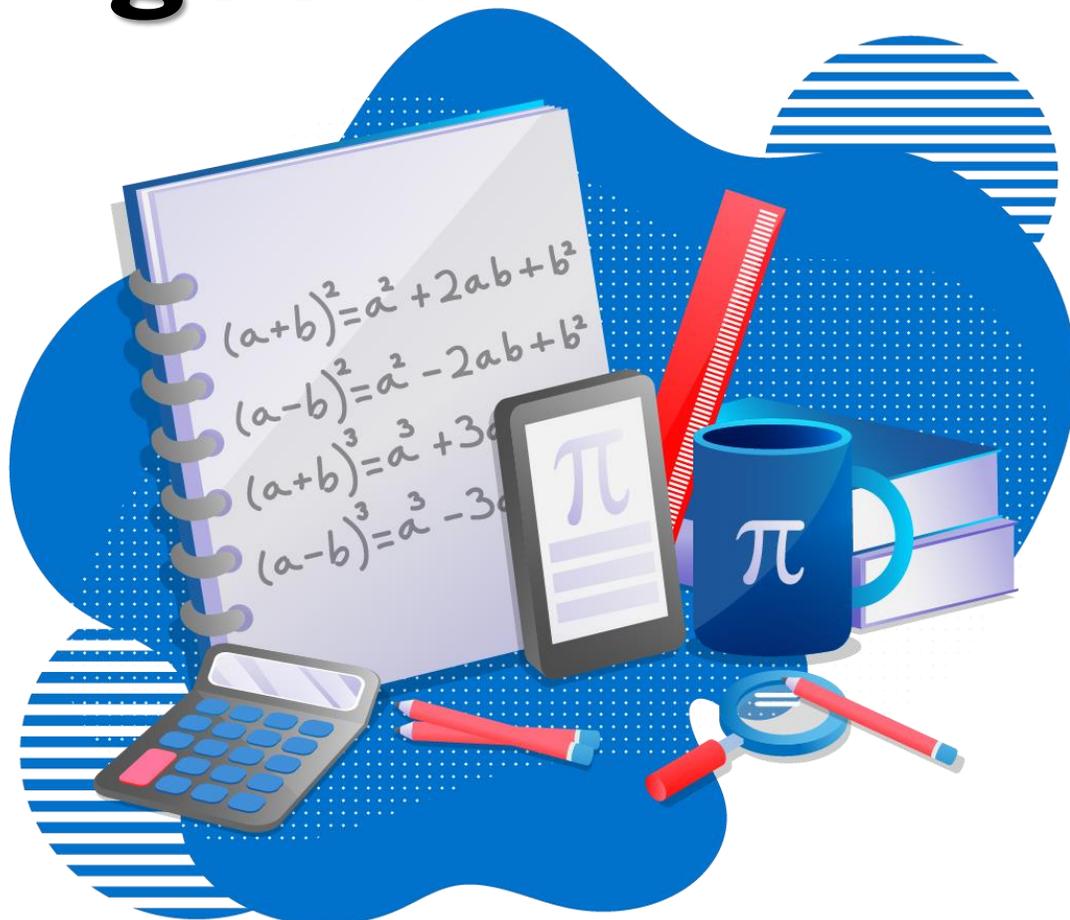


LEARNING MODULE

Mathematics | G8 | Q1

Patterns and Algebra



NOTICE TO THE SCHOOLS

This learning module (LM) was developed by the Private Education Assistance Committee under the GASTPE Program of the Department of Education. The learning modules were written by the PEAC Junior High School (JHS) Trainers and were used as exemplars either as a sample for presentation or for workshop purposes in the JHS In-Service Training (INSET) program for teachers in private schools.

The LM is designed for online learning and can also be used for blended learning and remote learning modalities. The year indicated on the cover of this LM refers to the year when the LM was used as an exemplar in the JHS INSET and the year it was written or revised. For instance, 2017 means the LM was written in SY 2016-2017 and was used in the 2017 Summer JHS INSET. The quarter indicated on the cover refers to the quarter of the current curriculum guide at the time the LM was written. The most recently revised LMs were in 2018 and 2019.

The LM is also designed such that it encourages independent and self-regulated learning among the students and develops their 21st century skills. It is written in such a way that the teacher is communicating directly to the learner. Participants in the JHS INSET are trained how to unpack the standards and competencies from the K-12 curriculum guides to identify desired results and design standards-based assessment and instruction. Hence, the teachers are trained how to write their own standards-based learning plan.

The parts or stages of this LM include Explore, Firm Up, Deepen and Transfer. It is possible that some links or online resources in some parts of this LM may no longer be available, thus, teachers are urged to provide alternative learning resources or reading materials they deem fit for their students which are aligned with the standards and competencies. Teachers are encouraged to write their own standards-based learning plan or learning module with respect to attainment of their school's vision and mission.

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MATHEMATICS 8

Module 1: Patterns And Algebra

LESSON 1. Special Products and Factoring

INTRODUCTION AND FOCUS QUESTION(S):



Have you ever wondered how the architect of your school came up with its structure? Is the structure of your school the same as before? How is it possible for school buildings to be constructed without tearing down the old structures to preserve its historical essence? Or maybe, did you ever think why your school needs to be remodeled this way?

What essential skills did the architect use in designing your school structure? What were the preparations made and the tools which were used? How many revisions could have been made in the initial plan until the desired structure was attained?

In this module, you will discover how important it is to plan well and utilize essential mathematical skills to be able to accomplish several tasks and come up with the desired output.

As you go through this module, think of this question: How can unknown quantities in geometric problems be solved?

LESSON COVERAGE:

In this lesson, you will examine this question when you take the following topics:

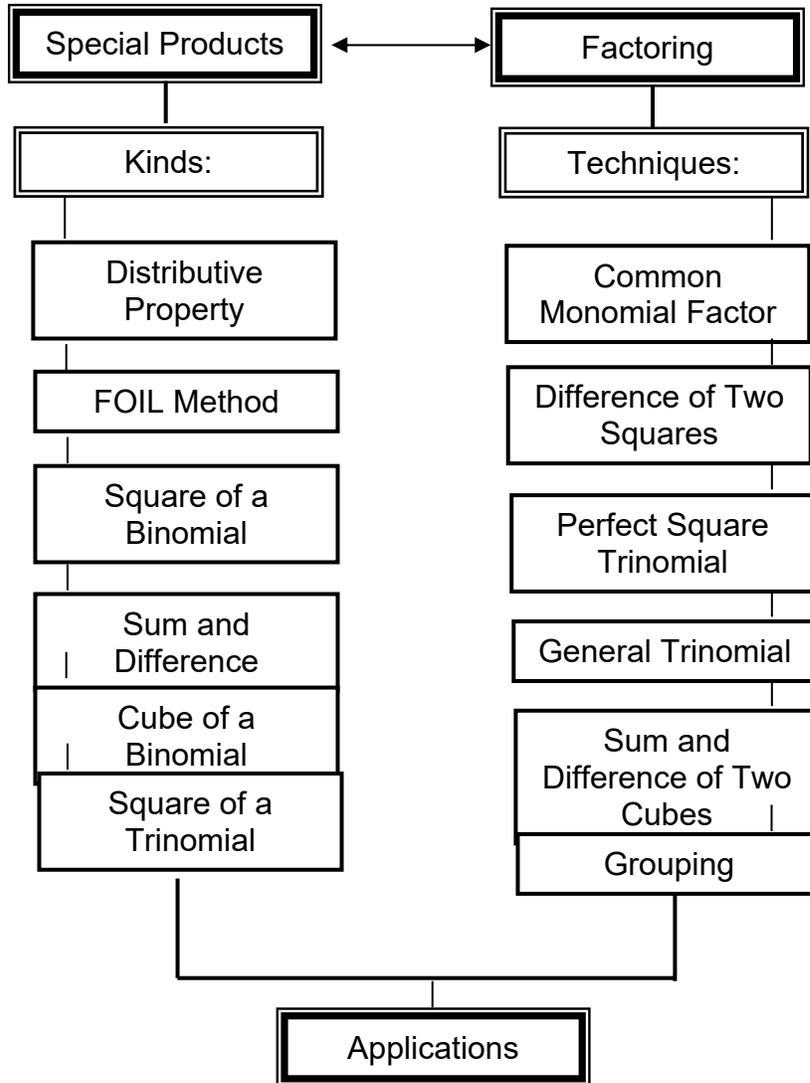
- 1.1: Special Products
- 1.2: Factoring
- 1.3: Applications

In these lessons, you will learn the following:

<p>1.1 <i>Special Products</i></p>	<p>Special products</p> <ul style="list-style-type: none"> • identify polynomials which are special products • find special products of certain polynomials • use special products to solve geometric problems
<p>1.2 <i>Factoring</i></p>	<p>Factoring</p> <ul style="list-style-type: none"> • relate special products with factoring • factor polynomials • use factoring to model and solve geometric problems
<p>1.3 <i>Applications</i></p>	<p>Applications</p> <ul style="list-style-type: none"> • apply special products and factoring in solving real-life problems

MODULE MAP:

Here is a simple map of the above lessons you will cover:



EXPECTED SKILLS:

To do well in this module, you need to remember and do the following:

1. follow the directions carefully
2. master the prerequisite skills such as operations on integers, laws of exponents, and basic geometry formulae
3. solve with speed and accuracy

PRE-ASSESSMENT



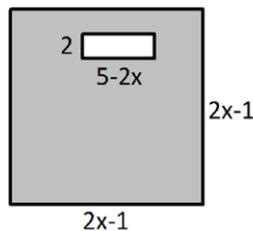
Let's find out how much you already know about this module. Click on the letter that you think correctly answers the question. Please answer all items. After taking the test, you will see your score. Take note of the items that you were not able to correctly answer and look for the right answer as you go through this module.

- Which mathematical statement is correct?
 - $(2x - y)(3x - y) = 6x^2 - 5x^2y^2 + y^2$
 - $(4x - 5)(4x - 5) = 16x^2 + 25$
 - $(3x - 4)(2x + 7) = 6x^2 + 13x - 28$
 - $(2x + 5)^2 = 4x^2 + 25$
- Which of the following DOES NOT belong to the group?
 - $\frac{1}{4}x^4 - 1$
 - $x^2 - 0.0001y^4$
 - $1.6(x - 1)^2 - 49$
 - $(x + 1)^4 - 4x^6$
- Which of the following factors gives a product of $x^2 + 5x + 4$?
 - $(x + 1)(x + 4)$
 - $(x + 2)(x + 2)$
 - $(x + 5)(x - 1)$
 - $(x + 2)^2$
- A polynomial expression is evaluated for the x- and y-values shown in the table below. Which expression was evaluated to give the values shown in the third column?

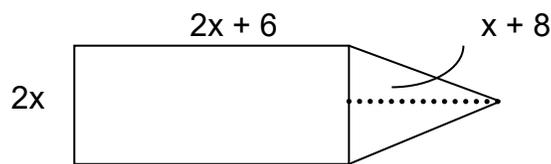
X	y	Value of the Expression
0	0	0
-1	-1	0
1	1	0
1	-1	4

- $x^2 - y^2$
 - $x^2 + 2xy + y^2$
 - $x^2 - 2xy + y^2$
 - $x^3 - y^3$
- Find the missing term: $(x + \underline{\quad})(3x + \underline{\quad}) = 3x^2 + 27x + 24$
 - 6, 4
 - 4, 6

- C. 8, 3
D. 12, 2
6. The length of a box is five inches less than twice the width. The height is 4 inches more than three times the width. The box has a volume of 520 cubic inches. Which of the following equations can be used to find the height of the box?
- A. $W(2L - 5)(3H + 4) = 520$
 B. $W(2W + 5)(3W - 4) = 520$
 C. $W(2W - 5)(3W - 4) = 520$
 D. $W(2W - 5)(3W + 4) = 520$
7. One of the factors of $2a^2 + 5a - 12$ is $a + 4$. Which is the other factor?
- A. $2a - 3$
 B. $2a + 3$
 C. $2a - 8$
 D. $2a + 8$
8. The area of a square is $4x^2 + 12x + 9$ square units. Which expression represents the length of the side?
- A. $(3x + 2)$ units
 B. $(2x + 3)$ units
 C. $(4x + 9)$ units
 D. $(4x + 3)$ units
9. The side of a square is x cm long. The length of a rectangle is 5 cm longer than a side of this square and the width is 5 cm shorter. Which statement is true?
- A. The area of the square is greater than the area of the rectangle.
 B. The area of the square is less than the area of the rectangle.
 C. The area of the square is equal to the area of the rectangle.
 D. The relationship cannot be determined from the given information.
10. A square piece of land was rewarded by a master to his servant. They agreed that a portion of it, represented by the rectangle inside, should be used to construct a grotto. How large is the area of the land that is available for the other uses?



- A. $4x^2 - 9$
 B. $4x^2 + x + 9$
 C. $4x^2 - 8x - 9$
 D. $4x^2 + 9$
11. Which of the following values will make the largest area of the square with a side of $3x + 2$?
- A. $-\frac{3}{4}$
 B. 0.4
 C. $\frac{1}{3}$
 D. -0.15
12. Which strategy could not be used to solve for the area of the figure below?



- A.
- $$A = 2x(2x + 6) + \frac{1}{2}(2x)(x + 8)$$
- $$A = 4x^2 + 12x + x^2 + 8x$$
- $$A = 5x^2 + 20x$$
- B.
- $$A = 2x(3x + 14) - 2\left(\frac{1}{2}\right)(x)(x + 8)$$
- $$A = 6x^2 + 28x - x^2 - 8x$$
- $$A = 5x^2 + 20x$$
- C.
- $$A = [2x(2x + 6) + (x + 8)(2x)] - 2\left(\frac{1}{2}\right)(x)(x + 8)$$
- $$A = [(4x^2 + 12x) + (2x^2 + 16x) - (x^2 + 8x)]$$
- $$A = 6x^2 + 28x - x^2 - 8x$$
- $$A = 5x^2 + 20x$$
- D.
- $$A = 2x(2x + 6) + \left(\frac{1}{2}\right)(2 + x)(x + 8)$$
- $$A = 4x^2 + 12x + x^2 + 8x$$
- $$A = 5x^2 + 20x$$

13. In your quiz, you were asked to square $(2x - 3)$. Your classmate answered $4x^2 - 9$. Is your classmate's answer correct?
- A. Yes, because squaring a binomial always produces a binomial product.
 - B. Yes, because product rule is correctly applied.
 - C. No, because squaring a binomial always produces a trinomial product.
 - D. No, because the answer must be $4x^2 + 9$.
14. Expression A: $4x^2 - 81$
Expression B: $(2x-9)(2x+9)$
If $x = 2$, which statement is true about the given expressions?
- A. The value of expression A is greater.
 - B. The value of the expression B is greater.
 - C. The values of the two expressions are equal.
 - D. The relationship cannot be determined from the given information.
15. Your sister plans to remodel her closet. She hired a carpenter to do the task. What could be the best way that your sister can do so that the carpenter can do the task according to what she wants to happen?
- A. Show a replica of a closet.
 - B. Download a picture from the internet.
 - C. Leave everything to the carpenter.
 - D. Provide the lay out drawn to scale.
16. Which of the following standards would best apply in checking the carpenter's work?
- A. accuracy of measurements and wise utilization of materials
 - B. accuracy of measurements and workmanship
 - C. workmanship and artistic design
 - D. workmanship and wise utilization of materials

For 17 – 18 use the problem below.

Your dad decided to fence the rectangular garden with an area of $(x^2 + 13x + 42)$ ft². The width is $(x + 6)$ ft. and the cost of fencing is P100/ft.

17. How much will you spend if you want to surround the garden with fences if $x = 5$?
- A. ₱13,200
 - B. ₱4,600
 - C. ₱1,840
 - D. ₱5,280
18. If the cost of fertilizer is ₱40.00/ft², how much will you then spend if you want to fertilize the entire garden?
- A. ₱13,200
 - B. ₱4,600
 - C. ₱1,840
 - D. ₱5,280

For 19 – 20, use the information below.

19. A factory makes rectangular cartons for its flower vases. If the volume of each carton is given by $V = y^3 + 14y^2 + 48y$, which are the possible dimensions of the carton?
- A. $(y^2 + 8)(y + 6)$
 - B. $y(y + 16)(y + 3)$
 - C. $y(y + 12)(y + 4)$
 - D. $y(y + 6)(y + 8)$
20. Based from your answer in the previous number, which are the dimensions of the carton if $y = 4$?
- A. 4 x 24 x 10
 - B. 4 x 20 x 7
 - C. 4 x 10 x 12
 - D. 4 x 16 x 8

Lesson 1.1: Special Products

LESSON PRE-ASSESSMENT



Let's find out how much you already know about this lesson. Click on the letter that you think correctly answers the question. Please answer all items. After taking the test, you will see your score. Take note of the items that you were not able to correctly answer and look for the right answer as you go through this lesson.

- Which law of exponents states that when a power is raised to another power, keep the base and multiply the exponents?
A. product B. quotient C. power D. negative exponent
- Which equation is true?
A. $(x - 2)(x + 2) = x^2 - 4x + 4$
B. $(x - 2)(x + 2) = x^2 - 4$
C. $(x + 2)(x + 2) = x^2 + 4$
D. $(x - 2)(x - 2) = x^2 + 4$
- Which expression is equivalent to $(3a + b)(2a + 4b)$?
A. $5a + 3b$
B. $5a^2 + 10ab + 5ab^2$
C. $6a^2 + 4b^2$
D. $6a^2 + 14ab + 4b^2$
- Which is the area of a square that has a side of $\frac{1}{2}x - 1$?
A. $\frac{1}{4}x^2 - x + 1$
B. $\frac{1}{4}x^2 - x - 1$
C. $\frac{1}{4}x^2 + x + 1$
D. $\frac{1}{4}x^2 - 2x + 1$
- Express $(r^2s^3)(rs^4)$ using exponents.
A. r^3s^7
B. r^7s^3
C. r^2s^{12}
D. r^7s^7



EXPLORE



How long would it take you to solve the following multiplication expressions below without using pen and paper? Give yourself three minutes to solve the following expressions.

PART 1:

$$97 \times 103$$

$$26 \times 26 =$$

$$99 \times 99 \times 99 =$$

Let us now see if your answers are correct. Click the answer button.

How many answers did you get correctly?

Now, answer the following process questions:

1. What do you notice about the given expressions?

2. Did you solve them easily?

3. What technique/s did you use?

4. What hindered you from solving the expressions easily?



The given multiplication expressions show us how that with the use of patterns, the operation can be performed easily. Click the box below to see the patterns used.

Solutions:

$$\begin{aligned}
 97 \times 103 &= (100 - 3)(100 + 3) = 10000 - 9 = 9991 \\
 26 \times 26 &= (20 + 6)(20 + 6) = 400 + 2(20)(6) + 36 = 676 \\
 99 \times 99 \times 99 &= (100 - 1)^3 = 100^3 + 3(100)^2(-1) + 3(100)(-1)^2 + (-1)^3 = \\
 &= 1000000 - 30000 + 300 - 1 = 970299
 \end{aligned}$$

Now, answer the following process questions:

1. What did you realize regarding the different solutions?

2. What was used in order to arrive at the products easily?

Part 2:



Let us now see whether you can make a connection between what you did in part one with the problems that you see below.

1. One side of the square below measures 5.5 m. How will you compute for its area?



2. If the side of the square will be increased by 3.2 m, what will be the new area?

3. If six squares of the same dimension in number two will be put together to form a box, what would be the volume?

Now, answer the following process questions:

1. How long did you solve the answers to the questions in Part 2?

2. Were you able to make use of the knowledge that you gained in Part 1?

3. How did you arrive easily at the answers?



The problems you answered show one of the many situations where we can apply knowledge of special products. As you go through all the activities in this module, think of this question: How can unknown quantities in geometric problems be solved?

Part 3:



*In this lesson, you will do varied activities which will help you answer this topical focus question: **What makes a product special?***

Let's begin by answering the "I" portion of the IRF Worksheet that you see below. Fill it up by writing your initial answer to the topical focus question:

IRF Worksheet
Initial Answer
Revised Answer
Final Answer



Well, those were your thoughts and ideas about our lesson. Let's start a new activity to further explore on the important key concepts about special products.

ACTIVITY 1. WHAT PATTERNS CAN DO!

In this activity, you will work with other examples which will allow you to apply the patterns that were presented in the previous activity.

$52 \times 49 =$	$(x + 5)(x - 5) =$
$29 \times 29 =$	$(x + 5)(x + 5) =$
$43 \times 43 \times 43 =$	$(x + 5)(x + 5)(x +$

$52 \times 49 =$	
------------------	--

$29 \times 29 =$	
------------------	--

$43 \times 43 \times 43 =$	
----------------------------	--

$(x + 5)(x - 5)$ =	
-----------------------	--

$$\begin{array}{r} (x + 5) (x + 5) \\ - \end{array}$$

$$\begin{array}{r} (x + 5) (x + 5) \\ (x + 5) = \end{array}$$

Questions to Answer:

1. How long did it take you to answer the questions?

2. Were you able to make use of patterns in solving?

3. How did the patterns facilitate ease in solving?

4. In real life, what patterns have you used to make your tasks easier?

5. What makes a product special?

END OF EXPLORE:



You just tried finding out how products can be found easily through the use of patterns. Let us now find out what the answer is by doing the next part. What you will learn in the next sections will also enable you to do the final project which involves the making of a packaging box using the concepts of special products.



FIRM-UP



Your goal in this section is to learn and understand key concepts of special products.

We will start by doing the next activity.

ACTIVITY 2. THE MYSTERY BEHIND SPECIAL PRODUCTS

In this activity, you will read an article which will explain what Special products are all about by visiting this website,
<http://konigcruz.wikispaces.com/file/view/Special+Products.pdf>

Questions to Answer:

1. What are special products?

2. Discuss the processes involved when using special products.

3. Why do you need a rule when finding the product of binomials when you already know the FOIL method?

4. Can you formulate a rule for finding the product of $(x + 2y + 3)^2$?

ACTIVITY 3. SQUARING TRINOMIALS ON VIDEO

Let us now extend our lesson on special products by learning how to square trinomials. In this activity, you will watch a video which will teach you how trinomials are squared. The video is available on this link: <http://www.youtube.com/watch?v=-8DFxTI0Ls>.

Questions to Answer:

1. Describe the form of the new special product which you just learned.

2. Name the steps involved when squaring a binomial.

3. How can you relate this new concept with the other special products that you learned?

4. From the new pattern that you learned, can you think of a pattern which you could use to square a polynomial with four terms?

For additional information regarding how multiplication could be linked with special products, you can visit this link:

<http://hanlonmath.com/pdfFiles/199.SpecialProductsBH.pdf>

ACTIVITY 4. IRF Worksheet



Now that you have learned the different special products, using the “R” portion of the IRF Worksheet, answer the topical focus question: “What makes a product special?”

IRF Worksheet
Initial Answer
Revised Answer
Final Answer

ACTIVITY 5. DRAG AND DROP

Now, that you have learned the various special products, you will now do an interactive activity which will allow you to drag sets of factors and drop them beside special products. The activity is available in this website: http://www.media.pearson.com.au/schools/cw/au_sch_bull_gm12_1/dnd/2_spec.html.

Questions to Answer:

1. What special products did you use in the activity?

2. Name some techniques which you used to make the work easier.

3. What generalizations about solving for special products can you draw out of the examples shown?

4. Given the time constraint, how could you do the task quickly yet accurately?

ACTIVITY 6. PRACTICE MAKES PERFECT

Let us now practice deriving special products. In this activity, you will be asked to complete a worksheet that contains problems where students could use special products.



Solve for the following special products.

1. $(3a + 2b)(3a + 2b)$

2. $(a - 8)(a - 6)$

3. $(2x + 5)^3$

4. $(12k^3 - 6)(12k^3 + 6)$

5. $(7x + y)(7x - y)$

6. $(3x - 2y)^3$

7. $(8x - 5 + 2y)^2$

8. $(4c^2 + 2d)(4c^2 - d)$

9. $(g^3 + h^2)^2$

10. $(2x + 3y)(2x - 3y)$

ANSWERS:

ACTIVITY 7. ERROR RECOGNITION

After practicing how to solve for special products, let us now find out how well you can detect errors in the way special products are solved. You will be given equations containing factors on one side and special products on the other. Terms of the product are labeled with letters. You will click the letter which corresponds to the part of the product that is erroneous. If the product does not contain an error, click letter E.

- ____ 1. $(2x + 5y)^2 = \frac{4x^2}{A} + \frac{10xy}{B} + \frac{25y^2}{D}$
- ____ 2. $(8x + 4)(4 - 8x) = \frac{16}{A} + \frac{64x^2}{C}$
- ____ 3. $(9z - 5)^3 = \frac{27z^3}{A} - \frac{1215z^2}{B} + \frac{225z}{C} - \frac{125}{D}$
- ____ 4. $(-10x + y^2 + 2z)^2 = \frac{100x^2}{A} + y^4 + \frac{4z^2}{B} - \frac{20xy^2}{C} - \frac{20xz^2}{D} + 4y^2z$
- ____ 5. $(R + 2S)(R - 2S) = \frac{R^2}{A} - \frac{4S^2}{C}$
- ____ 6. $(ab^2 + c)(a^2b - c^2) = \frac{a^3b^3}{A} - \frac{ab^2c^2}{B} + \frac{a^2bc}{C} - \frac{c^3}{D}$
- ____ 7. $(9v^3 + w^2)(9v^3 + w^2) = \frac{81v^9}{A} + \frac{w^4}{C}$
- ____ 8. $(7 + 5a - b)^2 = 49 + \frac{25a^2}{A} - \frac{b^2}{B} + 70a - \frac{14b}{C} - \frac{10ab}{D}$
- ____ 9. $(-8x + 5)^3 = \frac{-512x^3}{A} + \frac{960x^2}{B} - \frac{600x}{C} + \frac{75}{D}$
- ____ 10. $(6z^2 + 9)(6z^2 + 9) = \frac{36z^4}{A} + \frac{54z^2}{C} + \frac{81}{D}$

After taking the exercise, click the answer button to check your answer.

Questions to Answer:

1. How did you determine the errors of each special product?

2. Name the techniques that you used to make the work easier to perform.

3. Why is error recognition important in learning special products? What does this indicate?

4. What new idea did you learn from this activity?

ACTIVITY 8. 3-2-1 CHART

In this activity, you will be asked to complete the 3-2-1 Chart regarding the special products that you have discovered.

3-2-1 Chart

Three things you found out:

1. _____

2. _____

3. _____

Two interesting things:

1. _____

2. _____

One question I still have:

1. _____

ACTIVITY 9. MAKING CONNECTIONS

Let us now connect special products with real life problems, specifically on geometry and measurement. In this activity, you will read about how special products can be used to solve the perimeter, area and volume of figures. Recall the formulae that could be used to find such measurements. Click this link: <http://www.gub.ac.uk/keyskills/WN/Numeracyperimeter.html>. Read the discussion in the link. After which, answer the questions below.

Questions to Answer:

1. Differentiate among perimeter, area and volume.

2. What operation/s is/are involved in deriving perimeter, area and volume?

3. What formulae are needed to solve for each?

4. How can special products be used to solve for these measures?

ACTIVITY 10. ADVENTURES INTO THE UNKNOWN

From the previous activity, you learned how perimeter, area and volume could be solved using various operations and formulae. In this next activity, you will find out how unknown dimensions in geometric figures could be derived by using algebraic expressions. After finding these dimensions, you will also learn to see how special products can facilitate the solving of perimeter, area and volume.

To do this activity, proceed to the link:

WORKSHEET:

1. The rectangular garden below has a length that is two more than three times the width.



Source: www.rotarygardens.org

Use algebraic expressions to represent the dimensions of the garden in terms of x .

What is the length? _____

What is the width? _____

Based from the previous activity, what formula will you use to solve for the perimeter in algebraic form? _____

What is the perimeter of the figure?

Based from the previous activity, what formula will you use to solve for the area in algebraic form? _____

What is the area of the figure?

2. Your mother wants to purchase a shell grotto for your house. When she visited a store, she found a shell grotto that is shaped like a square as shown in figure 1. It has a side which measures $x + 2$ cm. However, when she

looked at the wall, where it would be placed, she noticed that it was small so she requested the store to add three more centimeters to each side.



www.flickr.com



www.flickr.com

Based from the previous activity, what formula will you use to solve for the perimeter in algebraic form? _____

Express in algebraic form the perimeter of the

- a. Smaller shell grotto - _____
- b. larger shell grotto - _____

Based from the previous activity, what formula will you use to solve for the area in algebraic form? _____

Express in algebraic form the area of the

- a. Smaller shell grotto - _____
- b. larger shell grotto - _____

3. The circular swimming pool below has a diameter which measures $2x + 4$.

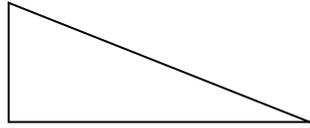


Source: www.susancohangardens.com

Based from the previous activity, what formula will you use to solve for the area in algebraic form? _____

Express in algebraic form the area of the pool. _____

4. A piece of land is triangular in shape. The hypotenuse measures three more than twice the height. The base measures two more than twice the height.



Based from the previous activity, what formula will you use to solve for the perimeter in algebraic form? _____

Express in algebraic form the perimeter of the land. _____

5. Try solving this problem on your own using the processes that you performed in the previous number.

A packaging box has a length that is two more than the width. The height is 3 more than the length. Express in algebraic form the volume of the packaging box.



Laundry Box Available: 2, 3, 5, 7, 9, 10 1/2 inch

www.alibaba.com

After solving, click on the link below to be able to see the correct answer:

LINK:

$$\begin{aligned}
 V &= L \times W \times H \\
 \text{Length} &= x + 2, \text{ width} = x \text{ and height} = x + 5 \\
 V &= (x + 2)(x)(x + 5) \\
 V &= x^3 + 7x^2 + 10x
 \end{aligned}$$

Questions to Answer:

1. What did you discover from the activity?

2. How did you apply special products in solving for the various measurements?

3. What procedures are usually involved or done?

4. Can the procedures be changed or altered? In what way?

5. What special product did you use the most?

6. What new insights did you gain?

7. How can unknown quantities in geometric problems be solved?

END OF FIRM UP:



In this section, the discussion focused on special products and how they are easily derived through the use of patterns. Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

Now that you know the important ideas about this topic, let us go deeper by moving on to the next section.



EXPLORE



Your goal in this section is to take a closer look at some aspects of the topic.

ACTIVITY 11. EXTENDING ONE’S KNOWLEDGE

After learning the different special products, you will now learn how to apply the special products in order to expand higher powers for binomials and trinomials and solve non-routine problems involving the concepts that you have learned. Click the link below to the guided worksheet which will assist you on how to solve non-routine problems on special products.

GUIDED WORKSHEET:

Your knowledge on special products could be used to solve each problem below. Study the following examples below.

1. $(x + 2y)^2 + (x - 2y)^2$

Step 1: $(x + 2y)^2 = x^2 + 4xy + 4y^2$

Step 2: $(x - 2y)^2 = x^2 - 4xy + 4y^2$

Step 3: $(x^2 + 4xy + 4y^2) + (x^2 - 4xy + 4y^2) = 2x^2 + 8y^2$

What special products were used to solve the problem? _____

Aside from multiplication, what other algebraic processes did you use to solve the problem? _____

2. $[(3x)(2x) - (2y)(-4y)]^2$

Step 1: $(3x)(2x) = 6x^2$

Step 2: _____

Step 3: _____

Step 4: _____

Multiply the subtrahend.

Combine like terms.

Square the binomial.

What special products were used to solve the problem? _____

Aside from multiplication, what other algebraic processes did you use to solve the problem? _____

3. $(3x - 2)^4$

Step 1: $(3x - 2)^2 (3x - 2)^2$

Step 2: $(3x - 2)^2 = 9x^2 - 12x + 4$

Step 3: $(9x^2 - 12x + 4)^2$

Step 4: _____ Derive the square of a trinomial.

What special products were used to solve the problem? _____

Aside from multiplication, what other algebraic processes did you use to solve the problem? _____

4. $[2h - k + 3]^3$

Step 1: $[(2h + k) + 3]^3$

Step 2: $(2h + k)^3 + 3(2h + k)^2(3) + 3(2h + k)(3)^2 + 3^3$

Step 3: $8h^3 + 12h^2k + 6hk^2 + k^3 + 36h^2 + 36hk + 9k^2 + 54h + 27k + 27$

What special products were used to solve the problem? _____

Aside from multiplication, what other algebraic processes did you use to solve the problem? _____

5. $[8a - 7 + 3b][8a + 7 - 3b]$

Step 1: $[8a - (7 - 3b)][8a + (7 - 3b)]$

Step 2: $[(8a)^2 - (7 - 3b)^2]$

Step 3: $64a^2 - (49 - 42b + 9b^2)$

Step 4: _____ Remove the grouping symbol.

What special products were used to solve the problem? _____

Aside from multiplication, what other algebraic processes did you use to solve the problem? _____

Questions to Answer:

1. In solving the problems above, how many special products did you utilize?

2. What algebraic processes did you perform aside from multiplying polynomials?

3. How are the special products related?

4. What can you generalize regarding solutions to the non-routine problems that you discussed?

5. Compare the solutions that you used in these problems with the routine problems that you solve before. In what way are they similar? different?

ACTIVITY 12. MUDDIEST POINT

In this activity, you will complete the journal below.

The part of the lesson that I still find confusing is _____
because _____.

Let us see if by doing the next activity, the muddiest point will be given clarification.

ACTIVITY 13. SKILL BOOSTER

In this activity, you will solve non-routine problems similar to the ones given in the guided worksheet. Justify each step that you have performed by citing a special product used or any algebraic processes performed.

WORKSHEET:

Simplify the following expressions below. Justify each step that you perform.

1. $(6x - 7y + 9z)^3$

STEPS

REASONS

_____	_____
_____	_____
_____	_____
_____	_____

2. $[m + p + 5][m + p - 5]$

STEPS

REASONS

_____	_____
_____	_____
_____	_____
_____	_____

3. $(3 + x)(2x + 5) + (3x + 5)^2$

STEPS

REASONS

_____	_____
_____	_____
_____	_____
_____	_____

4. $(m - p)^5$

STEPS

REASONS

_____	_____
_____	_____
_____	_____
_____	_____
_____	_____

5. $(2x + y + z - 1)^2$

STEPS

REASONS

_____	_____
_____	_____
_____	_____
_____	_____

Questions to Answer:

1. After answering the worksheet, what new knowledge did you find out regarding special products?

2. Why was there a need to cite the reasons for performing each step of the simplification process?

3. Which part do you still find confusing?

4. In which particular algebraic process did you commit the most error?

After answering the process questions, compare the new knowledge that you have gained in the previous part of the lesson. What new realizations did you gain? What new connections have you made for yourself?

ACTIVITY 14. IRF Worksheet



Now that you have learned the different special products, using the “F” portion of the IRF Worksheet, answer the topical focus question: “What makes a product special?”

IRF Worksheet
Initial Answer
Revised Answer
Final Answer

ACTIVITY 15. MANAGING COMPLEXITY

Let us now extend our knowledge on solving for the measurement of complex figures. In this activity, you will watch a video about finding the area of composite figures. Click this link: <http://www.youtube.com/watch?v=9HE3kfu8K1I>.

Questions to Answer:

1. What kind of figures did you see?

2. How were the figures shown dealt with?

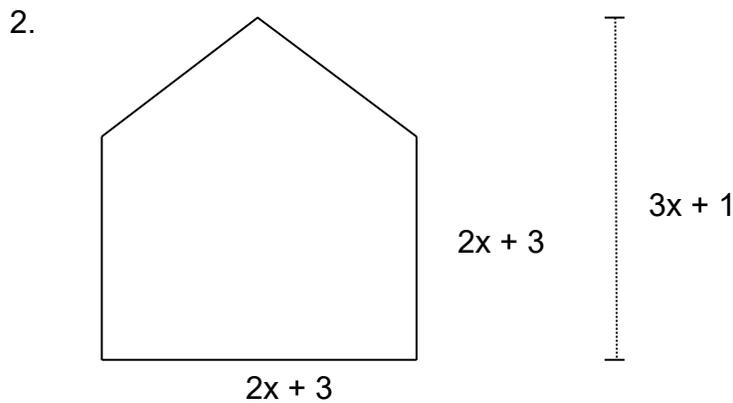
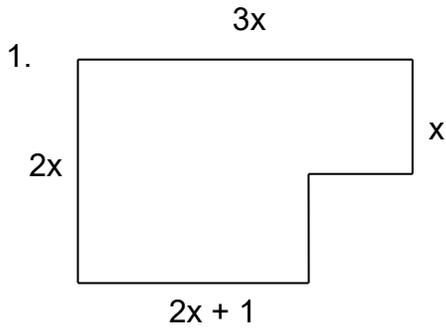
3. Name the steps performed to get the area of the entire figure. Explain why such steps need to be done.

4. What would happen if the dimensions are not given in numerical form.

ACTIVITY 16. PART BY PART

You will now be asked to practice solving on your own. In this activity, you will complete the worksheet on solving for the area of composite figures. Click on the link _____

WORKSHEET:



Questions to Answer:

1. How did you solve each given problem?

2. What did you find difficult?

3. How did you use special products?

4. What new insights did you gain?

5. How can unknown quantities in geometric problems be solved?

END OF DEEPEN:



In this section, the discussion was about using the concepts of special products to be able to solve non routine problems which will involve higher order polynomials derived from the use of a variety of special products and algebraic processes.

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.



TRANSFER



Your goal in this section is to apply your learning to real life situations. You will be given a practical task which will demonstrate your understanding.

ACTIVITY 17. SCAFFOLD FOR TRANSFER 1 (SCALE DRAWING)

In preparation for your performance task, you will watch this video on scale drawing: <http://www.youtube.com/watch?v=u5LaVILWzx8>.

Questions to Answer:

1. What is a scale drawing?

2. How is a scale made?

3. How do you construct a scale drawing?

4. Why is using scale drawing important?

5. How can you use scale drawing in your daily tasks?

ACTIVITY NO. 18: SCAFFOLD FOR TRANSFER 1 (MODEL MAKING)

Aside from scale drawing, another skill that you need to learn for the packaging box construction is model making. This next video will help you learn how geometric models are constructed. Click this link: <http://www.youtube.com/watch?v=er2-q86kJ9s>.

Questions to Answer:

1. What model was constructed in the video?

2. What procedures were used in the construction?

3. How did you find the scaffolding activities?

4. How did the task help you see the real world use of the topic?

END OF TRANSFER:



In this section, your task was to look at how the skills needed to accomplish the performance task were done.

You have completed this lesson. Before you go to the next lesson, you have to answer the following post-assessment.

LESSON POST-ASSESSMENT



It's now time to evaluate your learning. Click on the letter of the answer that you think best answers the questions. If you do not do well, you may move on to the next lesson. If your score is not at the expected level yet, you have to go back and take the lesson again.

- Which is the error in the product below?

$$(2x^3 - 5)^3 = \frac{8x^9}{A} - \frac{60x^6}{B} + \frac{150x^3}{C} + \frac{75}{D}$$
- Which of the following is a perfect square trinomial?
 A. $x^2 + 2xyz^2 - y^2z^4$
 B. $x^2 + 2xyz + y^2z^4$
 C. $x^2 - 2xyz - y^2z^4$
 D. $x^2 - 2xyz^2 + y^2z^4$
- The side of a square is x meters long. The length of a rectangle is 3 meters longer than a side of the square and the width of the rectangle is 3 meters shorter than the side of the square. Which has greater area and by how much?
 A. the rectangle has larger area by 6 units
 B. the rectangle has larger area by 9 units
 C. the square has larger area by 6 units
 D. the square has larger area by 9 units
- The length of a side of a square window in Jessica's bedroom is represented by $2x - 1$. Which expression represents the area of the window?
 A. $2x^2 + 1$
 B. $4x^2 + 4x - 1$
 C. $4x^2 - 4x + 1$
 D. $4x^2 + 1$
- Which equation is true?
 A. $(a + 2)^3 = a^3 + 8$
 B. $(3 - 2y)^2 = 9 - 12y + 4y^2$
 C. $(m^2 - 2)(m^2 + 2) = m^4 + 4$
 D. $(2m + 5)(2m - 5) = 4m^2 + 25$

Lesson 1.2: Factoring

LESSON PRE-ASSESSMENT



Let's find out how much you already know about this lesson. Click on the letter that you think correctly answers the question. Please answer all items. After taking the test, you will see your score. Take note of the items that you were not able to correctly answer and look for the right answer as you go through this lesson.

