PEAC
PRIVATE EDUCATION ASSISTANCE COMMITTEE
Trustee of the Fund for Assistance to Private Education

DepED
DEPARTMENT OF EDUCATION

2017

LEARNING MODULE

Mathematics G9 | Q1

Quadratic Functions

PRIVATE EDUCATION ASSISTANCE COMMITTEE
Trustee of the Fund for Assistance to Private Education

PEAC National Secretariat
Units 2505-2507, 25th Floor Philippine AXA Life Centre
1286 Sen. Gil Puyat Avenue corner Tindalo St., Makati City
Tel No.: (02) 8-570-7322
Fax No.: (02) 8-511-7695

NOTICE TO THE SCHOOLS

This learning module (LM) was developed by the Private Education Assistance Committee under the GASTPE Program of the Department of Education. The learning modules were written by the PEAC Junior High School (JHS) Trainers and were used as exemplars either as a sample for presentation or for workshop purposes in the JHS In-Service Training (INSET) program for teachers in private schools.

The LM is designed for online learning and can also be used for blended learning and remote learning modalities. The year indicated on the cover of this LM refers to the year when the LM was used as an exemplar in the JHS INSET and the year it was written or revised. For instance, 2017 means the LM was written in SY 2016-2017 and was used in the 2017 Summer JHS INSET. The quarter indicated on the cover refers to the quarter of the current curriculum guide at the time the LM was written. The most recently revised LMs were in 2018 and 2019.

The LM is also designed such that it encourages independent and self-regulated learning among the students and develops their 21st century skills. It is written in such a way that the teacher is communicating directly to the learner. Participants in the JHS INSET are trained how to unpack the standards and competencies from the K-12 curriculum guides to identify desired results and design standards-based assessment and instruction. Hence, the teachers are trained how to write their own standards-based learning plan.

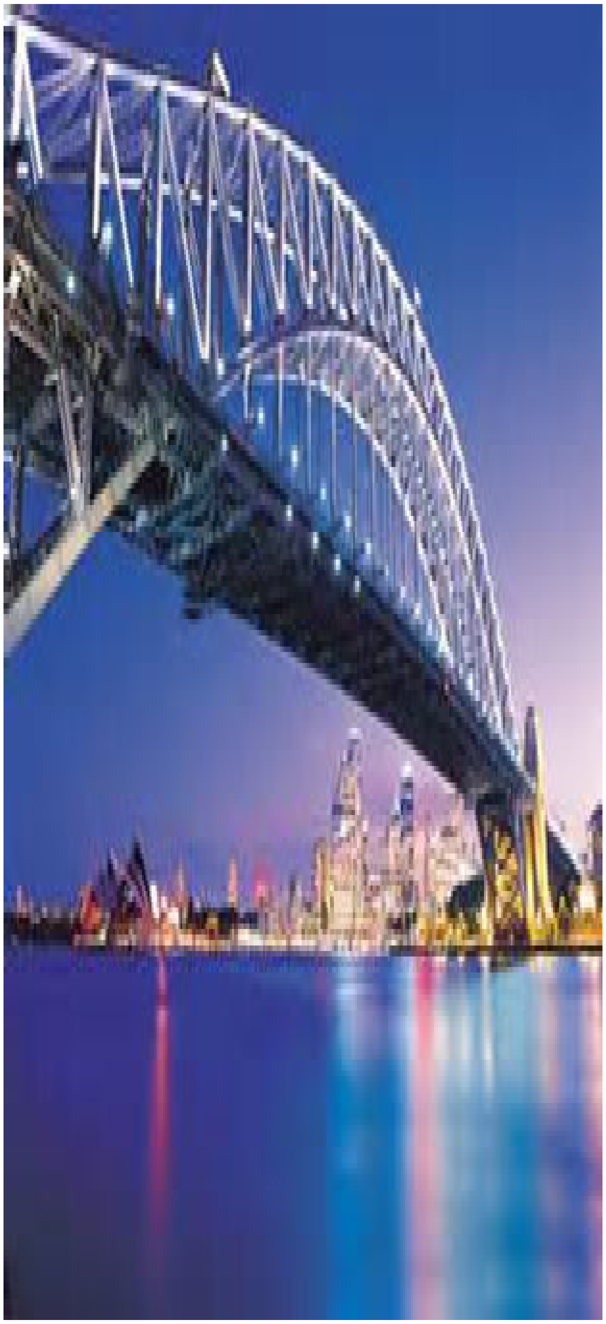
The parts or stages of this LM include Explore, Firm Up, Deepen and Transfer. It is possible that some links or online resources in some parts of this LM may no longer be available, thus, teachers are urged to provide alternative learning resources or reading materials they deem fit for their students which are aligned with the standards and competencies. Teachers are encouraged to write their own standards-based learning plan or learning module with respect to attainment of their school's vision and mission.

