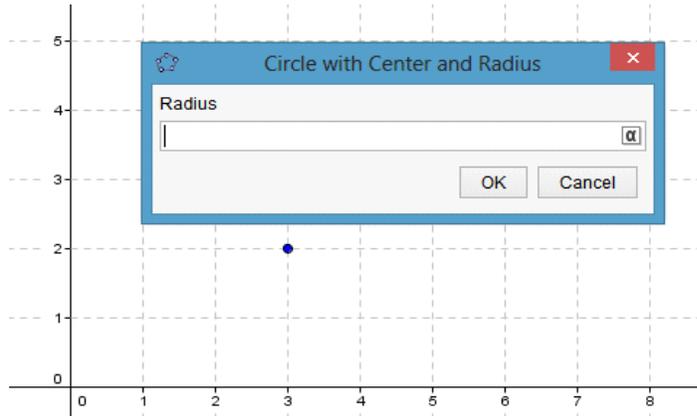
   h. Input the radius as 2, then click "OK".
   i. You should be able to see the graph of the circle by now. It should be the same with the circle drawn in the previous page.

j. Now try to graph a circle with center (0,1) and radius 3 units. Your graph should look like this:
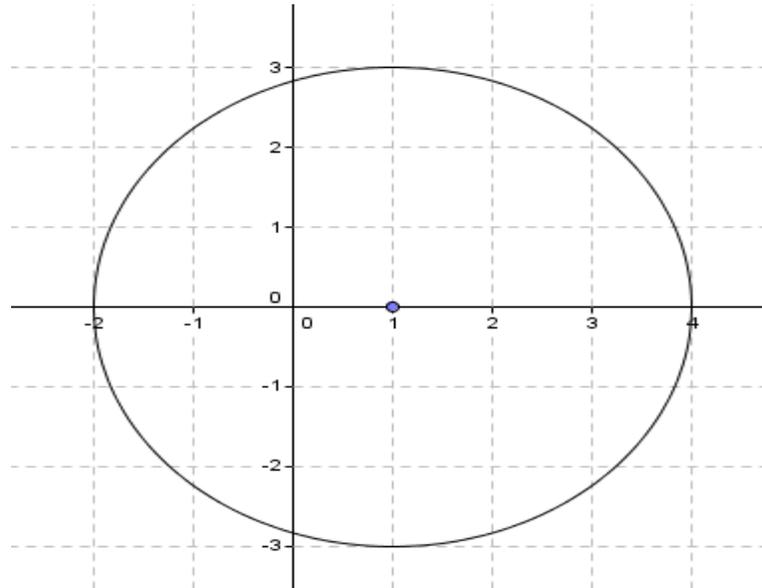
If you need more help in using Geogebra to draw circles, do Activity 4 again. If you are already confident you know how to use it, proceed to Activity 5. You can also access the about graphing circles by clicking the website below.
http://www.dummies.com/how-to/content/how-to-graph-a-circle.html
An article about graphing circles

ACTIVITY 6.   Graphing Circles through Geogebra

DESCRIPTION: Graph the following circles, with the specified center and radius, using Geogebra. Email your output to your teacher.
1. Center (1,3); radius: 3 units
2. Center (0,0); radius: 1 unit
3. Center (2, 0); radius: 2 units
4. Center (-2, 1); radius: 4 units
5. Center (5-1); radius: 3.5 units

Using a software makes drawing circles very easy. However, drawing circles in a paper can be challenging. But with the help of a compass, you should be able to draw circles perfectly. Do the next activity to develop your skills in drawing circles using a compass. You can always compare your drawing with the drawing using Geogebra.

ACTIVITY 7.   Graphing Circles with a Compass

DESCRIPTION: Graph the following circles in a graphing paper, with the specified center and radius, using a compass. Access the website below to watch a video on how to draw circles using a compass.

http://www.youtube.com/watch?v=oYQbj0MWt2k
A video showing how to draw a circle in a paper using a compass.

http://www.sophia.org/tutorials/video-two-how-to-draw-circle-using-a-compass-and-r
A video showing how to draw a circle in a paper using a compass.

http://www.youtube.com/watch?v=-B1VtI6ZoE8
A video showing how to draw a circle in a paper using a compass.

Scan then email your output to your teacher.
1. Center (3,1); radius: 2 units
2. Center (4,0); radius: 3 units
3. Center (-1, -2); radius: 4 units
4. Center (0,0); radius: 5 units
5. Center (-3, 0); radius: 1 unit

Before passing your work to your teacher, you can compare the graph you made with the graph using the interactive websites below:

http://www.mathwarehouse.com/geometry/circle/interactive-circle-equation.php
An interactive website used for graphing circles

http://www.webmath.com/gcircle.html
An interactive website used for graphing circles
Now that you know how to graph circles using Geogebra and compass, revisit your previous ideas by updating the generalization table you completed in Activity 4.

ACTIVITY 8.   GENERALIZATION TABLE REVISITED

DESCRIPTION: Fill in the unshaded columns of the generalization table below by stating your findings and corrections, supporting evidence and qualifying conditions on the question, then save your answer.

How can challenging problems involving geometric figures be analyzed and solved?

My Initial Thoughts	My Findings and Corrections	Supporting Evidence	Qualifying Conditions	My Generali-zations

Now that you're finished in this section, let's proceed to the next part to deepen your understanding about circles. Click **this link** to revisit the required competencies for circles. When ready, proceed to the next activity.

End of FIRM UP:

In this section, the discussion was about graphing circles

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision? What new learning goal should you now try to achieve?

> *Your goal in this section is to take a closer look about circles. Click this link to check on the expected competencies for joint and combined variations. As you go on, be focused on how can challenging problems be analyzed and solved?*

DEEPEN

In the previous lesson, you learned about the distance formula which is used in determining the distance between two points. In the previous topic, you have learned to draw a circle with Geogebra software and compass if the coordinates of the center and radius are given. However, how are you going to draw a circle if the center and radius are not given? How do we form equations of circles? Before we discover how to derive equations of circles, answer this activity.