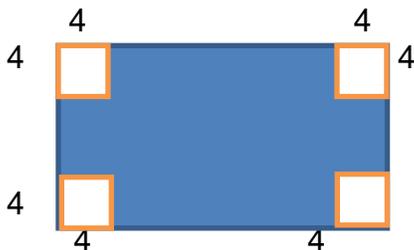
- Which of the following gives the product $3x^2 - 5x - 2$?
 - $(3x + 1)(x - 2)$
 - $(3x - 1)(x + 2)$
 - $(3x + 2)(x - 1)$
 - $(x + 2)(3x - 3)$
- How is $4x^2 - 36$ be expressed as product of factors?
 - $4(x - 6)(x + 6)$
 - $4(x - 3)(x + 3)$
 - $(2x - 6)(2x + 6)$
 - $4(x^2 - 9)$
- What should you multiply to $3m + n$ to obtain the product of $9m^2 - n^2$?
 - $3m - 1$
 - $3m - n^2$
 - $3m - n$
 - $3m^2 - 1$
- What are factors of $-72b^2$ such that their sum is $-34b$?
 - $-36b$ and $2b$
 - $-72b$ and b
 - $36b$ and $-2b$
 - -36 and $2b^2$
- Which of the following is a perfect square trinomial?
 - $25m^2 + 6m + 1$
 - $36x^2 + 72x + 81$
 - $4a^2 + 12a + 36$
 - $8x^2 + 16x + 16$
- What is the missing term in $4s^2 + \underline{\hspace{1cm}} + 121$ so that the expression becomes a perfect square trinomial?
 - $44s$
 - $88s$

- C. 22s
- D. 48s

7. The area of a rectangle is $2x^2 + x - 190$. What may be its length and width?

- A. $length = x - 10, width = 2x + 19$
- B. $length = 2x + 10, width = x - 19$
- C. $length = 2x - 10, width = x + 19$
- D. $length = x + 10, width = 2x - 19$

8. The length of a rectangular piece of cardboard is twice its width. A 4- cm square is cut out of each corner, and the sides are turned up to make a box. The volume of the box is 616 cm^3 . How long and wide was the card board originally?



- A. 15 cm by 30 cm
 - B. 9 cm by 18 cm
 - C. 81 cm by 85 cm
 - D. 3 cm by 15 cm
9. Which of the following is NOT true?
- A. $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
 - B. $x^2 + y^2 = (x + y)(x + y)$
 - C. $x^2 - y^2 = (x + y)(x - y)$
 - D. $x^3 - y^3 = (x - y)(x^2 - xy + y^2)$
10. A rectangular floor has an area of $2x^2 + 13x + 15 \text{ m}^2$. If the width is $(x + 5) \text{ m}$, how long is it if the actual width is 12 m?
- A. 27 m
 - B. 17 m
 - C. 19 m
 - D. 10 m
11. How many tiles of 1 meter by 1 meter would be needed to cover the floor?
- A. 58 pieces
 - B. 116 pieces

- C. 204 pieces
D. 104 pieces
12. If Mr. Reyes bought 11 boxes of 1 m by 1 m tile and each box contains 20 pieces of tiles, how many excess would there be if there is any?
A. 36 pieces
B. 104 pieces
C. 24 pieces
D. 16 pieces
13. Which of the following is equal to $4x^2 - 28x + 49$?
A. $(2x - 7)(2x - 7)$ or $(2x - 7)^2$
B. $(7x + 2)(2x - 7)$
C. $(2x + 7)^2(2x - 7)^2$
D. $4x^2 - 49$
14. The area of a square is defined by $25a^2 + 20a + 4$ sq. cm. Which of the following represents the length of its side?
A. $(5a + 4)$ cm
B. $5a$ cm
C. $(5a + 2)$ cm
D. $(10a + 2)$ cm
15. The actual area of the square in item # 14 is 144 sq. ft. when $a = 2$ ft. How much difference would the area be if the value of a is doubled?
A. 144 sq. ft.
B. 340 sq. ft.
C. 288 sq. ft.
D. 140 sq. ft.



EXPLORE



<http://www.treehugger.com/corporate-responsibility/cadburys-dairy-milk-chocolate-bar-goes-fairtrade.html>



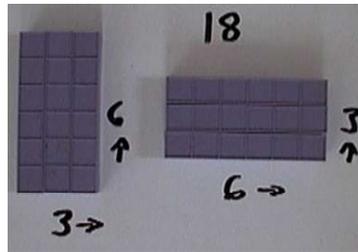
Have you ever thought of why these chocolate bars contain small squares inside? Why are most chocolates designed as small bars? Can you relate it with the previous lesson on special products? Write down your answers on the box provided below.

To give you a glimpse of our next lesson, let us do Activity 1.

ACTIVITY 1. Measure Me!

This activity will lead you to think of the reverse process of Special Products.

Given a rectangle with an area of 18 sq. units, what are other dimensions that you can think of aside from what is shown below? Write in the box provided as many answers as you can think of.



<http://www.crewtonramoneshouseofmath.com/factoring-polynomials.html>



What do you consider in looking for the other dimensions? Suppose, one side is increased by an unknown quantity; example, x , how would you represent the new dimensions?

How can unknown quantities in geometric problems be solved?

The following activities would somehow lead us to answer this question. Let us do the next activity.

ACTIVITY 2. IRF Sheet

This activity will check the prior knowledge on factoring.

The IRF sheet contains polynomial expressions in the first column. You are going to write an equivalent expression that is the product form of each of the polynomial expressions in the second column under "My Initial Answer". You may apply the ideas that you gained from your Lesson on Special Products.

The IRF Sheet			
Polynomial Expressions	The factored Form		
	My Initial Answer	My Revised Answer	My Final Answer
1. $8x^6 - 12x^5 + 36x^3$			
2. $m^2 - 81n^2$			
3. $144 - 9a^2b^4$			
4. $9m^2 + 30m + 25$			
5. $4x^2 - 36x + 81$			
6. $x^2 - 2x - 63$			

7. $6x^2 + 31x + 35$			
8. $20x^2 + 7x - 3$			
9. $8x^2 + 27y^3$			
10. $ax + bx - 2a - 2b$			

END OF EXPLORE:



Let us find out if your initial answers are correct after performing the next activities. What you will learn in the next activities will also enable you to do the final project which involves model and lay – out making of a packaging box.



FIRM-UP



In order for you to gain knowledge on factoring polynomials, watch the videos through the different links and answer the practice exercises after each of the discussions about factoring and be ready to complete the graphic organizer.

ACTIVITY 3. Video Watching A

This activity will provide a direct instruction on Factoring Polynomials by GCF (Greatest Common Factor).

Click the following link below for the complete discussion of the first technique of factoring polynomials: <http://www.youtube.com/watch?v=3RJIPvX-3vg>. This site discusses the first technique in factoring polynomials, factoring by GCF)

Questions to Answer:

1. What factoring technique was presented in the video clip?

2. How was the process of factoring done?

3. How will it be useful in finding the factors of a certain polynomial?

Now, let us do a practice exercise to check on your mastery of the first technique of factoring polynomials.

Assessment 1: Practice Exercise A

This activity will check on your retention and mastery on the first method of factoring polynomials, by GCF.

Click the button that corresponds to the product form of the given polynomial:

- | | |
|-------------------------------------|--|
| 1. $3x^4 + 9x^3$ | $3x^3(x + 3)$
$3x^4(1 + 3x)$
$3x(x^3 + 3x^2)$
$x^3(3x + 9)$ |
| 2. $3abc - 2ab^2c + 5a^2b^3c^2$ | $abc(b + 5ab^2c)$
$abc(3 - 2b + 5abc)$
$abc(3 - 2b + 5ab^2c)$
$abc(1 + 5ab^3c)$ |
| 3. $12x^3y^2z^2 + 24x^4y^3z^2$ | $12x^3y^2z(1 + 2xyz)$
$12x^3y^2z^2(1 + 2xy)$
$12x^2y^2z^2(1 + 2xy)$
$12x^3y^2z^2(1 + 2xyz)$ |
| 4. $13m^4n^2 + 39m^3n^3 - 26m^2n^4$ | $13m^2n^2(m^2 + 3mn - 2n^2)$
$13m^2n^2(m^2 + 3mn - 2n)$
$13m^2n(m^2 + 3mn - 2n^2)$
$13m^2n^2(m + 3mn - 2n^2)$ |
| 5. $x(a + 2) - y(a + 2)$ | $(x + y)(a - 2)$
$xy(a + 2)$
$-xy(a + 2)$ |

$$(x - y)(a + 2)$$

The exercises that you have just performed involved only one technique of factoring polynomials.

What if a polynomial expression does not have a GCF, for example $x^2 + 3x + 2$, or $x^2 - 9$? Will you immediately conclude that the expression is prime or non-factorable? How can these expressions be written in product form?

Let's now learn another factoring technique. Click on the following link.

ACTIVITY 4. Video Watching B

This activity will provide direct instructions on the two techniques of Factoring Polynomials: Factoring Trinomials of the form $ax^2 + bx + c$ and Factoring Difference of Two Squares.

<http://www.onlinemathlearning.com/algebra-factoring-2.html>

(This site contains videos that introduce two factoring techniques; factoring trinomials and factoring difference of two squares.)

Questions to Answer:

1. What factoring techniques did you encounter in the presentation?

2. What forms of polynomials are dealt with?

3. When can we use GCF in these forms?



The videos that you have watched discuss two techniques of factoring; factoring quadratic trinomials and factoring difference of two squares. Before you will take the next test, you will watch another video which focuses on the process of factoring difference of two squares. Take note of how the process is done so you can answer the next test easily.

ACTIVITY 5. Video Watching C

This activity will provide more discussion on Factoring Difference of Two Squares.

<http://www.youtube.com/watch?v=Vpol-yyi43c&feature=related>

(This site contains follow up discussion on factoring difference of two squares).

Now, let us check your mastery on the two factoring techniques that you have just studied. Take the next test.

Assessment 2: Practice Exercise B

This activity will check on your retention and mastery on factoring quadratic trinomials and difference of two squares.

Click the button that corresponds to the product form of the given polynomial:

1. $x^2 + 13x + 42$
 - $(x + 6)(x + 7)$
 - $(x + 21)(x + 2)$
 - $(x + 10)(x + 3)$
 - $(x - 21)(x + 2)$

2. $2x^2 + x - 3$
 - $(2x - 3)(x + 1)$
 - $(2x + 3)(x - 1)$
 - $(2x + 1)(x - 3)$
 - $(2x - 1)(x + 3)$

3. $12x^2 - 11x + 2$
 - $(6x - 1)(2x - 2)$
 - $(4x - 1)(3x - 2)$
 - $(4x - 2)(3x - 1)$
 - $(6x - 2)(2x - 1)$

4. $3x^2 - 17x - 6$
 - $(3x + 1)(x - 6)$
 - $(3x - 2)(x + 9)$
 - $(x - 9)(3x + 2)$
 - $(3x - 1)(x + 6)$

5. $42x^2 + 19x - 45$
 - $(7x - 5)(6x + 9)$
 - $(21x + 9)(2x - 5)$
 - $(6x + 5)(7x - 9)$
 - $(7x + 9)(6x - 5)$

6. $x^2 - 64$
 - $(x + 6)(x - 6)$
 - $(x - 8)(x - 8)$

- $(x - 8)(x + 8)$
 $(x + 8)(x + 8)$
7. $4x^2 - 9$ $(2x + 3)(2x - 3)$
 $(4x - 3)(x + 3)$
 $(4x - 9)(x + 1)$
 $(4x - 1)(x + 9)$
8. $100 - 81x^2$ $(100 - 9x)(1 + 9x)$
 $(10 - 9x)(10 + 9x)$
 $(10 + 9x)(10 + 9x)$
 $(100 - x)(1 - 81x)$
9. $a^4b^2 - 121c^6$ $(a^2b - 11c^3)(a^2b + 11c^3)^*$
 $(a^2b + 12c^3)(a^2b - 11c^3)$
 $(a^2b - 11c^3)(a^2b - 11c^3)$
 $(a^2b + 12c^3)(a^2b - 12c^3)$
10. $\frac{4}{25}x^4 - \frac{16}{49}y^2$ $\left(\frac{2}{5}x^2 - \frac{4}{7}y\right)\left(\frac{2}{5}x^2 - \frac{4}{7}y\right)$
 $\left(\frac{2}{5}x^2 - \frac{2}{7}y\right)\left(\frac{2}{5}x^2 + \frac{8}{7}y\right)$
 $\left(\frac{2}{5}x^2 - \frac{4}{7}y\right)\left(\frac{2}{5}x^2 + \frac{4}{7}y\right)$
 $\left(\frac{4}{5}x^2 - \frac{4}{7}y\right)\left(\frac{4}{5}x^2 + \frac{4}{7}y\right)$



Not all polynomials can be factored by using the techniques that you have been exposed to. Some of these are polynomials of the form $x^3 + y^3$ and $x^3 - y^3$. What polynomials that you have multiplied before gave the product of these forms? The following link will bring you to a page that will teach you on how to factor sum and difference of two cubes.

ACTIVITY 6. Video Watching D

Let us now look at another factoring technique that we could use. This activity will provide discussion on factoring sum and difference of two cubes.

<http://www.youtube.com/watch?v=8c7B-UaKI0U>

(This site will bring you to another technique in factoring which is factoring the sum and difference of two cubes.)

Questions to Answer:

1. What factoring technique was presented in the video clip?

2. How was the process done?

3. What important skill will you need to do this kind of factoring easy?



Your familiarization of the perfect cube numbers and correct identification of expressions that are perfect cubes are very important in finding for the factors of the polynomials of the form sum of two cubes and difference of two cubes. So that it would be easy for you to identify the cube of a number, try to have a list of the first 10 perfect cube numbers. You just cube every counting number from 1 up to 10. You can then continue from 11 and so on.

Let us now check your mastery on this factoring technique by answering the next test.

Assessment 3: Practice Exercise C

This activity will check the retention and mastery on factoring sum of two cubes and difference of two cubes.

Click the button that corresponds to the product form of the given polynomial:

1. $8a^3 + 27b^3$ $(2a + 3b)(4a^2 - 6ab + 9b^2)$
 $(2a - 3b)(4a^2 + 6ab + 9b^2)$
 $(2a + 3b)(4a^2 + 6ab + 9b^2)$
 $(2a - 3b)(2a^2 - 12ab + 9b^2)$

2. $125x^3 - 64$ $(5x - 4)(25x^2 - 20x + 16)$

$$(5x - 4)(25x^2 + 20x + 16)$$

$$(5x + 4)(25x^2 - 20x + 16)$$

$$(15x - 8)(5x^2 + 40x + 64)$$

3. $1 - 216m^3$
- $$(1 - 6m)(1 - 6m + 36m^2)$$
- $$(1 - 6m)(1 + 6m + 36m^2)$$
- $$(1 + 6m)(1 - 6m + 36m^2)$$
- $$(1 + 6m)(1 + 6m + 36m^2)$$
4. $64m^6 + 27n^6$
- $$(4m^2 + 3n^2)(16m^4 - 12m^2n^2 + 9n^4)$$
- $$(4m^2 - 3n^2)(16m^4 + 12m^2n^2 + 9n^4)$$
- $$(4m^2 + 3n^2)(16m^4 + 12m^2n^2 + 9n^4)$$
- $$(4m^2 - 3n^2)(16m^4 - 12m^2n^2 + 9n^4)$$
5. $a^3 - b^9$
- $$(a + b^3)(a^2 + ab^3 + b^6)$$
- $$(a - b^3)(a^2 + ab^3 + b^6)$$
- $$(a - b^3)(a^2 - ab^3 + b^6)$$
- $$(a + b^3)(a^2 - ab^3 + b^6)$$

You have learned several techniques of factoring polynomial expressions. Remember that when you factor polynomial expressions you change the expressions from terms to product of factors.

The next link below will show you another factoring technique. This technique is similar to factoring GCF but requiring you first to group/regroup terms. Watch the video closely as you will learn how this process is done. Click the link below.

<http://www.youtube.com/watch?v=-hiGJwMNNsM>

(This site will teach you how to do factoring by grouping/ regrouping of terms)

Questions to Answer:

1. When is grouping of terms necessary?

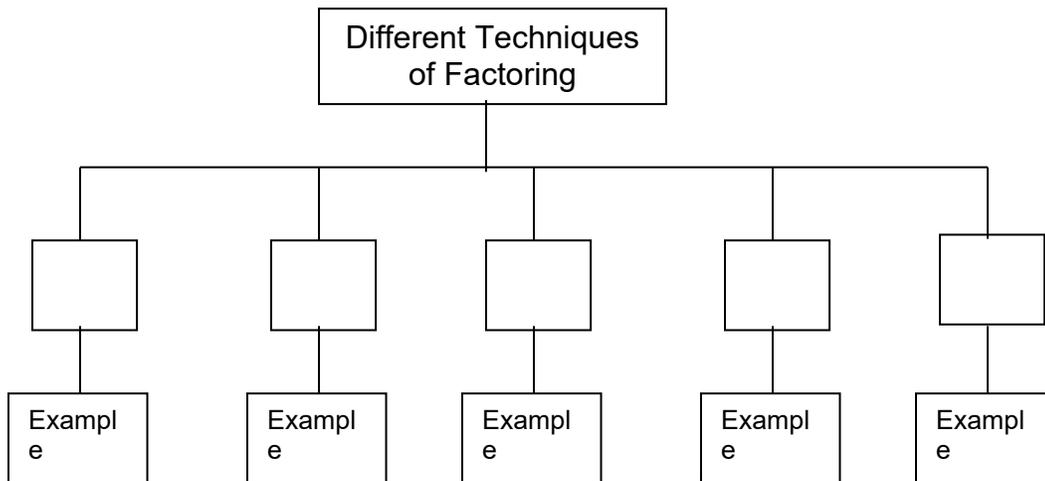
2. What should you consider when grouping/regrouping terms?

3. To summarize your learning, accomplish the graphic organizer below.

ACTIVITY 7. Graphic Organizer

This activity will summarize the different factoring techniques learned from the previous activities.

Accomplish the graphic organizer below by filling in the boxes of things you have learned from the different sources.



ACTIVITY 8. Skill Booster

This activity provides a link that is interactive which will enhance the mastery on factoring polynomials.

Perform all the exercises on the different types of factoring provided in this web site. Click the link below to answer some exercises.

http://www.khanacademy.org/math/algebra/polynomials/e/factoring_polynomials_1

(This site will check on your mastery in factoring general trinomial)

After doing some practice exercises and visiting different websites, you have learned already the different techniques of factoring polynomials.

ACTIVITY 9. IRF Revisit

This activity will give a chance to revise the answers of the student.

Pull back now the IRF Sheet that you accomplished prior to learning the different techniques from the different sources and answer the next column under “My Revised Answer”.

The IRF Sheet			
Polynomial Expressions	The factored Form		
	My Initial Answer	My Revised Answer	My Final Answer
1. $8x^6 - 12x^5 + 36x^3$			
2. $m^2 - 81n^2$			
3. $144 - 9a^2b^4$			
4. $9m^2 + 30m + 25$			
5. $4x^2 - 36x + 81$			
6. $x^2 - 2x - 63$			
7. $6x^2 + 31x + 35$			
8. $20x^2 + 7x - 3$			
9. $8x^2 + 27y^3$			
10. $ax + bx - 2a - 2b$			

Questions to Answer:

- How many items in your initial answer column did you change?

- Why do you think did you change them?

ACTIVITY 10. Algebra Tiles

In the previous exercises, you were factoring expressions that were actually representing area of plane figures. Let us now see in this part how factoring relates to solving for the area of figures.

Let us now see how the concept of factoring can be used also. The next sites that you will visit will show you another way of factoring polynomial. This activity will somehow prepare you to do your final output or project.

This website where you can also see factoring using algebra tiles is this site <http://illuminations.nctm.org/ActivityDetail.aspx?ID=216> . This site will give you exercises on factoring polynomials through the use of tiles.

After learning how the tiles are used in factoring polynomials, you may create also your own algebra tiles and factor different polynomials. Try it for yourself.

Questions to Answer:

1. Where can you apply the learning that you got from this activity?

2. How do special products and factors relate?

3. How can unknown quantities in geometric problems be solved?

END OF FIRM UP:



Now that you know the important ideas about this topic, let us go deeper by moving on to the next section.



DEEPEN



Your goal in this section is to take a closer look at some aspects of the topic.

The following activity will check your masterful understanding about factoring polynomials.

ACTIVITY 11. Spotting Errors

This activity will check how to associate between product and its factors.

Which of the following can be factored using difference of two squares?
Click the button corresponding to your answer and click as many as you can see.

- * $x^2 - 81$
- * $4a - 121b$
- $3x^2 - 4x^4$
- * $100 - 25m^6$
- * $3x^3 - 12x$
- * $36x^2 - 49y^4$
- $9b^2 - 144c^3$
- $a^4 - 64b^3$
- * $20a^2 - 5b^4$

How would we know that the expression is in a form difference of two squares? Write your answer in the box.

Your classmate asserted that $x^2 - 4x - 12$ and $12 - 4x - x^2$ has the same factors. Is your classmate correct? Prove by showing your solution in the box.

Make a generalization for the errors found in the following polynomials. Write it the box provided in each item.

1. $x^2 + 4 = (x + 2)(x + 2)$
2. $1.6x^2 - 9 = (0.4x - 3)(0.4x + 3)$
3. $3x^2 - 27$ is not factorable or prime

Are all polynomial expressions factorable? Cite examples to defend your answer. Write your answer in the box.

After having been exposed to different activities of factoring polynomials, answer the worksheet that follows. This will check on your masterful understanding of the different techniques of factoring.

ACTIVITY 12. Interactive Activity

This will check on the mastery of student on the different techniques of factoring polynomials.

Click on this link <http://www.guia.com//rr/36611.html>.

Rags to Riches: Answer questions in a quest fame and fortune. This is an interactive factoring game like “Who Wants to be A Millionaire?” The students will have to answer multiple type of test. If they answered wrongly, they lose all their money and they will go back to start. A student wins upon answering all questions correctly.

After practicing how to factor polynomials, let us find out if you can solve some application problems below.

1. The sugarcane plantation below has an area of $(x^2 + 5x + 6)$ hm². What are its possible dimensions?



www.123rf.com

2. If the length will be increased by 3 hm, what will be the new area?

After learning more on factoring polynomials through the website that you explored and some application problems that you solved, you now go back to the IRF sheet and see if your answers under column 2 are all correct.

ACTIVITY 13. IRF Revisit

This activity will allow the student to check their previous answers and allow to make changes for some of the items they think are incorrect.

Pull back your IRF sheet and revise your answer by filing in column 3 under “My Final Answer”.

The IRF Sheet			
Polynomial Expressions	The factored Form		
	My Initial Answer	My Revised Answer	My Final Answer
1. $8x^6 - 12x^5 + 36x^3$			
2. $m^2 - 81n^2$			
3. $144 - 9a^2b^4$			
4. $9m^2 + 30m + 25$			
5. $4x^2 - 36x + 81$			
6. $x^2 - 2x - 63$			
7. $6x^2 + 31x + 35$			
8. $20x^2 + 7x - 3$			
9. $8x^2 + 27y^3$			
10. $ax + bx - 2a - 2b$			

Questions to Answer:

1. What have you observed with your answers in your initial column? Is there a big difference?

2. What realization can you make with regards to the relationship of special products and factors?

3. What new realizations do you have about the topic? What new connections have you made for yourself?

END OF DEEPEN:



Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.



TRANSFER

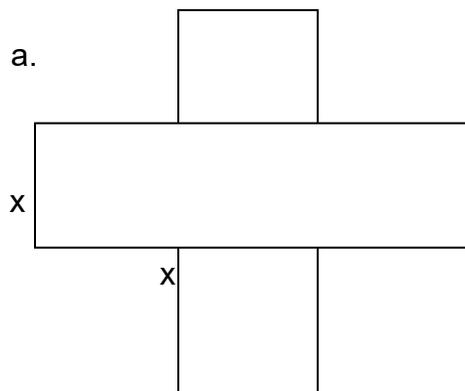


Your goal in this section is apply your learning to real life situations. You will be given a practical task which will demonstrate your understanding.

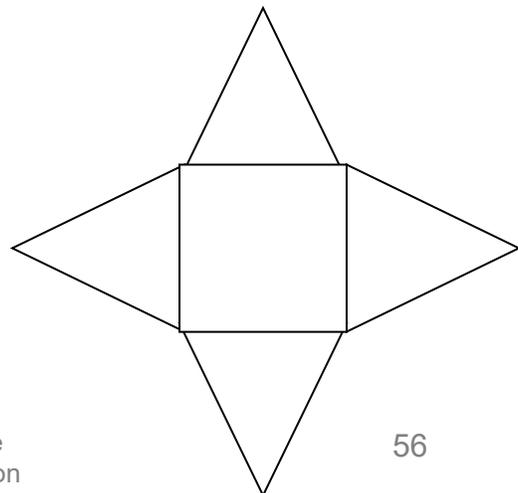
ACTIVITY 14. Model Making

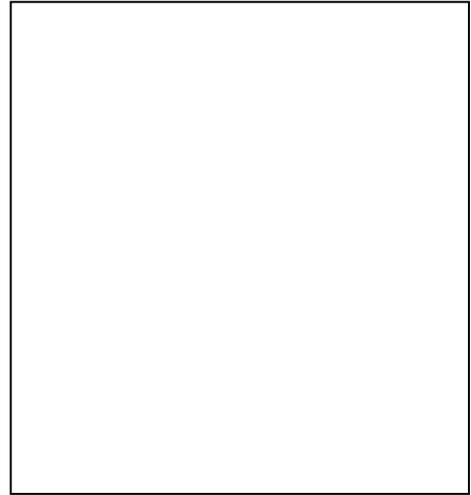
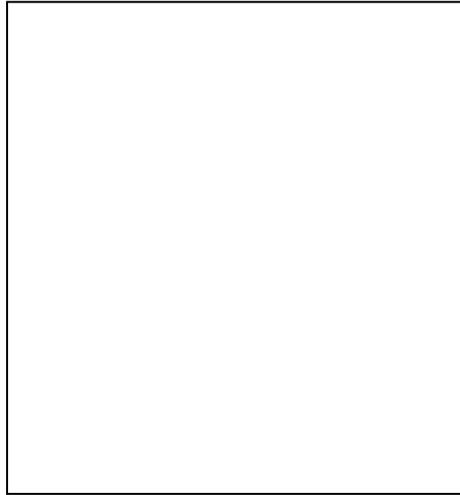
Create a space figure out of a plane figure/s provided.

1. Create a space figure that you can produce using the plane figure below.

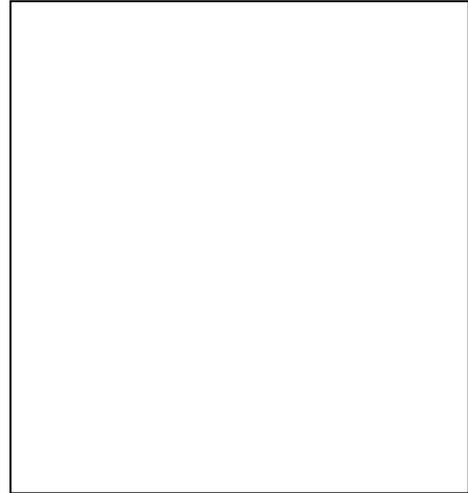
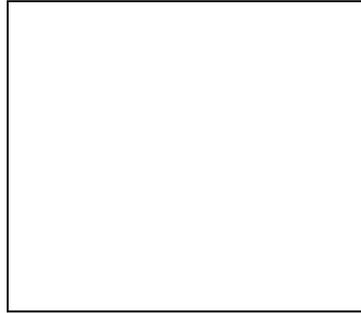


b.





c.



2. The length of each side of the plane figures above is x units. How would you determine the volume of the resulting figure in terms of x ? Express this in factored form.

Questions to Answer:

1. What have you learned about the activity?

2. How do you relate factoring in finding volume of space figures?

3. How can unknown quantities in geometric problems be solved?

End of TRANSFER:



How did you find the performance task? How did the task help you see the real world use of the topic?

You have completed this lesson. Before you go to the next lesson, you have to answer the following post-assessment.

LESSON POST-ASSESSMENT



It's now time to evaluate your learning. Click on the letter of the answer that you think best answers the questions. If you do not do well, you may move on to the next lesson. If your score is not at the expected level yet, you have to go back and take the lesson again.

1. Which of the following gives the product $3x^2 - 5x - 2$?
 - A. $(3x + 1)(x - 2)$
 - B. $(3x - 1)(x + 2)$
 - C. $(3x + 2)(x - 1)$
 - D. $(x + 2)(3x - 3)$

2. How is $4x^2 - 36$ be expressed as product of factors?
 - A. $4(x - 6)(x + 6)$
 - B. $4(x - 3)(x + 3)$
 - C. $(2x - 6)(2x + 6)$
 - D. $4(x^2 - 9)$

3. What should you multiply to $3m + n$ to obtain the product of $9m^2 - n^2$?
 - A. $3m - 1$
 - B. $3m - n^2$
 - C. $3m - n$
 - D. $3m^2 - 1$

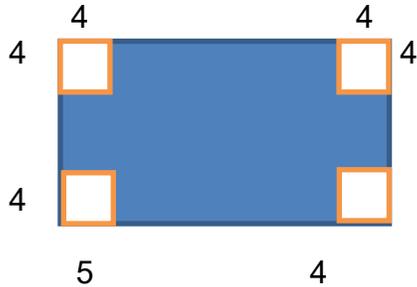
4. What are factors of $-72b^2$ such that their sum is $-34b$?
 - A. $-36b$ and $2b$
 - B. $-72b$ and b
 - C. $36b$ and $-2b$
 - D. -36 and $2b^2$

5. Which of the following is a perfect square trinomial?
 - A. $25m^2 + 6m + 1$
 - B. $36x^2 + 72x + 81$
 - C. $4a^2 + 12a + 36$
 - D. $8x^2 + 16x + 16$

6. What is the missing term in $4s^2 + \underline{\hspace{1cm}} + 121$ so that the expression becomes a perfect square trinomial?
 - A. $44s$
 - B. $88s$
 - C. $22s$
 - D. $48s$

7. The area of a rectangle is $2x^2 + x - 190$. What may be its length and width?
 - A. *length* = $x - 10$, *width* = $2x + 19$
 - B. *length* = $2x + 10$, *width* = $x - 19$
 - C. *length* = $2x - 10$, *width* = $x + 19$
 - D. *length* = $x + 10$, *width* = $2x - 19$

8. The length of a rectangular piece of cardboard is twice its width. A 4- cm square is cut out of each corner, and the sides are turned up to make a box. The volume of the box is 616 cm^3 . How long and wide was the card board originally?



- A. 15 cm by 30 cm
B. 9 cm by 18 cm
C. 81 cm by 85 cm
D. 3 cm by 15 cm
9. Which of the following is NOT true?
A. $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
B. $x^2 + y^2 = (x + y)(x + y)$
C. $x^2 - y^2 = (x + y)(x - y)$
D. $x^3 - y^3 = (x - y)(x^2 - xy + y^2)$
10. A rectangular floor has an area of $2x^2 + 13x + 15 \text{ m}^2$. If the width is $(x + 5) \text{ m}$, how long is it if the actual width is 12 m?
A. 27 m
B. 17 m
C. 19 m
D. 10 m
11. How many tiles of 1 meter by 1 meter would be needed to cover the floor?
A. 58 pieces
B. 116 pieces
C. 204 pieces
D. 104 pieces
12. If Mr. Reyes bought 11 boxes of 1 m by 1 m tile and each box contains 20 pieces of tiles, how many excess would there be if there is any?
A. 36 pieces
B. 104 pieces
C. 24 pieces
D. 16 pieces
13. Which of the following is equal to $4x^2 - 28x + 49$?
A. $(2x - 7)(2x - 7)$ or $(2x - 7)^2$
B. $(7x + 2)(2x - 7)$
C. $(2x + 7)^2(2x - 7)^2$
D. $4x^2 - 49$

14. The area of a square is defined by $25a^2 + 20a + 4$ sq. cm. Which of the following represents the length of its side?
- A. $(5a + 4)$ cm
 - B. $5a$ cm
 - C. $(5a + 2)$ cm
 - D. $(10a + 2)$ cm
15. The actual area of the square in item # 14 is 144 sq. ft. when $a = 2$ ft. How much difference would the area be if the value of a is doubled?
- A. 144 sq. ft
 - B. 340 sq. ft.
 - C. 288 sq. ft
 - D. 140 sq. ft

Lesson 1.3: Application of Special Products and Factors

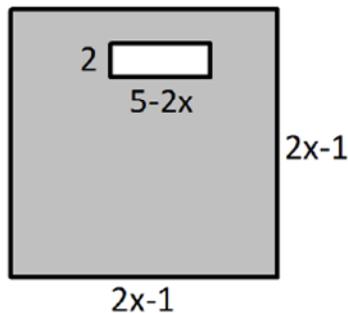


Let's find out how much you already know about this lesson. Click on the letter that you think correctly answers the question. Please answer all items. After taking the test, you will see your score. Take note of the items that you were not able to correctly answer and look for the right answer as you go through this lesson.

1. The area of a rectangle is given by $x^2 - 4x - 21$. Which expressions represent the dimensions of the rectangle?
 - A. $x - 7$ and $x - 3$
 - B. $x + 7$ and $x - 3$
 - C. $x - 21$ and $x + 1$
 - D. $x - 7$ and $x + 3$

2. You deposit ₱1000 in a savings account that has an annual interest rate r . At the end of three years, the value of your account will be $1000(1 + r)^3$ pesos. Find the amount of money in the account if the interest rate is 3%.
 - A. ₱1092.73
 - B. ₱3090.00
 - C. ₱2060.00
 - D. ₱2197.00

3. A square piece of land was rewarded by a master to his servant. They agreed that a portion of it, represented by the rectangle inside, should be used to construct a grotto. Find the area of the land that is available for other uses.



- A. $4x^2 - 9$
- B. $4x^2 + x + 9$
- C. $4x^2 - 8x - 9$
- D. $4x^2 + 9$

4. Your sister wants to enlarge her closet that has an area of $(x^2 + 3x + 2)$ ft². The length is $(x + 2)$ ft. After construction, the area will be $(x^2 + 8x + 15)$ ft² with a length of $(x + 3)$ ft. By how many feet will the length and the width increase after construction?
- The length will increase by four feet and the width will increase by one feet.
 - The length will increase by one foot but the width will remain the same.
 - The length and the width will increase by two feet each.
 - The length will increase by one foot and the width will increase by four feet.
5. When your long test papers were returned by your Math teacher, your classmate approached you to check the item in the test found below where he made a mistake.

Factor the polynomial completely.

$$\begin{aligned} & x^2 - 8x + 16 - 36y^4 \\ & = (x^2 - 8x) + (16 - 36y^4) \\ & = x(x - 8) + 4(4 - 9y^4) \\ & = \text{prime} \end{aligned}$$

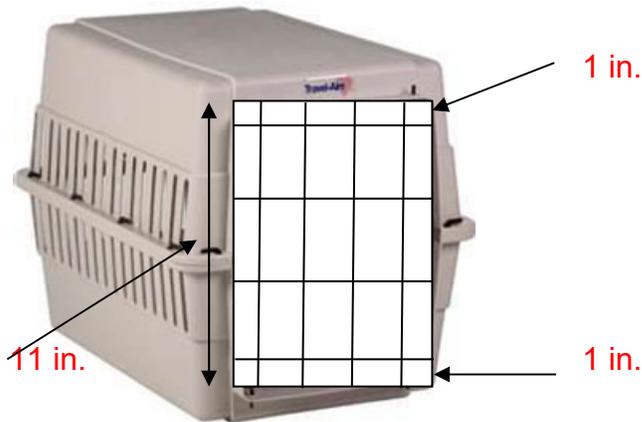
Your classmate asked you to explain why his solution was marked wrong by the teacher. How would you explain to him the correct way of factoring?

- The choice of grouping is wrong. The first three terms should be grouped to form a perfect square trinomial and if combined with the last term will form a difference of two squares.
- The choice of grouping is wrong. The first and third terms must be grouped and the second and fourth terms. The second and fourth terms have a greatest common factor.
- The choice of grouping is wrong. The first and the last terms must be grouped and the second and third terms. The first and last terms form a difference of two squares. The second and third terms have a greatest common factor.
- The choice of grouping is correct. The factoring in the third step is not completed. There is still a difference of two squares.



EXPLORE

You are shopping for a travel kennel. Your new puppy can just barely squeeze through a square opening that is $3\frac{1}{2}$ inches on each side. Are the grille openings in the kennel below too large to hold your puppy?



www.edgw365.com

Let us now see if your explanation is correct. Click the answer button.
Now, answer the following process questions:

1. What did you find out regarding the problem?

2. How did you arrive at your answer?

3. What algebraic processes did you use?



The problem you solved shows one of the many situations where we can apply knowledge of special products and factors in Geometry. In this lesson, you will do varied activities which will help you answer the essential question: “How can unknown quantities in geometric problems be solved?”

Let’s begin by answering the “I” portion of the IRF Worksheet that you see below. Fill it up by writing your initial answer to the topical focus question:

IRF Worksheet
Initial Answer
Revised Answer
Final Answer

END OF EXPLORE:



You just tried finding out how special products and factors can be used to solve real life problems. Let us now find out what the answer is by doing the next part. What you will learn in the next sections will also enable you to do the final project which involves the making of a packaging box using the concepts of special products.



FIRM-UP



Your goal in this section is to learn and understand key concepts of special products.

ACTIVITY 1. DECISION, DECISION, DECISION!

Help each person decide what to do by applying your knowledge on special products to each given situations.

1. Jem Boy wants to make his pool rectangular in shape by increasing its length by 2 m and decreasing its width by 2 m. Jem Boy asked your expertise to help him decide on certain matters.



<http://www.oyster.com/las-vegas/hotels/luxor-hotel-and-casino/photos/square-pool-north-luxor-hotel-casino-v169561/#>

- a. What will be the new dimensions of Jem Boy's pool be?

- b. What will be the new area of Jem Boy's pool be? What special product will be used?

- c. If Jem Boy does not want the area of his pool to decrease, will he pursue his plan? Explain your answer.

2. Emmanuel wants to tile his rectangular floor by choosing between two tiles, one of which is 3 dm larger than the other. Emmanuel hired your services to help him decide which tile to use.

- a. What is the area that will be covered by the small tile? bigger tile?

- b. If the rectangular floor has dimensions of 15 dm by 40 dm, how many small tiles with side 2 dm are needed to cover it?

- c. How many bigger tiles are needed to cover the rectangular floor?

- d. If the smaller tile cost Php 10.00 and the bigger tile cost Php 60.00, which tile should Emmanuel use to have a lesser expenses in tiling his floor? Explain why.