The learning modules developed by PEAC are aligned with the K to 12 Basic Education Curriculum of the Department of Education. Public school teachers may also download and use the learning modules.

MATHEMATICS GRADE 9

Module 1: Quadratic Functions

☑ INTRODUCTION AND FOCUS QUESTION(S):



Have you ever thought of how a businessman projects his or her sales? When does a businessman know how much he or she should produce to maximize his or her profits? When does he or she know that he or she needs to stop production? How can he or she determine the break even point?

Have you also at a certain point asked yourself why airplanes are curved? Have you ever wondered why a football travels in an arch or how far it would go before it hits the ground? Have you ever wondered how long a dolphin can stay in the air after jumping out of the water?

In this module, you will discover how important it is to utilize essential mathematical skills to be able to understand these questions that arise in various real-life situations that we encounter everyday and use these skills wisely to be able to come up with the desired output.

As you go through this module, think of this question: **How can various real-life situations involving maximum and minimum values be solved and analyzed?**

☑ MODULE LESSONS AND COVERAGE:

In this module, you will examine this question when you take the following lessons:

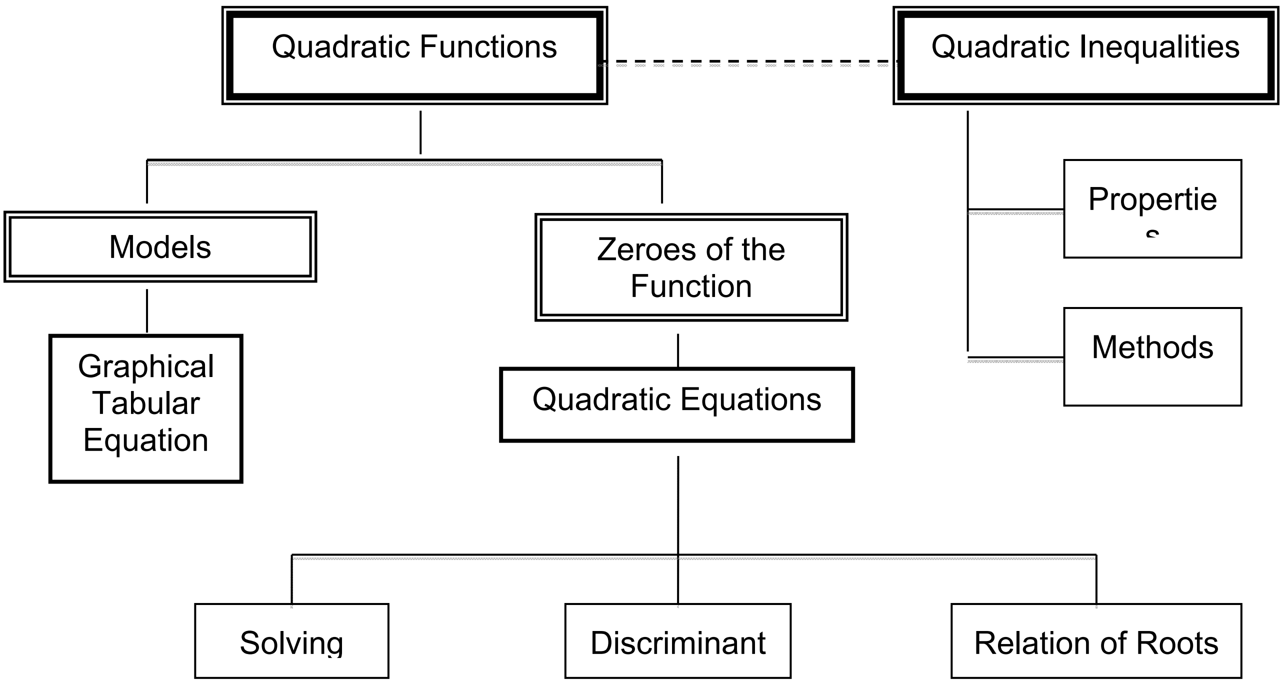
Lesson 1 – Quadratic Equations
Lesson 2 – Quadratic Inequalities
Lesson 3 – Quadratic Functions and Applications