ACTIVITY 9.   ANTICIPATION – REACTION GUIDE

DESCRIPTION: Determine whether the statement is true or false under the INITIAL column by placing a check mark. After the discussion, you will be asked to revisit your answer and revised if necessary. Click "SAVE" when you're finished.

INITIAL		STATEMENT	REVISED	
True	False		True	False
		1. The center – radius form of the equation of a circle is $x^2 + y^2 = r$.		
		2. The standard form of a circle is $x^2 + y^2 + Dx + Ey + F = 0$		
		3. The general form of a circle is $(x – h)^2 + (y – k)^2 = r^2$		
		4. The circle with the equation $x^2 + y^2 = 4$ has a radius of 4.		
		5. The center of the circle with the equation $x^2 + y^2 = 4$ is at (0,0).		
		5. The general form of $x^2 + y^2 = 4$ is $x^2 + y^2 = 2^2$.		
		6. The center of the circle with the equation $(x – 2)^2 + (y – 3)^2 = 8^2$ is at (–2, –3).		
		7. The center of the circle with the equation $(x + 2)^2 + (y + 3)^2 = 8^2$ is at (2,3).		

Every circle can be represented by an equation. The equation of a circle can be derived through the distance formula. To know more about deriving the formula, do the next activity.

ACTIVITY 10.   DERIVING THE EQUATION OF A CIRCLE

DESCRIPTION: Access the websites to learn about deriving the equation of a circle. In the next activity, you will be asked to summarize your thoughts.

http://hotmath.com/hotmath_help/topics/equation-of-a-circle.html
An article showing how the equation of the circle was derived .

http://www.sophia.org/tutorials/equations-of-a-circle
The website contains articles and videos showing how the equation of the circle was derived.

ACTIVITY 11.   THE EQUATION OF A CIRCLE

DESCRIPTION: After reading the articles and watching videos about the equation of a circle. Use your Evernote account in www.evernote.com to take notes . Then summarize your thoughts in the table below. **Read aloud** what you've written in the box. Click "SAVE" when you're finished.

Distance formula	Distance between the points (h,k) and (x,y) with radius r	Distance between the point of origin (0,0) and (x,y) with radius r
Equation of a circle in **General Form**	Equation of a circle with center (h,k) and radius r (**standard form**)	Equation of a circle with center at the origin

Example	Example	Example

PROCESS QUESTIONS:

1. What is the difference of the standard form and the general form of a circle?

2. What is the advantage of writing the equation of a circle in standard form with that of the general form?

Now that you know the derivation and different forms of the equations of a circle, let us try to solve some problems.

Example 1
Write the equation of a circle with center at (0,0) and a radius of 9 units.

Solution
The equation of a circle with its center at (0,0) is $x^2 + y^2 = r^2$, where r is the radius.
Thus:
$x^2 + y^2 = 9^2$ or
$x^2 + y^2 = 81$

Example 2
Write the equation of a circle with center at the origin and a radius of ½ units.
Write the equation in general form.

Solution
The equation of a circle with its center at the origin is $x^2 + y^2 = r^2$, where r is the radius. Thus:
$x^2 + y^2 = (\frac{1}{2})^2$ or
$x^2 + y^2 = \frac{1}{4}$

Converting it to general form,
$x^2 + y^2 - \frac{1}{4} = 0$

Example 3
Find the equation of the circle of radius 11 which is centered at (-5,-3). Write the equation in general form.

Solution
If the center of a circle is at $(-5, -3)$ and the radius is 11, the equation of the circle to be used is $(x - h)^2 + (y - k)^2 = r^2$ with $(h,k)=(-5,-3)$ and $r=11$. Thus, substituting the values yields:
$(x - (-5))^2 + (y - (-3))^2 = 11^2$
$(x + 5)^2 + (y + 3)^2 = 121$

Converting it to general form,
$(x^2 + 10x + 25) + (y^2 + 6y + 9) = 121$
$x^2 + y^2 + 10x + 6y - 87 = 0$

Example 4
Find the equation of the circle of radius $\sqrt{5}$ which is centered at $(2 - 6)$. Write the equation in general form.

Solution
If the center of a circle is at $(2, -6)$ and the radius is $\sqrt{5}$, the equation of the circle to be used is $(x - h)^2 + (y - k)^2 = r^2$ with $(h,k)=(-5,-3)$ and $r=11$. Thus, substituting the values yields
$(x - 2)^2 + (y - (-6))^2 = \sqrt{5}^2$
$(x - 2)^2 + (y + 6)^2 = 5$

Converting it to general form,
$(x^2 - 4x + 4) + (y^2 + 12y + 36) = 5$
$x^2 + y^2 + - 4x + 12y\ 35 = 0$

Example 5
Determine the center and radius of the circle with the equation
$x^2 + y^2 - 8x - 4y + 4 = 0$

Solution
To determine the center and radius, the equation should be converted into the center-radius form.
First isolate the constant on the right side $x^2 + y^2 - 8x - 4y = -4$
Second, group terms with common variables $(x^2 - 8x) + (y^2 - 4y) = -4$
Third, add a term to make a perfect square trinomial, don't forget to add the same values on the right side
$(x^2 - 8x + 16) + (y^2 - 4y + 4) = -4 + 16 + 4$
Fourth, factor each perfect square trinomial then simplify the right side
$(x - 4)^2 + (y - 2)^2 = 16$ or $(x - 4)^2 + (y - 2)^2 = 4^2$

Therefore: center (4,2); radius = 4 units
Activity 11 shows more examples on how to convert the general form of a circle to standard form.