ACTIVITY 2. MUDDIEST POINT

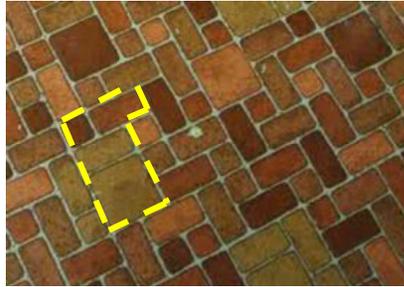
In this activity, you will complete the journal below.

The part of the lesson that I still find confusing is _____
because _____.

Let us see if by doing the next activity, the muddiest point will be given clarification.

ACTIVITY 3. BEAUTY IN MY TILE!

See different tile pattern on a building and calculate the area of the region bounded by the broken lines, then answer the questions on it sides.



1.

<http://www.apartmenttherapy.com/tile-vault-midcentury-rec-room-39808>

- What is the area represented by the big square? small square? rectangles?
- What is the total area bounded by the region?
- What special product is present in this tile design?
- Why do you think the designer of this room designed it as such?

ANSWERS:



<http://onehouseonecouple.blogzam.com/2012/03/master-shower-tile-progress/>

- What is the area represented by the big square? small square decoration?
- What is the sum of all areas of small squares?
- If the small square decorations were to remove, how are you going to represent the area that will be left?

ANSWERS:

Questions to Answer:

1. What difficulties did you find in doing the activity?

2. How did you use special products in this activity?

3. What new insights did you gain?

4. How can unknown quantities in geometric problems be solved?

ACTIVITY 4. WHERE IS THE PATTERN?

Take a picture/sketch a figure around you that you can see special products were applied. Upload it on the links found below.

Paste picture here:

Process Questions:

1. Did you find difficulty in looking for patterns where special products were applied?

2. What special products were applied in your illustration?

3. What realization can you make in this activity?

ACTIVITY 5. IRF Worksheet

Using the “R” portion of the IRF Worksheet, answer the essential question: **How can unknown quantities in geometric problems be solved?**

IRF Worksheet

Initial Answer
Revised Answer
Final Answer

END OF FIRM UP:

In this section, the discussion focused on ways in which special products and factors could be used to solve geometric problems.



Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

Now that you know the important ideas about this topic, let us go deeper by moving on to the next section.



DEEPEN



Your goal in this section is to take a closer look at some aspects of the topic.

ACTIVITY 6. The ALGEBRA in BIOLOGY

In this activity, you will learn how to make connections between algebra and biology by studying how special products could be used in the use of Punnett Squares. Click the link to learn more about Punnett Squares:

<http://www.youtube.com/watch?v=prkHKjfUmMs>

From the video that you watched, study the illustration found below:

A pink snapdragon has two genes – one white (W) and one red (R) that determines its color. The punnett square found below illustrates the possible results of crossing two pink snapdragons. Since each parent snapdragon passes along only one gene for color to an offspring and pollination is at random), the snapdragon offspring will be approximately 25% red (RR), 50% pink (RW) and 25% white (WW).



www.internetfamilyfun.com



		R	W
R		RR	RW
W		RW	WW

Each of the two pink parent snapdragons has half red genes and half white genes. You can model the genetic makeup of each parent as $50\%R + 50\%W = 0.5R + 0.5W$. When the two parents are crossed, the genetic makeup of the offspring can be modeled by the product $(0.5R + 0.5W)^2$.

$$\begin{aligned} (0.5R + 0.5W)^2 &= (0.5R)^2 + 2(0.5R)(0.5W) + (0.5W)^2 \\ &= 0.25R^2 + 0.5RW + 0.25W^2 \end{aligned}$$

Thus, 25% of the offspring should be red, 50% should be pink and 25% should be white.

Now, try solving the problem below:

In tigers, the normal color gene C, is dominant and the albino color A, is recessive. This means that tigers whose color genes are CC, CA or AC will have normal coloring. A tiger whose color genes are AA will be an albino. The Punnett square at the right shows the possible results of crossing two tigers that have recessive albino genes. What percent of the offspring of two such tigers will be the normal color? What percent will be albino? Use the model $(0.5C + 0.5A)^2 = 0.25CC + 0.5CA + 0.25AA$ to answer the questions.

PROCESS QUESTIONS:

1. How did this particular Biology lesson make use of Algebra?

2. What algebraic processes did you use?

3. What insights did you find out from doing this activity?

ACTIVITY 7. The ALGEBRA in BUSINESS

In this activity, you will learn how to make connections between algebra and business by studying how special products could be used to solve for compound interest. Click on the links below to learn more about compound interest:

1. <http://www.youtube.com/watch?v=DB-goEkQGOo>
2. <http://www.youtube.com/watch?v=ISfyTzVxfMo>

From the videos that you watched, study the illustration found below:

You invested ₱2000 in a savings account that earns interest compounded annually at a rate of 6%. Find the savings account balance after the two-year period.

Using the formula, $A = P(1+r)^t$

$$A = 2000(1 + 0.06)^2$$

$$A = 2000(1 + 0.12 + 0.0036)$$

$$A = 2000 + 2000(0.12) + 2000(0.0036)$$

$$A = ₱ 2247.20$$

Questions to Answer:

1. What special product was used to derive the equation?

2. How could Algebra be used in business?

To help you see more connections, study the illustration found below:

You invested ₱2000 in a savings account that earns interest compounded annually at a rate of 6%. Find the savings account balance after the three-year period.

Using the formula, $A = P(1+r)^t$

$$A = 2000(1 + 0.06)^3$$

$$A = 2000(1 + 0.18 + 0.0108 + 0.000216)$$

$$A = 2000 + 2000(0.18) + 2000(0.0108) + 2000(0.00216)$$

$$A = \text{₱ } 2382.03$$

Questions to Answer:

1. What special product was used to derive the equation?

2. How is the process different from the previous illustration?

3. Did you discover an easier way to solve the problems?

4. What new insights did you learn?

Now, try solving the problem below:

You invested ₱10000 in a savings account that pays compound interest at 4% per annum. Find the total amount in the savings account after a period of 2 years? 3 years?

END OF DEEPEN:



In this section, the discussion was about using the concepts of special products and factors to be able to solve problems related to Biology and Business.

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.



TRANSFER



Your goal in this section is apply your learning to real life situations. You will be given a practical task which will demonstrate your understanding.

ACTIVITY 8. I BRING MY TRASH HOME

Perform the activity in preparation for your final output in this module.

TASK

In response to the school’s environmental advocacy, you are required to make cylindrical containers for your trash. This is in support of the “I BRING MY TRASH HOME!” project of our school. You will present your output to your homeroom adviser and it will be graded according to the following criteria: explanation of the proposal, accuracy of computations, and utilization of the resources and appropriateness of the models.

Questions to Answer:

1. Did you experience any difficulty in making containers for your trash? If so, how did you overcome those difficulties?

2. How did you apply the concept of special product and factoring in making containers?

3. How many containers did you make?

4. What will happen if you only produce one container for your trash?

5. How does the “I bring my trash home” project of the school help preserve the environment?

ACTIVITY 9. PACKAGING ACTIVITY

After learning special products and factoring and their varied applications to real life situations, it is now time to showcase your learning in this module. You will assume the role of a member of a designing team that will present in a packaging company.

TASK

The RER packaging company is in search for the best packaging for a new dairy product that they will introduce in the market. (R) You are a member of the design department of RER Packaging Company. (G) Your company is tapped to create the (P) best packaging box using a provided square shaped material with an area of 10 in^2 . (A) You are to present the design proposal for the box and cylinder to the Chief Executive Officer of the dairy company and head of the RER Packaging department. (S) The design proposal is evaluated according to the following: explanation of the proposal, accuracy of computations, utilization of the resources and appropriateness of the model.

Use the rubric below as your guide in completing your final project:

CRITERIA	Outstanding 4	Satisfactory 3	Developing 2	Beginning 1	RATING
Explanation of the Proposal (20%)	Explanations and presentation of the lay-out is detailed and clear.	Explanations and presentation of the lay-out is clear.	Explanations and presentation of the lay-out is a little difficult to understand but includes some critical components.	Explanations and presentation of the lay –out is difficult to understand and is missing several components.	
Accuracy of Computations (30%)	The computations done are accurate and show understanding of the concepts of special products and factoring. There is an explanation for every computation made.	The computations done are accurate and show proper use of the concepts of special products and factoring.	The computations done are erroneous and show incorrect use of the concepts of special products and factoring.	The computations done are erroneous and do not show wise use of the concepts of special products and factoring.	
Utilization of Resources (20%)	Resources are efficiently utilized with less than 10% excess.	Resources are fully utilized with less than 10%-25% excess.	Resources are utilized but with a lot of excess.	Resources are not utilized properly.	
Appropriateness of the Model (30%)	The model is well-crafted and useful for understanding the design proposal. It showcases the desired product and is artistically done.	The model is well-crafted and useful for understanding the design proposal. It showcases the desired product.	The diagram and model are less useful in understanding the design proposal.	The diagram and model are not useful in understanding the design proposal.	
				OVERALL RATING	

Questions to Answer:

1. How did you find the performance task?

2. How did the task help you see the real world use of the topic?

ACTIVITY 10. IRF Worksheet

Now that you have learned completed doing the various activities in the module, using the “F” portion of the IRF Worksheet, answer the essential question: **How can unknown quantities in geometric problems be solved?**

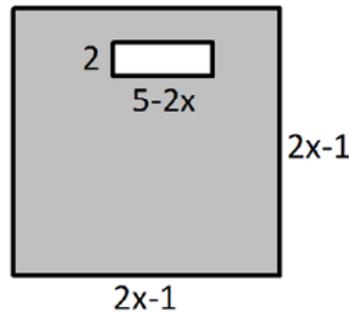
IRF Worksheet

Initial Answer
Revised Answer
Final Answer

Now, that you have completed your final project, let us now complete the synthesis journal found below.

2. You deposit ₱1000 in a savings account that has an annual interest rate r . At the end of three years, the value of your account will be $1000(1 + r)^3$ pesos. Find the amount of money in the account if the interest rate is 3%.
 - A. ₱1092.73
 - B. ₱3090.00
 - C. ₱2060.00
 - D. ₱2197.00

3. A square piece of land was rewarded by a master to his servant. They agreed that a portion of it, represented by the rectangle inside, should be used to construct a grotto. Find the area of the land that is available for other uses.



- A. $4x^2 - 9$
 - B. $4x^2 + x + 9$
 - C. $4x^2 - 8x - 9$
 - D. $4x^2 + 9$

4. Your sister wants to enlarge her closet that has an area of $(x^2 + 3x + 2)$ ft². The length is $(x + 2)$ ft. After construction, the area will be $(x^2 + 8x + 15)$ ft² with a length of $(x + 3)$ ft. By how many feet will the length and the width increase after construction?
 - A. The length will increase by four feet and the width will increase by one feet.
 - B. The length will increase by one foot but the width will remain the same.
 - C. The length and the width will increase by two feet each.
 - D. The length will increase by one foot and the width will increase by four feet.

5. When your long test papers were returned by your Math teacher, your classmate approached you to check the item in the test found below where he made a mistake.

Factor the polynomial completely.

$$\begin{aligned}
 & x^2 - 8x + 16 - 36y^4 \\
 = & (x^2 - 8x) + (16 - 36y^4) \\
 = & x(x - 8) + 4(4 - 9y^4) \\
 = & \text{prime}
 \end{aligned}$$

Your classmate asked you to explain why his solution was marked wrong by the teacher. How would you explain to him the correct way of factoring?

- A. The choice of grouping is wrong. The first three terms should be grouped to form a perfect square trinomial and if combined with the last term will form a difference of two squares.
- B. The choice of grouping is wrong. The first and third terms must be grouped and the second and fourth terms. The second and fourth terms have a greatest common factor.
- C. The choice of grouping is wrong. The first and the last terms must be grouped and the second and third terms. The first and last terms form a difference of two squares. The second and third terms have a greatest common factor.
- D. The choice of grouping is correct. The factoring in the third step is not completed. There is still a difference of two squares.

MODULE: POST-ASSESSMENT



It's now time to evaluate your learning. Click on the letter of the answer that you think best answers the questions. If you do not do well, you may move on to the next lesson. If your score is not at the expected level yet, you have to go back and take the module again.

1. Which statement is true?
 - A. The square of a binomial is also a binomial.
 - B. The product of a sum and difference of two terms is a binomial.
 - C. The product of a binomial and a trinomial is the square of a trinomial.
 - D. The terms of the cube of a binomial are all positive.

2. Which of the following is NOT a difference of two squares?
 - A. $\frac{1}{4}x^4 - 1$
 - B. $x^2 - 0.0001y^4$
 - C. $1.6(x - 1)^2 - 49$
 - D. $(x + 1)^4 - 4x^6$

3. Which of the following can be factored?
 - A. $0.08x^3 - 27y^3$
 - B. $1.44(x^2 + 1) - 0.09$
 - C. $24xy(x - y) + 5(x + y)$
 - D. $0.027(x^2 + 1)^3 - 8$

4. A polynomial expression is evaluated for the x- and y-values shown in the table below. Which expression was evaluated to give the values shown in the third column?

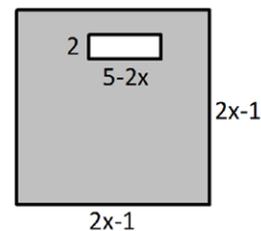
x	y	Value of the Expression
0	0	0
-1	-1	0
1	1	0
1	-1	4

- A. $x^2 - y^2$
 - B. $x^2 + 2xy + y^2$
 - C. $x^2 - 2xy + y^2$
 - D. $x^3 - y^3$

5. Evaluate: $99^3 + 3(99)^2 + 3(99) + 1$.
 - A. 1,000,001
 - B. 1,000,000

- C. 999,999
D. 999,991
6. The length of a box is five inches less than twice the width. The height is 4 inches more than three times the width. The box has a volume of 520 cubic inches. Which of the following equations can be used to find the height of the box?
- A. $W(2L - 5)(3H + 4) = 520$
 B. $W(2W + 5)(3W - 4) = 520$
 C. $W(2W - 5)(3W - 4) = 520$
 D. $W(2W - 5)(3W + 4) = 520$
7. One of the factors of $2a^2 + 5a - 12$ is $a + 4$. Which is the other factor?
- A. $2a - 3$
 B. $2a + 3$
 C. $2a - 8$
 D. $2a + 8$
8. Factor completely: $3s^2 - 48$
- A. $3(s^2 - 16)$
 B. $3(s^2 - 4)(s^2 + 4)$
 C. $3(s - 4)(s + 4)$
 D. $3(s - 2)(s + 2)(s + 4)$
9. The side of a square is x cm long. The length of a rectangle is 5 cm longer than a side of this square and the width is 5 cm shorter. Which statement is true?
- A. The area of the square is greater than the area of the rectangle.
 B. The area of the square is less than the area of the rectangle.
 C. The area of the square is equal to the area of the rectangle.
 D. The relationship cannot be determined from the given information.
10. A square piece of land was rewarded by a master to his servant. They agreed that a portion of it, represented by the rectangle inside, should be used to construct a grotto. How large is the area of the land that is available for the other uses?

- A. $4x^2 - 9$
 B. $4x^2 + x + 9$
 C. $4x^2 - 8x - 9$
 D. $4x^2 + 9$



11. Which of the following values will make the largest area of the square with a side of $3x + 2$?

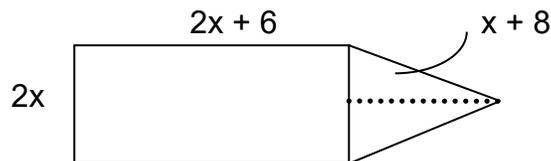
A. $-\frac{3}{4}$

B. 0.4

C. $\frac{1}{3}$

D. -0.15

12. Which strategy could not be used to solve for the area of the figure below?



A.

$$A = 2x(2x + 6) + \frac{1}{2}(2x)(x + 8)$$

$$A = 4x^2 + 12x + x^2 + 8x$$

$$A = 5x^2 + 20x$$

B.

$$A = 2x(3x + 14) - 2\left(\frac{1}{2}\right)(x)(x + 8)$$

$$A = 6x^2 + 28x - x^2 - 8x$$

$$A = 5x^2 + 20x$$

C.

$$A = [2x(2x + 6) + (x + 8)(2x)] - 2\left(\frac{1}{2}\right)(x)(x + 8)$$

$$A = [(4x^2 + 12x) + (2x^2 + 16x)] - (x^2 + 8x)$$

$$A = 6x^2 + 28x - x^2 - 8x$$

$$A = 5x^2 + 20x$$

D.

$$A = 2x(2x + 6) + \left(\frac{1}{2}\right)(2 + x)(x + 8)$$

$$A = 4x^2 + 12x + x^2 + 8x$$

$$A = 5x^2 + 20x$$

13. In your quiz, you were asked to square $(2x - 3)$. Your classmate answered $4x^2 - 9$. Is your classmate's answer correct?
- Yes, because squaring a binomial always produces a binomial product.
 - Yes, because product rule is correctly applied.
 - No, because squaring a binomial always produces a trinomial product.
 - No, because the answer must be $4x^2 + 9$.
14. Expression A: $4x^2 - 81$
 Expression B: $(2x-9)(2x+9)$
 If $x = 2$, which statement is true about the given expressions?
- The value of expression A is greater.
 - The value of the expression B is greater.
 - The values of the two expressions are equal.
 - The relationship cannot be determined from the given information.
15. Your sister plans to remodel her closet. She hired a carpenter to do the task. What could be the best way that your sister can do so that the carpenter can do the task according to what she wants to happen?
- Show a replica of a closet.
 - Download a picture from the internet.
 - Leave everything to the carpenter.
 - Provide the lay out drawn to scale.
16. Which of the following standards would best apply in checking the carpenter's work?
- accuracy of measurements and wise utilization of materials
 - accuracy of measurements and workmanship
 - workmanship and artistic design
 - workmanship and wise utilization of materials

For 17 – 18 use the problem below.

Your dad decided to fence the rectangular garden with an area of $(x^2 + 13x + 42)$ ft². The width is $(x + 6)$ ft. and the cost of fencing is P100/ft.

17. How much will you spend if you want to surround the garden with fence if $x = 5$?
- ₱13,200
 - ₱4,600
 - ₱1,840
 - ₱5,280

18. If the cost of fertilizer is P40.00/ft², how much will you then spend if you want to fertilize the entire garden?
- A. ₱13,200
 - B. ₱4,600
 - C. ₱1,840
 - D. ₱5,280

For 19 – 20, use the information below.

19. A factory makes rectangular cartons for its flower vases. If the volume of each carton is given by $V = y^3 + 14y^2 + 48y$, which are the possible dimensions of the carton?
- A. $(y^2 + 8)(y + 6)$
 - B. $y(y + 16)(y + 3)$
 - C. $y(y + 12)(y + 4)$
 - D. $y(y + 6)(y + 8)$
20. Based from your answer in the previous number, which are the dimensions of the carton if $y = 4$?
- A. 4 x 24 x 10
 - B. 4 x 20 x 7
 - C. 4 x 10 x 12
 - D. 4 x 16 x 8

GLOSSARY OF TERMS USED IN THIS LESSON:

AREA – the amount of surface contained by a figure

COMPOSITE FIGURE – a figure that is made from two or more geometric figures

FACTOR – an exact divisor of a number.

GEOMETRY – the branch of mathematics that deals with the nature of space and the size, shape, and other properties of figures as well as the transformations that preserve these properties

PATTERN – constitutes a set of numbers or objects in which all the members are related with each other by a specific rule

PERIMETER – the distance around a polygon

PRODUCT – the answer in a multiplication equation

SCALE DRAWING – a reduced or enlarged drawing whose shape is the same as an actual object that it represents

VOLUME – the measure of space occupied by a solid body

REFERENCES AND WEBSITE LINKS USED IN THIS LESSON:

BOOKS

Bautista, Evangeline P. *Experiencing Mathematics Series: Intermediate Algebra*. Quezon City: Vibal Publishing House, Inc., 2008. Print.

Bellman, Allan. *Algebra: Tools for a Changing World*. Philippines: C and E Publishing House, 2005. Print.

Chua, Simon L, et al. *Soaring 21st Century Mathematics*. Philippines: Phoenix Publishing House, 2009, Print.

Demana, Franklin, et. al. *Intermediate Algebra: A Graphing Approach*. United States: Addison-Wesley Publishing Company, Inc. , 1994. Print.

Larson, Roland E., et al. *Course One: Integrating Algebra, Data Analysis, Functions, Geometry*. United States: D.C. Heath and Company, 1999. Print.

Lial, Margaret L. *Beginning Algebra, Ninth Edition*. United States: Addison Wesley. 2004. Print.

Nocon, Rizaldi C. *Math for Engaged Learning: Intermediate Algebra*. Quezon City: SIBS Publishing, 2011. Print.

Oronce, Orlando A. and Mendoza, Marilyn O. *E-math: Intemediate Algebra*. Manila: Rex Publishing House, 2010. Print.

IMAGES

LESSON 1: SPECIAL PRODUCTS

www.rotarygardens.org

www.flickr.com

www.susancohangardens.com

www.alibaba.com

LESSON 2: FACTORING

<http://www.treehugger.com/corporate-responsibility/cadburys-dairy-milk-chocolate-bar-goes-fairtrade.html>

<http://www.crewtonramoneshouseofmath.com/factoring-polynomials.html>

LESSON 3: APPLICATIONS

www.edgw365.com

<http://www.oyster.com/las-vegas/hotels/luxor-hotel-and-casino/photos/square-pool-north-luxor-hotel-casino-v169561/#>

<http://www.apartmenttherapy.com/tile-vault-midcentury-rec-room-39808>

<http://onehouseonecouple.blogzam.com/2012/03/master-shower-tile-progress/>
www.internetfamilyfun.com

WEBSITES

Lesson 1: SPECIAL PRODUCTS

SPECIAL PRODUCTS

<http://konigcruz.wikispaces.com/file/view/Special+Products.pdf>

SQUARING TRINOMIALS

http://www.youtube.com/watch?v=-_8DFxTIOLs.

MULTIPLICATION and SPECIAL PRODUCTS

<http://hanlonmath.com/pdfFiles/199.SpecialProductsBH.pdf>

DRAG and DROP

http://www.media.pearson.com.au/schools/cw/au_sch_bull_gm12_1/dnd/2_spec.html.

GEOMETRIC PROBLEMS

<http://www.qub.ac.uk/keyskills/WN/Numeracyperimeter.html>

AREA OF COMPOSITE FIGURES

<http://www.youtube.com/watch?v=9HE3kfu8K1I>.

SCALE DRAWING

<http://www.youtube.com/watch?v=u5LaVILWzx8>

MODEL MAKING

<http://www.youtube.com/watch?v=er2-q86kJ9s>

LESSON 2: FACTORING

COMMON MONOMIAL FACTOR

<http://www.youtube.com/watch?v=3RJIPvX-3vg>

FACTORING TRINOMIALS

<http://www.onlinemathlearning.com/algebra-factoring-2.html>

FACTORING DIFFERENCE OF TWO SQUARES

<http://www.youtube.com/watch?v=Vpol-yyi43c&feature=related>

FACTORING SUM AND DIFFERENCE OF TWO CUBES

<http://www.youtube.com/watch?v=8c7B-UaKI0U>

FACTORING BY GROUPING

<http://www.youtube.com/watch?v=-hiGJwMNNsM>

FACTORING EXERCISES

http://www.khanacademy.org/math/algebra/polynomials/e/factoring_polynomials

1

FACTORING USING TILES

www.allenteachers.com/resources/algebra-tiles/

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=216>

FACTORING EXERCISES

http://www.wtamu.edu/anns/mps/math/mathlab/int_algebra/int_alg_tut29_specfact.htm

LESSON 3: APPLICATIONS

THE ALGEBRA IN BIOLOGY

<http://www.youtube.com/watch?v=prkHKjfUmMs>

THE ALGEBRA IN BUSINESS

<http://www.youtube.com/watch?v=DB-goEkQGOo>
<http://www.youtube.com/watch?v=ISfyTzVxfMo>

Lesson 2: Rational Algebraic Expressions

☑ INTRODUCTION AND FOCUS QUESTION(S):

You learned about special products and factoring polynomials in the first module. What did you do when a polynomial is divided by another polynomial? What will you do if the polynomial involves zero and negative exponents? Which strategies can be used in simplifying and operating rational algebraic expressions?

In this module, you will learn a lot about rational algebraic expressions. In the end, you should be able to answer the question, **“How can rate – related problems be solved?”**

☑ LESSON COVERAGE:

In this lesson, you will examine this question when you take the following topics:

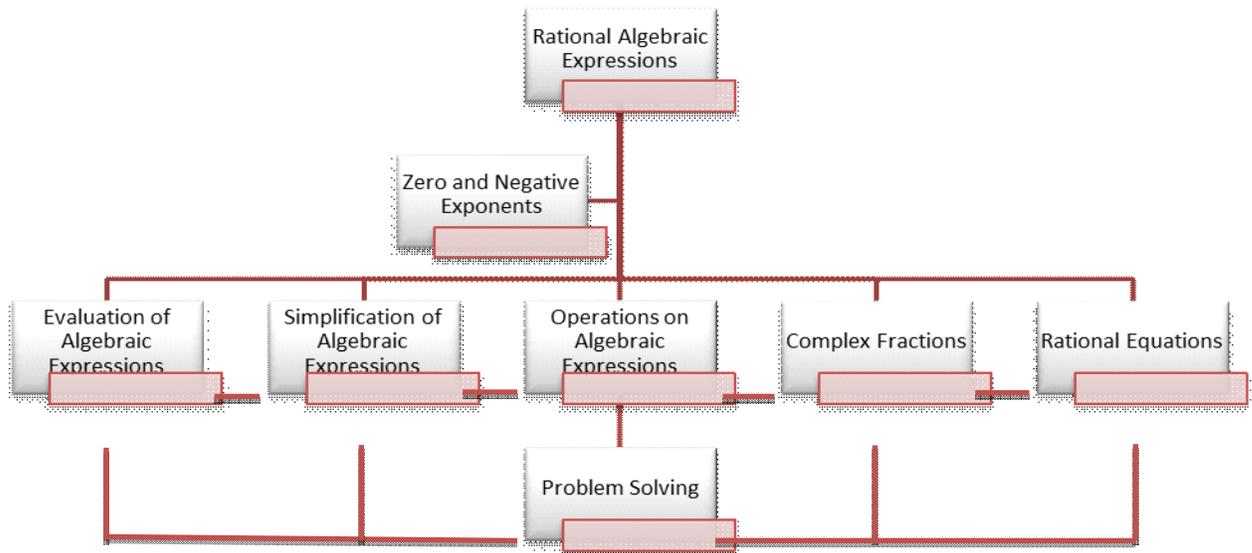
- 2.1 – Rational Algebraic Expression
- 2.2 – Operations on Rational Algebraic Expressions
- 2.3 – Applications of Rational Algebraic Expressions

In these topics, you will learn the following:

2.1	<ul style="list-style-type: none"> • Describe and illustrate rational algebraic expressions. • Interpret zero and negative exponents. • Evaluate algebraic expressions involving integral exponents. • Simplify rational algebraic expressions.
2.2	<ul style="list-style-type: none"> • Multiply, divide, add and subtract rational algebraic expressions. • Simplify complex fractions.
2.3	<ul style="list-style-type: none"> • Solve rational equations. • Solve problems involving rational algebraic expressions.

LESSON MAP:

Here is a simple map of the above topics you will cover:



EXPECTED SKILLS:

To do well in this lesson, you need to remember and do the following:

1. Follow the instructions provided for each activity.
2. Review and evaluate your work using the rubric provided before submission.
3. Complete all exercises.
4. Be mindful of the meaning of unfamiliar words you encounter in this module. A glossary of terms is provided in the last part of this module.
5. Maximize the use of online resources in each lesson. Online resources can be accessed multiple times. The summary of online resources is provided in the end of the module.

PRE-ASSESSMENT:


 Let's find out how much you already know about this module.
 Answer the pre-test below.

Click on the letter that you think best answers the question. Please answer all items. After taking this short test, you will see your score. Take note of the items that you were not able to correctly answer and look for the right answer as you go through this module.

1. The following are rational numbers except _____.
 - A. - 14
 - B. 36
 - C. $\frac{6}{17}$
 - D. $\sqrt{10}$

2. The expression $\frac{4^5}{4^3}$ can be simplified like this: $\frac{4^5}{4^3} = 4^{5-3} = 4^2$. What math principle was applied?
 - A. Definition of exponents
 - B. Division of real numbers
 - C. Quotient Law of exponents
 - D. Subtraction of exponents

3. To find the speed of a car, use the expression $\frac{d}{t}$ where d represents the distance and t represents time. Find the speed of a car that travels from Baguio to Manila in 6 hours and covered a total distance of 200 kilometers.
 - A. 0.03 km per hour
 - B. 3 km per hour
 - C. 33 km per hour
 - D. 33.33 km per hour

4. What value/s of k will make the $\frac{5k}{k^2-25}$ undefined?
 - A. k = 0
 - B. k = $\frac{1}{5}$
 - C. k = 25
 - D. k = 5 and k = - 5

5. In the rectangle below, what is the length if the width is $2k$?

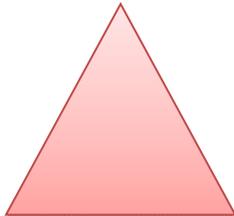
$$\text{Area} = 10k^2 - 8k$$

- A. $5k + 4k$
 B. $5k - 4k$
 C. $5k + 4$
 D. $5k - 4$
6. When asked to simplify the fraction $\frac{6}{6+8}$, Jerry just cancelled 6 in both numerator and denominator and said that the simplest form is $\frac{1}{8}$. Is Jerry correct?
 A. Yes, he is correct.
 B. No. Since both 6 in the numerator and denominator are cancelled, the final answer is 8.
 C. No. $\frac{6}{6+8} = \frac{6}{14}$. Therefore, the correct answer is $\frac{6}{14}$.
 D. No. $\frac{6}{6+8} = \frac{6}{14} = \frac{3}{7}$. Therefore, the correct answer is $\frac{3}{7}$.

7. What is the sum of $\frac{6x-13}{x-2}$ and $\frac{3x-5}{x-2}$?

- A. $\frac{9x-18}{x-2}$
 B. $\frac{9x-18}{2x-4}$
 C. $x - 2$
 D. 9
8. When $\frac{3d^2+2d-1}{5d^2-9d-2}$ is divided by $\frac{2d^2-d-3}{10d^2-13d-3}$ what will be the quotient?
 A. $\frac{3d+1}{3d-1}$
 B. $\frac{d+2}{3d-1}$
 C. $\frac{d+2}{3d+1}$
 D. $\frac{d-2}{3d-1}$

9. The triangle below has a height that measures $\frac{2x+8}{3}$ cm and a base of $\frac{6}{x+4}$ cm. What is the area of 10 similar triangles with the one below?



- A. $\frac{4}{3x+12}$ cm²
- B. $\frac{12x+48}{3x+12}$ cm²
- C. $\frac{4}{3(x+4)}$ cm²
- D. 20 cm²

10. If $A = \frac{1}{x+6}$ and $B = \frac{-4}{x^2+8x+12}$, what is $A - B$.

- A. $\frac{1}{x+6}$
- B. $\frac{1}{x+2}$
- C. $x+6$
- D. $x+2$

11. What is the simplified form of $\frac{\frac{x}{y}}{\frac{x}{y+1}}$?

- A. 1
- B. y
- C. $\frac{y}{y+1}$
- D. $\frac{y+1}{y}$

12. If $P = \frac{5}{x-3}$; $Q = \frac{2}{x}$; $R = \frac{1}{x}$; and $S = \frac{2}{x-3}$, then what is the equivalent value of $\frac{P-Q}{R+S}$?

- A. -3
- B. 3
- C. $\frac{x+2}{x-2}$
- D. $\frac{x-2}{x+2}$

13. What is the solution of the equation $\frac{6}{x} - \frac{2}{3} = -\frac{4}{x}$?

- A. 15
- B. 3
- C. -15
- D. 6

14. What are all the solutions of the equation $\frac{-6}{x+7} = \frac{x}{2}$?

- A. 3,4
- B. -3
- C. -4, -3
- D. -3, 4

15. If the same number is added to both the numerator and denominator of the fraction $\frac{3}{5}$, the result is $\frac{4}{5}$. Find the number.
- A. 3
 - B. 4
 - C. 5
 - D. 6
16. Ann takes 2 hours to plant 50 flower bulbs. Cesar takes 3 hours to plant 45 flower bulbs. Working together, how long should it take them to plant 150 bulbs?
- A. 3.75 hours
 - B. 3 hours
 - C. 2 hours
 - D. 4.25 hours
17. James and John are planning to paint a house together. John thinks that if he worked alone, it would take him 3 times as long as it would take Joe to paint the entire house. Working together, they can complete the job in 24 hours. How long would it take each of them, working alone, to complete the job?
- A. James = 30 hours; John = 90 hours
 - B. James = 24 hours; John = 72 hours
 - C. James = 16 hours; John = 48 hours
 - D. James = 32 hours; John = 96 hours
18. A water tank has two outlet pipes. When the first pipe is left open, it takes 4 hours to drain the tank while if only the second pipe is open, it takes 6 hours to drain. How long should it take to drain the tank if both pipes are left open simultaneously?
- A. 1.5 hours
 - B. 3 hours
 - C. 2.4 hours
 - D. 3.25 hours
19. The formula $\frac{1}{t} = \frac{1}{A} + \frac{1}{B}$ gives the total time (t) for some job to be done by workers (A and B). If the total time necessary for worker A and worker B to do a particular job is 6 hours, and the individual time for worker A is 10 hours, what is the time for worker B?
- A. 7 hrs
 - B. 12 hr.
 - C. 15 hrs
 - D. 18 hrs
20. With spraying equipment, Mateo can paint the woodwork in a small house in 8 hr. His assistant, Chet, need 14 hr. to complete the same job painting

by hand. If Mateo and Chet work together, how long will it take them to paint the woodwork?

- A. $5 \frac{1}{11}$ hrs
- B. 5 hrs
- C. 6 hrs
- D. 11 hrs

LESSON 2: RATIONAL ALGEBRAIC EXPRESSIONS

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In this lesson, you will examine this question when you take the following topics:

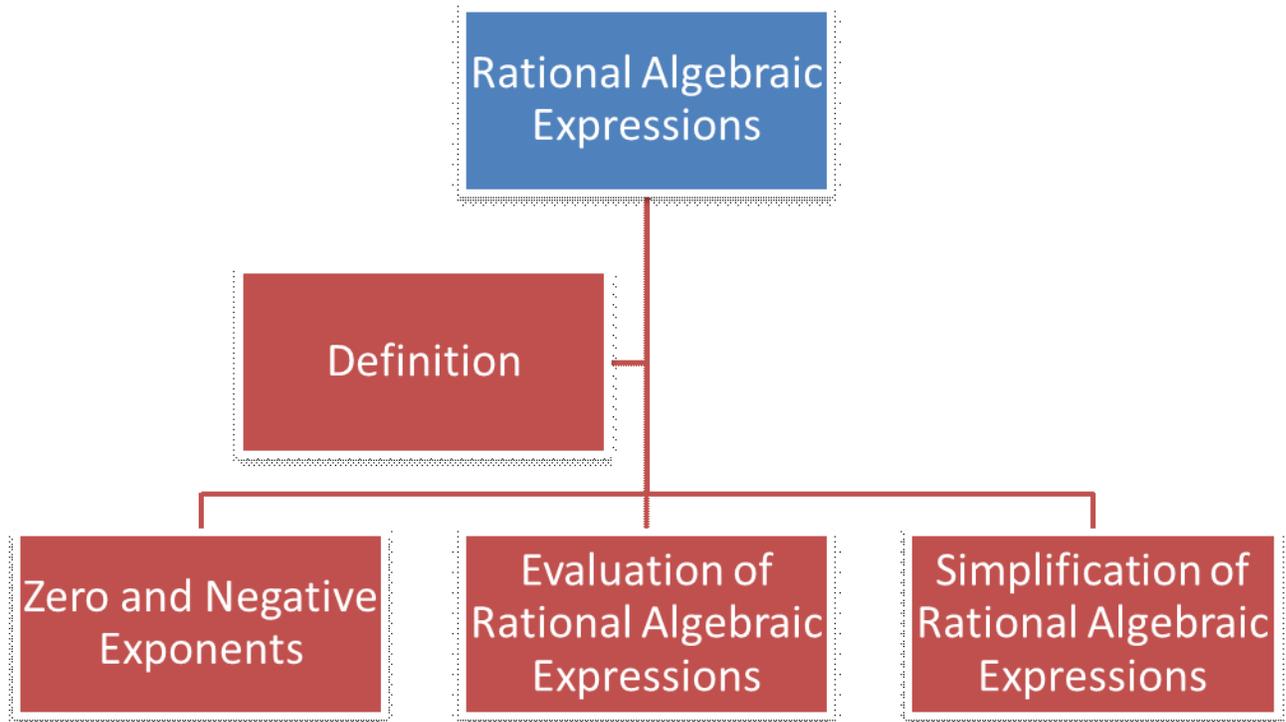
- 2.1 – *Definition of Rational Algebraic Expressions*
- 2.2 – *Zero and Negative Exponents*
- 2.3 – *Evaluation of Rational Algebraic Expressions*
- 2.4 – *Simplification of Rational Algebraic Expressions*

In these topics, you will learn the following:

2.1	<ul style="list-style-type: none"> • <i>Describe and illustrate rational algebraic expressions.</i>
2.2	<ul style="list-style-type: none"> • <i>Interpret zero and negative exponents.</i>
2.3	<ul style="list-style-type: none"> • <i>Evaluate algebraic expressions involving integral exponents.</i>
2.4	<ul style="list-style-type: none"> • <i>Simplify rational algebraic expressions.</i>

MODULE MAP:

Here is a simple map of the above lesson you will cover:



EXPECTED SKILLS:

To do well in this lesson, you need to remember and do the following:

1. Follow the instructions provided for each activity.
2. Review and evaluate your work using the rubric provided before submission.
3. Complete all activities and exercises.
4. Be mindful of the meaning of unfamiliar words you encounter in this module. A glossary of terms is provided in the last part of this module.
5. Maximize the use of online resources in each lesson. Online resources can be accessed multiple times. The summary of online resources is provided in the end of the module.



EXPLORE



Let's start the module by exploring activities related to work.

ACTIVITY 1. HOW FAST

DESCRIPTION: When you printed your 30--page research paper, you observed that the printer A in the Internet shop finished printing in 2 minutes. How long will it take printer A to print 150 pages? How long will it take printer A to print p pages? If printer B can print x pages per minute, how long will it take to print p pages?

ACTIVITY 2. KWLH CHART

DESCRIPTION: Write your ideas on Rational Algebraic Expressions. Fill – in the first two columns. You will revisit and complete this chart in the end of the lesson. When you're done, click "SAVE".

Rational Algebraic Expressions

What I Know	What I Want to Find Out	What I Learned	How Can I Learn More



PROCESS QUESTIONS:

1. How did you determine the number of pages that can be printed by each printer? What operation is involved?
2. What can you recall about rational numbers?

3. How can rate – related problems be solved?

End of Explore:



You gave your initial ideas on rational algebraic expressions. Let's now find out what the answer is by doing the next part. What you learn in the next sections will also enable you to do the final project which involves rate – related problems.



FIRM-UP



Your goal in this section is to learn and understand key concepts about rational algebraic expressions.

In Activity 1, to find out how long Printer A can print 150 pages, determine first the rate of the printer. The rate of the printer is the number of pages printed per minute (ppm).

So, $\frac{30 \text{ pages}}{2 \text{ minutes}} = 15 \text{ ppm}$. Therefore, Printer A can print 15 pages per minutes. If it will print 150 pages, then it will take $\frac{150 \text{ pages}}{15} = 10 \text{ minutes}$.