In these lessons, you will learn the following:

Lesson 1	Quadratic Equations • illustrates quadratic equations • solves quadratic equations by: (a) extracting square roots; (b) factoring; (c) completing the square and (d) using the quadratic formula • characterizes the roots of a quadratic equation using the discriminant • describes the relationship between the coefficients and the roots of a quadratic equation • solves equations transformable to quadratic equations (including rational algebraic equations) • solves problems involving quadratic equations and rational algebraic equations
Lesson 2	Quadratic Inequalities • illustrates quadratic inequalities • solves quadratic inequalities • solves problems involving quadratic inequalities
Lesson 3	Quadratic Functions and Applications • models real-life situations using quadratic functions • represents a quadratic functions using (a) tables of values; (b) graph and (c) equation • transforms the quadratic functions defined by $y = ax^2 + bx + c$ into the form $y = a(x – h)^2 + k$ • graphs a quadratic functions: (a) domain, (b) range, (c) intercepts, (d) axis of symmetry, (e) vertex, (f) directions of the opening of the parabolas • analyzes the effects of changing the values of a, h and k in the equation $y = a(x – h)^2 + k$ of a quadratic function on its graph • determines the equation of the quadratic function given: (a) a table of values, (b) graph and (c) zeros • solves problems involving quadratic functions

☑ MODULE MAP:

Here is a simple map of the above lessons you will cover:



☑ EXPECTED SKILLS:

To do well in this module, you need to remember and do the following:

1. follow the directions carefully
2. master the prerequisite skills such as factoring, graphing and solving linear equations
3. solve with speed and accuracy

MODULE: PRE-ASSESSMENT

Let's find out how much you already know about this module. Click on the letter that you think best answers the question. Please answer all items. After taking this short test, you will see your score. Take note of the items that you were not able to correctly answer and look for the right answer as you go through this module.

1. Which of the following is NOT a quadratic equation?
   A. $2x^2 = \sqrt{3}$
   B. $2x + 5 = 7$
   C. $4(x^2 – 5) = 7x$
   D. $\frac{x^2}{2} = 4\sqrt{5}$

2. A quadratic equation in one variable is an equation of the form $ax^2 + bx + c = 0$. Which could <u>NOT</u> have a value of 0?
   A. a
   B. b
   C. c
   D. a and b

3. A quadratic equation has two distinct unreal roots if $b^2 – 4ac$ is ______.
   A. less than zero
   B. less than or equal to zero
   C. greater than zero but not a perfect square
   D. greater than zero and a perfect square

4. Which number should be added on both sides of the equation $2x^2 + 8x + 5 = 0$ if completing the square will be used?
   A. 64
   B. 5
   C. 16
   D. 4