Example 6
Find the radius of the circle below. Write the equation in center – radius form and general form.
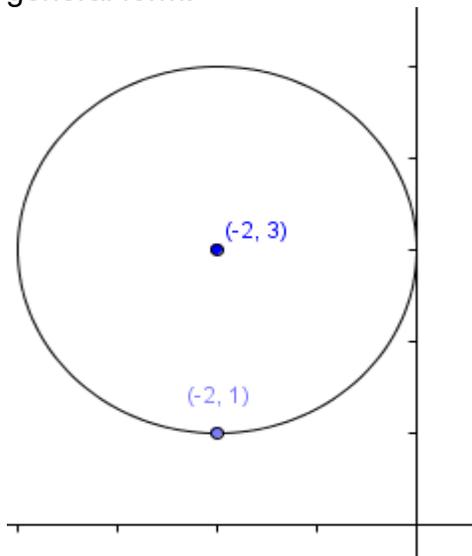
Solution
Since the coordinates of the center (h,k)=(–2, 3) and point
(x,y)= (–2, 1) in the circle are provided, just substitute the values in $(x - h)^2 + (y - k)^2 = r^2$ to solve the radius. Thus:
$(x - h)^2 + (y - k)^2 = r^2$
$(-2 - (-2))^2 + (1 - 3)^2 = r^2$
$(-2 + 2)^2 + (2)^2 = r^2$
$0 + 4 = r^2$
$4 = r^2$; note that r^2 means radius squared, so get the square root
$\sqrt{4} = \sqrt{r^2}$
r = 2 units

If you still have some questions about the derivation and different forms of the equations of a circle, you can click this to email your teacher and ask for help. Then, proceed to the next activity to discover how to convert equations in general form to standard form.

ACTIVITY 12. CONVERTING FROM GENERAL FORM TO STANDARD FORM

DESCRIPTION: Access the websites to learn more about converting the equations of circles from general form to standard form. Use your Evernote account in www.evernote.com to take notes.

http://www.purplemath.com/modules/sqrcircle.htm
The website shows how to convert the general form to standard form

http://www.regentsprep.org/regents/math/algtrig/atc1/circlelesson.htm
The website shows how to convert the general form to standard form

http://www.youtube.com/watch?v=XoXUiFMSJ9M
The video shows how to convert the general form to standard form

ACTIVITY 13. QUIZ ABOUT CIRCLES

DESCRIPTION: Answer the following problems. Show your complete solution. Click "SUBMIT" when you're finished.

1. Write the equation of the circle that satisfies the given conditions.
   a. Center (3,4), radius 5

   b. Center (0,8), radius 6

   c. Center (5, – 12), radius 7

   d. Center (– 1, –6), radius 5

   e. Center $(\frac{3}{5}, \frac{3}{10})$, radius 2

   f. The center is at (1,–3) and the circle passes through (–3, 5).

2. Determine the center – radius form of the following then determine the center and radius.

   a. $x^2 + y^2 + 6x - 4y - 12 = 0$

   b. $x^2 + y^2 - 8x - 2y + 1 = 0$

   c. $x^2 + y^2 - 3x - 4y = 0$

3. Determine whether each equation represents a circle or not. If it is draw the circle. If not, explain why.

   a. $x^2 + y^2 - x = 0$ b. $x^2 + y^2 - 4x - 4y + 9 = 0$

Now that you have tried answering problems regarding circles, revisit your thoughts about circles.

ACTIVITY 14.   ANTICIPATION – REACTION GUIDE REVISITED

DESCRIPTION: This time, let us see if you mastered already the concept about the equation of circles by revisiting a previous activity.

Determine whether the statement is true or false under the REVISED column by placing a check mark. Your initial answers are displayed for your reference. Click "SUBMIT" when you're finished.

INITIAL		STATEMENT	REVISED	
True	False		True	False
		1. The center – radius form of the equation of a circle is $x^2 + y^2 = r$.		
		2. The standard form of a circle is $x^2 + y^2 + Dx + Ey + F = 0$		
		3. The general form of a circle is $(x – h)^2 + (y – k)^2 = r^2$		
		4. The circle with the equation $x^2 + y^2 = 4$ has a radius of 4.		
		5. The center of the circle with the equation $x^2 + y^2 = 4$ is at (0,0).		
		5. The general form of $x^2 + y^2 = 4$ is $x^2 + y^2 = 2^2$.		
		6. The center of the circle with the equation $(x – 2)^2 + (y – 3)^2 = 8^2$ is at (–2, –3).		
		7. The center of the circle with the equation $(x + 2)^2 + (y + 3)^2 = 8^2$ is at (2,3).		

? PROCESS QUESTIONS:

1.   In which items did you change your initial answer?

[ ]

2. What made you change your answer? Explain.

```
┌─────────────────────────────────────────────┐
│                                             │
│                                             │
│                                             │
│                                             │
│                                             │
└─────────────────────────────────────────────┘
```

Now that you revised your ideas about your initial thoughts about circles, summarize your ideas in the next activity.

ACTIVITY 15.   FRAYER'S MODEL

DESCRIPTION: Complete the Frayer's model. Click "SUBMIT" when you're finished.
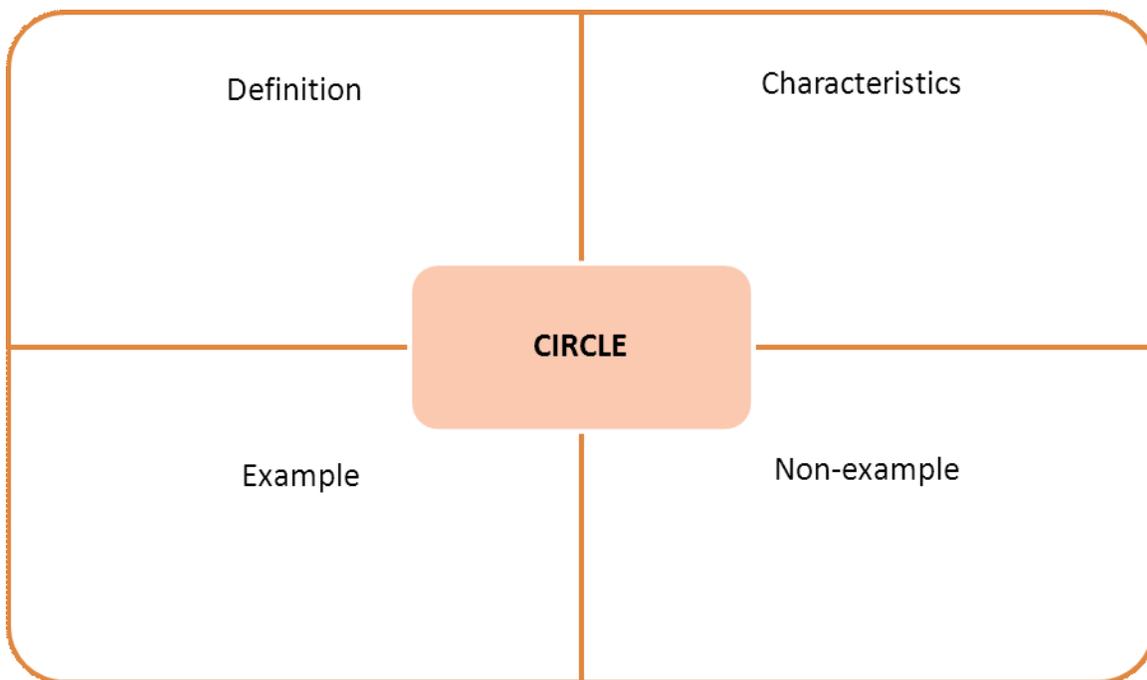
Definition	Characteristics
CIRCLE	
Example	Non-example