If there will be p pages, then it will take $\frac{p}{15} \text{ minutes}$.

On the other hand, Printer B will take $\frac{p}{x} \text{ minutes}$ to print p pages.

What type of numbers are $\frac{30}{2}$ and $\frac{150}{15}$?

Yes, they are fractions or rational numbers. Fractions are produced by dividing an integer by another integer.

How about $\frac{p}{15}$ and $\frac{p}{x}$? What are they called?

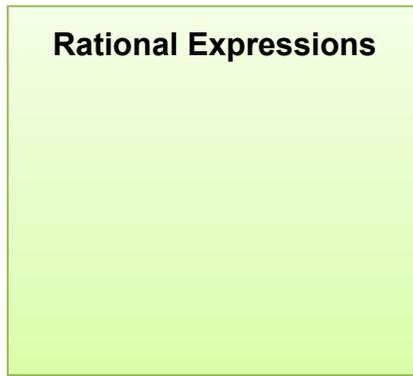
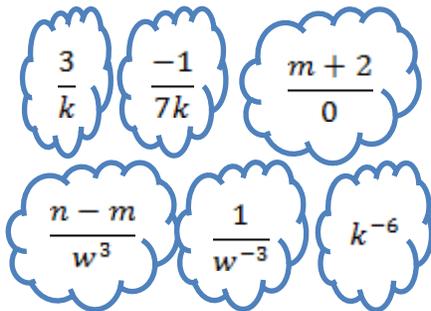
Yes, they are called algebraic expressions. They are similar to fractions since they have a numerator and a denominator. Did you notice that the numerators and denominators are polynomials? Thus, specifically, they are called **algebraic fractions** or **rational algebraic expressions** or simply **rational expressions**.

Definition: A rational expression is an expression in the form

$\frac{P}{Q}$ where P and Q are polynomials and $Q \neq 0$.

ACTIVITY 3. IS IT OR IS IT NOT?

DESCRIPTION: Determine whether the expression is a rational expression or not. If it is, drag and drop the expression under **Rational Expressions**; otherwise, drag and drop under **Not Rational Expressions**. When you're done, click "SUBMIT".



PROCESS QUESTIONS:

1. What is your basis in identifying rational expressions from non – rational expressions?
2. Why do you think the denominator in a rational expression should not be zero?
3. What is the meaning of a negative exponent?

Recall that an exponent is a positive number or symbol that tells how many times the base is used as a factor.

Let's do it: What is the meaning of the following?

1. 5^3
2. $(-4m)^2$
3. $\left(\frac{1}{2d}\right)^4$
4. p^m

Solution:

1. $5^3 = 5(5)(5)$
2. $(-4m)^2 = (-4m)(-4m)$
3. $\left(\frac{1}{2d}\right)^4 = \left(\frac{1}{2d}\right)\left(\frac{1}{2d}\right)\left(\frac{1}{2d}\right)\left(\frac{1}{2d}\right)$
4. p^m means p is multiplied to itself m times

If $5^3 = 5(5)(5)$, how about 5^{-3} ? Does it mean 5 is used as a factor negative three times? What will be the expanded form?

Recall that in the quotient law of exponents, $\frac{x^m}{x^n} = x^{m-n}$

so that $\frac{x^5}{x^2} = x^{5-2} = x^3$.

Let's do it: Simplify using the Quotient Law. Verify by expansion.

1. $\frac{7^3}{7^2}$ 2. $\frac{2^2}{2^3}$ 3. $\frac{d^2}{d^5}$ 4. $\frac{10^3}{10^3}$

Solution:

1. $\frac{7^3}{7^2} = 7^{3-2} = 7$

by expansion: $\frac{7^3}{7^2} = \frac{7(7)(7)}{7(7)} = 7$

2. $\frac{2^2}{2^3} = 2^{2-3} = 2^{-1}$ *What is the meaning of 2^{-1} ?*

by expansion: $\frac{2^2}{2^3} = \frac{2(2)}{2(2)(2)} = \frac{1}{2}$

Therefore, $\frac{2^2}{2^3} = 2^{-1} = \frac{1}{2}$

3. $\frac{d^2}{d^5} = d^{2-5} = d^{-3}$ *What is the meaning of d^{-3} ?*

by expansion: $\frac{d^2}{d^5} = \frac{d(d)}{d(d)(d)(d)(d)} = \frac{1}{d(d)(d)} = \frac{1}{d^3}$

Therefore, $\frac{d^2}{d^5} = d^{-3} = \frac{1}{d^3}$

4. $\frac{10^3}{10^3} = 10^{3-3} = 10^0$ *What is the meaning of 10^0 ?*

by expansion: $\frac{10^3}{10^3} = \frac{10(10)(10)}{10(10)(10)} = \frac{1000}{1000} = 1$

Therefore, $\frac{10^3}{10^3} = 10^0 = 1$

Notice that if the exponent $m > n$, then $m - n$ is positive (item 1). However, when $m < n$, $m - n$ is negative (items 2 and 3) and when $m = n$, then $m - n$ is zero (item 4).

Note: 0^0 is not 1 but instead "*indeterminate*".

ACTIVITY 4. WHO'S RIGHT?

DESCRIPTION: John and Kelly were asked to simplify $\frac{k^2}{k^{-3}}$. Their solutions are below together with their explanation.

John's Solution	Kelly's Solution
$\frac{k^2}{k^{-3}} = k^{2-(-3)} = k^5$ <p>I used the quotient law of exponents.</p>	$\frac{k^2}{k^{-3}} = \frac{k^2}{\frac{1}{k^3}} = k^2 \frac{k^3}{1} = k^5$ <p>I made the negative exponent positive first. Then I multiplied the reciprocal of the denominator with the numerator.</p>

Who is right? Write your explanation then click "SUBMIT".

ACTIVITY 5. MATCHED!

DESCRIPTION: Determine how each rational expression will be simplified through the quotient law and then determine the simplified form. Just drag and drop the expressions under the right places. When you're done, click "SUBMIT".

Rational Expression	Using the quotient law	Final answer
1. $\frac{p}{p^3}$		

2. $\frac{8}{8^{-2}}$		
3. $\frac{8^{-2}}{8^{-3}}$		
4. $\frac{p^{-1}}{p^{-3}}$		

$$p^2$$

$$8$$

$$8^{-2-(-3)} = 8^{-2+3}$$

$$8^{1-(-2)} = 8^{1+2} = 8^3$$

$$p^{-1-(-3)} = p^{-1+3}$$

$$p^{1-3} = p^{-2}$$

$$\frac{1}{p^2}$$

$$512$$

ACTIVITY 6. ONLINE ACTIVITY 1

DESCRIPTION: Click the hyperlink below to open the website.

Rational Expressions

<http://www.shmoop.com/polynomial-division-rational-expressions/rational.html>

The article on this website is about rational expressions. At the last part of the article are some examples and non – examples of rational expressions. Re-evaluate your answers in Activity 3.



PROCESS QUESTIONS:

1. What do you need to remember when dealing with negative exponents?
2. Why is there a need to simplify negative exponents?
3. Do you have a shorter way to simplify expressions with negative exponents? Explain.
- 4. How can rate – related problems be solved?**

Evaluation of Rational Expressions

ACTIVITY 7. SWIM PENGUIN SWIM

DESCRIPTION: A Gentoo penguin can swim at a rate of 17 miles per hour.

How many hours will it take a penguin to swim a distance of

a) 51 miles? _____ hours

b) 136 km? _____ hours

Use the expression $\frac{d}{r}$, where d represents distance and r represents rate.

Write your answer in the blank. Then, click "SOLUTION" to display the complete solution.

SOLUTION

a) $\frac{d}{r} = \frac{51 \text{ miles}}{17 \text{ miles per hour}} = 3 \text{ hours}$

b) Convert first 136 km to miles. 1 mile = 1.6 km;

So, $(136\text{km}) \frac{1 \text{ mile}}{1.6 \text{ km}} = 85 \text{ miles}$

$\frac{d}{r} = \frac{85 \text{ miles}}{17 \text{ miles per hour}} = 5 \text{ hours}$



PROCESS QUESTIONS:

1. What did you do to the assigned values of ***d*** and ***r***?
2. What does it mean to evaluate rational expressions?
3. What are the steps in evaluating rational expressions?

When a number is substituted for the variable in a rational expression, the rational expression takes a numerical value. Finding the value is called ***evaluating the rational expression***.

There are two steps involved in evaluating a rational expression:

1. Replace the variable by the given value; and
2. Perform the indicated operations.

Let's do it. Evaluate the rational expression $\frac{40}{y}$ for

a. $y = 5$

b. $y = 100$

Solution:

a. $\frac{40}{y} = \frac{40}{5} = 8$

b. $\frac{40}{y} = \frac{40}{100} = \frac{2}{5}$

Let's do it. Evaluate the rational expression $\frac{m^2}{5+m}$ for

a. $m = 4$

b. $m = -8$

Solution:

a. $\frac{m^2}{5+m} = \frac{4^2}{5+4} = \frac{16}{9}$

b. $\frac{m^2}{5+m} = \frac{8^2}{5+(-8)} = \frac{64}{-3} = -\frac{64}{3}$

Let's do it. Evaluate the rational expression $\frac{p}{p^2-9}$ for

a. $p = 3$

b. $p = -3$

Solution:

a. $\frac{p}{p^2-9} = \frac{3}{3^2-9} = \frac{3}{0}$

b. $\frac{p}{p^2-9} = \frac{3}{(-3)^2-9} = \frac{3}{0}$

Can you remember the definition of a rational expression? It says that the denominator ***must not be equal to zero*** because any number or expression divided by zero is undefined. Therefore, values that will make the denominator zero must be avoided. In the example above, 3 and -3 should not be used to evaluate the rational expression $\frac{p}{p^2-9}$ because in both cases, the expression will be undefined.

Note: $\frac{0}{0}$ is not called as undefined but instead "*indeterminate*".

Let's do it. Find the value/s for x for which each rational expression is undefined.

a. $\frac{2x}{x+4}$

b. $\frac{x-1}{x^2+9x+20}$

Solution:

a. $\frac{2x}{x+4} = \frac{2(-4)}{(-4)+4} = \frac{-8}{0}$

Identify the value of the variable that will make the denominator zero. The rational expression is undefined for $x = -4$ because the expression becomes $\frac{-8}{0}$. Thus, any number may be used to replace x in $\frac{2x}{x+4}$ except -4 .

b. To find the value/s of x that will make the denominator zero:

- 1) Equate the denominator to zero
- 2) Factor the denominator.
- 3) Find the value that will make each factor equal to zero.

$$x^2 + 9x + 20 = 0$$

$$(x + 4)(x + 5) = 0$$

$$x + 4 = 0$$

$$x = -4$$

$$x + 5 = 0$$

$$x = -5$$

Thus, the expression is undefined for $x = -4$ and $x = -5$. You can of course verify this by evaluating the denominator using the values of x .

if $x = -4$

$$x^2 + 9x + 20 = 0$$

$$(-4)^2 + 9(-4) + 20 = 0$$

$$16 - 36 + 20 = 0$$

$$0 = 0$$

if $x = -5$

$$x^2 + 9x + 20 = 0$$

$$(-5)^2 + 9(-5) + 20 = 0$$

$$25 - 45 + 20 = 0$$

$$0 = 0$$

Thus, the denominator is zero when the value of x is -4 or -5 .

ACTIVITY 8. ONLINE ACTIVITY 2

DESCRIPTION: Open the following websites about evaluating rational expressions.

Evaluating Rational Expressions

<http://www.shmoop.com/polynomial-division-rational-expressions/evaluating.html>

This website provides a step by step guide in evaluating rational expressions.

Evaluating Rational Expressions

<http://www.teacherweb.com/NY/Arlington/AlgebraProject/U7L1EvaluatingRationalExpressions.pdf>

This website has a detailed explanation and varied activities for evaluating rational expressions. Practice by answering the activities.

Evaluating Rational Expressions and finding restrictions

http://2012books.lardbucket.org/books/elementary-algebra/section_10_01.html

This is a website that provides examples for both evaluating and finding the restrictions for a rational expression.

Evaluating Rational Expressions and finding restrictions

<http://www.openalgebra.com/2012/11/simplifying-rational-expressions.html>

This is another website provide that shows examples for both evaluating and finding the restrictions for a rational expression.

ACTIVITY 9. FIND ME

DESCRIPTION: Answer the following. When you're done, click "SUBMIT". Click "ANSWERS" to show the answers.

For items 1 – 2, find the value of the rational expressions for

- a) $m = 3$ b) $m = -2$ c) $m = \frac{1}{2}$

1. $\frac{6m}{4m+9}$

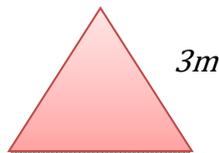
- a) _____ b) _____ c) _____

2. $\frac{4m^2-1}{2m+2}$

- a) _____ b) _____ c) _____

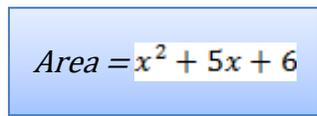
For items 3 – 5, find the value of the following.

3. The ratio of one side to the perimeter of the equilateral triangle if $m = 6$.



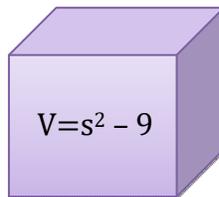
Answer: _____

4. The length of the rectangle if $x = 4$.



width = $x+2$ Answer: _____

5. The area of the base of the prism if $s = 5$ meters.



height = $s - 3$ Answer: _____

For items 6 – 10, find the value of the variable in which the rational expression is undefined.

6. $\frac{3k}{k-3}; k \neq$ _____

9. $\frac{d^2+2d+3}{d^2+2d-8}; d \neq$ _____ and _____

7. $\frac{p^2+2p-12}{5p}; p \neq$ _____

10. $\frac{w^2+6w+9}{w^2+16}; w \neq$ _____

8. $\frac{16-m}{m^2-16}; m \neq$ _____ and _____



PROCESS QUESTIONS:

1. What does it mean to “evaluate rational expressions”?
2. are the steps in evaluating a rational expression?
3. Why is it that any number that will make the rational expression be avoided?
4. How can rate – related problems be solved?

End of FIRM UP:



In this section, the discussion was about zero and negative exponents and evaluating rational expressions.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

Now that you know the important ideas about rational expressions, let’s go deeper by moving on to the next section.



DEEPEN



Your goal in this section is to take a closer look at some aspects of simplifying rational expressions.

ACTIVITY 10. IRF WORKSHEET

DESCRIPTION: Complete the first part of the Initial Answer section of the worksheet below. You will revisit this worksheet as you progress in this topic. Click "SAVE" when you're done.

Initial Answer
To simplify a rational expression, _____ _____.
A rational expression is in simplest form if _____ _____.
Revised Answer
Final Answer

ACTIVITY 11.

ACTIVITY 12. FRACTIONS SIMPLIFIED

DESCRIPTION: Complete the factors () then write the fraction in simplest form $\frac{\quad}{\quad}$. When you're done, click "SAVE" then click "ANSWERS" to show the answer for each item. Compare your answers.

$$1. \frac{16}{24} = \frac{2(8)}{3(\quad)} = \frac{\quad}{\quad}$$

$$2. \frac{35}{21} = \frac{5(\quad)}{3(\quad)} = \frac{\quad}{\quad}$$

$$3. \frac{-36}{40} = \frac{-4(\quad)}{4(\quad)} = \frac{\quad}{\quad}$$

$$4. \frac{132}{-144} = \frac{11(\quad)}{12(\quad)} = -\frac{\quad}{\quad}$$



PROCESS QUESTIONS:

1. What are the steps in simplifying fractions?
2. When can you say that a fraction is in simplest form?
3. How are rational expressions simplified?

Simplification of Rational Expressions

In the activity FRACTIONS SIMPLIFIED, the first step is to factor both numerator and denominator. Then, divide any common factors. Simplifying rational expressions is similar to simplifying fractions. The steps you need to remember to simplify a rational expression to lowest term are:

1. Factor the numerator and denominator completely.
2. Divide (cancel) any common factors.

Let's do it. Reduce to lowest terms.

a. $\frac{5b^5}{20b^2}$

b. $\frac{-7n^{-4}}{14}$

c. $\frac{3k^{-5}}{-15k^{-2}m^{-2}}$

Solution:

a. $\frac{5b^5}{20b^2} = \frac{\cancel{5} \cdot \cancel{b} \cdot \cancel{b} \cdot \cancel{b} \cdot \cancel{b} \cdot b}{4 \cdot \cancel{5} \cdot \cancel{b} \cdot b} = \frac{b^3}{4}$

Factor the numerator and denominator then divide.

b. $\frac{-7n^{-4}}{14} = \frac{-\cancel{7}n^{-4}}{2 \cdot \cancel{7}n^4} = \frac{-1}{2n^4}$

Simplify negative exponents. Factor the numerator and denominator then divide.

c. $\frac{3k^{-5}m^0}{-15k^{-2}m^{-2}} = \frac{3k^2m^2}{-15 \cdot k^5} = \frac{\cancel{3} \cdot \cancel{k} \cdot \cancel{k} \cdot m \cdot m}{-3 \cdot \cancel{5} \cdot \cancel{k} \cdot \cancel{k} \cdot \cancel{k} \cdot k \cdot k} = -\frac{3m^2}{5k^3}$ or $\frac{-3m^2}{5k^3}$

Simplify all negative exponents. Take note that $m^0=1$. Factor the numerator and denominator then divide.

Avoid leaving a negative number in the denominator. In letter c, after dividing common factors, what is left is $\frac{3m^2}{-5k^3}$ which is not considered to be in simplest form. To simplify, write as $-\frac{3m^2}{5k^3}$ or $\frac{-3m^2}{5k^3}$. This is because the fraction $\frac{3}{-5} = -\frac{3}{5}$ or $\frac{-3}{5}$.

Can you still recall about factoring polynomials in module 1? You will use factoring polynomials to simplify rational expressions.

Let's do it. Reduce to lowest terms.

a. $\frac{3k+6}{4k^2+8k}$ b. $\frac{x^2-9}{x^2+7x+12}$ c. $\frac{k^2+k-20}{16-k^2}$

Solution:

a. $\frac{3k+6}{4k^2+8k} = \frac{3(k+2)}{4k(k+2)} = \frac{3}{4k}$ Factor the numerator and denominator completely. Divide common factors.

b. $\frac{x^2-9}{x^2+7x+12} = \frac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}(x+4)} = \frac{x-3}{x+4}$ Factor the numerator and denominator completely. Divide common factors.

c. $\frac{k^2+k-20}{16-k^2} = \frac{(k+5)(k-4)}{(4-k)(4+k)} = \frac{(k+5)\cancel{(k-4)}}{(-1)\cancel{(k-4)}(4+k)} = -\frac{k+5}{4+k}$ OR $\frac{-k-5}{4+k}$
 Factor the numerator and denominator completely. Divide common factors.

In letter c, observe that in $\frac{(k+5)(k-4)}{(4-k)(4+k)}$, $(k-4)$ and $(4-k)$, were not divided directly because they are not similar factors. $(4-k)$ was factored as $(-1)(k-4)$ since $(-1)(k-4) = -k+4 = 4-k$. Then $\frac{\cancel{(k-4)}}{(-1)\cancel{(k-4)}} = -1$.

Let's do it. Which of the following expression is equal to - 1?

a. $\frac{m-6}{m+6}$ b. $\frac{m-6}{6-m}$ c. $\frac{m-6}{-6+m}$

Solution:

The rational expression in (a) is already in simplest form. Take note, you can only divide (cancel) common factors.

The rational expression in (c) is equal to 1 since $\frac{m-6}{-6+m} = \frac{\cancel{m-6}}{\cancel{m-6}} = 1$.

The rational expression in (b) is equal to - 1 since $\frac{m-6}{6-m} = \frac{m-6}{-1(-6+m)} = \frac{\cancel{m-6}}{-1\cancel{(m-6)}} = -1$

ACTIVITY 13. SIMPLIFIED – 1

DESCRIPTION: Evaluate the following rational expressions. Observe the resulting value. Let $a = 2$; $b = 5$. When you're done, click "SUBMIT" then click "ANSWERS" to show the answers.

1. $\frac{a-b}{a-b} = \underline{\hspace{2cm}}$

2. $\frac{a-b}{a+b} = \underline{\hspace{2cm}}$

3. $\frac{a-b}{b-a} = \underline{\hspace{2cm}}$

4. $\frac{a^3-b}{b-a^3} = \underline{\hspace{2cm}}$

5. $\frac{a^2-b^2}{b^2-a^2} = \underline{\hspace{2cm}}$

1. $\frac{a-b}{a-b} = \frac{2-5}{2-5} = \frac{-3}{-3} = 1$

2. $\frac{a-b}{a+b} = \frac{2-5}{2+5} = \frac{-3}{7}$

3. $\frac{a-b}{b-a} = \frac{2-5}{5-2} = \frac{-3}{3} = -1$

4. $\frac{a^3-b}{b-a^3} = \frac{2^3-5}{5-2^3} = \frac{8-5}{5-8} = \frac{3}{-3} = -1$

5. $\frac{a^2-b^2}{b^2-a^2} = \frac{2^2-5^2}{5^2-2^2} = \frac{4-25}{25-4} = \frac{-21}{21} = -1$

Therefore the value of a rational expression is - 1 if

_____.



PROCESS QUESTIONS:

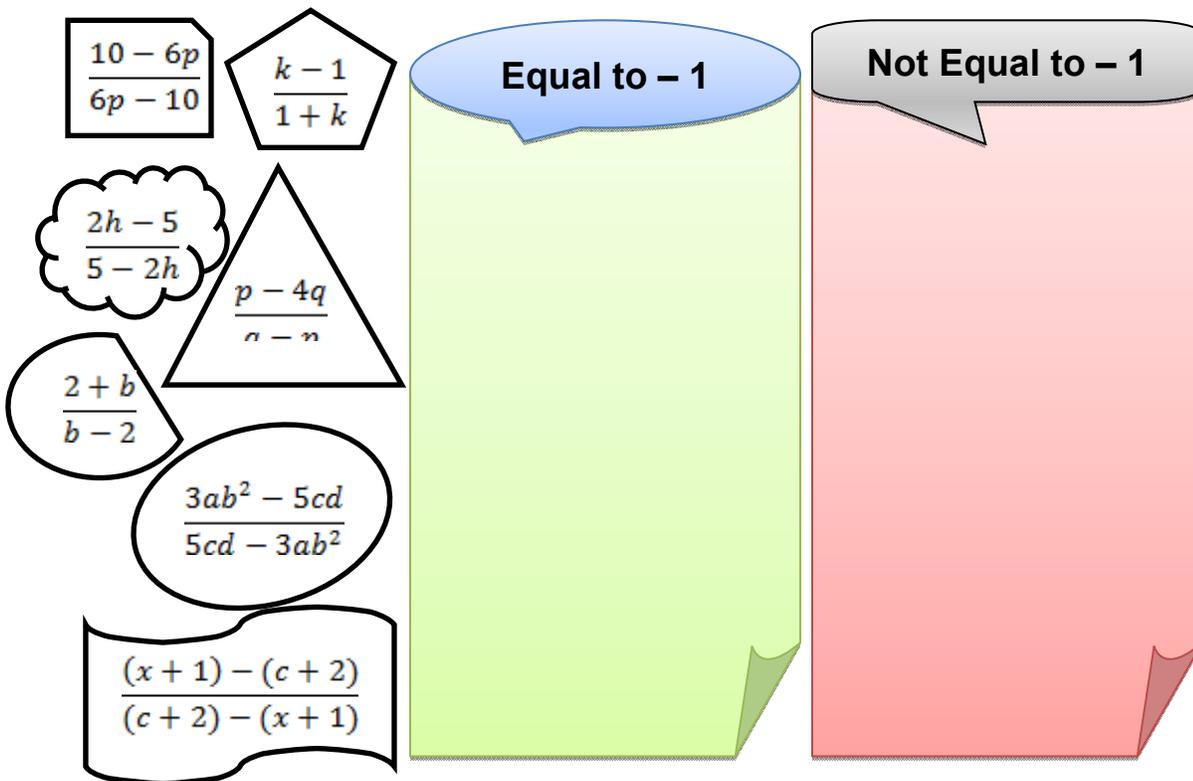
1. What is your observation?
2. When is a rational expression equal to - 1?
3. When can you say that a rational expression is in simplest form?

The value of a rational expression is -1 only when the factor in the numerator and the factor in the denominator are exactly opposite in sign.

In symbols, for all monomials a and b where $a \neq b$, $\frac{a-b}{b-a} = -1$.

ACTIVITY 14. 1 ONLY

DESCRIPTION: Determine which of the following rational expressions is equal to -1 . Just drag and drop the expressions. When you're done, click "SUBMIT".



$\frac{10 - 6p}{6p - 10}$

$\frac{k - 1}{1 + k}$

$\frac{2h - 5}{5 - 2h}$

$\frac{p - 4q}{a - n}$

$\frac{2 + b}{b - 2}$

$\frac{3ab^2 - 5cd}{5cd - 3ab^2}$

$\frac{(x + 1) - (c + 2)}{(c + 2) - (x + 1)}$

Equal to - 1

Not Equal to - 1

ACTIVITY 15. IRF WORKSHEET

DESCRIPTION: Complete the Revised Answer part of the worksheet below. You will revisit this worksheet again for your final thoughts. Click "SAVE" when you're done.

Initial Answer

Revised Answer

To simplify a rational expression, _____

A rational expression is in simplest form if _____

Final Answer



PROCESS QUESTIONS:

1. What is common among rational expressions that are equal to -1 ?
2. What made you revised your initial ideas?
3. Why is there a need to simplify rational expressions?

In simplifying rational expressions, factoring the numerator and denominator completely, and dividing common factors are the basic steps. However, as you have encountered in this lesson, simplifying does not only involve the two steps.

You should know how to perform the different operations following the order of operations (PEMDAS). Rational expressions involve also exponents and when dealing with exponents, you should know how to expand as well as interpret the meaning of zero and negative exponents.

Based again on the definition, a rational expression is a ratio of two polynomials, so don't be surprised that the skills you learned with polynomials, like factoring, special products, and operations on polynomials are very helpful in simplifying rational expressions.

Let's do it. Simplify the following.

a. $\frac{5(k+1)-5}{10(k+1)-10}$

b. $\frac{m(m+2)-3}{m(m+4)+3}$

Solution:

a. $\frac{5(k+1)-5}{10(k+1)-10} = \frac{5k+5-5}{10k+10-10} = \frac{5k}{10k}$
 $= \frac{\cancel{5}(k)}{2(\cancel{5})(k)} = \frac{1}{2k}$

Apply the Distributive Property then combine similar terms if possible.

Factor the numerator and denominator completely then divide.

b. $\frac{m(m+2)-3}{m(m+4)+3} = \frac{m^2+2m-3}{m^2+4m+3}$
 $= \frac{(m-1)(\cancel{m+3})}{(m+1)(\cancel{m+3})} = \frac{m-1}{m+1}$

Apply the Distributive Property then combine similar terms if possible.

Factor the numerator and denominator completely then divide.

ACTIVITY 16. ONLINE ACTIVITY 3

DESCRIPTION: Check out the following websites about simplifying rational expressions.

Simplifying Mixed Exponential Expressions

<http://www.youtube.com/watch?v=guqtvxoj8a0>

The video is about simplifying expressions by using the laws of exponents.

Simplifying Rational Expressions

<http://www.onlinemathlearning.com/simplify-rational-expressions.html>

The website shows some examples of simplifying rational expressions. It also has videos about simplifying rational expressions.

Rational Expressions

http://www.cpm.org/pdfs/skillBuilders/GC/GC_Extra_Practice_Section_30.pdf

The website has a detailed explanation of simplifying rational expressions. There are also a lot rational expressions you can practice to simplify then compare with answers provided.

ACTIVITY 17. WHERE DID I GO WRONG?

DESCRIPTION: Paula simplified a rational expression below. Brenda, her classmate said, it was wrong. Can you explain why? Click "SUBMIT" when you're done.

$$\frac{2(d+3)-2}{3(d+3)-15} = \frac{2\cancel{(d+3)}-2}{3\cancel{(d+3)}-15} = \frac{2-2}{3-15} = \frac{0}{-12} = 0$$

Paula is wrong because



PROCESS QUESTIONS:

1. What makes the solution of Paula wrong?
2. What could have been the correct answer?
3. In reducing/simplifying rational expressions, what can be divided, common factors or common terms? Explain.
4. Why is there a need to simplify rational expressions?

ACTIVITY 18. IT'S NOT SO SIMPLE

DESCRIPTION: John was asked to evaluate and simplify the rational expressions below. Describe and correct the error in simplifying the rational expression or in stating the excluded values.

1. $\frac{2x^2 - x - 3}{2x^2 - 11x + 12}$

2. $\frac{2(x - 5)}{(x - 5)(x + 2)}$

Solution:

$$\begin{aligned} \frac{2x^2 - x - 3}{2x^2 - 11x + 12} &= \frac{(x+1)(2x-3)}{(2x-3)(x-4)} \\ &= \frac{(x+1)\cancel{(2x-3)}}{\cancel{(2x-3)}(x-4)} \\ &= \frac{x+1}{x-4} \end{aligned}$$

The excluded value is 4.

Solution:

$$\begin{aligned} \frac{2(x - 5)}{(x - 5)(x + 2)} &= \frac{\cancel{2(x - 5)}}{\cancel{(x - 5)}(x + 2)} \\ &= \frac{2}{x + 2} \\ &= \frac{1}{x + 1} \end{aligned}$$

The excluded value is - 2 and 5.

Watch the video about rational expression: writing in lowest terms, by clicking:

http://www.youtube.com/watch?v=x6XyP2_RfA4

Then revisit your answer in the activity above.



PROCESS QUESTIONS:

1. What is the error committed by John in number 1?
2. When finding the excluded values, is there a need to simplify the rational expression?
3. What is the error committed by John in number 2?
4. What can be divided, common factors or common terms?
5. What is the difference of evaluating rational expressions and simplifying rational expressions?

ACTIVITY 19. LET'S MAKE IT SIMPLE

DESCRIPTION: Simplify each rational expression.

1. $\frac{27b^{-2}}{30b^5}$

2. $\frac{14a^{-3}b^2}{28a^4b^{-5}}$

3. $\frac{2m-6}{3-4m}$

4. $\frac{3c+33}{c+11}$

5. $\frac{-48p}{16p^2-40p}$

6. $\frac{12m^2+24m}{3m^2-12}$

7. $\frac{n^2+16n+64}{n^2+7n-8}$

8. $\frac{5t^2+20t+15}{5t^2-5}$

9. $\frac{2x^2-4xy+2y^2}{x^2-y^2}$

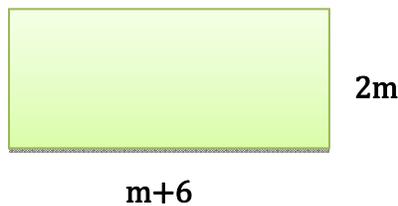
10. $\frac{2(d-4)-2}{7(d-4)-7}$

Write and simplify a rational expression for the ratio of the perimeter of the given figure to its area; $\frac{\text{perimeter}}{\text{area}}$.

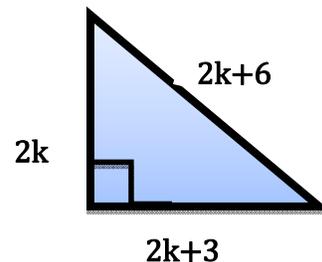
11. Square



12. Rectangle



13. Triangle



Solve the following problems.

14. A train travels $x^2 - 9$ mi in $x + 3$ hours. Find the speed (rate). Use the formula: $speed = \frac{distance}{time}$.

15. A machine has a work output of $14ab^4$ J (joules) and a work input of $7a^2b^5$ J. What is the efficiency of the machine?

$$efficiency \text{ in } \% = \frac{work \text{ output in joules}}{work \text{ input in joules}}$$

16. The perimeter of a hexagon is $6a^2s$ feet. The perimeter of a triangle is $3a^3b^2s$ feet. Find the ratio of the perimeter of the hexagon to the perimeter of the triangle.



PROCESS QUESTIONS:

1. What does it mean to simplify rational expressions?
2. In what ways rational expressions be used in real – life?
3. How can rate – related problems be solved?



End of DEEPEN:

In this section, the discussion was about simplifying rational expressions.

What new realizations do you have about rational expressions?
What new connections have you made for yourself?

ACTIVITY 20. IRF WORKSHEET

DESCRIPTION: Complete the Final Answer of the worksheet below.
Click "SUBMIT" when you're done.

Initial Answer

Revised Answer

Final Answer

To simplify a rational expression, _____

 _____.

A rational expression is in simplest form if _____

 _____.

Now that you have a deeper understanding of rational expressions, you are ready to do the tasks in the next section.



TRANSFER



Your goal in this section is apply your learning to real life situations. You will be given a practical task which will demonstrate your understanding.

You might have noticed that the repeating question after each topic is “How can rate – related problems be solved?” Well, the case in Activity 1. is a rate – related problem. Now, let’s solve more rate – related problems, particularly work problems. Pay attention on how the problems are solved.

Work problems are one of the rate – related problems and usually deal with persons or machines working at different rates or speed. The first step in solving these problems involves determining how much of the work an individual or machine can do in a given unit of time called the **rate**.

Let’s do it.

- a. A painter can paint the wall in 4 hours. What part of the wall is painted in 3 hours?

Solution:

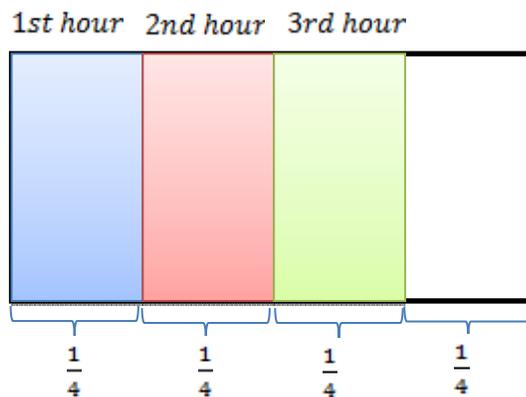
Since the painter can paint the wall in 4 hours, then in 1 hour the painter can paint $\frac{1}{4}$ of the wall. The painter’s rate of work is $\frac{1}{4}$ of the wall each hour. The rate of work is the part of a task that is completed in 1 unit of time.

Therefore, in 3 hours, the painter will be able to paint $3 \cdot \frac{1}{4} = \frac{3}{4}$ of the wall.

You can also solve the problem by using a table. Examine the table below.

Rate of work (wall painted per hour)	Time worked	Work done (Wall painted)
$\frac{1}{4}$	1 hour	$\frac{1}{4}$
$\frac{1}{4}$	2 hours	$\frac{2}{4} = \frac{1}{2}$
$\frac{1}{4}$	3 hours	$\frac{3}{4}$

You can also illustrate the problem.



So after 3 hours, the painter only finished painting $\frac{3}{4}$ of the wall.

- b. Pipe A can fill a tank in 30 minutes. Pipe B can fill the tank in x minutes. What part of the tank is filled if either of the pipes are opened in twenty minutes?

Solution:

Pipe A fills $\frac{1}{30}$ of the tank in 1 minute. Therefore, the rate is $\frac{1}{30}$ of the tank per minute. So after 20 minutes,
 $20 \cdot \frac{1}{30} = \frac{2}{3}$ of the tank is full.

Pipe B fills $\frac{1}{x}$ of the tank in x minutes. Therefore, the rate is $\frac{1}{x}$ of the tank per minute. So after x minutes,
 $20 \cdot \frac{1}{x} = \frac{20}{x}$ of the tank is full.

In summary, the basic equation that is used to solve work problem is:

$$\text{Rate of work} \cdot \text{time worked} = \text{work done.}$$

$$r \cdot t = w$$

ACTIVITY 21. LET'S WORK IT OUT

DESCRIPTION: The AEA Printing Press has two photocopying machines. M1 can print a box of booklets in 5 hours while M2 can print a box of booklets in $15x + 20$ hours.

- How many boxes of booklets are printed by M1 in 10 hours? In 25 hours? in 65 hours?
- How many boxes of booklets can M2 print in 10 hours? in $120x + 160$ hours? in $30x^2 + 40x$ hours?

Follow the steps below:

For M1:

- What is the rate of M1? The rate is the number of boxes of booklets printed per hour.
- To get the number of boxes in 10 hours:
 - Multiply the rate with 10.
 - Simplify the expression. That's the number of boxes.
- To get the number of boxes in 25 hours:
 - Multiply the rate with 25.
 - Simplify the expression. That's the number of boxes.
- To get the number of boxes in 65 hours:
 - Multiply the rate with 65.
 - Simplify the expression. That's the number of boxes.

For M2:

- What is the rate of M2? The rate is the number of boxes of booklets printed per hour.
- To get the number of boxes in 10 hours:
 - Multiply the rate with 10.
 - Simplify the expression. That's the number of boxes.
- To get the number of boxes in $120x + 160$ hours:
 - Multiply the rate with $120x + 160$.
 - Simplify the expression. That's the number of boxes.
- To get the number of boxes in $30x^2 + 40x$ hours:
 - Multiply the rate with $30x^2 + 40x$ hours.
 - Simplify the expression. That's the number of boxes.



PROCESS QUESTIONS:

1. How can you represent rate of people or machines?
2. What do you need to remember when solving work problems?
3. How can rate – related problems be solved?

ACTIVITY 22. KWLH

DESCRIPTION: Complete the KWLH Chart. Notice the changes in your initial thoughts. Click "SUBMIT" when you're finished.

Rational Algebraic Expressions

(To the programmer, kindly show the answers of the student in the first two columns based on Activity 2).

What I Know	What I Want to Find Out	What I Learned	How Can I Learn More

End of TRANSFER:



In this section, your task was about rate – related problems.

How did you find the activity? How did the task help you see the real world use of rational expressions?

You have completed this lesson. Before you go to the next lesson, you have to answer the following post-assessment.

GLOSSARY OF TERMS USED IN THIS LESSON:

Algebraic fractions - an expression in the form $\frac{P}{Q}$ where P and Q are polynomials and $Q \neq 0$. Synonymous with rational algebraic expression and rational expression.

Cancel - in mathematics, it means to divide common factors

Evaluating rational expressions - the process of finding the value of the rational expression by substituting the assigned value of the variables

Exponent - is a positive number or symbol that tells how many times the base is used as a factor. When the exponent is negative or zero, it is not simplified.

Factors - the expressions being multiplied

Indeterminate - a term referring to 0^0 or $\frac{0}{0}$

Rate - amount of work an individual or machine can do in a given unit of time

Rational algebraic expression - an expression in the form $\frac{P}{Q}$ where P and Q are polynomials and $Q \neq 0$. Synonymous with algebraic fraction and rational expression.

Rational expression - an expression in the form $\frac{P}{Q}$ where P and Q are polynomials and $Q \neq 0$. Synonymous with algebraic fraction and rational algebraic expression.

Simplest form of a rational expression - the lowest term of the rational expression where the only common factor of the numerator and denominator is 1 or - 1.

Simplifying rational expressions - the process of putting the rational expression in lowest terms so that the only common factor of the numerator and denominator is 1 or - 1.

Undefined - any number (except 0) or expression divided by zero

REFERENCES AND WEBSITE LINKS USED IN THIS LESSON:

Merle Alferez and Ma. Cecilia Duro. (2004). **MSA Elementary Algebra**. MSA Publishing House, Quezon City, Philippines.

Rational Expressions

<http://www.shmoop.com/polynomial-division-rational-expressions/rational.html>

The article on this website is about rational expressions. At the last part of the article are some examples and non – examples of rational expressions

Evaluating Rational Expressions

<http://www.shmoop.com/polynomial-division-rational-expressions/evaluating.html>

This website provides a step by step guide in evaluating rational expressions.