5. If $x^2 - 13x + 42 = 0$, then x is equal to _____.
   A. 6 and 7
   B. -3 and 14
   C. 3 and -14
   D. -6 and -7

6. Which of the following illustrates a quadratic inequality?
   A. $2x(x + 3) > 2x^2$
   B. $(x + 2) (x - 3) < 0$

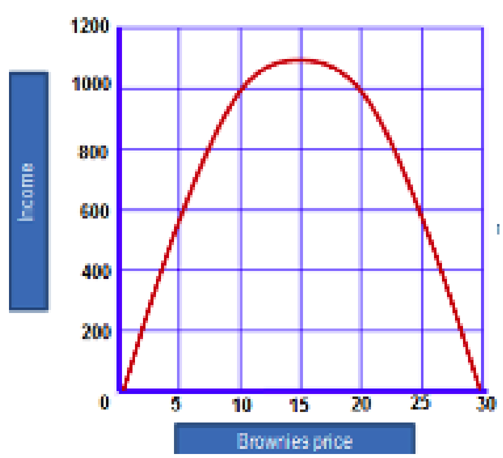
C. $x(x-5)(x-4) \leq 4$
D. $x^3 + x^2 \leq 2x^2 + x^3 - 4$

7. Solve the quadratic inequality $x^2 - x - 12 > 0$. Express your answer in interval notation.
   A. $(-\infty, 3)$ $(-4, \infty)$
   B. $(-\infty, -3)$ $(4, \infty)$
   C. $(-\infty, 4)$ $(-3, \infty)$
   D. $(-\infty, -4)$ $(3, \infty)$

8. A stuntman will jump off a 20 m building. A high-speed camera is ready to film him between 15 m and 10 m above the ground. When should the camera film him? The camera should film him from
   A. 1.0 to 1.4 seconds after jumping.
   B. 1.2 to 1.6 seconds after jumping.
   C. 1.4 to 1.8 seconds after jumping.
   D. 1. 6 to 2.0 seconds after jumping.

9. The unit cost in dollars for manufacturing n starters is given by $C = 0.004n^2 - 3.2n + 660$. For what number of starters is the unit cost at a minimum?
   A. 100
   B. 200
   C. 300
   D. 400

10. Pauleen is selling delicious brownies in school as part of their entrepreneurship activity in their Home Economics Subject that will last for one month. By adjusting the price of the brownies for several days, she created the scatter plot below for the weekly income.



What price should Pauleen charge to maximize the weekly income?
   A. 10 pesos
   B. 15 pesos

C. 20 pesos
D. 30 pesos

11. The income I (in pesos) earned by a company on the manufacture of s liters of an acid solution is I = $s^2 - 95s - 500$. Compute for its profit or loss if it manufactured 50 liters of acid solution.
    A. (P 2750)
    B. P2750
    C. P3000
    D. (P3000)

12. The profit (P) in dollars on each cellphone manufactured by Clear Lines Inc., is related to the number of cell phones produced each day according to the equation $P = 0.82x^2 + 4.25x + 45$ where x is the number of cellphones produced each day (in hundreds) and $2 \leq x \leq 8$. About how many cell phones should be produced each day to make a profit of $70 per phone?
    A. 4
    B. 3
    C. 5
    D. 7

13. If the total costs are $C(x) = 500 + 90x$, and total revenues are $R(x) = 150x - x^2$.
    Find the break- even point(s). At break-even, there is no profit, the costs equal the revenue, so *R(x) = C(x).*
    A. 20 and 25
    B. 50 and 10
    C. 5 and 100
    D. 4 and 125

14. At what value of x will profit be at a maximum?
    A. 15
    B. 20
    C. 25
    D. 30

15. You are hired one of the consultants of a fast food chain in your city. The owner asked you to come up with a business plan proposal to increase its sales and maximize its profits. Which should be the possible characteristics of your proposal?
    A. presentable, accurate and organized
    B. practical, authentic and accurate
    C. authentic, grammatically correct and organized
    D. presentable, realistic and neat

16. You are a crew from an entertainment who launches fireworks at angle of 60°

from the horizontal. The height of one particular type of display can be approximated by the function, $h(t) = -16t^2 + 160t$ where h(t) is measured in feet and t is measured in seconds. How long will it take the fireworks reach the maximum height?