Now that you know the derivation and different forms of the equations of a circle, let us try to solve some real – life problems involving the equation of circles. One of the problems you will encounter is about the orbit of communication satellites. A communication satellite revolves around the Earth. As shown in Activity 2, it follows a circular path in which Earth is the center while the distance from Earth to the communication satellite is the radius.

Example 1
What is the equation of the circle that represents an orbiting communication satellite 45 000 km from the center of Earth?

Solution
The situation can be illustrated as:
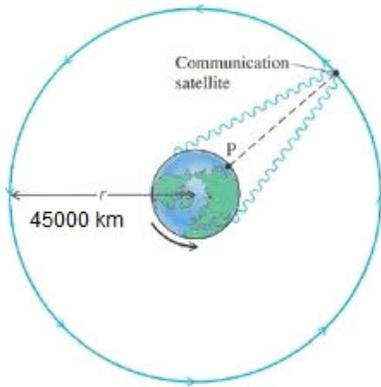
Since the center of the Earth is the center of the circle and origin, the equation to be used will be $x^2 + y^2 = r^2$. Substituting the value of the radius yields $x^2 + y^2 = 45000^2$ or
$x^2 + y^2 = 2,025,000,000$

Example 2
A stone is dropped into a pond and sends out a circular ripple whose radius increases by 5 cm/s. find the equation of the circle 10 seconds after the stone is dropped.
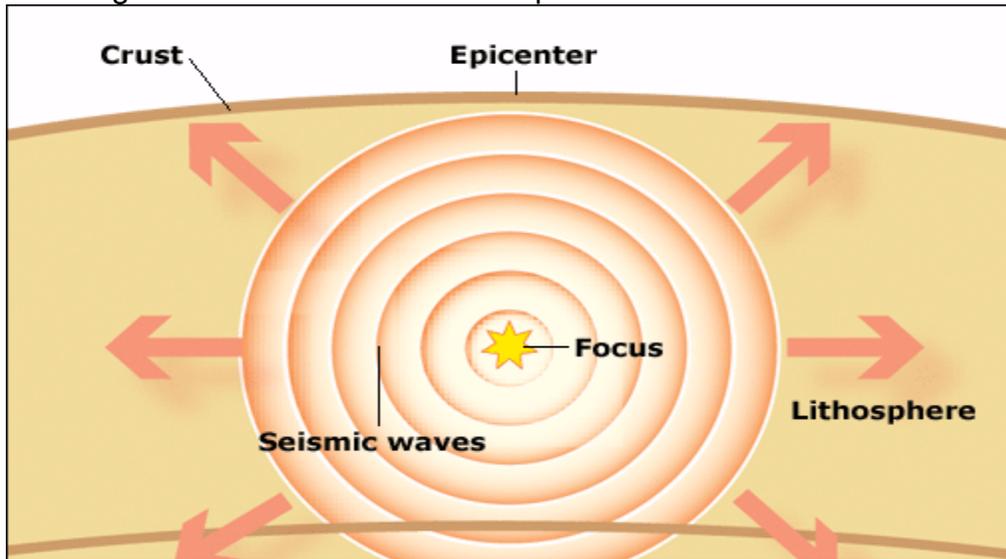
Solution

A circle with center at the origin can be used as a graphical model of the ripple. If the ripple grows by 5cm/s, then after 10 seconds, the radius is 10(5) = 50cm. The equation of the circle at this time is
$x^2 + y^2 = 50^2$ or $x^2 + y^2 = 2500$

Example 3
Write the equation of a graph that models the shock wave (seismic wave) from an earthquake, 30 seconds after the quake. Assume that the wave travels 6 km/s.

Solution
The diagram below shows how earthquakes occur

A circle with center at the origin can be used as a graphical model of the shock wave. If the shock wave travels 6 km/s, then after 30 seconds, the radius is 6(30) = 180 km. The equation of the circle at this time is
$x^2 + y^2 = 180^2$ or $x^2 + y^2 = 32,400$
Meaning, after 30 seconds, the shock wave can be felt 180 km away from the epicenter (focus) of the earthquake.

Now that you have practiced solving problems involving circles, it's time for you to revisit your answers in a previous activity.

After learning so much about circles, it is time for you now to analyze problems related to real – life phenomena by using the concepts of circles.

ACTIVITY 16.   RIPPLES, SHOCK WAVES AND SATELLITES

DESCRIPTION: Answer the following problems. Show your complete solution. Click "SUBMIT" when you're finished.

1. What is the equation of the circle that represents an orbiting communication satellite 21 000 km from the center of Earth?

2. Write the equation of a graph that models the shock wave from an earthquake, 15 seconds after the quake. Assume that the wave travels 9 km/s.

3. Two satellites are orbiting Earth. The path of one has the equation $x^2 + y^2 = 2{,}250{,}000$. The orbit of the other is 1,250 km from the center of the Earth. How much farther is the first satellite's orbit than the second? In one orbit, how much farther does the first satellite travel than the second one?