Evaluating Rational Expressions

<http://www.teacherweb.com/NY/Arlington/AlgebraProject/U7L1EvaluatingRationalExpressions.pdf>

This website has a detailed explanation and varied activities for evaluating rational expressions. Practice by answering the activities.

Evaluating Rational Expressions and finding restrictions

http://2012books.lardbucket.org/books/elementary-algebra/section_10_01.html

This is a website that provides examples for both evaluating and finding the restrictions for a rational expression.

Evaluating Rational Expressions and finding restrictions

<http://www.openalgebra.com/2012/11/simplifying-rational-expressions.html>

This is another website provide that shows examples for both evaluating and finding the restrictions for a rational expression.

Simplifying Rational Expressions

<http://www.onlinemathlearning.com/simplify-rational-expressions.html>

The website shows some examples of simplifying rational expressions. It also has videos about simplifying rational expressions.

Rational Expressions

http://www.cpm.org/pdfs/skillBuilders/GC/GC_Extra_Practice_Section_30.pdf

The website has a detailed explanation of simplifying rational expressions. There are also a lot rational expressions you can practice to simplify then compare with answers provided.

Writing Rational Expressions in Lowest Terms

http://www.youtube.com/watch?v=x6XyP2_RfA4

The video is about writing rational expressions in lowest terms.

LESSON 2.2: OPERATIONS ON RATIONAL EXPRESSIONS

INTRODUCTION AND FOCUS QUESTION(S):

You learned special products and factoring polynomials in the first module. What happens when a polynomial is divided by another polynomial? What will also happen if the polynomial involves zero and negative exponents? Which strategies can be used in simplifying and operating rational algebraic expressions?

LESSON AND COVERAGE:

In this lesson, you will examine this question when you take the following topics:

Topic 1 – Multiplying Rational Expressions

Topic 2 – Dividing Rational Expressions

Topic 3 – Adding and Subtracting Rational Expressions

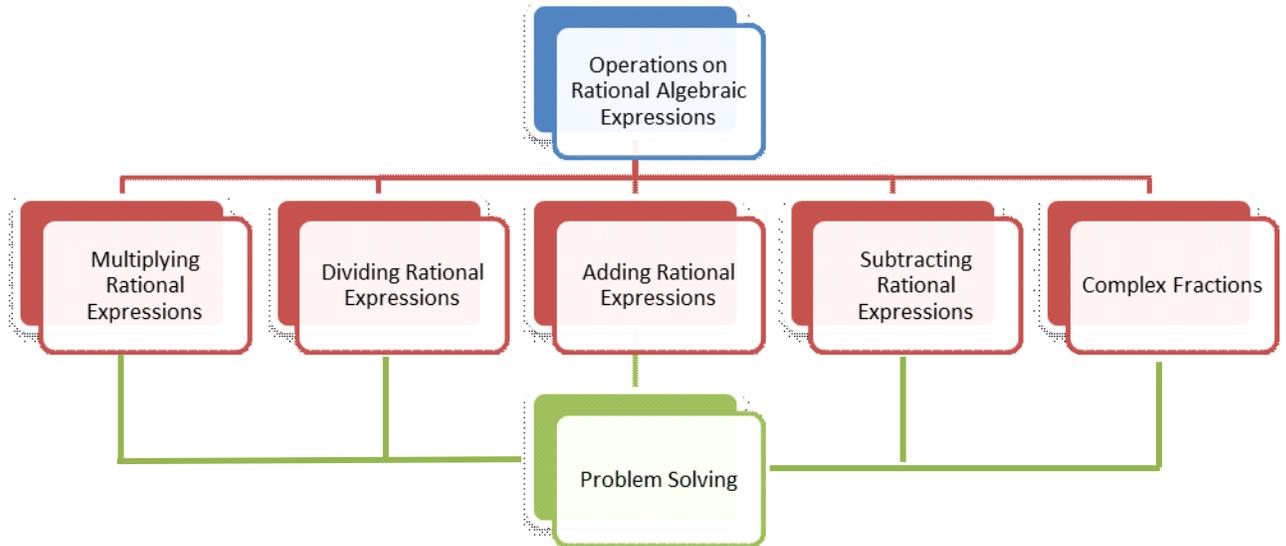
Topic 4 – Complex Rational Expressions

In these topics, you will learn the following:

<i>Topic 1</i>	<ul style="list-style-type: none"> • <i>Multiply rational expressions.</i>
<i>Topic 2</i>	<ul style="list-style-type: none"> • <i>Divide rational expressions.</i>
<i>Topic 3</i>	<ul style="list-style-type: none"> • <i>Add and subtract rational expressions.</i>
<i>Topic 4</i>	<ul style="list-style-type: none"> • <i>Simplify complex rational expressions.</i>

MODULE MAP:

Here is a simple map of the above topics you will cover:



EXPECTED SKILLS:

To do well in this lesson, you need to remember and do the following:

1. Follow the instructions provided for each activity.
2. Review and evaluate your work using the rubric provided before submission.
3. Complete all activities and exercises.
4. Be mindful of the meaning of unfamiliar words you encounter in this module. A glossary of terms is provided in the last part of this module.
5. Maximize the use of online resources in each lesson. Online resources can be accessed multiple times. The summary of online resources is provided in the end of the module.



EXPLORE



Let's begin by gathering your thoughts about the operations on rational algebraic expressions.

Can you still remember how to add and subtract fractions? What mathematical concept plays a vital role in adding and subtracting fraction? How about multiplying and dividing fractions?

ACTIVITY 1. Anticipation Reaction Guide

There are sets of rational algebraic expressions in the table below. Check AGREE if column I is the same as column II and check DISAGREE if the two columns are not the same. Click "SAVE" when you're finished.

I	II	Agree	Disagree
$\frac{x^2 - xy}{x^2 - y^2} \cdot \frac{x + y}{x^2 - xy}$	$x^{-1} - y^{-1}$		
$\frac{6y - 30}{y^2 + 2y + 1} \div \frac{3y - 15}{y^2 + y}$	$\frac{2y}{y + 1}$		
$\frac{5}{4x^2} + \frac{7}{6x}$	$\frac{15 + 14x}{12x^2}$		
$\frac{a}{b - a} - \frac{b}{a - b}$	$\frac{a + b}{b - a}$		
$\frac{a + b}{b} - \frac{b}{a + b}$	$\frac{a^2}{a + b}$		
$\frac{1}{b} + \frac{2}{a}$			

ACTIVITY 2. Picture Analysis

Take a close look at this picture. Describe what you see.



http://www.portlandground.com/archives/2004/05/volunteers_buil_1.php

? PROCESS QUESTIONS:

1. What will happen if one of them will not do his job?
2. What will happen when there are
3. people working together? Will they finish the job earlier or later?
4. How can rate – related problems be solved?



End of Explore:

The picture above shows how the operations on rational algebraic expressions can be applied to a real-life scenario. You'll get to learn more rate-related problems and how operations on rational algebraic expression associate to rate-related problems. You will also find out the answer to the question "How can rate-related problems be solved?"



FIRM-UP



Your goal in this section is to learn and understand key concepts in the operations on rational algebraic expressions.

As the concepts of operations on rational algebraic expressions become clear to you through the succeeding activities, do not forget to think about how to apply these concepts in solving real-life problems. How can rate-related problems be solved?

This time, take a look at how the ancient Egyptians used special rules in their fraction. When they had 5 loaves for 8 persons, they did not divide them immediately by 8. Instead, they used the concept of unit fraction. A unit fraction is a fraction with 1 as its numerator. Egyptian fractions used unit fractions without repetition except $\frac{2}{3}$. Like 5 loaves for 8 persons, they had to cut the 4 loaves into two and the last one would be cut into 8 parts. In short:

$$\frac{5}{8} = \frac{1}{2} + \frac{1}{8}$$

ACTIVITY 3. Egyptian Fractions

To understand more about Egyptian fractions, access the websites below. Then, answer the items that follow.

<http://www.mathcats.com/explore/oldegyptianfractions.html>

This interactive website allows you to come up with a specified sum by adding unit fractions.

<http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fractions/egyptian.html>

This website discusses the Egyptian Mathematics and the use of unit fractions. It also identifies some reasons why we use unit fractions today.

Now, be like an Ancient Egyptian. Give the equivalent unit fractions of the fractions below.

1. $\frac{7}{10}$ using 2 unit fractions.

6. $\frac{13}{12}$ using 3 unit fractions.

2. $\frac{8}{15}$ using 2 unit fractions.

7. $\frac{11}{12}$ using 3 unit fractions.

3. $\frac{3}{4}$ using 2 unit fractions.

8. $\frac{31}{30}$ using 3 unit fractions.

4. $\frac{11}{30}$ using 2 unit fractions.

9. $\frac{19}{20}$ using 3 unit fractions

5. $\frac{7}{12}$ using 2 unit fractions.

10. $\frac{25}{28}$ using 3 unit fractions



PROCESS QUESTIONS:

1. What did you do to determine the unit fraction?
2. Do you like the way Egyptians add fractions?
3. What difficulties did you encounter in giving unit fraction?
4. What will you do to overcome these difficulties?

After working with unit fractions, let us deal with rational expressions. Examine and analyze the illustrative examples below about multiplying rational expressions. Pause once in a while to answer the check—up questions.

Illustrative example 1: Find the product of $\frac{5t}{8}$ and $\frac{4}{3t^2}$.

$$\frac{5t}{8} \cdot \frac{4}{3t^2} = \frac{5t}{2^3} \cdot \frac{2^2}{3t^2}$$

$$= \frac{(5t)(2^2)}{(2^3)(3t^2)}$$



Express the numerators and denominators as prime factors as possible.

$$= \frac{5}{(2)(3t)}$$

$$= \frac{5}{6t}$$



Simplify rational expression using laws of exponents.

Illustrative example 2: Multiply $\frac{4x}{3y}$ and $\frac{3x^2y^2}{10}$.

$$\frac{4x}{3y} \cdot \frac{3x^2y^2}{10} = \frac{(2^2)x}{3y} \cdot \frac{3x^2y^2}{(2)(5)}$$

$$= \frac{(2^2)(x)(3)(x^2)(y^2)}{(3)(y)(2)(5)}$$

$$= \frac{(2^2)(3)(x^3)(y^2)}{(3)(y)(2)(5)}$$

$$= \frac{(2)(x^3)(y)}{(5)}$$

$$= \frac{2x^3y}{5}$$



What laws of exponents were used in these steps?

Illustrative example 3: What is the product of $\frac{x-5}{4x^2-9}$ and $\frac{4x^2+12x+9}{2x^2-11x+5}$?

$$\begin{aligned} \frac{x-5}{4x^2-9} \cdot \frac{4x^2+12x+9}{2x^2-11x+5} &= \frac{x-5}{(2x-3)(2x+3)} \cdot \frac{(2x+3)^2}{(2x-1)(x-5)} \\ &= \frac{(x-5)(2x+3)^2}{(2x-3)(2x+3)(2x-1)(x-5)} \\ &= \frac{2x+3}{(2x-3)(2x-1)} \\ &= \frac{2x+3}{4x^2-8x+4} \end{aligned}$$



What factoring methods were used in this step?



What are the rational algebraic expressions equivalent to 1 in this step?



The product of two rational expressions is the product of the numerators divided by the product of the denominators. In symbols,

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}, \quad bd \neq 0$$

ACTIVITY 4. Multiplying Rational Expressions

Find the product of the following rational expressions. When you're finished, click "ANSWERS" to see the correct answers.

- $\frac{10uv^2}{3xy^2} \cdot \frac{6x^2y^2}{5u^2v^2}$
- $\frac{a^2-b^2}{2ab} \cdot \frac{a^2}{a-b}$
- $\frac{x^2-3x}{x^2+3x-10} \cdot x^2 - 4x^2 - x - 6$

Answers:

$$\begin{aligned} 1. \quad & \frac{10uv^2}{3xy^2} \cdot \frac{6x^2y^2}{5u^2v^2} = \frac{5 \cdot 2(u)(v^2)(x^2)(y^2)}{3 \cdot 5(u^2)(v)(x)(y^2)} = \frac{2vx}{3u} \\ 2. \quad & \frac{a^2-b^2}{2ab} \cdot \frac{a^2}{a-b} = \frac{(a-b)(a+b)(a)(a)}{2ab(a-b)} = \frac{a(a+b)}{2b} \\ 3. \quad & \frac{x^2-3x}{x^2+3x-10} \cdot \frac{x^2-4}{x^2-x-6} = \frac{x(x-3)(x-2)(x+2)}{(x+5)(x-2)(x-3)(x+2)} = \frac{x}{x+5} \end{aligned}$$

ACTIVITY 5. Multiplying Rational Expressions One More Time

DESCRIPTION: Study the website below about multiplying rational expressions.

Multiplying Rational Expressions

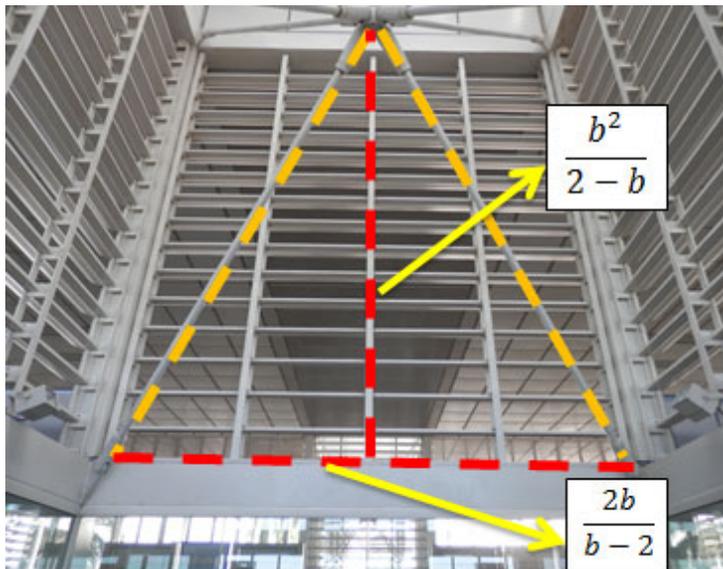
<http://www.onlinemathlearning.com/multiplying-rational-expressions-help.html>

The article on this website is about multiplying rational expressions.

ACTIVITY 6. What's My Area?

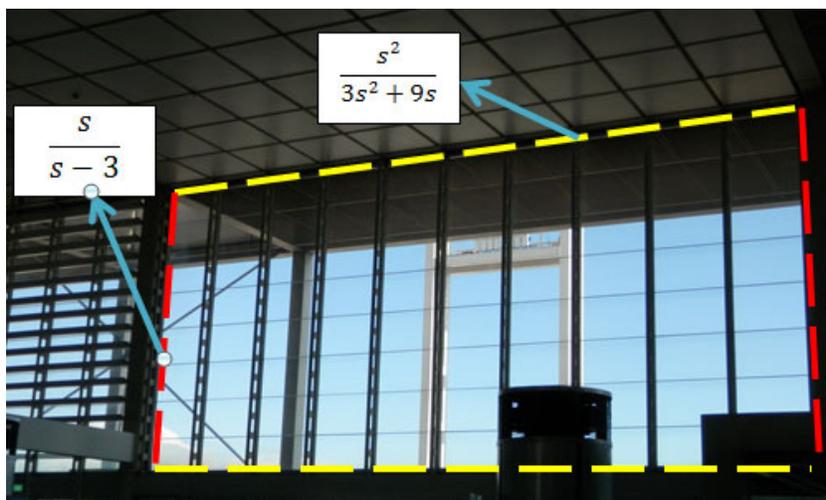
Find the area of the plane figures below. Write your answer in the box provided. Click "SUBMIT" when you're done to see the answers.

1.



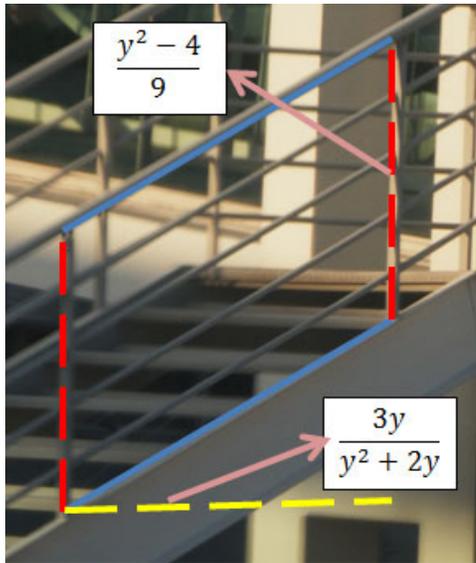
ANSWER:

2.



ANSWER:

3.



ANSWER:

ANSWERS:

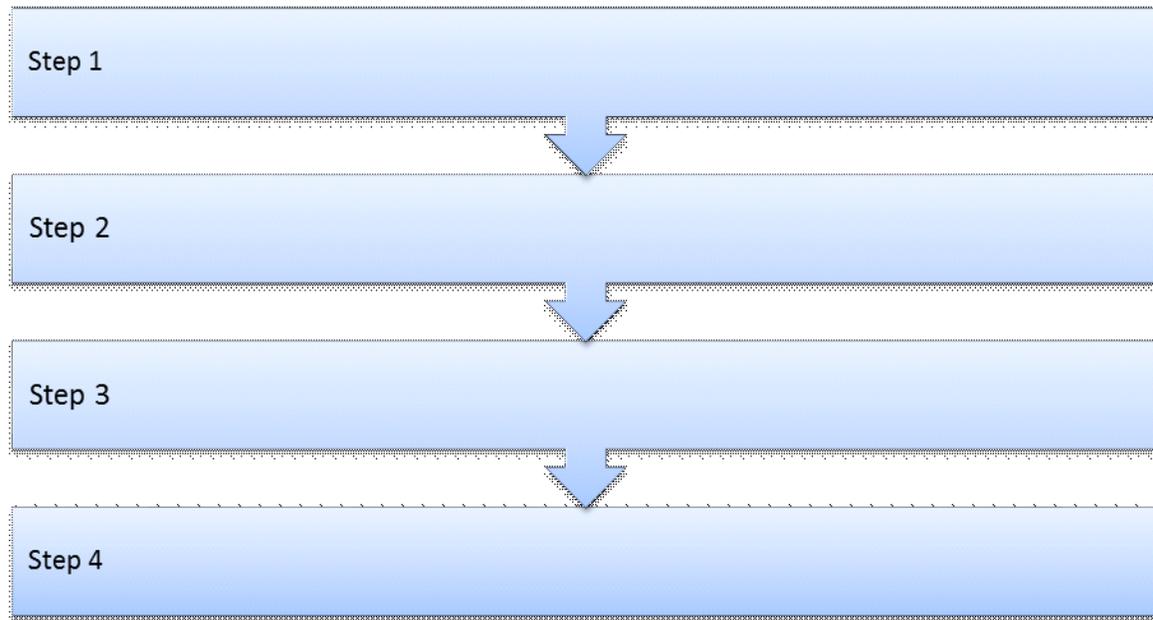
- $$A = \frac{bh}{2} = \frac{\frac{2b}{b-2} \cdot \frac{b^2}{2-b}}{2} = \frac{2b}{b-2} \cdot \frac{b^2}{2-b} \cdot \frac{1}{2} = \frac{b^3}{(b-2)(2-b)} = \frac{b^3}{-(b-2)(b-2)} = -\frac{b^3}{(b-2)^2}$$
- $$A = lw = \frac{s^2}{3s^2+9s} \cdot \frac{s}{s-3} = \frac{s^2}{3s(s+3)} \cdot \frac{s}{s-3} = \frac{s^2}{3(s+3)(s-3)} \text{ or } \frac{s^2}{3(s^2-9)}$$
- $$A = bh = \frac{y^2-4}{9} \cdot \frac{3y}{y^2+2y} = \frac{(y-2)(y+2)}{9} \cdot \frac{3y}{y(y+2)} = \frac{y-2}{3}$$

ACTIVITY 7. How Did I do it?

Based on what you've done in the previous activity, make a step by step guide when multiplying rational algebraic expressions. Write the procedure or important concepts in every step. Click "Submit" when you're done.

- ?

 1. How did you find the area of the figures?
 2. What are your steps in answering the area of the figures?



1. Does every step have a mathematical concept involved?
2. What makes that mathematical concept important to every step?
3. How can rate—related problems be solved?

Examine and analyze the illustrative examples below about dividing rational expressions. Pause once in a while to answer the check – up questions.

Illustrative example 4: Find the quotient of $\frac{6ab^2}{4cd}$ and $\frac{9a^2b^2}{8dc^2}$.

$$\begin{aligned} \frac{6ab^2}{4cd} \div \frac{9a^2b^2}{8dc^2} &= \frac{6ab^2}{4cd} \cdot \frac{8dc^2}{9a^2b^2} \\ &= \frac{(2)(3)ab^2}{(2^2)cd} \cdot \frac{(2^3)dc^2}{(3^2)a^2b^2} \\ &= \frac{(2^4)(3)ab^2dc^2}{(2^2)(3^2)cda^2b^2} \\ &= \frac{(2^2)c}{(3)a} \end{aligned}$$

Multiply the dividend by the reciprocal of the divisor.

Follow the steps in multiplying rational algebraic expressions.

$$= \frac{4c}{3a}$$

Illustrative example 5: Divide $\frac{2x^2+x-6}{2x^2+7x+5}$ by $\frac{x^2-2x-8}{2x^2-3x-20}$.



Why do we need to factor out the numerators and denominators?

$$\begin{aligned} \frac{2x^2+x-6}{2x^2+7x+5} \div \frac{x^2-2x-8}{2x^2-3x-20} &= \frac{2x^2+x-6}{2x^2+7x+5} \cdot \frac{2x^2-3x-20}{x^2-2x-8} \\ &= \frac{(2x-3)(x+2)}{(x+5)(x+1)} \cdot \frac{(x-4)(2x+5)}{(x+2)(x-4)} \end{aligned}$$



What mathematical principle behind the concept of "cancellation"?

$$\begin{aligned} &= \frac{(2x-3)(x+2)(x-4)(2x+5)}{(2x+5)(x+1)(x+2)(x-4)} \\ &= \frac{(2x-3)}{(x+1)} \\ &= \frac{2x-3}{x+1} \end{aligned}$$



The quotient of two rational algebraic expressions is the product of the dividend and the reciprocal of the divisor. In symbols,

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}, \quad bc \neq 0$$

ACTIVITY 8. Dividing Rational Expressions

Find the quotient of the following rational algebraic expressions. When you're done, click "ANSWERS" to see the correct answers.

$$1. \frac{81xz^3}{36y} \div \frac{27x^2z^2}{12xy}$$

$$2. \frac{2a+2b}{a^2+ab} \div \frac{4}{a}$$

$$3. \frac{16x^2-9}{6-5x-4x^2} \div \frac{16x^2+24x+9}{4x^2+11x+6}$$

ANSWERS:

$$1. \frac{81xz^3}{36y} \div \frac{27x^2z^2}{12xy} = \frac{81xz^3}{36y} \cdot \frac{12xy}{27x^2z^2} = \frac{3^4xz^2 \cdot 3 \cdot 2^2xy}{2^23^2y \cdot 3^3x^2z^2} = z$$

$$2. \frac{2a+2b}{a^2+ab} \div \frac{4}{a} = \frac{2a+2b}{a^2+ab} \cdot \frac{a}{4} = \frac{2(a+b)(a)}{a(a+b)(4)} = \frac{1}{2}$$

$$3. \frac{16x^2-9}{6-5x-4x^2} \div \frac{16x^2+24x+9}{4x^2+11x+6} = \frac{16x^2-9}{6-5x-4x^2} \cdot \frac{4x^2+11x+6}{16x^2+24x+9}$$

$$= \frac{(4x-3)(4x+3)}{-(4x-3)(x+2)} \cdot \frac{(4x+3)(x+2)}{(4x+3)(4x+3)} =$$

ACTIVITY 9. Dividing Rational Expressions One more Time

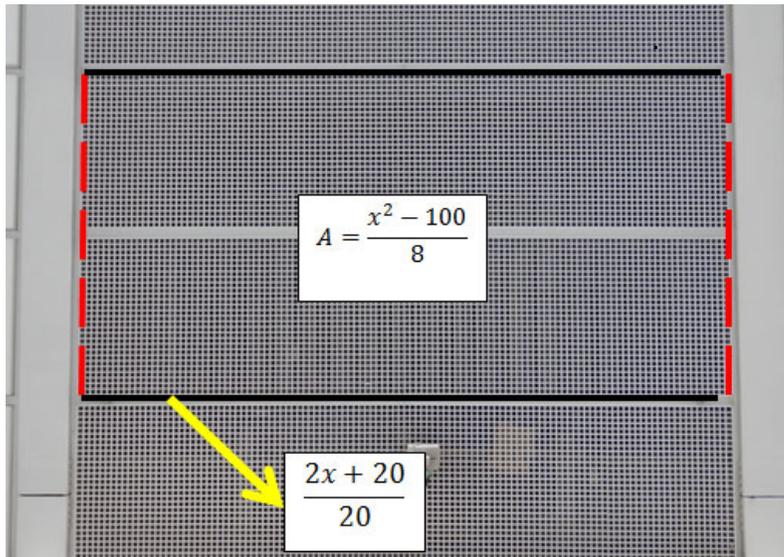
Description: The website below is about dividing rational expressions. Study the content of the website, then do the activity that follows.

<http://www.onlinemathlearning.com/dividing-rational-expressions-help.html>

ACTIVITY 10. The Missing Dimensions

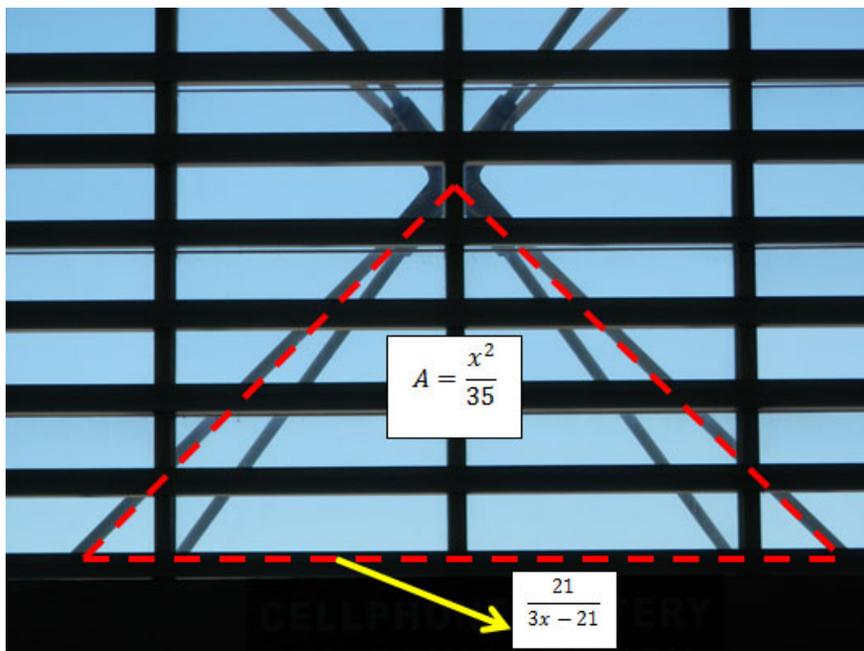
Find the missing length of the figures. Input your answer in the blank. Click "SUBMIT" when you're done to see the answers.

1. The height of the rectangle



ANSWER:

2. The height of the triangle



ANSWER:

ANSWERS:

1. $A = bh$ thus $h = \frac{A}{b}$

$$h = \frac{\frac{x^2 - 100}{8}}{\frac{2x + 20}{20}} = \frac{x^2 - 100}{8} \cdot \frac{20}{2x + 20} = \frac{(x - 10)(x + 10) \cdot (2^3)(5)}{(2^3) \cdot (2)(x + 10)}$$

$$= 5(x - 10)$$

2. $A = \frac{bh}{2}$ thus $h = \frac{2A}{b}$

$$h = \frac{2 \cdot \frac{x^2}{35}}{\frac{21}{3x - 21}} = \frac{2x^2}{35} \cdot \frac{3x - 21}{21} = \frac{2x^2}{5 \cdot 7} \cdot \frac{3(x - 7)}{3(7)} = \frac{2x^2(x - 7)}{245}$$



1. How did you find the area of the figures?
2. What are your steps in answering the area of the figures?

ACTIVITY 11. Chain Reaction

Use the Chain Reaction Chart to sequence your steps in dividing rational algebraic expressions. Write the process or mathematical concepts used in each step in the chamber. Add another chamber if necessary.

The diagram shows a 'Chain Reaction Chart' consisting of four vertical rectangular chambers arranged horizontally. Each chamber is light blue with rounded corners and contains a circular area at the top with a white box inside. The boxes are labeled 'Chamber 1', 'Chamber 2', 'Chamber 3', and 'Chamber 4' from left to right. Below each label, there are several horizontal lines for writing. A large, light blue arrow with a white outline points from left to right across the bottom of the four chambers, indicating the sequence of steps.

- 
 1. Does every step have a mathematical concept involved? Explain.
 2. What makes that mathematical concept important to every step?
 3. Is there another method in dividing rational algebraic expressions? Explain.
 4. How can rate—related problems be solved?

Examine and analyze the illustrative examples below about adding/subtracting rational expressions. Pause in a while to answer the check – up questions.

Illustrative example 6: Add $\frac{x^2-2x-7}{x^2-9}$ and $\frac{3x+1}{x^2-9}$

$$\frac{x^2 - 2x - 7}{x^2 - 9} + \frac{3x + 1}{x^2 - 9} = \frac{x^2 - 2x + 3x - 7 + 1}{x^2 - 9}$$

$$= \frac{x^2 + x - 6}{x^2 - 9}$$



Combine like terms in the numerator.

$$= \frac{(x+3)(x-2)}{(x-3)(x+3)}$$



Factor out the numerator and denominator.

$$= \frac{(x - 2)}{(x + 3)}$$

$$= \frac{x - 2}{x + 3}$$



Do we always factor out the numerator and denominator? Why yes or why no?

Illustrative example 7: Subtract $\frac{-10-6x-5x^2}{3x^2+x-2}$ from $\frac{x^2+5x-20}{3x^2+x-2}$.



Why do we need to multiply the subtrahend

by -1 in the numerator?

$$\begin{aligned}
 \frac{x^2 + 5x - 20}{3x^2 + x - 2} - \frac{-10 - 6x - 5x^2}{3x^2 + x - 2} &= \frac{x^2 + 5x - 20 - (-10 - 6x - 5x^2)}{3x^2 + x - 2} \\
 &= \frac{x^2 + 5x - 20 + 10 + 6x + 5x^2}{3x^2 + x - 2} \quad \Rightarrow \\
 &= \frac{x^2 + 5x^2 + 5x + 6x - 20 + 10}{3x^2 + x - 2} \\
 &= \frac{6x^2 + 11x - 10}{3x^2 + x - 2} \\
 &= \frac{(3x-2)(2x+5)}{(3x-2)(x+1)} \quad \Rightarrow \quad \text{Factor out the numerator and denominator.} \\
 &= \frac{2x + 5}{x + 1}
 \end{aligned}$$



In adding or subtracting similar rational expressions, add or subtract the numerators and write it in the numerator of the result over the common denominator. In symbols,

$$\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}, \quad b \neq 0$$

ACTIVITY 12. Adding and subtracting similar rational expressions

Find the sum/difference of the following rational algebraic expressions. When you're done, click "ANSWERS" to compare your work with the correct answers.

1. $\frac{6}{a-5} + \frac{4}{a-5}$

2. $\frac{x^2+3x-2}{x^2-4} + \frac{x^2-2x+4}{x^2-4}$

3. $\frac{7}{4x-1} - \frac{5}{4x-1}$

4. $\frac{x^2+3x+2}{x^2-2x+1} - \frac{3x+3}{x^2-2x+1}$

ANSWERS:

1. $\frac{6}{a-5} + \frac{4}{a-5} = \frac{6+4}{a-5} = \frac{10}{a-5}$
2. $\frac{x^2+3x-2}{x^2-4} + \frac{x^2-2x+4}{x^2-4} = \frac{x^2+3x-2+x^2-2x+4}{x^2-4} = \frac{2x^2+x+2}{(x-2)(x+2)}$
3. $\frac{7}{4x-1} - \frac{5}{4x-1} = \frac{7-5}{4x-1} = \frac{2}{4x-1}$
4. $\frac{x^2+3x+2}{x^2-2x+1} - \frac{3x+3}{x^2-2x+1} = \frac{(x^2+3x+2)-(x+3)}{x^2-2x+1} = \frac{x^2+3x+2-x-3}{x^2-2x+1} = \frac{x^2+2x-1}{(x-1)(x-1)}$

Examine and analyze the following illustrative examples about adding/subtracting dissimilar rational expressions. Pause in a while to answer the check—up questions.

Illustrative example 8: Find the sum of $\frac{5}{18a^4b}$ and $\frac{2}{27a^3b^2c}$.

$$\frac{5}{18a^4b} + \frac{2}{27a^3b^2c} = \frac{5}{(3^2)(2)a^4b} + \frac{2}{(3^3)a^3b^2c}$$



Express the denominators as prime factors.

LCD of $\frac{5}{(3^2)(2)a^4b}$ and $\frac{2}{(3^3)a^3b^2c}$
 $(3^2)(2)a^4b$ and $(3^3)a^3b^2c$
 The LCD is $(3^3)(2)(a^4)(b^2)(c)$



Denominators of the rational algebraic expressions



Take the factors of the denominators. When the same factor is present in more than one denominator, take the factor with the highest exponent.



The product of these factors is the **LCD**. Find a number equivalent to 1 that should be multiplied to the rational algebraic expressions so that

$$\begin{aligned}
 &= \frac{5}{(3^2)(2)a^4b} \cdot \frac{3bc}{3bc} + \frac{2}{(3^3)a^3b^2c} \cdot \frac{2a}{2a} \\
 &= \frac{(5)(3)bc}{(3^3)(2)a^4b^2c} + \frac{(2^2)a}{(3^3)(2)a^4b^2c} \\
 &= \frac{15bc}{54a^4b^2c} + \frac{4a}{54a^4b^2c} \\
 &= \frac{15bc + 4a}{54a^4b^2c}
 \end{aligned}$$

the denominators are the same with the **LCD**.

Illustrative example 9: Subtract $\frac{t+3}{t^2-6t+9}$ by $\frac{8t-24}{9-t^2}$

$$\frac{t+3}{t^2-6t+9} - \frac{8t-24}{t^2-9} = \frac{t+3}{(t-3)^2} - \frac{8t-24}{(t-3)(t+3)}$$



Express the denominators as prime factors.

$$= \frac{t+3}{(t-3)^2} \cdot \frac{t+3}{t+3} - \frac{8t-24}{(t-3)(t+3)} \cdot \frac{t-3}{t-3}$$



What property of equality is illustrated in this step?

$$= \frac{(t+3)(t+3)}{(t-3)^2(t+3)} - \frac{(8t-24)(t-3)}{(t-3)^2(t+3)}$$

$$= \frac{t^2 + 6t + 9}{t^3 - 9t^2 + 27t - 27} - \frac{8t^2 - 48t + 72}{t^3 - 9t^2 + 27t - 27}$$



What special products are illustrated in this step?

$$= \frac{t^2 + 6t + 9 - (8t^2 - 48t + 72)}{t^3 - 9t^2 + 27t - 27}$$

$$= \frac{t^2 + 6t + 9 - 8t^2 + 48t - 72}{t^3 - 9t^2 + 27t - 27}$$

$$= \frac{-7t^2 + 54t - 63}{t^3 - 9t^2 + 27t - 27}$$

LCD of $\frac{t+3}{(t-3)^2}$ and $\frac{8t-24}{(t-3)(t+3)}$
 $(t-3)^2$ and $(t-3)(t+3)$
 The LCD is $(t-3)^2(t+3)$

Illustrative example 10. Find the sum of $\frac{2x}{x^2+4x+3}$ and $\frac{3x-6}{x^2+5x+6}$

$$\frac{2x}{x^2+4x+3} + \frac{3x-6}{x^2+5x+6} = \frac{2x}{(x+3)(x+1)} + \frac{3x-6}{(x+3)(x+2)}$$



What factoring methods were used in this step?

LCD of $\frac{2x}{(x+3)(x+1)}$ and $\frac{3x-6}{(x+3)(x+2)}$
 $(x+3)(x+1)$ and $(x+3)(x+2)$
 The LCD is $(x+3)(x+1)(x+2)$

$$= \frac{2x}{(x+3)(x+1)} \cdot \frac{x+2}{x+2} + \frac{3x-6}{(x+3)(x+2)} \cdot \frac{x+1}{x+1}$$



What property of equality was used in this step?

$$= \frac{(2x)(x+2)}{(x+3)(x+1)(x+2)} + \frac{(3x-6)(x+1)}{(x+3)(x+2)(x+1)}$$

$$= \frac{2x^2+4x}{x^3+6x^2+11x+6} + \frac{3x^2-3x-6}{x^3+6x^2+11x+6}$$

$$= \frac{2x^2+3x^2+4x-3x-6}{x^3+6x^2+11x+6}$$

$$= \frac{5x^2+x-6}{x^3+6x^2+11x+6}$$



In adding or subtracting dissimilar rational expressions, change the rational algebraic expressions into similar rational algebraic expressions using the least common denominator or **LCD** and proceed as in adding similar fractions.

ACTIVITY 13. Adding and subtracting dissimilar Rational Expressions

Find the sum/difference of the rational algebraic expressions below. When you're done, click "ANSWERS" to compare your work with the correct answers.

1. $\frac{3}{x+1} + \frac{4}{x}$

2. $\frac{x+8}{x^2-4x+4} + \frac{3x-2}{x^2-4}$

3. $\frac{2x}{x^2-9} - \frac{3}{x-3}$

4. $\frac{3}{x^2-x-2} - \frac{2}{x^2-5x+6}$

ANSWERS:

1. $\frac{3}{x+1} + \frac{4}{x} = \frac{3(x)+4(x+1)}{x(x+1)} = \frac{3x+4x+4}{x(x+1)} = \frac{7x+4}{x(x+1)}$

$$\frac{x+8}{x^2-4x+4} + \frac{3x-2}{x^2-4} = \frac{x+8}{(x-2)(x-2)} + \frac{3x-2}{(x-2)(x+2)} = \frac{(x+2)(x+8)+((x-2)(3x-2))}{(x-2)(x-2)(x+2)} =$$

$$\frac{(x+2)(x+8)+(x-2)(3x-2)}{(x-2)^2(x+2)} = \frac{(x^2+10x+16)+(3x^2-8x+4)}{(x-2)^2(x+2)} = \frac{4x^2+2x+20}{(x-2)^2(x+2)} =$$

$$\frac{2(2x^2+x+10)}{(x-2)^2(x+2)}$$

2. $\frac{2x}{x^2-9} - \frac{3}{x-3} = \frac{2x}{(x-3)(x+3)} - \frac{3}{x-3} = \frac{2x-3(x+3)}{(x-3)(x+3)} = \frac{2x-3x-9}{(x-3)(x+3)} =$

3. $\frac{-x-9}{(x-3)(x+3)}$ or $\frac{-(x+9)}{(x-3)(x+3)}$

4. $\frac{3}{x^2-x-2} - \frac{2}{x^2-5x+6} = \frac{3}{(x-2)(x+1)} - \frac{2}{(x-2)(x-3)} = \frac{3(x-3)-2(x+1)}{(x+1)(x-2)(x-3)} =$

$$\frac{3x-9-2x-2}{(x+1)(x-2)(x-3)} = \frac{x-11}{(x+1)(x-2)(x-3)}$$

ACTIVITY 14. Adding and Subtracting Rational Expressions

One More Time

Description: Study the content of the websites below about adding and subtracting rational expressions, and then do the activity that follows.