A. 4 sec
B. 5 sec
C. 6 sec
D. 7 sec

17. You are a member of a group of students which starts a small business that sells used DVDs on the internet. The weekly profit of the business is given by the function $P(x) = -2x^2 + 100x – 800$ where x represents the number of DVDs produced. What is the vertex of the profit function and its meaning in the context of this problem?
    A. The vertex (25, 450) means a maximum weekly profit of 450 peso is obtained when 25 DVDs are sold.
    B. The vertex (25, 450) means a weekly loss of 450 peso when 25 DVDs are sold.
    C. The vertex (450, 25) means a maximum weekly profit of 25 peso selling 450 DVDs.
    D. The vertex (450, 25) means a weekly loss of 25 peso when 450 DVDs are sold.

18. Barangay Honesto is planning to fence their playground with an existing wall on one side. As an engineer of the Barangay, you are tasked to compute the maximum area that can be fenced using the available 200 feet fencing material. What should the maximum area be?
    A. 2 000 ft^2
    B. 3 000 ft^2
    C. 3 500 ft^2
    D. 5 000 ft^2

19. The number of bacteria in a refrigerated food is given by $n(t) = 5t^2 – 40t + 100$, where t is the temperature of the food in Celsius. As a microbiologist, you are tasked to answer the question: At what temperature will the number of bacteria be minimal?
    A. 4°C
    B. 5°C
    C. 6°C
    D. 50°C

20. The farmer predicted that the number of mango trees (x) planted in a farm could yield $y = -20x^2 + 2600x$ mangoes per year. As a mango grower, how

many trees should you plant to produce the maximum number of mangoes per year?

A. 130
B. 1300
C. 321 000
D. 331 000

LESSON NO. 1: QUADRATIC EQUATIONS

EXPLORE

When you were in Grade 8, you were able to talk about Linear Equations and Functions. You found out how these concepts could be used to model and solve real-life situations. In this section, you will be able to discover a new equation that could also solve more real-life problems. In this section, you will not only be introduced to the module but also be assessed on your initial ideas regarding the topic. Be reminded to take each activity seriously. Make a conscious effort to highlight the new concepts that you will be encountering.

The video you are about to see will flash before you some scenes that you encounter in real life. These are scenes that we never cease to appreciate for their aesthetic beauty. Have you ever wondered why they are constructed in a particular way?

ACTIVITY 1. PERFECT SHOTS OF MAN'S INGENUITY

Take a glimpse of man's ingenuity as you watch this video by clicking the link http://www.youtube.com/watch?v=D87sogoU_68. Pay attention to the commonalities that you see in the different shots flashed before you.

PROCESS QUESTIONS:

1. How many of the pictures shown are familiar to you?

2. What are the common things that you notice among the pictures?

3. What kind of paths are shown in the objects?

4. Can the scenes also use linear paths? Why? Why not?

5. If you were to draw the curved paths on a graphing paper, what characteristics could you name? Contrast this with a linear path.

The real-life situations that you saw in the video show you one of the many situations where we can encounter quadratic functions in real-life. As you go through all the activities in this module, think of this question: **How can various real-life situations involving maximum and minimum values be solved and analyzed?**

ACTIVITY 2. ANTICIPATION REACTION GUIDE

Let us begin by answering the Anticipation-Reaction Guide. You will be shown statements related to Quadratic Equations. Your task is to fill in the anticipation column by writing the letter A if you agree with the statement and letter D if you disagree with the statement.

Anticipation	Quadratic Equations	Reaction
	1. A equation of the form 0 = ax + by + c is a quadratic equation.	
	2. The standard form of a quadratic equation is $ax^2 + bx + c = 0$, were a, b, and c can be any real number.	
	3. All quadratic equations can be solved through factoring method.	
	4. In solving quadratic equations using completing the square method, there is a need to write the equation in standard form.	