4. Write the equation of a graph that models the shock wave (seismic wave) from an earthquake, 20 seconds after the quake. Assume that the wave travels 8 km/s. Interpret the equation.

PROCESS QUESTIONS:

1. What are some physical phenomena that can be modeled by a circle?

2. How does the concept of circles help in understanding these phenomena?

3. How can challenging situations involving geometric figures be analyzed and solved?

After learning so much about circles, it is time for you now to analyze problems related to real – life phenomena by using the concepts of circles. Refer to your solutions in Activity 16.

ACTIVITY 17.   GUIDED GENERALIZATION

	PROBLEM 1	PROBLEM 2	PROBLEM 3
ESSENTIAL QUESTION: **How can challenging problems involving geometric figures be analyzed and solved?**	What is the equation of the circle that represents an orbiting communication satellite 21 000 km from the center of Earth?	Write the equation of a graph that models the shock wave from an earthquake, 15 seconds after the quake. Assume that the	Two satellites are orbiting Earth. The path of one has the equation $x^2 + y^2 = 2,250,000$. The orbit of the other is 1,250 km from the center of the Earth. How much farther is the first satellite's orbit

		wave travels 9 km/s.	than the second? In one orbit, how much farther does the first satellite travel than the second one?
	What property was used to solve the given problem? Why? Show your solution/justification:	What property can be used to solve the given problem? Why? Show your solution/ justification:	What property can be used to solve the given problem? Why? Show your solution/justification:
	Complete the statement: Problems in real life(specify) can be analyzed and solved by _____ _____. The choice of properties to be used depends on _____ _____.	Complete the statement: Problems in real life can be analyzed and solved by _____ _____. The choice of properties to be used depends on _____ _____.	Complete the statement: Problems in real life can be analyzed and solved by _____ _____. The choice of properties to be used depends on _____ _____.

PROCESS QUESTIONS:
8. Look at your answers above; what do all the solutions have in common?
9. Were you able to answer all the problems?
10. How did you come up with correct property for each situation?
11. Have you encountered problems which can be solved with two or more ways? What made you choose the solution that you used?
12. How is each property useful in solving the problem?
13. If the situations will differ, how do you know which one to use? Explain.
14. Complete the statements below.

In general, problems in real life can be analyzed and solved by…..

In general, the choice of properties to be used depends on ……

How can challenging problems involving geometric figures be analyzed and solved?

After summing up everything, try solving the problem below. Try to see if the generalization that you have made can also be used in the new situation.

Write the equation of a graph that models the shock wave (seismic wave) from an earthquake, 20 seconds after the quake. Assume that the wave travels 8 km/s. Interpret the equation.

Process Questions:
1. What are some physical phenomena that can be modeled by a circle? What property can be used to solve the problem below? Why?
2. How does the concept of circles help in understanding these phenomena?
3. How can challenging situations involving geometric figures be analyzed and solved?

Now that you are familiar with the equations and real – life situations modeled by circles, it is time for you to analyze some of the devastating events in the pasts.

## ACTIVITY 18.   RISK ASSESSMENT AND INFORMATION DISSEMINATION

DESCRIPTION: During calamities and disasters, the government or media usually informs the public about the scope and extent of the damage/affected area through graphics.

Below are images of devastating events in the past. Analyze the pictures then answer the follow – up questions. You can visit the website of the images for further reading. Click "SUBMIT" when you're finished.
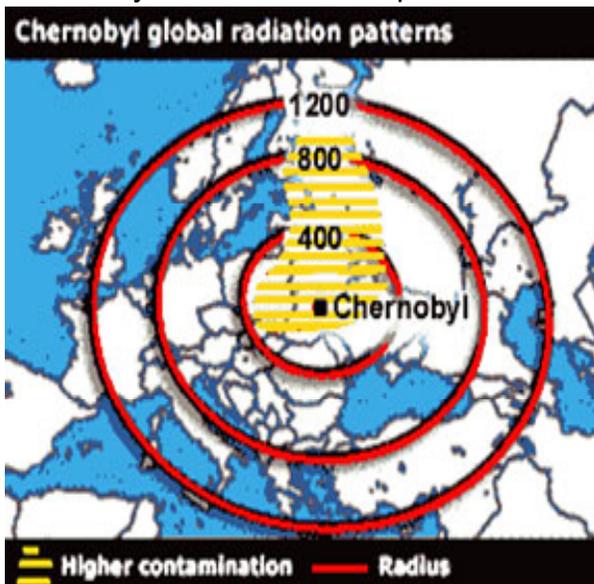
Typhoon Yolanda

http://weather.com.ph/announcements/super-typhoon-haiyan-yolanda-update-number-007
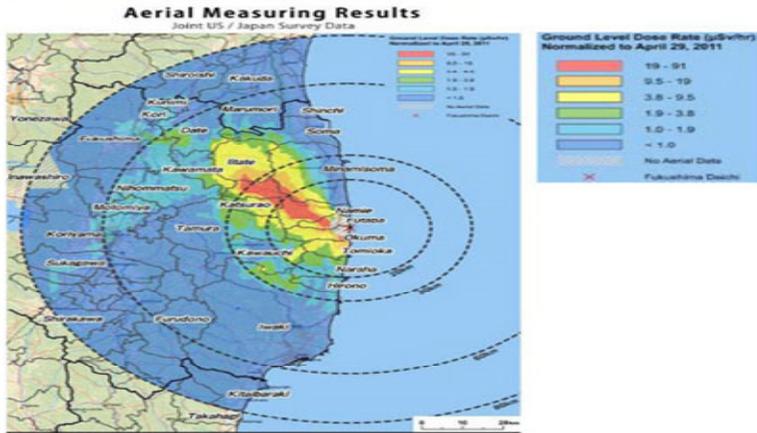
Aerial Measuring
Results of the
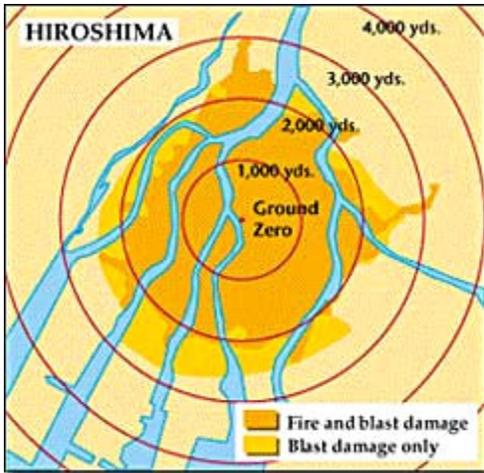Radiation Leak from
the Fukushima
Nuclear Power Plant

http://www.ucsusa.org/nuclear_power/making-nuclear-power-safer/preventing-nuclear-accidents/nuclear-reactor-crisis-faq.html