<http://www.onlinemathlearning.com/adding-rational-expressions-help.html>

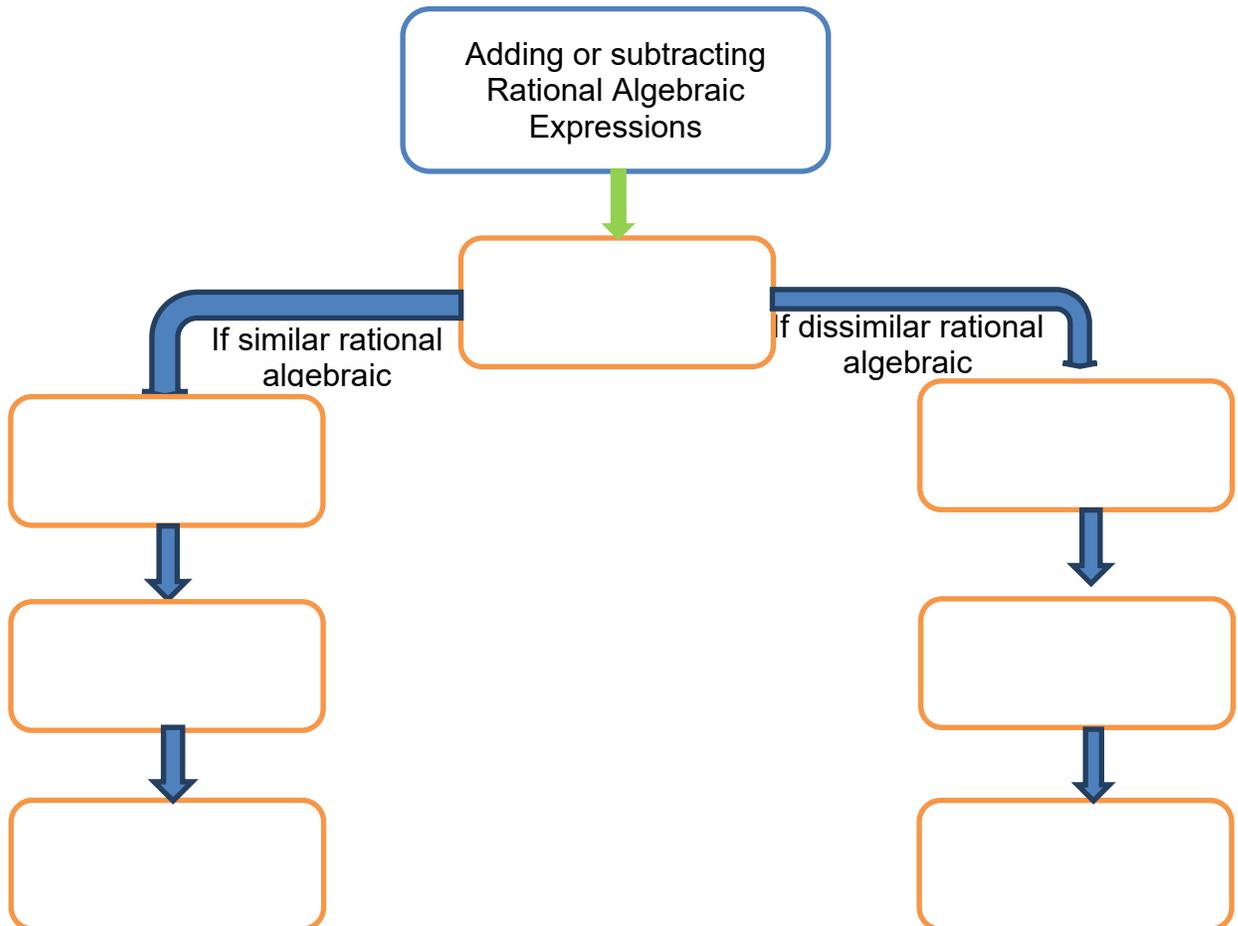
The website is about adding rational expressions.

<http://www.onlinemathlearning.com/subtracting-rational-expressions-help.html>

The website is about adding rational expressions.

ACTIVITY 15. This is How I Did It

Write every step in adding or subtracting rational algebraic expression in the boxes below. Add or erase a box if necessary.





1. Does every step has a mathematical concept involved? Explain.
2. What makes that mathematical concept important to every step?
3. Is there another method in adding or subtracting rational algebraic expressions? Explain.
4. How can rate-related problems be solved?

ACTIVITY 15. ACTIVITY 16. What Is Wrong with Me?

There are rational algebraic expressions below that were added or subtracted. The solutions in each rational algebraic expression are wrong. Write in the second box your explanation on what makes the solution wrong. In the third box, show how the rational algebraic expression should be operated.

Original	Explanation	The Correct
$\frac{2}{36-x^2} - \frac{1}{x^2-6x} = \frac{2}{(6-x)(6+x)} - \frac{1}{x(x+6)}$ $= \frac{2}{(x-6)(x+6)} - \frac{1}{x(x+6)}$ $= \frac{2}{(x-6)(x+6)} \cdot \frac{x}{x} - \frac{1}{x(x+6)} \cdot \frac{x-6}{x-6}$ $= \frac{2x}{x(x-6)(x+6)} - \frac{1(x-6)}{x(x+6)(x-6)}$ $= \frac{2x-(x-6)}{x(x-6)(x+6)}$ $= \frac{2x-x+6}{x(x-6)(x+6)}$ $= \frac{x+6}{x(x-6)(x+6)}$ $= \frac{1}{x(x-6)}$ $= \frac{1}{x^2-6x}$		

$\frac{2}{a-5} - \frac{3}{a} = \frac{2}{a-5} \cdot \frac{a}{a} - \frac{3}{a} \cdot \frac{a-5}{a-5}$ $= \frac{2a}{a-5(a)} - \frac{3(a-5)}{a(a-5)}$ $= \frac{2a}{a-5(a)} - \frac{3a-15}{a(a-5)}$ $= \frac{2a-3a-15}{a(a-5)}$ $= \frac{-a-15}{a^2-5a}$		
$\frac{3x}{2x-3} + \frac{9}{3-2x} = \frac{3x}{2x-3} + \frac{9}{(-1)(2x-3)}$ $= \frac{3x}{2x-3} - \frac{9}{2x-3}$ $= \frac{3x-9}{2x-3}$ $= \frac{3(x-3)}{2x-3}$ $= \frac{x-3}{2x}$		
$\frac{4}{b-2} + \frac{b^2-4b}{b-2} = \frac{b^2-4b+4}{b-2}$ $= \frac{(b-2)(b+2)}{b-2}$ $= b + 2$		

Examine and analyze the following illustrative examples about complex rational expressions. Pause in a while to answer the check – up questions.

Illustrative example 11. Simplify $\frac{\frac{2}{5} \frac{a}{b} - \frac{3}{6} \frac{b}{a}}{\frac{b}{a^2} + \frac{6}{a^2}}$.

$$\frac{\frac{2}{5} \frac{a}{b} - \frac{3}{6} \frac{b}{a}}{\frac{b}{a^2} + \frac{6}{a^2}}$$



Main fraction line – a line that separates the main numerator and main denominator.

$$= \frac{\left(\frac{2 \cdot b}{a \cdot b}\right) - \left(\frac{3 \cdot a}{b \cdot a}\right)}{\left(\frac{5 \cdot a^2}{b \cdot a^2}\right) + \left(\frac{6 \cdot b}{a^2 \cdot b}\right)}$$



Where does the $\frac{b}{b}$ and $\frac{a}{a}$ in the main numerator and the $\frac{a^2}{a^2}$ and $\frac{b}{b}$ in the main denominator come from?

$$\frac{\left(\frac{2b}{ab}\right) - \left(\frac{3a}{ab}\right)}{\left(\frac{5a^2}{a^2b}\right) + \left(\frac{6b}{a^2b}\right)}$$

$$= \frac{\frac{2b-3a}{ab}}{\frac{5a^2+6b}{a^2b}}$$



What happen to the main numerator and main denominator?

$$= \frac{2b-3a}{ab} \div \frac{5a^2+6b}{a^2b}$$



What principle is used in this step?

$$= \frac{2b-3a}{ab} \cdot \frac{a^2b}{5a^2+6b}$$

$$= \frac{(2b-3a)a^2b}{(5a^2+6b)ab}$$



Simplify the rational algebraic expression.

$$= \frac{(2b-3a)a}{(5a^2+6b)}$$



What laws of exponents are in this step?

$$= \frac{2ab - 3a^2}{5a^2 + 6b}$$

Illustrative example 12: Simplify $\frac{\frac{c}{c^2-4} - \frac{c}{c-2}}{1 + \frac{1}{c+2}}$

$$\begin{aligned}
 \frac{\frac{c}{c^2-4} - \frac{c}{c-2}}{1 + \frac{1}{c+2}} &= \frac{\frac{c}{(c-2)(c+2)} - \frac{c}{c-2}}{1 + \frac{1}{c+2}} \\
 &= \frac{\frac{c}{(c-2)(c+2)} - \frac{c}{c-2} \cdot \frac{(c+2)}{(c+2)}}{1 \cdot \frac{c+2}{c+2} + \frac{1}{c+2}} \\
 &= \frac{\frac{c}{(c-2)(c+2)} - \frac{c(c+2)}{(c-2)(c+2)}}{\frac{c+2}{c+2} + \frac{1}{c+2}} \\
 &= \frac{\frac{c}{(c-2)(c+2)} - \frac{c^2 + 2c}{(c-2)(c+2)}}{\frac{c+2}{c+2} + \frac{1}{c+2}} \\
 &= \frac{\frac{c - (c^2 + 2c)}{(c-2)(c+2)}}{\frac{c+2+1}{c+2}} \\
 &= \frac{\frac{-c^2 - 2c + c}{(c-2)(c+2)}}{\frac{c+2+1}{c+2}}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{-c^2 - c}{(c - 2)(c + 2)} \div \frac{c + 3}{c + 2} \\
 &= \frac{-c^2 - c}{(c - 2)(c + 2)} \cdot \frac{c + 2}{c + 3} \\
 &= \frac{(-c^2 - c)(c + 2)}{(c - 2)(c + 2)(c + 3)} \\
 &= \frac{-c^2 - c}{(c - 2)(c + 3)} \\
 &= \frac{-c^2 - c}{c^2 + c - 6}
 \end{aligned}$$



A rational algebraic expression is said to be in its simplest form when the numerator and denominator are polynomials with no common factors other than 1. If the numerator or denominator, or both numerator and denominator of a rational algebraic expression is also a rational algebraic expression, it is called a complex rational algebraic expression. To simplify the complex rational expression, it means to transform it into simple rational expression. You need all the concepts learned previously to simplify complex rational expressions.

ACTIVITY 16-ACTIVITY 17. _____ Complex Rational Expressions One More Time

DESCRIPTION: Open the following websites about complex fractions. This will further prepare you for the exercises that follow.

http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/col_alg_tut11_complexrat.htm

This website is a tutorial about complex fractions.

http://www.youtube.com/watch?v=-jli9PP_4HA

This is a video about simplifying complex fractions.

ACTIVITY 17-ACTIVITY 18. _____ Simplifying Complex Rational Expressions

Simplify the following complex rational expressions. When you're done, click "ANSWERS" to compare your work with the correct answers.

1.
$$\frac{\frac{1}{x} - \frac{1}{y}}{\frac{1}{x^2} + \frac{1}{y^2}}$$

2.
$$\frac{\frac{x-y}{x+y} - \frac{y}{x}}{\frac{y}{x+y} + \frac{x-y}{x}}$$

3.

$$\frac{b}{b-1} - \frac{2b}{b-2} - \frac{2b}{b-2} - \frac{3b}{b-3}$$

ANSWERS:

$$1. \frac{\frac{1}{x} - \frac{1}{y}}{\frac{1}{x^2} + \frac{1}{y^2}} = \frac{\frac{y-x}{xy}}{\frac{y^2+x^2}{x^2+y^2}} = \frac{\frac{y-x}{xy}}{1} = \frac{y-x}{xy}$$

$$2. \frac{\frac{x-y}{x+y} \cdot \frac{y}{x}}{\frac{x}{y} + \frac{x-y}{x+y}} = \frac{\frac{x(x-y)-y(x+y)}{x(x+y)}}{\frac{x(x+y)+y(x-y)}{y(x+y)}} = \frac{\frac{x^2-xy-xy-y^2}{x(x+y)}}{\frac{x^2+xy+xy-y^2}{y(x+y)}} =$$

$$\frac{x^2-xy-xy-y^2}{x(x+y)} \cdot \frac{y(x+y)}{x^2+xy+xy-y^2} =$$

$$\frac{(x^2-2xy-y^2)(y)(x+y)}{(x^2+2xy-y^2)(x)(x+y)} = \frac{y(x^2-2xy-y^2)}{x(x^2+2xy-y^2)}$$

$$3. \frac{\frac{b}{b-1} - \frac{2b}{3b}}{\frac{b-2}{2b} - \frac{b-3}{3b}} = \frac{\frac{b(b-2)-2b(b-1)}{(b-1)(b-2)}}{\frac{2b(b-3)-3b(b-2)}{(b-2)(b-3)}} = \frac{\frac{b^2-2b-2b^2+2b}{(b-1)(b-2)}}{\frac{2b^2-6b-3b^2+6b}{(b-2)(b-3)}} =$$

$$\frac{\frac{b^2-2b^2}{(b-1)(b-2)}}{\frac{2b^2-3b^2}{(b-2)(b-3)}} = \frac{b^2-2b^2}{(b-1)(b-2)} \cdot \frac{(b-2)(b-3)}{2b^2-3b^2} = \frac{-b^2}{(b-1)} \cdot \frac{(b-3)}{-b^2} =$$

$$\frac{b-3}{b-1}$$

ACTIVITY 18. ACTIVITY 19. My Guide



1. How did you simplify the complex rationale expressions?
2. What are your steps in simplifying complex rational algebraic expressions?

Make a conceptual map in simplifying complex rational expression using the vertical chevron list below. Write the procedure or important concepts in every step inside the circle. If necessary, add another chevron to complete your conceptual map. Click "SUBMIT" when you're finished.

Step 1

Step 2

Step 3

Step 4

ACTIVITY 19. ACTIVITY 20. Anticipation Reaction Guide: Revisited

Revisit Activity No.2. There are sets of rational algebraic expressions in the table below. Check agree if column I is the same as column II and check disagree if the two columns are not the same. Click “Submit” when you’re finished.

I	II	Agree	Disagree
$\frac{x^2 - xy}{x^2 - y^2} \cdot \frac{x + y}{x^2 - xy}$	$x^{-1} - y^{-1}$		
$\frac{6y - 30}{y^2 + 2y + 1} \div \frac{3y - 15}{y^2 + y}$	$\frac{2y}{y + 1}$		
$\frac{5}{4x^2} + \frac{7}{6x}$	$\frac{15 + 14x}{12x^2}$		
$\frac{a}{b - a} - \frac{b}{a - b}$	$\frac{a + b}{b - a}$		
$\frac{a + b}{b} - \frac{b}{a + b}$	$\frac{a^2}{a + b}$		
$\frac{1}{b} + \frac{2}{a}$			



1. Compare your answer of the anticipation reaction guide in Activity No 2 and Activity No.20.
2. Are there changes in your previous thoughts? Why is it so?
3. How can rate – related problems be solved?



End of FIRM UP:

In this section, the discussion was all about operations on rational algebraic expressions. How much of your initial ideas are found in the discussion? Which ideas are different and need revision? The skill in performing the operations on rational algebraic expressions is one of the key concepts in solving rate–related problems.



DEEPEN

 Your goal in this section is to relate the operations of rational expressions to real – life problems, especially rate type of problems.

ACTIVITY 20. ACTIVITY 21. Rate, Related, Solve

Read the problems below and answer the questions that follow.

1. Two vehicles travelled $(x+4)$ kilometers. The first vehicle travelled for $(x^2 - 16)$ hours while the second travelled for $\left(\frac{2}{x-4}\right)$ hours.

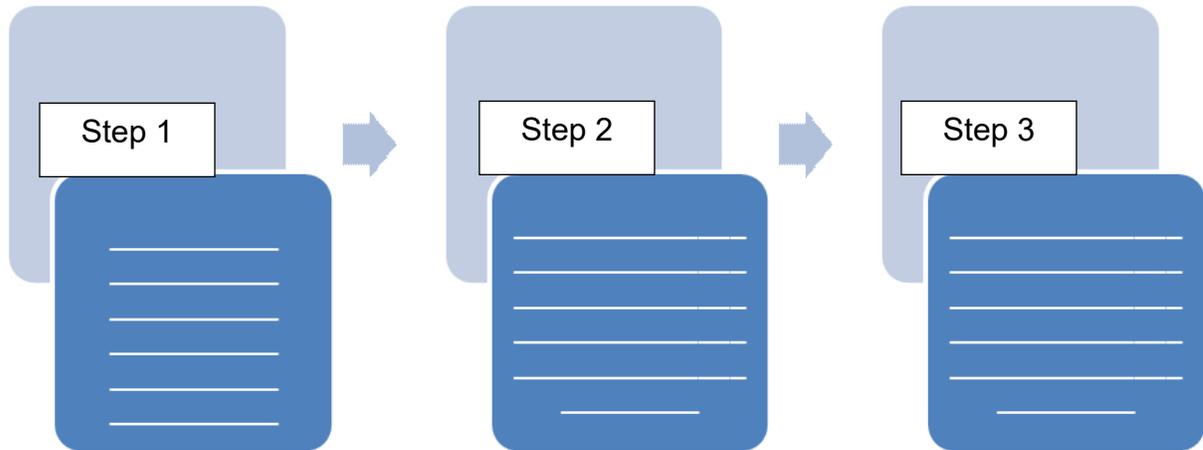
a. Complete the table below.

Vehicles	Distance	Time	Speed
Vehicle A			
Vehicle B			

- b. How did you compute the speed of the two vehicles?
 - c. Which of the two vehicles travelled faster? How did you find your answer?
2. Jem Boy and Roger were asked to fill the tank with water. Jem Boy can fill the tank in x minutes alone while Roger is slower by 2 minutes compared to Jem Boy if working alone.
 - a. What part of the job can Jem Boy finish in 1 minute?
 - b. What part of the job can Roger finish in 1 minute?
 - c. Jem Boy and Roger can finish filling the tank together in certain minutes. How will you represent algebraically, in simplest form, the job done by the two if they worked together?

ACTIVITY 21. ACTIVITY 22. Accent Process

List down the concepts and principles in solving problems involving operations of rational algebraic expressions in every step. You can add a box if necessary.



ACTIVITY 22. ACTIVITY 23. _____ **Time for check – up**

Answer the following. Kindly show your complete solution.

1. Perform the operations. Let:

$$A = x + 4$$

$$B = x^2 - 16$$

$$C = x - 4$$

$$D = 4x - 16$$

a. $\frac{A}{C} + \frac{C}{A}$

b. $\frac{B}{A} - \frac{D}{A}$

c. $\frac{A}{C} \div \frac{B}{D}$

d. $\frac{AC}{B} \div \frac{AD}{C}$

2. Solve the following:

a. If a printer can print $(x+3)$ pages per minute, then how long will it take 2 printers to print $(x^2 + 9x + 18)$ pages? If $x = 30$, what is the actual number of minutes?

b. What is the height of a triangle if its area = $\frac{m^2 - 3m - 10}{m + 3}$ and the

base = $\frac{2m^2 - 6m - 20}{m^2 - 9}$?



End of DEEPEN:

In this section, the discussion was about application in operations on rational algebraic expressions. It gives you a general picture of relation between the operations of rational algebraic expressions and rate – related problems.

What new realizations do you have about the topic? What new connections have you made for yourself? What questions do you still have? What new realizations do you have about rational expressions? What new connections have you made for yourself?



TRANSFER



Your goal in this section is apply your learning to real life situations. You will be given a practical task which will demonstrate your understanding.

ACTIVITY 23. ACTIVITY 24. _____ Let's Work Together



Present and discuss to the class the process of answering the questions below. Your output will be graded according to mathematical reasoning, accuracy, and presentation.

Alex can pour a concrete walkway in x hours alone while Andy can pour the same walkway in two more hours than Alex.

- a) How fast can they pour the walkway if they work together?
- b) If Emman can pour the same walkway in one more hours than Alex, and Roger can pour the same walkway in one hour less than Andy, who must work together to finish the job with the least time?

Rubrics for your output

CRITERIA	Outstanding 4	Satisfactory 3	Developing 2	Beginning 1	RATING
Mathematical reasoning	Explanation shows thorough reasoning and insightful justifications.	Explanation shows substantial reasoning.	Explanation shows gaps in reasoning.	Explanation shows illogical reasoning.	
Accuracy	All computations are correct and shown in detail.	All computations are corrects.	Most of the computations are correct.	Some of the computations are correct.	
Presentation	The presentation uses appropriate and creative visual materials. It is delivered in a clear and very convincing manner.	The presentation uses appropriate visual materials. It is delivered in a clear manner.	The presentation uses some visual materials. It is delivered in a disorganized manner.	The presentation does not use any visual materials. It is not delivered in a clear manner.	

 **End of TRANSFER:**
 In this section, your task was about rate – related problems. How did you find the activity? How did the task help you see the real world use of rational expressions?

How did you find the activity? How did the task help you see the real world use of rational expressions?

To summarize what you’ve learned about operations on rational expressions, do the activity below.

ACTIVITY 24. ACTIVITY 25. LAC Card

Fill-in the Learned, Affirmed, Challenged cards given below to evaluate what you’ve learned about operations on rational expressions. when you’re finished, click “SUBMIT”.

Learned	Affirmed	Challenged
<ul style="list-style-type: none">• What new realizations and learning do you have about the topic?• 1.• 2.• 3.	<ul style="list-style-type: none">• What new connections have you made? Which of your old ideas have been affirmed/confirmed?• 1.• 2.• 3.	<ul style="list-style-type: none">• What questions do you still have? Which areas seem difficult for you? Which do you want to explore?• 1.• 2.• 3.

You have completed this lesson. You can go to the next lesson.

GLOSSARY OF TERMS USED IN THIS LESSON:

LCD – also known as Least Common Denominator is the least common multiple of the denominators.

Rate – related problems – Problems involving rates (e.g., speed, percentage, ratio, work)

REFERENCES AND WEBSITE LINKS USED IN THIS LESSON:

<http://www.mathcats.com/explore/oldegyptianfractions.html>

This interactive website is about adding unit fractions.

<http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fractions/egyptian.html>

This website discusses the Egyptian Mathematics and the use of unit fractions. It also identifies some reasons why we use unit fractions today.

<http://www.onlinemathlearning.com/multiplying-rational-expressions-help.html>

The article on this website is about multiplying rational expressions.

<http://www.onlinemathlearning.com/dividing-rational-expressions-help.html>

The website below is about dividing rational expressions.

<http://www.onlinemathlearning.com/adding-rational-expressions-help.html>

The website is about adding rational expressions.

<http://www.onlinemathlearning.com/subtracting-rational-expressions-help.html>

The website is about adding rational expressions.

http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/col_alg_tut_11_complexrat.htm

This website is a tutorial about complex fractions.

http://www.youtube.com/watch?v=-jli9PP_4HA

This is a video about simplifying complex fractions.

Image credits

http://www.portlandground.com/archives/2004/05/volunteers_buil_1.php

LESSON 2.3: APPLICATION OF RATIONAL EXPRESSIONS

INTRODUCTION AND FOCUS QUESTION(S):

You learned special products and factoring polynomials in the first module then performing operations with rational expressions in the second module. This time, you will focus on the applications of rational algebraic expressions. In the end, you should be able to answer the question, "How can rate – related problems be solved?"

LESSON COVERAGE:

In this lesson, you will examine this question when you take the following topics:

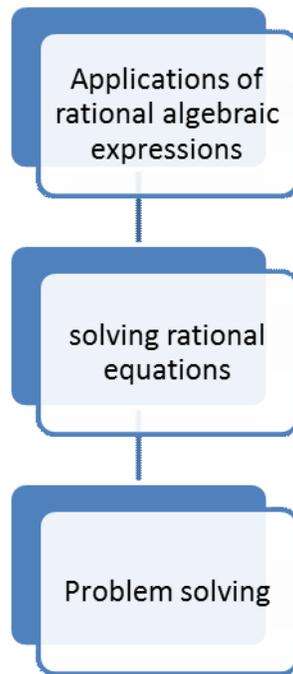
- *Solve rational equations*
- *problem solving involving rational algebraic expressions*

In the topics, you will learn the following:

- *Solve rational equations.*
- *Solve problems involving rational algebraic expressions.*

☑ **MODULE MAP:**

Here is a simple map of the above lesson you will cover:



☑ **EXPECTED SKILLS:**

To do well in this module, you need to remember and do the following:

1. Follow the instructions provided for each activity.
2. Review and evaluate your work using the rubric provided before submission.
3. Complete all activities and exercises.
4. Be mindful of the meaning of unfamiliar words you encounter in this module. A glossary of terms is provided in the last part of this module.
5. Maximize the use of online resources in each lesson. Online resources can be accessed multiple times. The summary of online resources is provided in the end of the module.



EXPLORE



In the previous lesson, you learned preliminary concepts regarding rational expressions. In this section, you will be able to find out how rational expressions can be used to be able to model and solve real life problems. Be reminded to take each activity seriously. Make a conscious effort to highlight the new concepts that you will be learning.

Let us begin this lesson by analyzing the problem below.

ACTIVITY 1. Working Machine

Analyze the situation below then answer the questions that follow. Write your answer in the space provided.

If it takes a machine to complete a job in 5 hours, how much of the work is done in 3 hours?

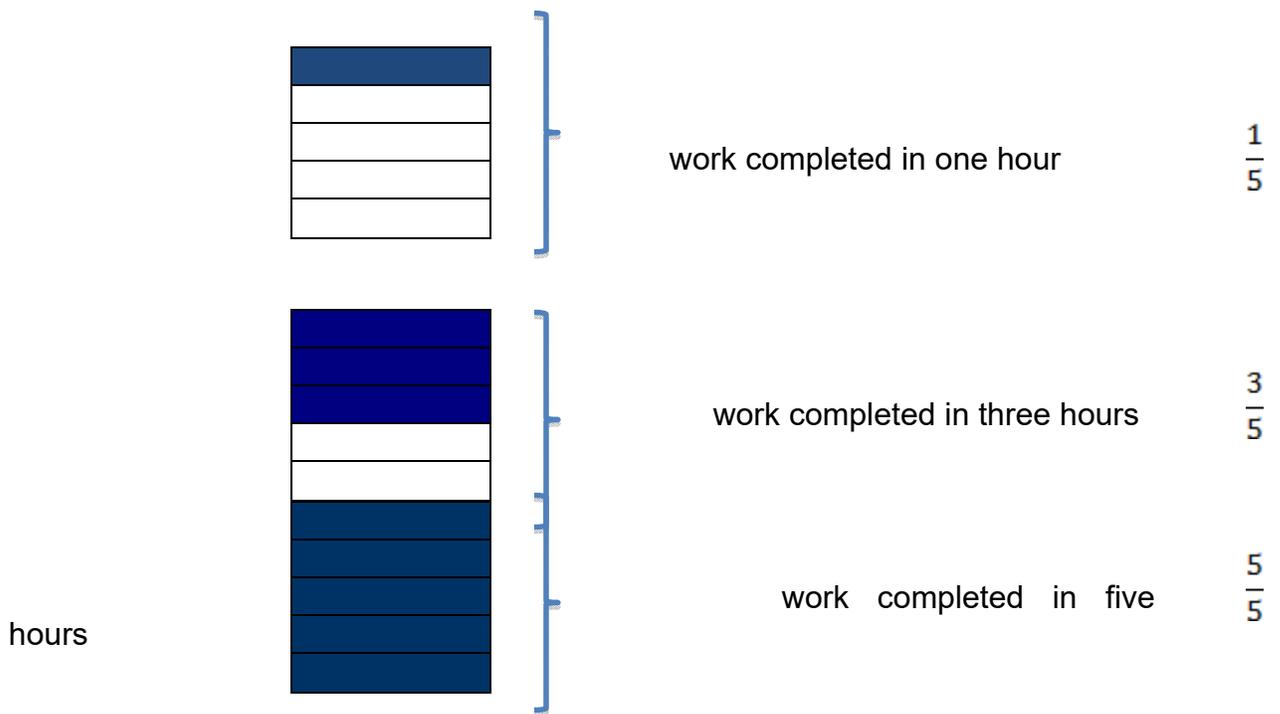


<http://www.ministryblue.com/risograph.html>

Let us now see if your explanation is correct. Click the answer button.

ANSWER

$$\text{WORK} = \frac{\text{-----}}{\text{hours to complete the job}} \quad 1$$



Now, answer the following process questions:

4. What did you find out regarding the problem?

5. How did you arrive at your answer?

6. What algebraic processes did you use?

Let us try to look at the situation below.

Suppose that you can trim your backyard in four hours. Then , after one hour, you will have moved $\frac{1}{4}$ of the lawn. After two hours, you will have mowed $\frac{2}{4}$ of the lawn. What generalizations can you make?

The relationship between problems involving work uses the formula $WORK = RATE \times TIME$. The fractional part of a job accomplished is equal to the rate of work multiplied by the time worked. In the lawn mowing exercise, after three hours, the fractional part of the job done is

$$\frac{1}{4} \cdot 3 = \frac{3}{4}$$

After four hours, $\frac{1}{4} \cdot 4 = 1$, the whole job has been done.



The situations that were presented show one of the many situations where we can apply knowledge of rational algebraic expressions in solving real life problems particularly on work. In this lesson, you will do varied activities which will help you answer the essential question: “How can rate-related problems solved?”

ACTIVITY 2. IRF Worksheet

To share your idea about the lesson, let’s begin by answering the “I” portion of the IRF Worksheet that you see below. Fill it up by writing your initial answer to the focus question: How can rate-related problems be solved?

IRF Worksheet
Initial Answer
Revised Answer

Final Answer

END OF EXPLORE:

	You just tried finding out how rational algebraic expressions can be used to solve real life problems. Let us now find out what the answer is by doing the next part. What you will learn in the next sections will also enable you to do the final project which involves the making of a manpower plan.
---	---



FIRM-UP

	Your goal in this section is to learn and understand key concepts of rational algebraic expressions.
---	--

ACTIVITY 3. WHAT'S NEW WITH RATIONAL ALGEBRAIC EQUATIONS?

In this activity, you will watch a video which will explain what rational algebraic equations are. Visit this website, <http://www.virtualnerd.com/algebra-1/rational-expressions-functions/solve/example-solutions/equation-definition> to watch the video. After watching, answer the process questions that follow.

1. What is a rational equation?

--

2. How is a rational equation different from a rational expression?

3. How do you think will you solve a rational equation?

ACTIVITY 4. FROM UNKNOWN TO KNOWN

In this activity, you will read an article which will explain how rational algebraic equations are solved about by clicking the link below,

<http://www.ssag.sk/SSAG%20study/MAT/Rational%20equations.pdf>.

1. Discuss the steps involved in solving rational expressions.

2. Why is there a need to get the least common denominator?

3. How are the terms containing rational expressions turned into polynomials?

4. What properties of equality did you utilize in solving?

5. Why is there a need to check your answer?

6. How many solution/s did you see?

ACTIVITY 5. BEWARE OF RESTRICTIONS

In Lessons 1 and 2, you have learned that rational expressions contain variables in the denominator. Thus, there are values that could not be used to replace these variables because the expression will become undefined. Because of this, it is possible that the root of an equation will also make the denominator zero. In this case, the root does not satisfy the original equation. Thus, it is called an extraneous root.

In this activity, you will look at examples of equations whose roots are extraneous. You will visit the website, http://www.mhhe.com/math/devmath/dugopolski/acs/student/olc/graphics/dugopolski02acs_s/ch06/others/ch06-5.pdf, and read page 367. Then, answer the following questions:

1. What is an extraneous root?

2. Is an extraneous root the same as a restricted value? Explain.

3. What conclusions can we make if the only root of an equation is considered an extraneous root?

4. Can one tell what the extraneous root is without solving a rational equation? How?

ACTIVITY 6. SKILL BUILDER

Now that you have found out how to solve rational algebraic equations and check your solutions, it is time to build your skills on the topic. Click on this link: <http://www.kutasoftware.com/FreeWorksheets/Alg2Worksheets/Solving%20Rational%20Equations.pdf>. After which, use the key answer at the bottom to check.

How many items did you get correctly?

ACTIVITY 7. PRACTICE MAKES PERFECT

For sure, after solving a number of problems, you are now ready to take a quiz on solving rational algebraic equations. Complete the worksheet below.

*****WORKSHEET (Please provide students with boxes where they could write their answers.)**

1. $\frac{1}{5} + \frac{3}{a} = \frac{1}{2}$

4. $\frac{a+5}{a} = \frac{a+5}{2}$

2. $\frac{1}{a-2} + \frac{1}{2a-4} = \frac{1}{4}$

5. $\frac{2}{a+3} + \frac{5}{a^2-9} = \frac{1}{a-3}$

3. $\frac{3}{a-3} + \frac{5}{a-4} = \frac{a^2-20}{a^2-7a+12}$

ANSWERS:

***** ANSWERS:**

1. $a = 10$
2. $a = 8$
3. $a = 1$ and $a = 7$
4. $a = 2$
5. $a = 4$

ACTIVITY 8. MUDDIEST POINT

In this activity, you will complete the journal below.

The part of the lesson that I still find confusing is _____
because _____.

Let us see if by doing the next activity, the muddiest point will be given clarification.

ACTIVITY 9. WORK CAN BE DONE WITH RATIONAL ALGEBRAIC EQUATIONS

After learning how to solve rational algebraic equations, let us see if the concept can be used to solve work-related problems. Click on the link and view the video about work related problems solved using rational algebraic equations:
<http://www.youtube.com/watch?v=QLhvLEeS08A>.

1. What kind of problem was presented in the video?

2. How did the table presented help you understand how to solve work-related problems?

3. Did the video make use of rational algebraic equations? How?

4. What other strategies were presented?

5. How can rate-related problems solved?

ACTIVITY 10. MAKING MORE CONNECTIONS

Let us see if you can apply what you have viewed in the video as you look at more examples on how work-related problems can be solved using rate-related problems. Click on this link and play the video: <http://www.tes.co.uk/teaching-resource/Rate-Problem-Involving-Rational-Equations-6198692/>. Answer the questions that follow.

1. What were the steps used to solve the rate-related problems?

2. How was the concept of rational algebraic equations used?

3. Is there a way you can use to solve these kinds of problems? Explain.

ACTIVITY 11. SKILL BOOSTER

After watching two videos that relate rational algebraic equations with rate related problems, let us see if you can solve the problems below on your own.

1. Joan can wallpaper a bathroom in 3 hours. Bonnie can wallpaper the same bathroom in 5 hours. How long would it take them if they worked together?

2. Bill's garden hose can fill the pool in 12 hours. His neighbor has a hose that can fill the pool in 15 hours. How long will it take to fill the pool using both hoses?
3. Norm and Cliff can paint the office in 5 hours working together. Being a professional painter, Norm can paint twice as fast as Cliff. How long would it take Cliff to paint the office by himself?
4. Joe can complete his yard work in 3 hours. If his son helps it will only take 2 hours working together. How long would the yard work take if his son was working alone?

ANSWERS:

ANSWERS:

1. 1 and 7/8 hours
2. 6 and 2/3 hours
3. 7 and 1/2 hours
4. 6 hours

ACTIVITY 12. IRF Worksheet

Now that you have learned how rational algebraic equations are solved, using the "R" portion of the IRF Worksheet, answer the focus question: How can rate-related problems be solved?

IRF Worksheet

Initial Answer
Revised Answer

Final Answer

END OF FIRM UP:



In this section, the discussion focused on ways in which rational algebraic expressions could be used to solve rate related problems.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

Now that you know the important ideas about this topic, let us go deeper by moving on to the next section.



DEEPEN



Now that you have learned the important skills needed to understand rational equations, your goal in this section is to look at some real-life situations where we can apply the concepts that you have learned.

In the next activities, you will now be asked to solve real life problems that make use of rational equations.

ACTIVITY 13. ALGEBRA MOVES (The Algebra in Physics)

In this activity, you will learn how to make connections between algebra and physics by studying how rational algebraic equations could be used to solve motion problems. Click the link to learn more about Motion problems:
<http://www.youtube.com/watch?v=737j3L-CX6s>

From the video that you watched, study the problem below:

The first leg of Mary's road trip consisted of 120 miles of traffic. When the traffic cleared she was able to drive twice as fast for 300 miles. If the total trip took 9 hours how long was she stuck in traffic?

SOLUTION:

Let x = Mary's speed in traffic

$2x$ = speed after traffic

	D	R	T
First leg	120	x	$120/x$
Second leg	300	$2x$	$300/2x$
			9 hours

$$\frac{120}{x} + \frac{300}{2x} = 9$$

$$240 + 300 = 18x$$

$$540 = 18x$$

$$30 = x$$

$$60 = 2x$$

$$\frac{120}{x} = \frac{120}{30}$$

She was in traffic for four hours.

Now, try solving the problem below:

1. A passenger train can travel 20mph faster than a freight train. If the passenger train can cover 390 miles in the same time it takes the freight train to cover 270 miles, how fast is each train?

2. Billy rode his skateboard 24 miles to his grandmother's house for the day. It was a rough ride so he borrowed his grandmother's bicycle for the return trip. Going twice as fast on the bicycle the return trip took 2 hours less time. What was his average speed on the bicycle?

Answer the following questions below:

4. How did this particular Physics lesson make use of Algebra?

5. What algebraic processes did you use?

6. What insights did you find out from doing this activity?

ACTIVITY 14. The ALGEBRA in BUSINESS

In this activity, you will learn how to make connections between algebra and business by studying rational algebraic equations could be used to problems related to business. Click on this link: http://www.youtube.com/watch?v=wV4JKv1_9iM

1. How was the rational equation formed?

2. How were rational algebraic equations used to solve problems related to business?

3. What new insights did you learn?

END OF DEEPEN:



In this section, the discussion was about using the concepts of rational algebraic expressions to be able to solve problems related to Physics and Business.

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.



TRANSFER



Our goal in this section is apply your learning to real life situations. You will be given a practical task which will demonstrate your understanding.

ACTIVITY 15. MANPOWER PLAN

Perform the activity in preparation for your final output in this module.

TASK

You are the foreman of a building construction. The project engineer assigned you to determine the needed number of workers for each phase of the construction. You know that if 10 skilled men and 16 unskilled men work together, they can complete a phase of the project in 10 days. The same job can be done by 30 skilled men in 8 days. The project has 9 phases. The daily wage of an unskilled worker is Php 260 while a skilled worker is Php 350. The completion of the project is 85 days. You will present the plan to the project engineer. The decision will be based on mathematical reasoning, accuracy, presentation, practicality and efficiency.

CRITERIA	Outstanding 4	Satisfactory 3	Developing 2	Beginning 1
Mathematical reasoning	Explanation shows thorough reasoning and insightful justifications.	Explanation shows substantial reasoning.	Explanation shows gaps in reasoning.	Explanation shows illogical reasoning.
Accuracy	All computations are correct and shown in detail.	All computations are correct.	Most of the computations are correct. There are some minor errors.	The computations show serious error.
Presentation	The presentation uses appropriate and creative visual materials. It is delivered in a convincing manner.	The presentation uses appropriate visual materials. It is delivered in a clear manner.	The presentation uses some visual materials. It is delivered in a disorganized manner.	The presentation does not use any visual materials. It is delivered in a haphazard manner.
Practicality	The proposed project will be completed ahead of time.	The proposed project will be completed on time.	The proposed project will be completed one week after the deadline.	The proposed project will be completed more than a week after the deadline.
Efficiency	The cost of the project is minimal.	The cost of the project is reasonable.	The cost of the project is expensive.	The cost of the project is unrealistic.