	5. The roots of a quadratic equation will always be a real numbers.	
	6. The completing the square method and the quadratic formula are used to solve quadratic equations which are not factorable.	
	7. The discriminant tells us the nature of the roots of a certain quadratic equation.	
	8. Imaginary numbers are possible roots of a quadratic equation.	
	9. The roots of a quadratic equation will always be distinct numbers.	
	10. In solving problems involving quadratic equations, it is possible to have one of the roots to be considered as extraneous.	

Well, those are your thoughts and ideas about our lesson. Let's start a new activity to further explore on the important key concepts about quadratic equations.

ACTIVITY 3. WINDSHIELD CHECK

This formative assessment will check on your own understanding as far as the focus question is concerned. Using the analogy of a windshield, you will now decide which of the following best describes what your understanding of the focus question.

How many bugs do you have on your windshield? What is making it hard to see clearly?

CLEAR = I get it! I thoroughly understand the concept.
BUGGY = I understand it for the most part, but a few things are still unclear.
MUDDY = I don't get it at all.

After making an assessment, write a short description for your choice and email your response to your teacher.

END OF EXPLORE:

You just tried finding out how quadratic equations can be used to model and solve real-life problems. It is now time to learn more about quadratic equations. What you will learn in the next sections will also enable you to do the final project, which involves looking at real-life situations that involve decision making and coming up with sound recommendations.

We will start by doing the next activity.

FIRM-UP

Your goal in this section is to learn and understand key concepts of quadratic equations. You will learn the important skills that are necessary in order to understand the succeeding lessons in this module. It is advised that for every new term that you encounter, be sure to define it in your own words aside from the definitions presented to you.

We will start by doing the next activity.

ACTIVITY 4. CONCEPT FORMATION (CLASSIFYING)

This activity will allow you to define a quadratic equation. Click on this link and read the first page: http://www.mth.msu.edu/~kadyrova/lectures/Lecture_04.pdf

After reading the article, can you now paraphrase what you have read by coming up with your own definition of a quadratic equation? Write your definition inside the box below.

Now that you are able to define what a quadratic equation is, let us now find out whether you can classify the equations inside the box into two, one for examples of quadratic equations and the other as non-examples of quadratic equations.

$-3x^3 - 2x = 5$	$2x - x^2 = -10$	$2x - 5 = x$
$8x - 2 = x$	$x(x - 1) = -1$	$\sqrt{2}\,x^{-2} = x + 3$
$(x - 2)(x + 4) = 7$	$\left(\sqrt{x}\right)^2 - 2x^2 = -4$	$100x^4 = 25$
$(x^3)^2 - x + 1 = 0$		

It is now time to check your answer.

QUADRATIC	NON QUADRATIC

From the examples which you listed down under non quadratic examples, can you explain why these are classified as non quadratic?

NON QUADRATIC	EXPLANATION
$-3x^3 - 2x = 5$	
$8x - 2 = x$	
$(x^3)^2 - x + 1 = 0$	
$2x - 5 = x$	
$\sqrt{2}\,x^{-2} = x + 3$	
$100x^4 = 25$	

PROCESS QUESTIONS:

1. What is a quadratic equation?

2. What are the characteristics of a quadratic equation?

3. How did you identify examples and non-examples of quadratic equations?

4. How is it different from other kinds of equations?

ACTIVITY 5. CONCEPT FORMATION (FORM AND COEFFICIENT)

After defining what a quadratic equation is, let us now look at how quadratic equations are written in standard form. In this activity, you will also identify the coefficients in the transformed equations.
Look at the example below:

An equation of the type $ax^2 + bx + c = 0$, where a, b and c are constants and $a \neq 0$, is called the standard form of a quadratic equation.

Take note that:
ax^2 = quadratic term or the squared term
bx = linear term
c = constant term
a = numerical coefficient of the quadratic term
b = numerical coefficient of the linear term

Quadratic equations can be classified into two: complete and incomplete. A complete quadratic equation is one where a, b and c have values while an incomplete quadratic equation is one where only two coefficients are not equal to zero.

The examples below show you how quadratic equations are written in standard form and how the real numbers a, b and c are identified.