Chernobyl Global radiation patterns

http://www.mindfully.org/Nucs/Chernobyl-Death-Zone.htm

Blast zone of the Atomic bomb dropped in Hiroshima

http://www.ushistory.org/us/51g.asp

What is common in the four images?	What did you learn?	How can you use what you learned?

End OF DEEPEN:

> In this section, the discussion was about the equations of circles and how to model real – life phenomena using equation of circles. You have learned these through reading and summarizing articles, watching videos while taking notes, revisiting your prior knowledge and solving lots of problems.

What new realizations do you have about the topic? What new connections have you made for yourself? What helped you make these connections?

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.

TRANSFER :

Your goal in this section is apply your learning to real life situations. You will be given a practical task which will demonstrate your understanding on variations.

ACTIVITY 19.   CCTV FOR THE BARANGAY

DESCRIPTION: Read and analyze the situation below. Answer completely using the skills you've learned in this lesson.

The barangay officials recently reported alarming news that the number of crimes in your community is drastically increasing. The barangay council decided to put up a CCTV camera in the area with the most number of crime incidents. You are a marketing officer of a CCTV company that will participate in the bidding to provide the barangay with the CCTV unit. You need to identify the best location of the CCTV unit in the area that is identified in your barangay. You will present your recommendation and justification to the barangay officials. Your work should have authentic data, accurate computation, should have practical recommendations, and with correct application of the concepts of the properties of   circles.

While doing this performance task you may have questions or you may need help from your teacher or peers. You may click on the email button to send your concern to your teacher or you can post your concern to our Discussion Forum to communicate with your peers.

After doing the task, evaluate your work using the rubric below.

Criteria	4 Excellent	3 Proficient	2 Progressing	1 Beginning	RATING
Authenticity of Data	The data used are authentic and updated. Data are taken from reliable resources.	The data used are authentic. Data are taken from reliable resources.	Some of the data used are not authentic.	1. Data are not authentic and updated. Derives inductively the relations among chords, arcs, central angles, and	

				geometric figures on the coordinate plane. 11. Solves problems involving geometric figures on the coordinate plane.	
Accuracy of the Compu-tations	Computa-tions are accurate and supported with correct and in-depth interpre-tation.	Computa-tions are accurate and supported with correct interpreta-tion	Computa-tions are correct but interpretation is incorrect.	Most of the computations and interpretations are erroneous.	
Practicality of Proposal	The cost of the CCTV unit is minimum and it works effectively in monitoring the area at all times.	The cost of the CCTV unit is reasonable and it works in monito-ring the area at all times.	The cost of the CCTV unit is expensive but it works in monitoring the area most of the time.	The cost of the CCTV unit is prohibitive, and it works in monitoring the area intermittently.	
Application of the Concepts of the properties of circles	The properties of circles are used appropria-tely, correctly, and elegantly.	The properties of circles are used appropria-tely and correctly.	The properties of circles are used appropriately but with some incorrect solution and explanation.	The properties of circles are used inappropriately and incorrectly.	
				OVERALL RATING	

End of TRANSFER:

In this section, your task was the application of circles.

How did you find the performance task? How did the task help you see the real world use of the topic?

ACTIVITY 20.   GENERALIZATION TABLE FINALIZED

DESCRIPTION: Now that you are finished with this module, finalize your thoughts by completing the last column, then click "SUBMIT".

How can challenging problems involving geometric figures be analyzed and solved?

My Initial Thoughts	My Findings and Corrections	Supporting Evidence	Qualifying Conditions	My Generali-zations

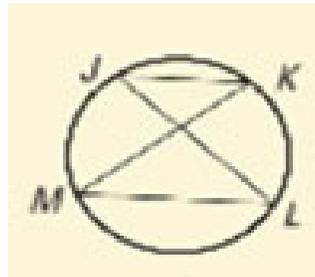*Congratulations! Take now the Post Assessment before you proceed to the next module.*

POST-ASSESSMENT:

It's now time to evaluate your learning. Click on the letter of the answer that you think best answers the question. Your score will only appear after you answer all items. If you do well, you may move on to the next module. If your score is not at the expected level, you have to go back and take the module again.

1. Which of the following is true about the measures of central angles in relation to the intercepted arc?
   A. The measure of the intercepted arc is half of the measure of the central angle.
   B. The measure of the intercepted arc is twice the measure of the central angle.
   C. The measure of the central angle is equal to the measure of the intercepted arc.
   D. The measure of the central angle is half of the measure of the intercepted arc.

2. Name two pairs of congruent angles using the figure below.
   A. $\angle JKM \cong \angle KJL$ & $\angle JLM \cong \angle KML$
   B. $\angle JLM \cong \angle KJL$ & $\angle JKM \cong \angle KML$
   C. $\angle JKM \cong \angle JLM$ & $\angle KJL \cong \angle KML$
   D. $\angle JLM \cong \angle KJL$ & $\angle JLM \cong \angle JKM$

3. A professional photographer has a camera that has a 90^0 field of vision and he wants to photograph the front of a statue. He moves to a spot where the statue is the only thing captured in your picture, as shown. After seeing this, you are also encouraged to capture the photograph of the statue but your camera has less than 90^0 field of vision. Which of the following should be your strategy to capture the whole picture?