Questions to Answer:

1. Did you experience any difficulty in making the manpower plan? If so, how did you overcome those difficulties?

2. How did you apply the concept of rational algebraic expressions in making the plan?

3. What realizations about planning in business did you gain from this activity?

ACTIVITY 16. Performance Task

After learning rational algebraic equations and its varied applications to real life situations, it is now time to showcase your learning in this module. You will assume the role of a project engineer who will design a manpower plan for a construction company.

TASK

AEA Construction Firm has just acquired a 23 – km road construction project. The completion of the project is 1 year. The project manager assigned you, being the project engineers, to prepare a manpower plan for the project. You are to present the manpower plan to the Project Team for approval. The decision of the team will be based on mathematical reasoning, accuracy, presentation, practicality and efficiency.

Use the rubric below as your guide in completing your final project:

CRITERIA	Outstanding 4	Satisfactory 3	Developing 2	Beginning 1
Mathematical reasoning	Explanation shows thorough reasoning and insightful justifications.	Explanation shows substantial reasoning.	Explanation shows gaps in reasoning.	Explanation shows illogical reasoning.
Accuracy	All computations are correct and shown in detail.	All computations are correct.	Most of the computations are correct. There are some minor errors.	The computations show serious error.
Presentation	The presentation uses appropriate and creative visual materials. It is delivered in a very convincing manner.	The presentation uses appropriate visual materials. It is delivered in a clear manner.	The presentation uses some visual materials. It is delivered in a disorganized manner.	The presentation does not use any visual materials. It is delivered in a vague manner.
Practicality	The proposed project will be completed ahead of time.	The proposed project will be completed on time.	The proposed project will be completed one	The proposed project will be completed more than a week

			week after the deadline.	after the deadline.
Efficiency	The cost of the project is minimal.	The cost of the project is reasonable.	The cost of the project is expensive.	The cost of the project is unrealistic.

Questions to Answer:

1. How did you find the performance task?

2. How did the task help you see the real world use of the topic?

ACTIVITY 17. IRF Worksheet

Now that you have learned completed doing the various activities in the module, using the “F” portion of the IRF Worksheet, answer the essential question:
How can rate-related problems be solved?

IRF Worksheet

Initial Answer
Revised Answer
Final Answer

Now, that you have completed your final project, let us now complete the synthesis journal found below.

ACTIVITY 18. Synthesis Journal

The unit's lesson was on _____.
One key idea was _____. This is important because _____.
Another key idea was _____.
This is also important because _____. In summary, the unit's lesson _____.

End of TRANSFER:



You have completed this lesson. Before you go to the next module, you have to answer the following post-assessment.

POST-ASSESSMENT:

1. It's now time to evaluate your learning. Click on the letter of the answer that you think best answers the question. Your score will only appear after you answer all items. If you do well, you may move on to the next module. If your score is not at the expected level, you have to go back and take the module again.

2.1. _____ The following are rational expressions except _____.

- A. $m^{\frac{1}{2}}$
- B. $\sqrt{4} mn$
- C. $\frac{2d}{5}$
- D. $\frac{2x+6}{x+1}$

3.2. _____ The simplified form of $\frac{4k^{-2}p^{-3}}{(8kp^3)^0}$ is _____.

- A. $\frac{1}{4k^2p^3}$
- B. $\frac{1}{2k^2p^3}$
- C. $\frac{4}{k^2p^3}$
- D. $\frac{4}{k^5p^6}$

4.3. _____ The average amount of a customer's purchase P applied to a credit card account at a toy store during the period 1998 – 2004 can be modeled by $P = \frac{2400+80x^2}{50+2x^2}$, where x is the number of years since 1998. Which expression shows how to compute the average amount of a customer's purchase in 2003?

- A. $\frac{2400+80(5)^2}{50+2(5)^2} = \frac{2400+80(10)}{50+2(10)} = 45.71$
- B. $\frac{2400+80(5)^2}{50+2(5)^2} = \frac{2400+(400)^2}{50+10^2} = 26.67$
- C. $\frac{2400+80(5)^2}{50+2(5)^2} = \frac{2400+80(25)}{50+2(25)} = 44$
- D. $\frac{2400+80(5)^2}{50+2(5)^2} = \frac{2400+80(25)}{50+2(25)} = 3.38$

5.4. _____ Dan simplified $\frac{2(x-5)}{(x-5)(x+2)}$ this way:

$$\frac{2(x-5)}{(x-5)(x+2)} = \frac{2(x-5)}{(x-5)(x+2)} = \frac{2}{x+2} = \frac{1}{x+1}$$

Clarisse said the way Dan simplified the rational algebraic expression is incorrect. What part of the solution of Dan is incorrect?

- A. The final answer should be $\frac{1}{x}$ since $\frac{\cancel{1}}{x+\cancel{1}} = \frac{1}{x}$.
- B. The final answer should be x since $\frac{\cancel{1}}{x+\cancel{1}} = x$.
- C. In $\frac{2}{x+2}$, it is incorrect to cancel 2 in the numerator and denominator since both are not factors. Thus, the final answer is $\frac{2}{x+2}$.
- D. There is nothing wrong with the way Dan simplified the rational algebraic expression.

6-5. What is the width of a rectangle if its area is $x^2 - 3x - 10$ and its length is $x - 2$?

- A. $(x + 5)(x - 2)$
- B. $\frac{(x+5)(x-2)}{x-2}$
- C. $\frac{x+5}{x-2}$
- D. $x + 5$

7-6. When asked to determine the excluded values in $\frac{2m^2 - m - 3}{2m^2 - 11m + 12}$,

Joseph presented this solution:

$$\frac{2m^2 - m - 3}{2m^2 - 11m + 12} = \frac{(m+1)(2m-3)}{(2m-3)(m-4)} = \frac{(m+1)(2m-3)}{(2m-3)(m-4)} = \frac{m+1}{m-4}$$

Since $m - 4 = 0$ when $m = 4$, then the excluded value is 4.

What is wrong with the way the excluded value were determined?

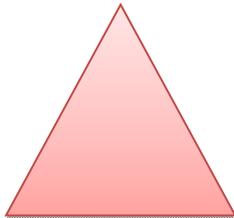
- A. There is nothing wrong with his solution. If 4 is substituted in the denominator, it becomes zero and will make the rational algebraic expression undefined.
- B. There is nothing wrong with his solution. But there are two excluded values, 4 and -4 .
- C. Joseph should have identified only the values that will make the denominator equal to zero.
- D. Joseph should only factor the denominator and equate it to zero. From $(2m - 3)(m - 4) = 0$, the excluded values are $\frac{3}{2}$ and 4.
7. What is the difference of $\frac{2w+5}{w-1}$ and $\frac{2w-4}{w-1}$?
- A. $\frac{4w+1}{w-1}$
- B. $\frac{-9}{w-1}$
- C. $\frac{9}{w-1}$

D. $w - 1$

8. When $\frac{m^2+4m-12}{2m^2+15m+18}$ is divided by $\frac{m^2-4}{8m+12}$ what will be the quotient?

- A. $\frac{1}{m}$
- B. $\frac{m}{2}$
- C. $\frac{m_4}{m+2}$
- D. $\frac{4(m-2)}{m+2}$

9. The triangle below has a height that measures $\frac{2m + 14}{m+3}$ cm and a base of $\frac{3m+9}{m+7}$ cm. What is the area of 20 similar triangles with the one below?



- A. 3 cm^2
- B. 30 cm^2
- C. 40 cm^2
- D. 60 cm^2

10. If $M = \frac{-2}{x-4}$ and $N = \frac{3x-9}{x^2-7x+12}$, what is $M + N$.

- A. $\frac{1}{x-3}$
- B. $\frac{1}{x-4}$
- C. $x - 3$
- D. $x - 4$

11. What is the simplified form of $\frac{\frac{2ab^2}{3}}{\frac{4a^2b^2}{6}}$?

- A. a
- B. $\frac{1}{a}$
- C. $\frac{ab}{4}$
- D. $\frac{4ab}{4ab}$

12. If $R = \frac{2}{b-2}$; $S = 2$; and $T = \frac{2}{b-2}$, then what is the equivalent value of $\frac{R}{S-T}$?
- 1
 - 3
 - $\frac{1}{b-3}$
 - $\frac{2}{b-3}$
13. What is the solution of the equation $\frac{5}{y} - \frac{1}{4} = \frac{3}{y}$?
- 6
 - 7
 - 8
 - 9
14. What are all the solutions of the equation $\frac{5}{m-2} = \frac{m}{3}$?
- 3, 5
 - 3, 5
 - 3, - 5
 - 3, -5
15. If the same number is added to both the numerator and denominator of the fraction $\frac{2}{7}$, the result is $\frac{6}{7}$. Find the number.
- 27
 - 28
 - 29
 - 30
16. Lorie takes 3 hours to plant 36 flower bulbs. Corrie takes 4 hours to plant 60 flower bulbs. Working together, how long should it take them to plant 162 bulbs?
- 3 hours
 - 4 hours
 - 5 hours
 - 6 hours
17. Jun and Jerry are planning to paint a house together. Jun thinks that if he worked alone, it would take him 2 times as long as it would take Jerry to paint the entire house. Working together, they can complete the job in 20 hours. How long would it take each of them, working alone, to complete the job?
- Jun = 24 hours; Jerry = 50 hours
 - Jun = 24 hours; Jerry = 60 hours

- C. Jun = 30 hours; Jerry = 50 hours
- D. Jun = 30 hours; Jerry = 60 hours

18. A water tank has two outlet pipes. When the first pipe is left open, it takes 4 hours to drain the tank while if only the second pipe is open, it takes 5 hours to drain. How long should it take to drain the tank if both pipes are left open simultaneously?

- A. 1 hour
- B. 2 hours
- C. 2.2 hours
- D. 2.6 hours

19. The formula $1/t = 1/A + 1/B$ gives the total time (t) for some job to be done by workers (A and B). If the total time necessary for worker A and worker B to do a particular job is 8 hours, and the individual time for worker A is 12 hours, what is the time for worker B?

- A. 16 hrs
- B. 20 hrs
- C. 24 hrs
- D. 28 hrs

20. With spraying equipment, Carl can paint the woodwork in a small house in 6 hr. His assistant, Dom, need 10 hour to complete the same painting job. If Carl and Dom work together, how long will it take them to paint the woodwork?

- A. 3 hours
- B. 3.5 hours
- C. 3.75 hours
- D. 4 hours

GLOSSARY OF TERMS USED IN THIS LESSON: (List in alphabetical order the key terms and give their definitions.)

EXTRANEIOUS ROOT – being a number obtained in solving an equation that is not a solution of the equation

RATIONAL EQUATION – is an equation containing at least one rational expression

REFERENCES AND WEBSITE LINKS USED IN THIS LESSON:

<http://www.virtualnerd.com/algebra-1/rational-expressions-functions/solve/example-solutions/equation-definition>
Examples of solving rational equations

<http://www.ssag.sk/SSAG%20study/MAT/Rational%20equations.pdf>
an article about rational equations

<http://www.kutasoftware.com/FreeWorksheets/Alg2Worksheets/Solving%20Rational%20Equations.pdf>
a worksheet about rational equations

<http://www.youtube.com/watch?v=QLhvLEeS08A>
a video about solving rationale quations

<http://www.tes.co.uk/teaching-resource/Rate-Problem-Involving-Rational-Equations-6198692/>
an article on how to solve rate-related problems

<http://www.youtube.com/watch?v=737j3L-CX6s>
Algebra applied in Physics

http://www.youtube.com/watch?v=wV4JKv1_9iM
Algebra applied in Business

Lesson 3: Linear Equations in Two Variables

☑ INTRODUCTION AND FOCUS QUESTION(S):



Have you ever wondered how businessmen nowadays make decisions? What guides businessmen in making these decisions? What are the tools that they use? How do they know when to increase or decrease production?

Have you ever wondered how fast things such as cars and trains can go? Have you ever thought about how their speeds are calculated? When a police officer gives someone a speeding ticket, how do they know for sure if the person was speeding?

In this module, you will discover how important it is to utilize essential mathematical skills to be able to understand these questions that arise in various real-life situations that we encounter everyday and use these skills wisely to be able to come up with the desired output.

As you go through this module, think of this question: **How can problems regarding trends and options be properly solved?**

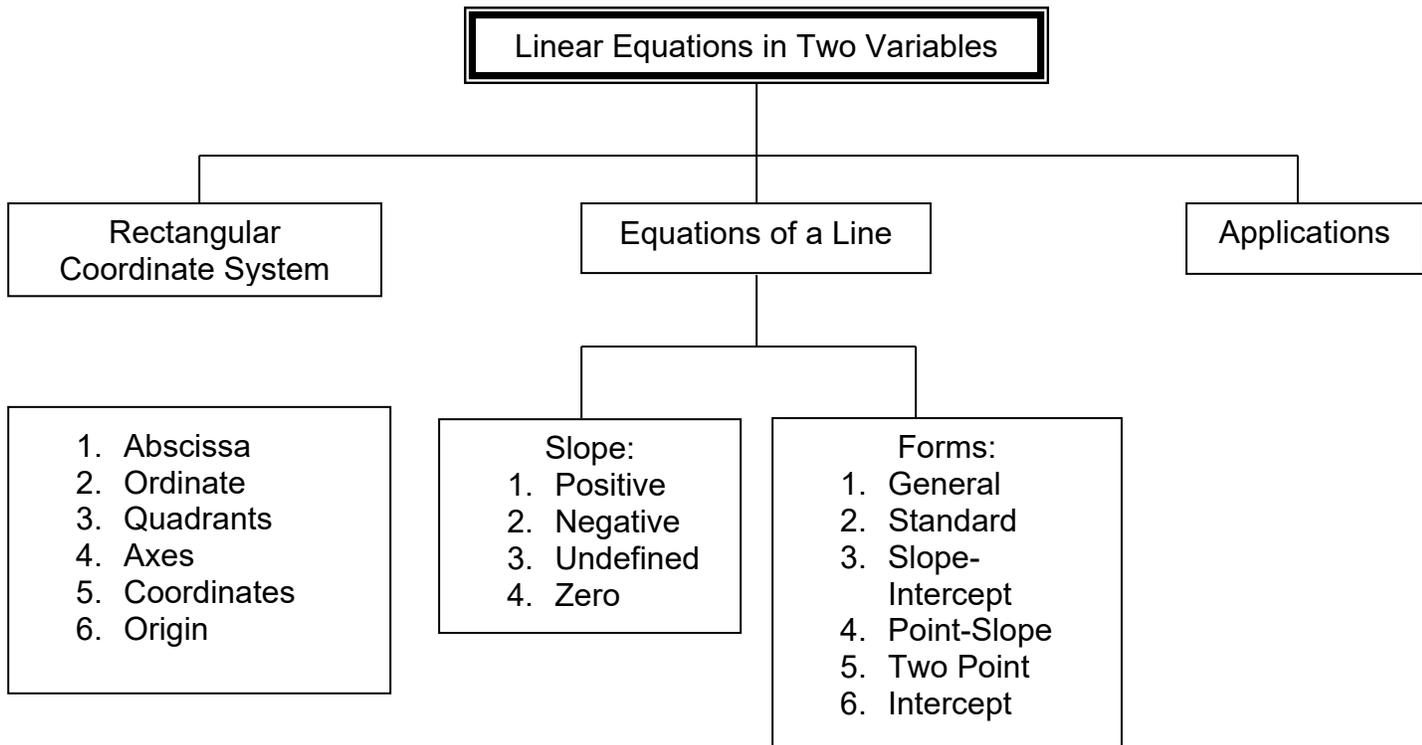
LESSON COVERAGE:

In this lesson, you will learn the following:

- Linear Equations in Two Variables**
- illustrates the rectangular coordinate system and its uses
 - illustrates linear equations in two variables
 - illustrates the slope of a line
 - finds the slope of a line given two points, an equation or a graph
 - finds a slope of lines given two points, equation and a graph
 - writes the linear equation $ax + by = c$ in the form $y = mx + b$
 - graphs a linear equations given (a) any two points, (b) the x and y intercepts and (c) the slope and a points on the line
 - describes the graph of a linear equation in terms of its intercepts and slope
 - finds the equation of a line given (a) two points, (b) a slope and a point, and (c) the slopes and its intercepts
 - solves problems involving linear equations in two variables

MODULE MAP:

Here is a simple map of the above lessons you will cover:



☑ **EXPECTED SKILLS:**

To do well in this module, you need to remember and do the following:

4. follow the directions carefully
5. master the prerequisite skill of solving linear equations in one variable
6. solve with speed and accuracy

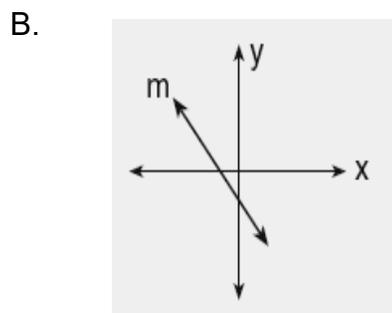
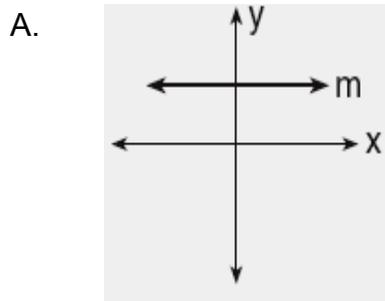
PRE-ASSESSMENT



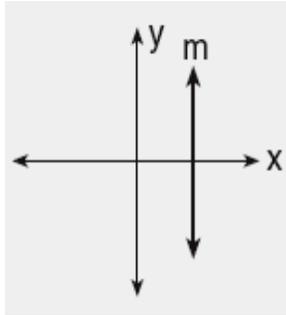
Let's find out how much you already know about this module. Click on the letter that you think best answers the question. Please answer all items. After taking this short test, you will see your score. Take note of the items that you were not able to correctly answer and look for the right answer as you go through this module.

1. Each point on a Cartesian plane is assigned a pair of coordinates called _____.
 - A. domain
 - B. ordinate
 - C. origin
 - D. ordered pair

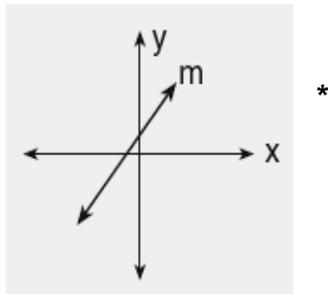
2. In which graph does line m have a positive slope?



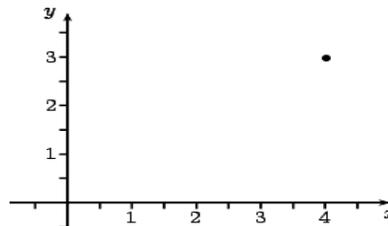
C.



D.



3. Which is the coordinate of the plotted point below?



- A. (4, 3)
 - B. (3, 4)
 - C. (4, 4)
 - D. (3, 3)
4. The graph of the line $x + 3y = 6$ intersects the y-axis at the point whose coordinates are _____.
- A. (0, 6)
 - B. (0, 2)
 - C. (6, 0)
 - D. (18, 0)

5. Which is an equation for the line that passes through the coordinates (2, 0) and (0, 3)?

A. $y = -\frac{3}{2}x - 3$

B. $y = -\frac{3}{2}x + 3$

C. $y = -\frac{2}{3}x - 2$

D. $y = -\frac{2}{3}x + 3$

6. The line $3x - 2y = 12$ has

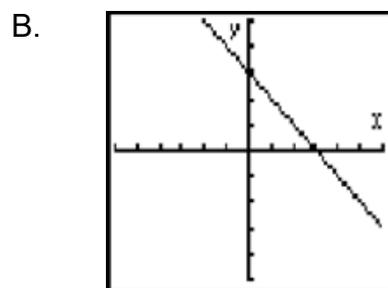
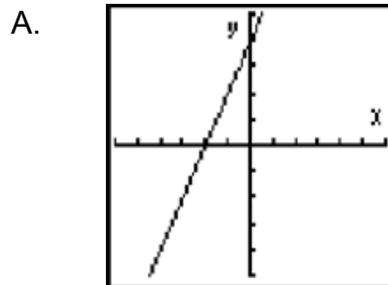
A. a slope of 3 and a y-intercept of -2.

B. a slope of $-\frac{3}{2}$ and a y-intercept of 6.

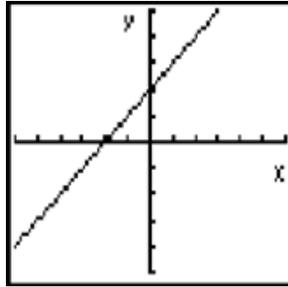
C. a slope of $\frac{3}{2}$ and a y-intercept of -6.

D. a slope of -3 and a y-intercept of -6.

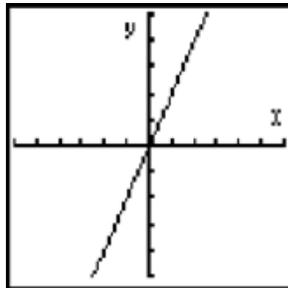
7. Which graph shows the line $y = 2x + 4$?



C.



D.

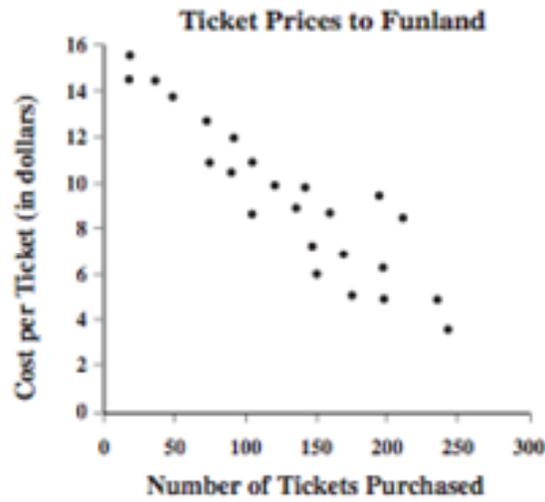


8. Given the equation $2x - 2y = 5$, complete table the table below.

x	-1	0	1
y		$-\frac{5}{2}$	

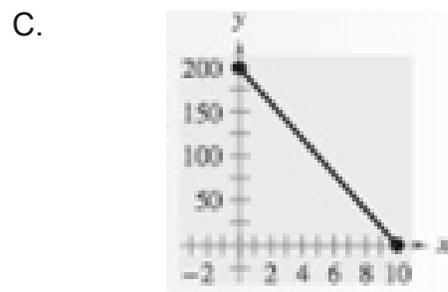
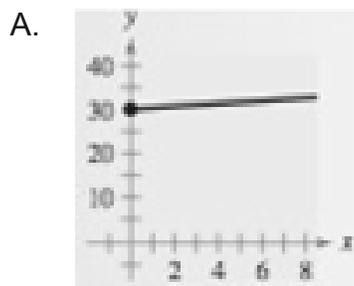
- A. $-\frac{7}{2}$ and $-\frac{3}{2}$
- B. $\frac{7}{2}$ and $-\frac{3}{2}$
- C. $-\frac{7}{2}$ and $\frac{3}{2}$
- D. $\frac{7}{2}$ and $\frac{3}{2}$
9. Mang Pedro sells lechon manok. The following linear equation describes the profit P he makes when he sells x orders of lechon manok: $P = 30x - 140$ where P is in pesos. How many orders of lechon manok should Mang Pedro sell in a day so he can earn a profit of P400?
- A. 10
- B. 13
- C. 16
- D. 18

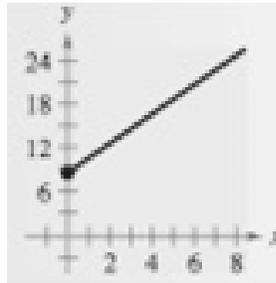
Use the graph below to answer number 10.



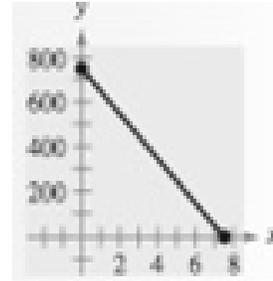
10. The cost of a ticket to Funland varies according to the season. Which of the following conclusions about the number of tickets purchased and the cost per ticket is BEST supported by the graph above?
- The cost per ticket increases as the number of tickets purchased increases.
 - The cost per ticket is unchanged as the number of tickets purchased increases.
 - The cost per ticket decreases as the number of tickets purchased increases.
 - There is no relationship between the cost per ticket and the number of tickets purchased.
11. Which graph is represented by the situation below?

A person is paying \$20 per week to a friend to repay a \$200 loan.



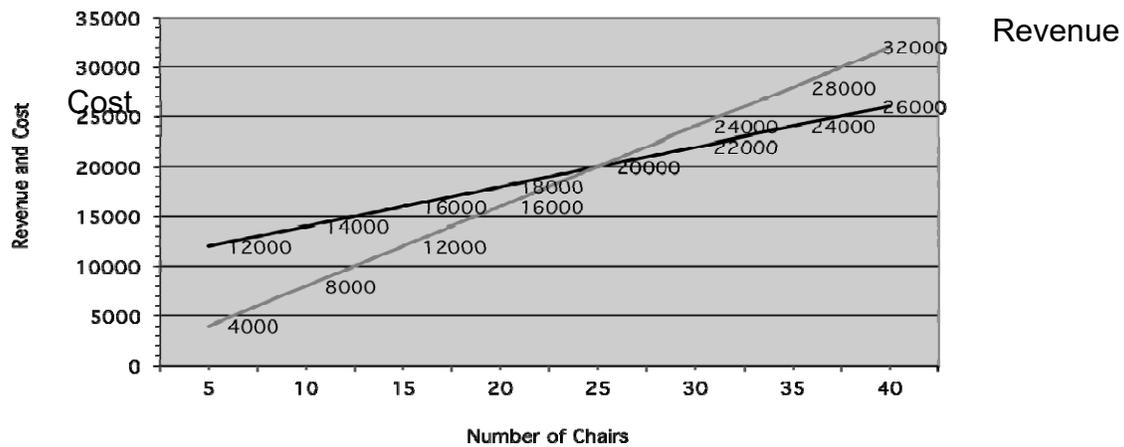


B. D.



Use the situation below to answer number 12 to 14.

Your father intends to launch a new line of lounging chairs. The graph below shows the cost to make the lounging chairs and the revenue he will receive from the sale.



12. Which equation corresponds to the cost of producing the lounging chairs?
 - A. $y = 10000 + 400x$
 - B. $y = 12000 + 2000x$
 - C. $y = 10000 - 2000x$
 - D. $y = 12000 - 200x$

13. Your father intends to supply 25 lounging chairs on the first month of operations. How much profit or loss should he expect?
 - A. He should expect a loss of P10 000 because producing 25 chairs will not allow him to recover his initial expenses.
 - B. He should expect a profit of P10 000 because producing 25 chairs will allow him to earn P20 000 less the initial expense of P10 000.

- C. He should expect a loss of P2 000 because producing 25 chairs will cost 18 000 but revenue is only P2 000.
 - D. He should not expect to gain any profit because producing 25 chairs will make the revenue equal to the cost.
14. Considering the trend, how much profit or loss should your father expect if he wishes to produce 100 lounging chairs in three months?
- A. a loss of P100 000
 - B. a profit of P30 000
 - C. a profit of 80 000
 - D. a loss of P50 000

Use the situation below to answer numbers 15 and 16.

You are helping to plan an awards banquet for your school. You need to rent tables to seat 180 people. Tables come in two sizes. Small tables seat four people and large tables seat six people. This situation can be modeled by $4x + 6y = 180$ where x is the number of small tables and y is the number of large tables.

15. What is the maximum number of large tables that you could rent?
- A. 20
 - B. 25
 - C. 30
 - D. 35
16. If at the last minute the number of attendees increase by 20, which possibilities for the number of each size table could you rent?
- A. (50, 0), (20, 20), (15, 23)
 - B. (23, 18), (8, 28), (12, 25)
 - C. (3, 12), (5, 15), (6, 20)
 - D. (2, 32), (5, 30), (20, 20)

Use the situation below to answer numbers 17 and 18.

Some providers of high-speed Internet service offers a choice of two plans. With plan A, you buy the modem and pay a monthly fee for Internet service. With plan B, the modem is free, but you pay a higher monthly fee than for plan A.

- Plan A: You pay P2000 for the modem and P600 per month for service.
- Plan B: The modem is free and you pay P700 per month for the service.

17. After how many months are the total costs for the plans the same?
- 10
 - 15
 - 20
 - 25
18. A friend seeks advice from you which plan he should avail of. Which could be a good advice to your friend?
- Plan A is a better deal if the subscription will be beyond twenty months.
 - Plan A will never become a better deal over plan B in all cases.
 - Plan B is a better deal if the subscription will last for more than two years.
 - Either plans will yield the same benefit for as long as the subscription will be less than twenty months.
19. You and your friend each want to buy a video game that costs P1200. You have no money saved but plan to save P120 per week. The table below shows the money your friend plans to save. Who will be able to buy the video game first and why?

Weeks	Amount Saved
0	P 200
1	P 300
2	P 400
3	P 500
4	P 600

- Which statement is true about the given situation?
- Your friend will be able to purchase the video game first because his initial savings of P250 is greater compared to you because you have not saved any amount at all.
 - You will be able to purchase the video game first because your weekly savings is greater than your friend's weekly savings by 20%.
 - You will be able to purchase the video game at the same time because each of you will be able to reach the desired amount at the end of the tenth week.
 - The given information is lacking so it is difficult to determine who will be able to purchase the video game first.
20. The first earth day was observed on April 22, 1970. Since then, the week of April 20 has been earth week, a time for showing support for environmental causes. Fan's Café is offering a reduced refill rate for softdrinks during Earth week for anyone purchasing a fan's mug. The mug costs \$2.95 filled with 16 ounces of softdrink. The refill price is 50¢. A 16-ounce drink in a disposable cup costs \$0.85.

Due to the reduced rate, many customers avail of the offer, which is a sign that your promo is effective. Given the scenario, if you were the owner of Fan's Café, would you continue the offer for a year?

- A. Yes, having more customers would mean more sales.
- B. Yes, but the quality of the drink should be controlled so that costs will be minimized.
- C. No, promos do not usually last for more than a month.
- D. No, even though revenues will increase but not as much as if it were sold at a higher price.



EXPLORE

When you were in Grade 7, you were able to talk about linear equations in one variable. You found out that these concepts could be used to model and solve real-life situations. In this section, you will be able to discover a new model for linear equations. This time, it will have two variables. You will find out how this new model could be used to solve more real-life problems involving the analysis of trends and options. In this section, you will not only be introduced to the module but also be assessed on your initial ideas regarding the topic. Be reminded to take each activity seriously. Make a conscious effort to highlight the new concepts that you will be encountering.

ACTIVITY 1. BUSINESS MIRROR

Let us see if you can analyze the problem below and determine the answers to the questions that follow:

A t-shirt company has expenses of P250,000 for equipment, t-shirts cost P50 each and there is a silk-screening expense of P25 per shirt. The company receives an initial revenue of P5,000 and charges P250 per shirt on wholesale basis.

- a. Define your variables.

$x =$ _____

$y =$ _____

- b. What are the equations for the following?

Income = _____

Expenses = _____

- c. Complete the table below. Then, graph the result.

Number of t-shirts	Expense	Income	Profit

PROCESS QUESTIONS:

1. How did you find the activity?

2. Which part did you find the most challenging?

3. What mathematical concepts did you use to analyze and solve the problem?

4. When you plotted the values into a graph, what picture did you see?



The real-life situation that you just analyzed and solved shows you one of the many situations where we can encounter and use linear equations in two variables to solve real-life problems. As you go through all the activities in this module, think of this question: **How can problems regarding trends and options be properly solved?**

ACTIVITY 2. IRF Sheet

Let's begin by answering the "I" portion of the IRF Worksheet that you see below. Fill it up by writing your initial answer to the topical focus question:

IRF Worksheet
Initial Answer
Revised Answer

Final Answer



Well, those are your thoughts and ideas about our lesson. Let's start a new activity to further explore on the important key concepts about linear equations in two variables.

ACTIVITY 3. WINDSHIELD CHECK

This formative assessment will check on your own understanding as far as the focus question is concerned. Using the analogy of a windshield, you will now decide which of the following best describes what your understanding of the focus question.

How many bugs do you have on your windshield? What is making it hard to see clearly?

CLEAR = I get it! I thoroughly understand the concept.

BUGGY = I understand it for the most part, but a few things are still unclear.

MUDDY = I don't get it at all.

After making an assessment, write a short description for your choice and email your response to your teacher.

END OF EXPLORE:



You just tried finding out how linear equations in two variables can be used to model and solve real-life problems. It is now time to learn more about it. What you will learn in the next sections will also enable you to do the final project which involves looking at real-life situations that involve decision making and coming up with sound recommendations.

We will start by doing the next activity.



FIRM-UP



Your goal in this section is to learn and understand key concepts of linear equations in two variables. You will learn the important skills that are necessary in order to understand the succeeding lessons in this module. It is advised that for every new term that you encounter, be sure to define it in your own words aside from the definitions presented to you.

We will start by doing the next activity.

ACTIVITY 4. A GENIUS IS BORN!

This activity will allow you to get to know the life and works of Rene Descartes, the man behind the rectangular coordinate system.

Click on this link: <http://www.youtube.com/watch?v=3SNmdNO5Bb4>

After watching the video, can you now write down some important notes about him? Write your answers inside the box below.

ACTIVITY 5. WEBSTER ATTACK!

It is now time to unlock the meaning of the important terms that you will be using in this module. Visit www.dictionary.com and look for the meaning of the words below. Write your answers in the second column of the box below.

Terms	Meaning
Axis	
Quadrant	
Abscissa	
Ordinate	
Origin	

ACTIVITY 6. IT IS NO ORDINARY SYSTEM

Let us now look at the work on Rene Descartes by clicking this link: <http://www.youtube.com/watch?v=T2-TO8XBNbU>. While watching the video, try to connect the definitions of the terms you have researched about in the previous activity.

PROCESS QUESTIONS:

1. Differentiate a one dimensional from a two dimensional coordinate system.

2. How do you identify the quadrants?

3. What is an ordered pair?

4. What does it mean to plot the points?

ACTIVITY 7. WHERE IS THE POINT?

Let us try to test your understanding about how points are located on the Cartesian coordinate system. Answer the following questions below:

1. A point is located three units to the left of the origin and six units up. What are the coordinates of the point?

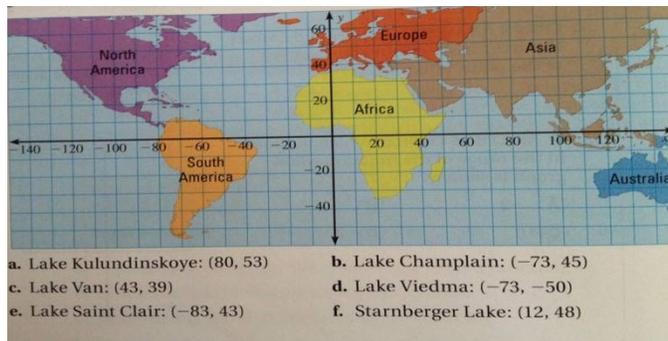
2. Point W (-6, 6) is six units to the left of the origin and six units up. Is the statement true or false?

- Plot the points $W(-4,-2)$, $X(-4,4)$, $Y(4,4)$ and $Z(4,-2)$ in a coordinate plane. Connect the points in order. Connect Z to W . Identify the resulting figure. Find its perimeter and area.

ACTIVITY 8. ASTRONAUT PHOTOGRAPHY

In this activity, you will be able to see how plotting and locating of points are useful in real life.

Astronauts use a coordinate system to describe the locations of objects they photograph from space. The x-axis is the equator, 0° latitude while the y-axis is the prime meridian, 0° longitude. The names and coordinates of some lakes photographed from space are given. Use the map to determine on which continent each lake is located.



ANSWERS:

- _____
- _____
- _____
- _____
- _____
- _____

ACTIVITY 9. MIND MAP

After finding out the different concepts which are important in understanding the rectangular coordinate system, put together the concepts you learned by making a concept map of the rectangular coordinate system. Go to this website: www.gliffy.com. Use the program to make a concept map. Send the file to your teacher afterwards.

ACTIVITY 10. 3-2-1 Chart

In this activity, you will be asked to complete the 3-2-1 chart on the rectangular coordinate system.

Rectangular Coordinate System	
<p>3 things you learned:</p>	<p>a. _____</p> <p>b. _____</p> <p>c. _____</p>
<p>2 things interesting things you found out:</p>	<p>a. _____</p> <p>b. _____</p>
<p>1 question you still have in mind:</p>	<p>a. _____</p>

ACTIVITY 11. LET IS PUT IT TO THE TEST

Let us find out how well you have mastered the rectangular coordinate system. Answer the quiz below.

KNOWLEDGE

Evaluate each statement. Write T if true and F if false. (5 points)

- ___ 1. A rectangular coordinate system is made up of two intersecting lines perpendicular to each other.
- ___ 2. In $P(-3, -5)$, -5 is called the abscissa.
- ___ 3. When the abscissa is 0, and the ordinate is not, the point lies on the x-axis.
- ___ 4. The x-axis is the vertical number line where the elements of x are found.
- ___ 5. In quadrant I, both the abscissa and ordinate are negative.

Identify the following terms described. (5 points)

- | | | |
|--|-----|---|
| | 6. | It is a pair of numbers, enclosed in parentheses and separated by a comma, and used to describe the location of a point in the plane. |
| | 7. | It is the other name for rectangular coordinate system. |
| | 8. | These are the four regions formed when the coordinate axes intersect. |
| | 9. | It is the other name for the y-coordinate. |
| | 10. | It is the point where the coordinate axes intersect. |

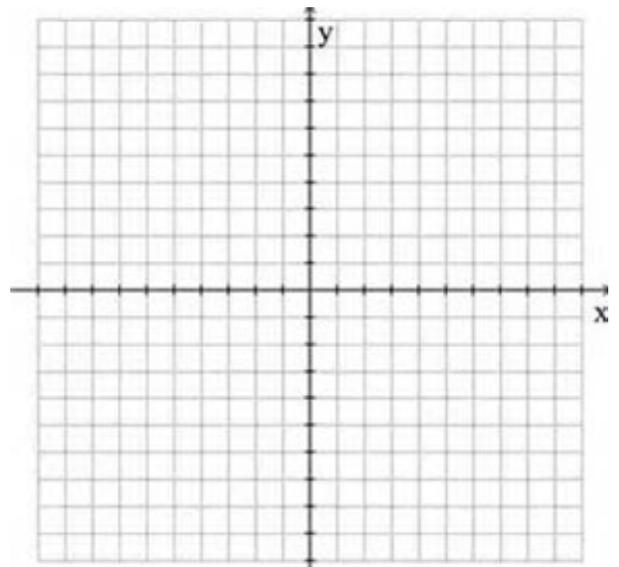
Without plotting the points, determine the quadrants where the given point is located. (5 points)

- ___ 11. (-5, -5)
- ___ 12. (-2, 6)
- ___ 13. (7, 3)
- ___ 14. (4, -5)
- ___ 15. (5, -3)

Plot the following ordered pairs on the Cartesian coordinate system and name the quadrant or axis where the point is located. Label properly. Two points each. (10 points)

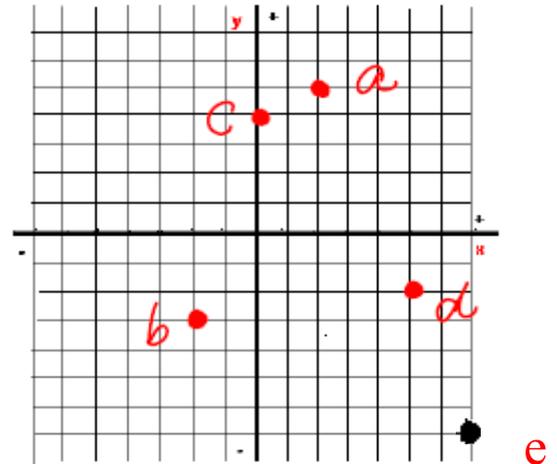
**Quadrant
or Axis**

- | | |
|--|----------------|
| | 16. A (3, 2) |
| | 17. B (-3, -1) |
| | 18. C (5, -7) |
| | 19. D (2, 0) |
| | 20. E (-5, 6) |



Give the coordinates of the following points. (5 points)

- _____ 21. a
- _____ 22. b
- _____ 23. c
- _____ 24. d
- _____ 25. e



PROCESS

Do the following below.