   A. You may move closer to the statue.
   B. You may move farther from the statue.
   C. You may move to the other side of the statue.
   D. Look for an elevated place to capture the statue.

4. You are a member of the board who will assess the different proposals who are applying for bidding for CCTV cameras to be installed in different places. Which among the following criteria should you consider most?
   A. Authenticity of data, accuracy, practicality of proposal
   B. Accuracy, organization, practicality of proposal
   C. Practicality, neatness, accuracy of proposal
   D. Organization, practicality, neatness of proposal

5. A surveillance camera is mounted on a corner of a building. It rotates clockwise and counterclockwise continuously between Wall A and Wall B at a rate of 15^0 per minute as illustrated in the figure. How long does it take the camera to survey the entire area once? Justify your answer.

   A. 3 minutes, since the measure of the angle is half of the intercepted arc.
   B. 6 minutes, since the angle and arc have equal measures.
   C. 12 minutes, since the measure of the angle is twice the measure of the intercepted arc.
   D. 18 minutes, the camera needs to survey about 135^0 of the circle.

6. On a map's coordinate grid, Brgy. Malaya is located at (2, 4) and Brgy. Pasong Tamo islocated at (2, 2). Brgy. Katapan is the in the middle of the two barangay. What is the distance from Brgy. Malaya to Brgy. Katapatan?

   A. 2km
   B. 4km
   C. 6km
   D. 8 km

7. Which of the following coordinates must added to A(2,3), B(4,4), D(-1,-1) to prove that the given figure is a kite?

   A. C(3,2)
   B. C(3,5)
   C. C(3,7)
   D. C(2,5)

8. The JDC Cargo ship is sailing from Zhanjiang China to Honolulu and then to Long Beach. The amount of diesel used of the cargo sheep in travelling is 100 gallons for every 10 miles. Using the information in the figure below, determine the number of gallons used by the cargo from Zhanjiang to Long Beach.
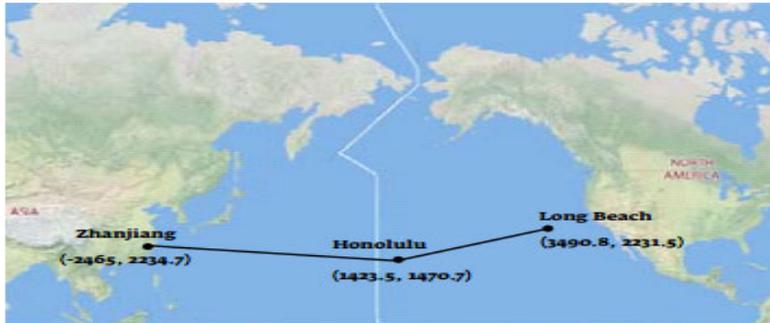
A. 616.56 gallons
B. 600 gallons
C. 700 gallons
D. 716.65 gallons

9. In the municipal vicinity map the coordinate of your barangay hall is (2, -1) and the school is (-2,2). How long will it take you to go to school from the barangay hall if the tricycle that you are riding is at 10km/hr?

   A. 30minutes
   B. 45 minutes
   C. 1 hour
   D. 1 hr and 10 minutes

10. Low – Earth orbit satellites occupies a region of space from 180 – 2000 kilometers above Earth. If the satellite is moving in a circular path with Earth as the point of origin, which circular equation represents the maximum path of the satellite?

    A. $x^2 + y^2 = 180$
    B. $x^2 + y^2 = 2000$
    C. $x^2 + y^2 = 32,400$
    D. $x^2 + y^2 = 4,000,000$

11. Based on the figure below, what equation that represents the circle?
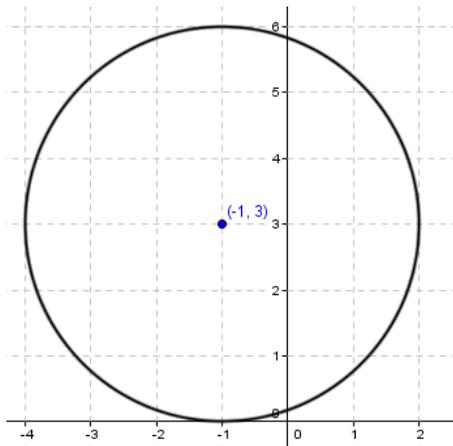
    I. $(x + 1)^2 + (y - 3)^2 = 9$

    II. $(x - 1)^2 + (y + 3)^2 = 9$

    III. $x^2 + y^2 + 2x - 6y + 1 = 0$

A. I only
B. II and III only
C. I and III only
D. I, II and III

To solve the radius, use the distance formula. So r = 3 . Completing $(x - h)^2$ + $(y - k)^2 = r^2$ results to $(x + 1)^2 + (y - 3)^2 = 9$ while rewriting to the general form results to $x^2 + y^2 + 2x - 6y + 1 = 0$

12. To determine the radius and coordinates of the center of the circle $x^2 + y^2 + 4x - 8y + 4 = 0$, Jerry and Akzie came up with the following solutions:

| Jerry's Solution	Akzie's Solution
$x^2 + y^2 + 4x - 8y + 4 = 0$ $x^2 + 4x + \_\_ + y^2 - 8y + \_\_ = -4$ $x^2 + 4x + 2^2 + y^2 - 8y + (-4)^2 = -4$ $(x + 2)^2 + (y - 4)^2 = -4$ Radius = $\sqrt{-4}$; center: $(-2, 4)$	$x^2 + y^2 + 4x - 8y + 4 = 0$ $x^2 + 6x + \_\_ + y^2 - 8y + \_\_ = -4$ $x^2 + 4x + 2^2 + y^2 - 8y + (-4)^2 = -4 + 4 + 16$ $(x + 2)^2 + (y - 4)^2 = 16$ Radius = 4; center: $(-2, 4)$

Which statement best describes the solutions of Jerry and Akzie?
    A. The solution of Jerry is correct.
    B. The solution of Akzie is correct.
    C. The solution of Jerry and Akzie are both correct.

D. The solution of Jerry and Akzie are both incorrect.

13. When delivering weather forecast during typhoons, weather forecasters uses diagrams and charts to illustrate the movement of the typhoon. The multiple colored circles indicate the effect of the typhoon in the area at a given time at. If you are the weather forecaster, which statement best describes typhoon Haiyan based on the illustration below?
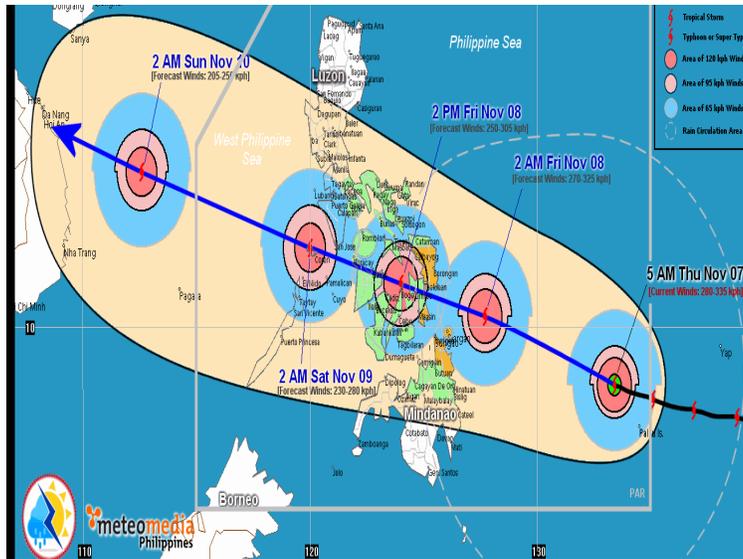
A. The area affected by 128 kph winds of Haiyan decreased from Nov. 7 – 10.
B. The area affected by 95 kph winds of Haiyan decreased from Nov. 7 – 10.
C. The area affected by 95 – 128 kph winds of Haiyan was maintained from Nov. 7 – 10.
D. The area affected by Haiyan decreased from Nov. 7 – 10.

14. The barangay officials recently reported alarming news that the number of crimes in your community is drastically increasing. The barangay council decided to put up a CCTV camera in the area with the most number of crime incidents. You are a marketing officer of a CCTV company that will participate in the bidding to provide the barangay with the CCTV unit. You need to identify the best location of the CCTV unit in the area that is identified in your barangay. Your work should have authentic data, accurate computation, should have practical recommendations, and with correct application of the concepts of the properties of circles. What should you do to convince the barangay officials?

A. You will present the CCTV to the barangay officials.

B. You will present the CCTV and layout to the barangay officials.

C. You will present your recommendation to the barangay officials.

D. You will present your recommendation and justification to the barangay officials.

GLOSSARY OF TERMS USED IN THIS LESSON:

Circle – the set of all points on a plane that are equidistant from a fixed point, called the center, on the plane.

Radius - and the distance from the center to any point on the circle

Standard form of a circle – refers to the center – radius form of the equation of the circle which is $(x - h)^2 + (y - k)^2 = r^2$

General form of the circle - refers to the equation of the circle in the form $x^2 + y^2 + Dx + Ey + F = 0$

REFERENCES AND WEBSITE LINKS USED IN THIS LESSON:

Zimmer, D. et.al.(2006). Mathematics 10. Thomson Learning Asia, Singapore

Fuller, Gordon and Dalton Tarwater. (1992). Analytic Geometry 7th Ed. Addison-Wesley Publishing Company, Philippines

https://www.youtube.com/watch?v=4jLN56GlNHQ
A video showing how to draw a perfect circle

http://www.esa.int/Our_Activities/Telecommunications_Integrated_Applications/Orbits
An image of a satellite orbiting Earth

http://weather.com.ph/announcements/tropical-storm-nari-santi-update-number-004
a graphical representation of the forecast of typhoon Santi given last October 10, 2013

http://www.geogebra.org/cms/en/download/
The website of geogebra

http://www.dummies.com/how-to/content/how-to-graph-a-circle.html
An article about graphing circles

http://www.youtube.com/watch?v=oYQbj0MWt2k
A video showing how to draw a circle in a paper using a compass.

http://www.sophia.org/tutorials/video-two-how-to-draw-circle-using-a-compass-and-r
A video showing how to draw a circle in a paper using a compass.

http://www.youtube.com/watch?v=-B1VtI6ZoE8
A video showing how to draw a circle in a paper using a compass.

http://www.mathwarehouse.com/geometry/circle/interactive-circle-equation.php
An interactive website used for graphing circles

http://www.webmath.com/gcircle.html
An interactive website used for graphing circles

http://hotmath.com/hotmath_help/topics/equation-of-a-circle.html
An article showing how the equation of the circle was derived .

http://www.sophia.org/tutorials/equations-of-a-circle
The website contains articles and videos showing how the equation of the circle was derived.

www.evernote.com
The website of evernote

http://www.purplemath.com/modules/sqrcircle.htm
The website shows how to convert the general form to standard form

http://www.regentsprep.org/regents/math/algtrig/atc1/circlelesson.htm
The website shows how to convert the general form to standard form

http://www.youtube.com/watch?v=XoXUiFMSJ9M
The video shows how to convert the general form to standard form

http://www.tattoopinners.com/earthquake-diagram/imaginethispromo.com*earthquake-diagram-217.gif/
An image showing how earthquake occurs

http://weather.com.ph/announcements/super-typhoon-haiyan-yolanda-update-number-007
An image of the weather forecast on Typhoon Yolanda

http://www.ucsusa.org/nuclear_power/making-nuclear-power-safer/preventing-nuclear-accidents/nuclear-reactor-crisis-faq.html
An image showing the aerial measurement of the radiation level from the Fukushima nuclear reactor leak

http://www.mindfully.org/Nucs/Chernobyl-Death-Zone.htm
An image showing the Chernobyl global radiation patterns

http://www.ushistory.org/us/51g.asp
An image showing the blast zone of the atomic bomb dropped in Hiroshima