26. Suppose the point (a, b) lies in the fourth quadrant. Describe the location of the

following points below. (3 points)

$(b, a) =$ _____ $(2a, -2b) =$ _____ $(-b, -a) =$ _____

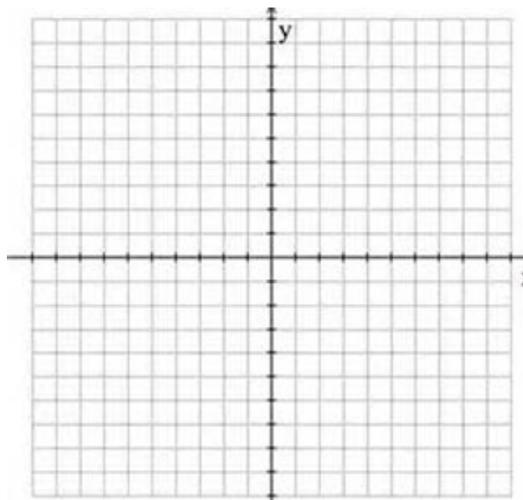
27. Plot the points $K(2, 1)$, $L(2, -1)$ and $M(5, -1)$ on the Cartesian plane below.

Then,

find a fourth point so that all four points will be vertices of a rectangle.

Sketch the

rectangle. (5 points)



28. How can you tell by looking at the coordinates of a point whether the point is on the x-axis? or y-axis? (4 points)

ACTIVITY 12. CONCEPT FORMATION

This activity will allow you to define a linear equation in two variables. Click on this link <http://schools.aqlasem.com/?p=38895> and read the article.

After reading the article, can you now paraphrase what you have read by coming up with your own definition of a linear equation in two variables? Write your definition inside the box below.

ACTIVITY 13. THE RIGHT OF SUFFRAGE

In this activity, you will discover how linear equations in two variables can be used to solve real life problems.

In 1920, the ratification of the 19th amendment to the United States Constitution gave women the right to vote. The table below shows the number (to the nearest million) of votes cast in the presidential elections both before and since women were able to vote.

Years before or since 1920	Votes (millions)
-12	15
-8	15
-4	19
0	27
4	29
8	37
12	40

PROCESS QUESTIONS:

1. Describe the trend in the number of votes cast.

2. When the points are connected, what figure is formed?

3. How can problems regarding trends and options be properly solved?

ACTIVITY 14. ARE LINES ALWAYS STRAIGHT?

In this activity, you will learn how to graph straight lines. Click on the link: <http://www.youtube.com/watch?v=F--060tUEk0>. Watch the video and answer the process questions below.

PROCESS QUESTIONS:

1. How do you identify a linear equation in two variables?

2. What is an intercept?

3. How many points can be used to draw a line?

4. How do you solve for x and y in a linear equation?

5. How do you graph straight lines?

ACTIVITY 15. PICTURE IT OUT!

In this activity, you will find out the relationship between the various kinds of linear equations and the graphs that they have. Complete the table below.

Forms of Linear Equations	Sample Equation	Graph	Characteristics of the Graph
	$3x - 6y = 12$		
	$3x = y$		
	$y - 4 = 0$		
	$x + 3 = 0$		

PROCESS QUESTIONS:

1. What forms of linear equations in two variables did you graph?

2. Describe the lines that you have drawn.

3. What generalizations can you make regarding the kind of equation that you have and the kind of graph that you get?

ACTIVITY 16. PICTURE PERFECT

After learning the various graphs of linear equations in two variables given some conditions, let us find out if you can graph a line satisfying the specified conditions:

a vertical line through the point (3, 4)	a line through (3, 4) and the origin
the x-coordinate of every point on this line is 4	for every point on the line, the y-coordinate is twice the x-coordinate

ACTIVITY 17. SELF-ASSESSMENT

In this activity, you will now gauge your own progress by completing the chart below. Mark the appropriate box that shows your progress.

Statements	I understand the concept and can apply accurately.	I understand the concept and can apply but makes errors in computation.	I understand the concept but not able to apply.	I do not understand.
<ul style="list-style-type: none"> I am able to write the variables of given linear equations. 				
<ul style="list-style-type: none"> I am able to graph linear equations in two variables correctly. 				
<ul style="list-style-type: none"> I am able to graph sketch the graph of a linear equation without using a table of values. 				

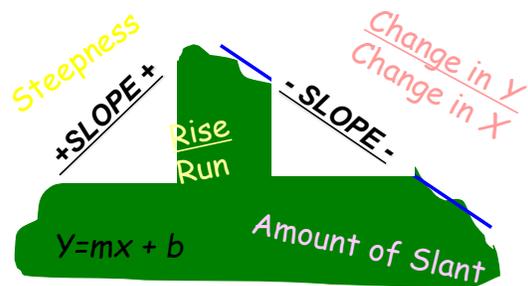
ACTIVITY 18. SLOPE

Now that you have learned how to graph straight lines, it is now time to look at another important property of straight lines which is the slope of a line. Study the Powerpoint presentations below and answer the process questions that follow.

7-1 Slope

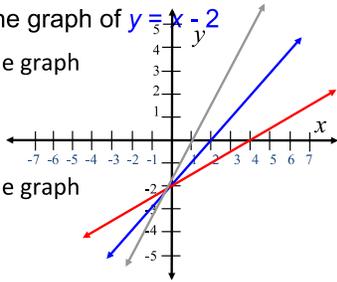
- Objectives: Find the slope of a line given the coordinates of two points on the line

What is Slope?



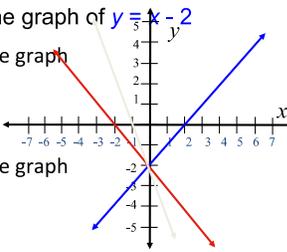
The Graph of $y = mx + b$

- Consider the graph of $y = x - 2$
- Compare to the graph of $y = \frac{1}{2}x - 2$
- Compare to the graph of $y = 2x - 2$

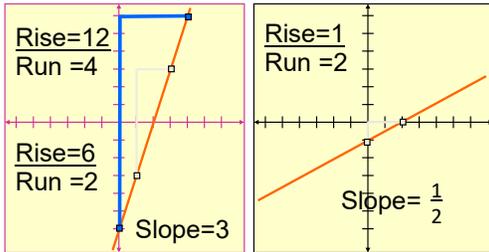


The Graph of $y = mx + b$

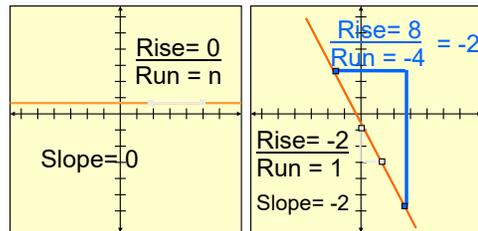
- Consider the graph of $y = x - 2$
- Compare to the graph of $y = -x - 2$
- Compare to the graph of $y = -2x - 2$



Determining Slope

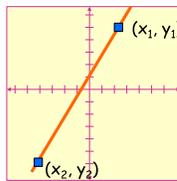


Determining Slope



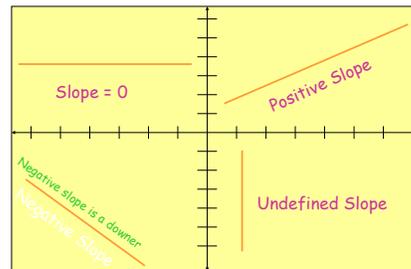
Determining Slope

In general, to find the slope given two points on a line:



- Subtract the Y-coordinates (rise)
- Subtract the X-coordinates (run)
- Write as a ratio ($\frac{\text{rise}}{\text{run}}$)

$$m = \frac{Y_2 - Y_1}{X_2 - X_1} = \frac{Y_1 - Y_2}{X_1 - X_2}$$



PROCESS QUESTIONS:

1. What is a slope?

2. How is the slope of a line determined?

3. What are the different values for the slope of a line?

4. What does it mean for the slope of a line to be undefined?

ACTIVITY 19. PRACTICE MAKES PERFECT

Let us see how far you have learned about the slope of a line. Visit this link and do some practice exercises on determining the slope of a line: http://www.quia.com/hm/163121.html?AP_rand=246985567.

PROCESS QUESTIONS:

1. How did you find the activity?

2. Which part did you find the most challenging?

3. How can you apply the concept of a slope in real life?

ACTIVITY 20. LET'S PRACTICE

This activity will allow you practice applying the knowledge and skills that you have learned regarding the slope of a line.

KNOWLEDGE

Fill in the blanks with the appropriate word/s or value/s. (5 points)

1. A slope is the ratio of rise to _____.
2. A _____ line has a slope that is undefined.
3. A line with a positive slope rises to the _____.
4. A line with a negative slope falls to the _____.
5. A horizontal line has a slope that is equal to _____.

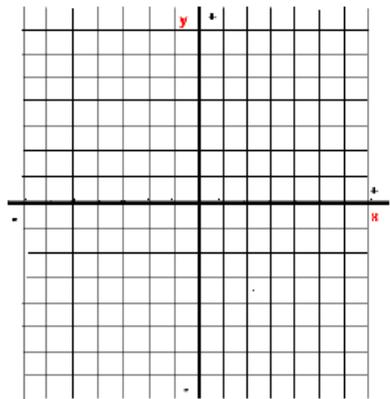
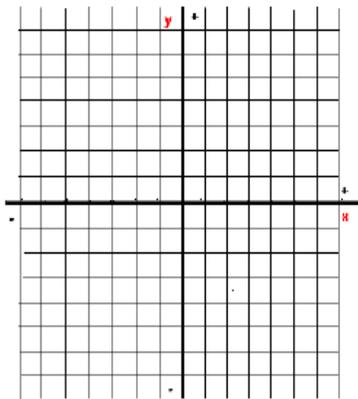
PROCESS

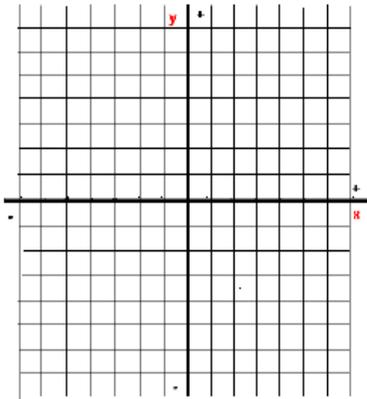
Solve for the slope of a line given two points. Two points each. (6 points)

6. (-2, -1) and (4, 5) 7. (-9, 1) and (1, 1) 8. (5, -3) and (5, 3)

Graph the line going through the given point and having the given slope. Two points each. (6 points)

9. (2, 5) and $m = 1/2$ 10. (0, 2) and $m = 7/4$ 11. (3, 5) and $m =$ undefined





UNDERSTANDING

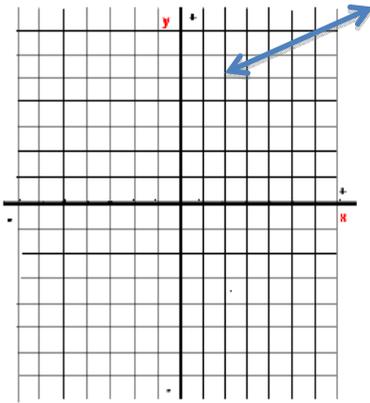
Read each situation below and answer that question/s that follow/s.

12. Describe and correct the error in calculating the slope of the line passing through the points (5, 3) and (2,6). (5 points)

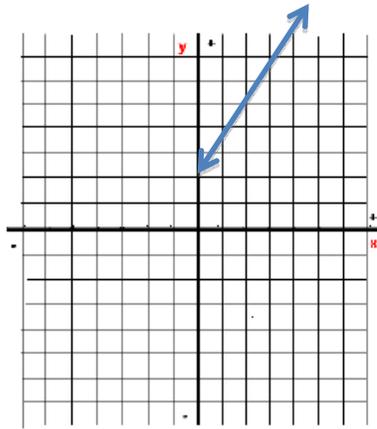
$$m = \frac{6-3}{5-2} = \frac{3}{3} = 1$$

13. The point (-1, 8) is on a line that has a slope of -3. Is the point (4, -7) on the same line? Explain your reasoning. (5 points)

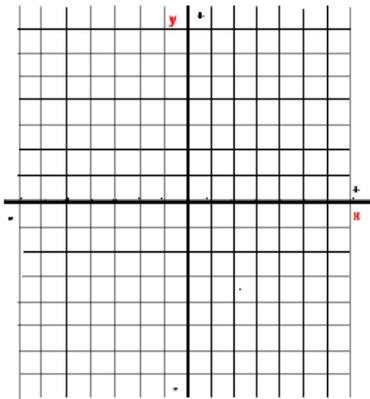
9.



10.



11.



12. $6 - 3/2 - 5 = 3/-3$ or -1

13. yes, if you continually add one to the x-coordinate and subtract 3 from the y-coordinate, one result will be $(4, -7)$

ACTIVITY 21. A VARIETY OF FORMS

After learning the different preliminary concepts on graphing straight lines, it is now time to learn the various equations of a line. You will be watching six videos on the various ways in which an equation of a line could be written.

As you watch each of these videos, pay attention to the following:

1. List down the steps involved in each method.

Forms	Description
General Form	

Standard Form	
Slope-Intercept Form	
Point-Slope Form	
Two-Point Form	
Intercept Form	

2. Highlight differences in the various forms.

You are not yet expected to master these forms. You are only expected to look at each form and make comparisons. Click on the different links below. You may key in your notes on the table provided below.

A. General Form

<http://www.youtube.com/watch?v=aLfZZ6guz88>

B. Standard Form:

<http://www.youtube.com/watch?v=AoJy7bsWKuc>

C. Slope-Intercept Form: <http://www.youtube.com/watch?v=3t7E8PTfey0>

D. Point-Slope

<http://www.youtube.com/watch?v=eHPTyYbNmx4>

E. Two-Point Form: <http://www.youtube.com/watch?v=4il4haYASys>

F. Intercept Form:

<http://www.youtube.com/watch?v=2mm7OfwIQkE>

PROCESS QUESTIONS:

1. What is involved in each form?

2. How is each form different from the rest?

3. What are the limitations of each form?

ACTIVITY 22. EVERNOTE

Go to www.evernote.com. Download the evernote program. You will be writing your notes on the forms of linear equations that you have viewed through this website. Take note that you will not be submitting the notes to the teacher for you will use these as you proceed with the other activities.

When you are writing your notes, pay attention to the following:

1. Describe each form.
2. Differentiate one from the other.
3. Discuss the limitations of each.

ACTIVITY 23. SKILL BOOSTER

Let us see whether you have understood the different ways to write the equation of a line. Click on each link below and do the activity.

- A. Slope-Intercept Form: <http://crtlessons.com/slope-intercept-game.html>
- B. Point-Slope Form: <http://www.quia.com/cb/79602.html>

ACTIVITY 24. MUDDIEST POINT

In this activity, you will complete the journal below.

The part of the lesson that I still find confusing is _____
because _____.

ACTIVITY 25. LET'S PUT IT TO THE TEST

This activity will test your understanding regarding the forms of linear equations in two variables.

Directions: Which of the following are linear equations in two variables? Write YES or NO.

- | | |
|-------|---------------------------------|
| _____ | 1. $5x + 4y = 3x - 6$ |
| _____ | 2. $x = 6xy - 1$ |
| _____ | 3. $3(x + 3) = 2y - 4x$ |
| _____ | 4. $2y + 4 = 3y^2$ |
| _____ | 5. $\frac{x-3}{4} + 5x = y + 4$ |

_____ 6. $2x = \frac{1}{3y}$

_____ 7. $\frac{2x}{5y} + 2 = 1$

_____ 8. $x^2 - 3x + 2 = 0$

Directions: Write each linear equation in general form and standard form.

9. $2x - 3y = 8x - 3x$
 GENERAL: _____
 STANDARD: _____

10. $6(x - y) = 4x + 5y$
 GENERAL: _____
 STANDARD: _____

11. $5x - 8 + 10y = 9 - 2y$
 GENERAL: _____
 STANDARD: _____

12. $4y - 5 + 5(x - y) = 2x$
 GENERAL: _____
 STANDARD: _____

Directions: Write each equation in slope intercept form. Identify the slope and the y-intercept.

13. $4x = 2y + 16$
 SLOPE: _____
 Y-INTERCEPT: _____

14. $10x + 14y = 28$
 SLOPE: _____
 Y-INTERCEPT: _____

15. $4x - 8y = 16$
 SLOPE: _____
 Y-INTERCEPT: _____

Directions: For each equation, find its slope and point on the graph.

16. $y + 7 = (x - 2)$
 SLOPE: _____
 POINT: _____

17. $y - 3 = -4(x + 5)$

SLOPE: _____
 POINT: _____

18. $y - 8 = (x - 6)$
 SLOPE: _____
 POINT: _____

19. $y + 2 = -3(x - 2)$
 SLOPE: _____
 POINT: _____

ACTIVITY 26. LEARNING LOG

After learning the preliminary concepts on linear equations in two variables, write a one paragraph essay on your progress in this module. What are the things that you need more assistance on?

End of FIRM UP:



In this section, the discussion was about linear equations in two variables and its forms. You were also given the opportunity to learn how some real life problems could be solved using the concept.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision? What new learning goal should you now try to achieve?



DEEPEN



Now that you have learned the important skills needed in solving quadratic equations, your goal in this section is to look at some real-life situations where we can apply the concepts that you have learned.

ACTIVITY 27. THE REAL LIFE PURPOSE (SCAFFOLD 1)

In this activity, you will view a video that will show real life applications of linear equations in two variables. Click on this link: <http://www.youtube.com/watch?v=8eXb-6wQUks>.

PROCESS QUESTIONS:

1. What real life was shown in the video?

2. How were linear equations in two variables used in solving real life problems?

3. Can you site any other real life situations that use linear equations in two variables?

4. How can problems regarding trends and options be properly solved?

ACTIVITY 28. YOU CAN DO IT (SCAFFOLD 2)

From the video presented, let us see if you can use your knowledge of linear equations to solve real life problems in business:

PROBLEM 1:

Mr. Garces, the owner of MIX, a newly built business in Bacolod that produces high quality printers, commissioned a Stat Company to conduct a survey on the demand of buyers for his products at different price levels. The Stat Company summarized the data gathered and reported the equation $Q_d = -0.5P + 800$.

Using the equation, complete the table of values below given the different price levels at which she could sell the printers.

Price	Quantity Demanded
0	
4000	
5000	
6000	
7000	
8000	

PROCESS QUESTIONS:

1. What concepts or skills did you use to complete the activity?

2. In the given equation, what is the slope? the y-intercept?

3. How did you arrive at the decision?

4. In business, why is it important to consider the demand of the buyers?

5. Will you be willing then to advise the businessman to set the price at zero to be able to gain more buyers?

6. Why is charging the highest price not favorable?

7. How can problems regarding trends and options be properly solved?

PROBLEM 2:

Mr. Soliman is looking for a job. After applying for the companies where his expertise can be applied and at the same time be improved, he finally has these two options to choose.

<p><i>Job A: Starting Salary: P3 000</i></p> <p><i>500</i></p> <p><i>Annual Increase: 500</i></p> <p><i>600</i></p>	<p><i>Job B: Starting Salary: 2</i></p> <p><i>Annual Increase:</i></p>
---	--

At the end of ten years, which job has the greater annual salary? Complete the table below.

Number of Years	Annual Salary (in pesos)	
	Job A	Job B
0	3 000	2 500
1		
2		
3		
10		

PROCESS QUESTIONS:

1. Where is the slope in the given problem?

2. What do the starting salaries represent?

3. Write an equation relating annual salary for each job.

4. If you were Mr. Soliman, which job would you choose? Why?

5. Is salary the only factor that will influence your choice? If not, what other factors would you consider?

6. How can problems regarding trends and options be properly solved?

PROBLEM 3:

You are the owner of Shape It. You charge your customers a membership fee of P900 per month after an initial membership fee. One of your customers paid a total of P6000 after six months. Write an equation that gives total cost as a function of the length of your gym membership (in months). Find the total cost after nine months.

PROCESS QUESTIONS:

1. What does P900 represent?

2. How will you use “six months” in your equation?

3. Write an equation relating the cost and the length of gym membership.

4. How much will a customer pay after nine months?

5. Can you use the equation to be able to make future decisions?

6. How can problems regarding trends and options be properly solved?

ACTIVITY 29. SUMMING EVERYTHING UP

You will now revisit the different situations that you have used to apply the concepts of linear equations in two variables. What are the common things that you notice among the various situations presented?

From the various real-life connections presented in the different activities, I noticed that the situations have the following things in common:

1. _____
2. _____
3. _____

What generalization can you make?

ACTIVITY 30. REVISITING THE IRF SHEET

It is now time to revisit the IRF Sheet. Fill it up by writing your revised answer to the topical focus question:

IRF Worksheet
Initial Answer
Revised Answer
Final Answer

End of DEEPEN:



In this section, the discussion was about using the concepts of quadratic equations to be able to solve real-life problems. Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.



TRANSFER



Your goal in this section is to apply your learning to real life situations by doing scaffold activities which will help you in the making of final project. You will be given a practical task which will demonstrate your understanding.

ACTIVITY 31. SCAFFOLD FOR TRANSFER 3

After looking at some real life applications of linear equations in two variables, it is now time to do the task below:

Foundation Week is fast approaching. One challenge that student leaders are facing is to come up with fund raising projects at a minimal cost. As members of the Executive Board of the Student Affairs Council, you are tasked to submit a proposal for an educational fund-raising project. It should include a description of the project, list of expenses and projected sales for two days. You are to come up with linear equations that you can use to analyze the potential profits of your business. Your work will be judged using the following criteria: explanation and presentation of the proposal, accuracy of computations, and demonstration of the understanding of key concepts of linear equations in two variables and systems of linear equations and inequalities in two variables.

ACTIVITY 32. THE FINAL TASK

Rogelio Loreda, who recently won in a lottery, is planning to open a business in your city. Since he lacks expertise, he is seeking assistance from RLB Accounting firm. As consultants of the firm, you are tasked to present a business plan which will include the varied salary schemes that could be used in paying the employees. Your work will be presented to the head of the accounting firm and to Rogelio Loreda. It will be judged according the following criteria: explanation and presentation and organization of the business plan, accuracy of computations, and demonstration of the understanding of key concepts of linear equations in two variables.

RUBRIC:

A. Oral Report (40%)

CRITERIA	Outstanding 4	Satisfactory 3	Developing 2	Beginning 1
Presentation (40%)	Presenters are confident and stimulating in delivery. They exert effort to engage the class. Preparation is evident. Presenters speak in clear voices.	Presenters are confident with their presentation. Presenters' voices are clear and audible.	Presenters show little evidence of confidence and preparedness. Presenters' voices are low.	Presenters demonstrate little evidence of planning prior to presentation. Presenters read from notes and/or make no eye contact.
Organization (30%)	The presentation shows logical and interesting sequence, is easy to follow and shows detailed evidence of preparation.	The presentation shows a logical sequence and shows evidence of preparation.	Parts of the presentation may still be resequenced to make the ideas clearer and easier to follow.	The presentation lacks logical sequence and has little evidence of preparation.
Visuals (30%)	Powerpoint presentation contains significant parts of the proposal and employ interesting visuals which make the content clear and easy to grasp. Slides are free of spelling and/or grammatical errors.	Powerpoint presentation contains significant parts of the proposal. Pictures are appropriate. Slides have less than two spelling and/or grammatical errors.	Powerpoint presentation contains adequate information. Pictures are presented but some of them are inappropriate. Slides have three or more spelling and/or grammatical errors.	Powerpoint presentation contains less important facts and inappropriate graphics. Slides have four or more spelling and/or grammar errors.

B. Written Report – 60%

CRITERIA	Outstanding 4	Satisfactory 3	Developing 2	Beginning 1
Mathematical Representation of Linear Equations in Two Variables (40%)	The linear models are detailed and clear.	The linear models are clear.	The linear models are difficult to understand but includes some critical components.	The linear models are difficult to understand and are missing several components.
Organization of Ideas (30%)	The report shows logical and interesting sequence, is easy to follow and shows detailed evidence of preparation.	The report shows a logical sequence and shows evidence of preparation.	Parts of the report may still be resequenced to make the ideas clearer and easier to follow.	The report lacks logical sequence and has little evidence of preparation.
Accuracy of Computations (30%)	The computations done are accurate and show understanding of the concepts of linear equations in two variables. There is an explanation for every computation made.	The computations done are accurate and show proper use of the concepts of linear equations in two variables.	The computations done are erroneous and show incorrect use of the concepts of linear equations in two variables.	The computations done are erroneous and do not show wise use of the concepts of linear equations in two variables.

PROCESS QUESTIONS:

1. What are the important factors that you consider important in contributing to the success of this task?

2. To what extent did your knowledge, skills and understanding of linear equations in two variables help you perform the task?

3. How can problems regarding trends and options be properly solved?

ACTIVITY 33. REVISITING THE IRF SHEET

It is now time to revisit the IRF Sheet. Fill it up by writing your revised answer to the topical focus question:

IRF Worksheet
Initial Answer
Revised Answer
Final Answer

ACTIVITY 34. REFLECTION LOG

After doing the final task, look back at your experience as you complete the log below:

1. What have I learned?

2. How do I feel with what I've learned?

3. What can I do with what I've learned?

Should you have any clarifications on this lesson, type your question and email this to the teacher or post it to the discussion forum.

ACTIVITY 35. SYNTHESIS JOURNAL

The unit's lesson was on _____.
 One key idea was _____.
 This is important because _____.
 Another key idea was _____.
 This is also important because _____.
 In summary, the unit's lesson _____

 _____.

In this section, your task was to look at how the skills needed to accomplish the performance task were done.

End of TRANSFER:



You have completed this lesson. Before you go to the next module, you have to answer the following post-assessment.

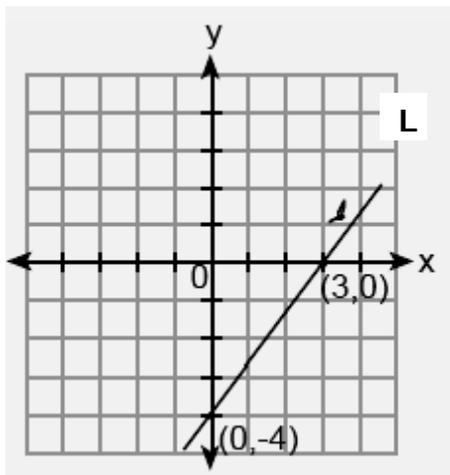
MODULE: POST-ASSESSMENT



It's now time to evaluate your learning. Click on the letter of the answer that you think best answers the question. Your score will only appear after you answer all items. If you do well, you may move on to the next module.

1. What is the intersection of the x-axis and y-axis in the coordinate plane?
 - A. origin
 - B. ordinate
 - C. abscissa
 - D. coordinate axes

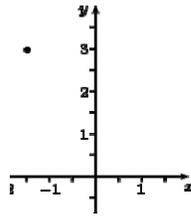
2. Which is the slope of line L as shown in the diagram below?



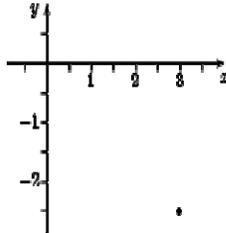
- A. $-\frac{3}{4}$
- B. $\frac{4}{3}$
- C. $\frac{3}{4}$
- D. $-\frac{4}{3}$

3. Which of the following plots the point $(-2.5, 3)$?

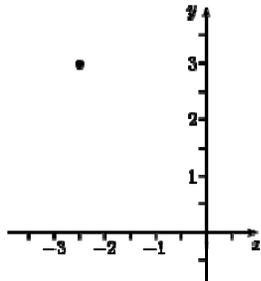
A.



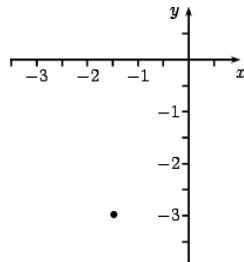
B.



C.



D.



4. What is the equation of a line whose x-intercept is 2 and whose y-intercept is $\frac{1}{3}$?

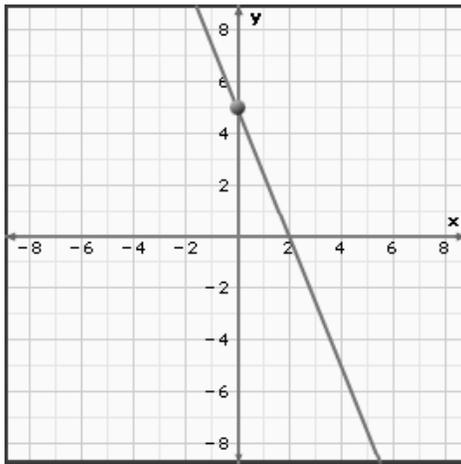
- A. $y = \frac{1}{2}x + \frac{1}{3}$
- B. $y = -\frac{1}{6}x + \frac{1}{3}$
- C. $y = -\frac{1}{2}x + \frac{1}{3}$
- D. $y = -\frac{1}{6}x - \frac{1}{3}$

5. Write the equation of the line containing $(4,0)$ and $(0,-5)$.

- A. $y = -\frac{4}{5}x$
- B. $5x - 4y = 20$
- C. $4x + 5y = 20^*$
- D. none of these

6. The line $2x - 3y = 12$ has
- a slope of 2 and a y-intercept of -12.
 - a slope of $-\frac{2}{3}$ and a y-intercept of 4.
 - a slope of $\frac{2}{3}$ and a y-intercept of -4.
 - a slope of -3 and a y-intercept of -3.

7. Which is the equation, in slope-intercept form, of the line graphed below?

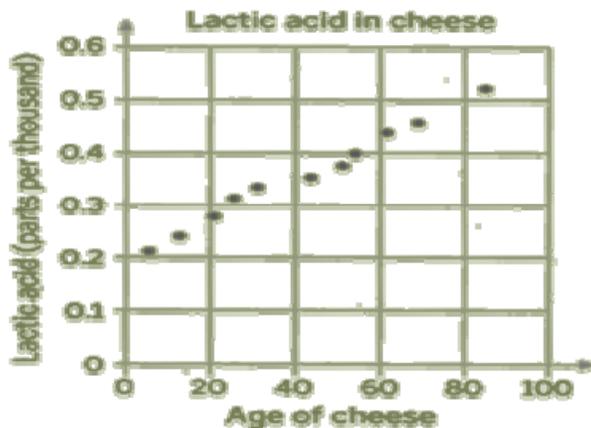


- $y = -2.5x + 2$
 - $y = -2x + 5$
 - $y = -5x + 2$
 - $y = -2.5x + 5$
8. Given the equation, $3x - y = 15$, complete table the table below.

X	1		3
Y		0	

- 10, 3, -5
 - 12, 2, -4
 - 5, 0, -5
 - 12, 5, -6
9. One side of a rectangular lot is four times longer than the other. Its perimeter fence is 70 meters long. Find the length and width of the lot.
- 7 meters wide and 28 meters long
 - 15 meters wide and 60 meters long
 - 28 meters wide and 7 meters long
 - 60 meters wide and 15 meters wide

10. The scatterplot on the right shows the time cheese has been aging and the amount of lactic acid present in the cheese. Which statement is *most* strongly supported by the scatterplot?

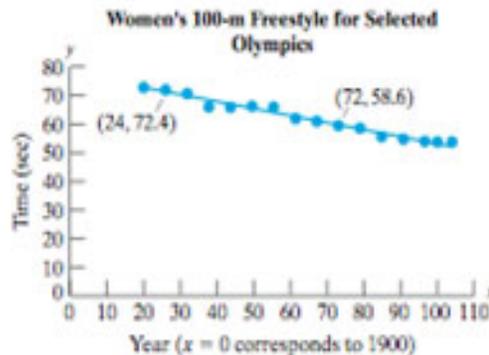


- A. The longer cheese ages, the more lactic acid is present.
 B. The longer cheese ages, the less lactic acid is present.
 C. The amount of lactic acid present remains constant as cheese ages.
 D. No relationship exists between the time cheese ages and the amount of lactic acid present.
11. The minimum hourly wage y (in dollars per hour) in the United States between 1960 and 2005 can be approximated by the equation: $y = 0.10x + 0.82$ where x is greater than or equal to 0 where x represents the number of years since 1960 ($x=0$ corresponds to 1960, $x=1$ corresponds to 1961, and so on).

Find the slope of the line and interpret the meaning of the slope in the context of this problem.

- A. The slope is 0.82. It means that the minimum hourly wage rose 0.82 dollars every year.
 B. The slope is 0.10. It means that in 1960, the hourly wage rose by 0.10 and doubled the succeeding years.
 C. The slope is 0.10 and indicates that minimum hourly wage rose an average of \$0.10 per year between 1960 and 2005.*
 D. The slope is 0.82 and indicates that the minimum hourly wage was 0.82 from 1960 to 2005.

12. The figure below represents the winning gold medal times for the women's 100-m freestyle swimming event for selected summer Olympics. Let y represent the winning time in seconds and let x represent the number of years since 1900 ($x = 0$ corresponds to 1900, $x = 1$ corresponds to 1901, and so on).



Would it be practical to use the linear model to predict the winning time in the year 2048?

- A. Yes, the graph shows a downward trend during the past years. Information is sufficient.
 - B. Yes, the trend has already been determined and the number of points is sufficient to make predictions.
 - C. No, data can change anytime. If the number of swimmers will increase, the trend can change.
 - D. No, if the trend will be followed the swimmers' time could be negative*.
13. Buffalo, New York, had 2 ft (24 in.) of snow on the ground before a snowstorm. During the storm, snow fell at an average rate of 5 in./hr. Write a linear equation to compute the total snow depth y after x hours of the storm.
- A. $y = 24x + 5/8$
 - B. $y = 5/8x + 24$
 - C. $y = 5/8x - 24$
 - D. $y = -5/8x + 24$
14. **Referring to the previous situation**, if the snow depth was 31.5 in. at the end of the storm, determine how long the storm lasted.
- A. 10 hours
 - B. 11 hours
 - C. 12 hours
 - D. 13 hours

15. A school has two soccer seasons. There are currently 30 students playing on the spring team and participation is increasing by two students per year. There are currently 19 students playing on the fall team, and this is increasing by three students per year. When will the teams have the same size?
- A. 12 years
 - B. 10 years
 - C. 8 years
 - D. 11 years

16. Some providers of high-speed Internet service offers a choice of two plans. With plan A, you buy the modem and pay a monthly fee for Internet service. With plan B, the modem is free, but you pay a higher monthly fee than for plan A.

Plan A: You pay P3000 for the modem and P500 per month for service.

Plan B: The modem is free and you pay P700 per month for the service.

After how many months are the total costs for the plans the same?

- A. 10
 - B. 15
 - C. 20
 - D. 25
17. A friend seeks advice from you which plan he should avail of. Which could be a good advice to your friend?
- A. Plan A is a better deal if the subscription will be beyond fifteen months.
 - B. Plan A will never become a better deal over plan B in all cases.
 - C. Plan B is a better deal if the subscription will last for more than three years.
 - D. Either plans will yield the same benefit for as long as the subscription will be less than fifteen months.

18. A college purchased exercise equipment worth \$12,000 for the new campus fitness center. The equipment has a useful life of 8 years. The salvage value at the end of 8 years is \$2000. Write a linear equation that describes the book value of the equipment each year.
- A. $V = -2000t + 12,000$
 - B. $V = -1250t + 12,000$
 - C. $V = -1250t - 12,000$
 - D. $V = 2000t + 12,000$

19. In a Physical Education class, the students start out with a grade of 100. For each day of absence from the class, a student gets a deduction of two points.

The equation that represents the grade y of a student who is absent for x days is _____.

- A. $y = 100 - x$
 - B. $y = 100 - 2x$
 - C. $y = 2(100 - x)$
 - D. $y = 2x - 100$
20. You are a sales representative and earns a base salary of \$1000 per month plus a 4% commission on your sales for the month. How much will you make if your sales for a given month is \$30,000?
- A. \$2000
 - B. \$2200
 - C. \$2400
 - D. \$2600

GLOSSARY OF TERMS USED IN THIS LESSON:

ABSCISSA - the coordinate representing the position of a point along a line perpendicular to the y -axis in a plane Cartesian coordinate system

INTERCEPT- the coordinate of a point at which a line, curve, or surface intersects a coordinate axis

LINEAR EQUATION - An algebraic equation, such as $y = 2x + 7$ or $3x + 2y - z = 4$, in which the highest degree term in the variable or variables is of the first degree. The graph of such an equation is a straight line if there are two variables

ORDERED PAIR - A pair of numbers used to locate a point on a coordinate plane

ORDINATE - The plane Cartesian coordinate representing the distance from a specified point to the x -axis, measured parallel to the y -axis

ORIGIN - the point whose coordinates are $(0,0)$ on the coordinate plane

RECTANGULAR COORDINATE SYSTEM - a coordinate system in which the coordinates of a point are its distances from a set of perpendicular lines that intersect at an origin, such as two lines in a plane or three in space.

QUADRANTS - Any of the four areas into which a plane is divided by the reference axes in a Cartesian coordinate system, designated first, second, third, and fourth, counting counterclockwise from the area in which both coordinates are positive

SLOPE - the ratio of the rise to the run in a linear equation.

REFERENCES AND WEBSITE LINKS USED IN THIS LESSON:

VIDEOS

<http://www.youtube.com/watch?v=3SNmdNO5Bb4>

This video is about the life and contributions of Rene Descartes.

<http://www.youtube.com/watch?v=T2-TO8XBNbU>.

This video will introduce the students to the rectangular coordinate system.

<http://www.youtube.com/watch?v=F--060tUEk0>.

This video is about graphing straight lines.

<http://www.youtube.com/watch?v=aLfZZ6guz88>

This video is about the general form of the equation of a line.

<http://www.youtube.com/watch?v=AoJy7bsWKuc>

This video is about the standard form of the equation of a line.

<http://www.youtube.com/watch?v=3t7E8PTfey0>

This video illustrates how the slope and y-intercept could be used to derived the equation of a line.

<http://www.youtube.com/watch?v=eHPTyYbNmx4>

This video illustrates how the slope and a point could be used to derived the equation of a line.

<http://www.youtube.com/watch?v=4il4haYASys>

This video illustrates how two points could be used to derived the equation of a line.

<http://www.youtube.com/watch?v=2mm7OfwIQkE>

This video illustrates how the two intercepts could be used to derived the equation of a line.

INTERACTIVE WEBSITES

http://www.quia.com/hm/163121.html?AP_rand=246985567

This contains exercises on solving for the slope of a line.

<http://www.quia.com/cb/79602.html>

This contains exercises on the use of point slope form.

<http://crctlessons.com/slope-intercept-game.html>

This contains exercises on the use of slope intercept form.

WEBSITES

www.dictionary.com

This is a dictionary that students can use to unlock the meaning of words.

www.evernote.com

This is a tool students can use to summarize the facts which they have learned.