1. $9x^2 - 5x = 5$
   $= 9x^2 - 5x - 5 = 0$

a	b	c
9	-5	-5

2. $\frac{1}{3x} - 10 = x$

   $= 1 - 30x = 3x^2$

   $= -3x^2 - 30x + 1 = 0$

a	b	c
-	-	1

Now, try the next examples below and see whether you know how to write quadratic equations in standard form and identify a, b and c.

3. $6 + 3x^2 = -3x$

a	b	c

4. $2x^2 = 9$

a	b	c

5. $(x + 1)(x + 2) = 0$

a	b	c

PROCESS QUESTIONS:

1. When is a quadratic equation written in standard form?

2. What are the things that you need to remember when identifying a, b and c?

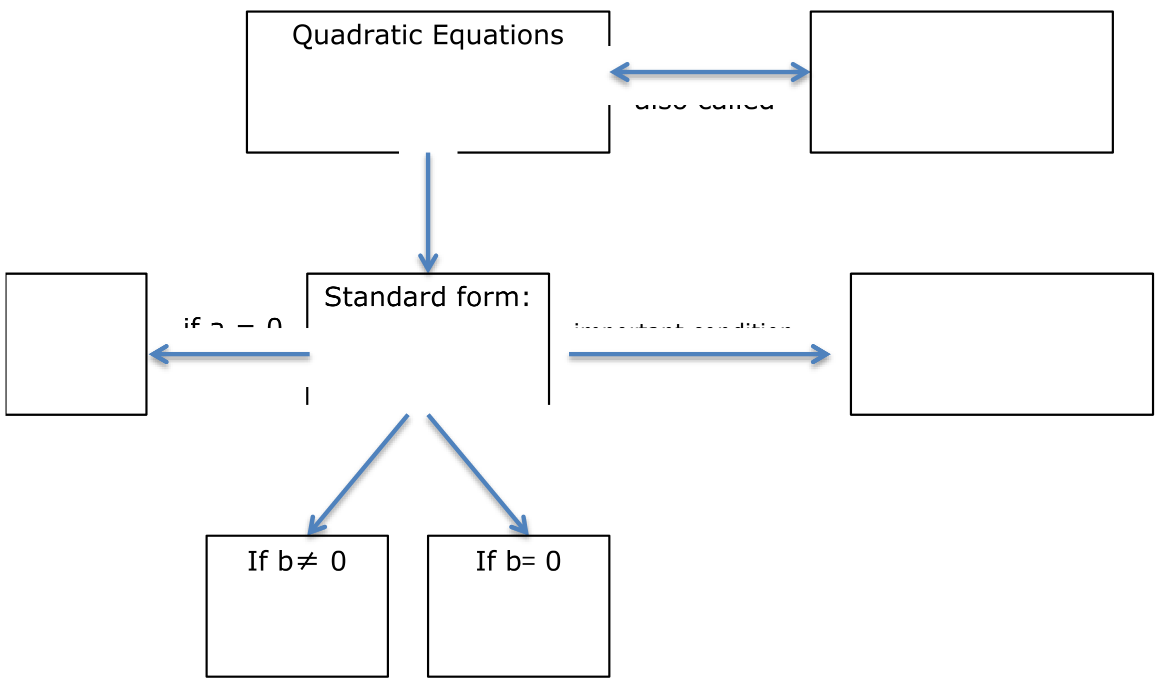
3. Go back to the exercise that you just did. Do all examples have values for all

of a, b and c?

4. How do you classify quadratic equations with complete set of coefficients?
   What about equations with incomplete coefficients?

ACTIVITY 6. MIND MAP

After finding out the different concepts which are important in understanding quadratic equations, let us now put together the concepts you learned by completing the mind map below.



ACTIVITY 7. THE ROOTS ARE NOT FAR BEYOND YOUR REACH

After learning the different preliminary concepts on quadratic equations, it is now time to learn how to solve for the roots of a quadratic equation. You will be watching four videos on the various methods that you can use to solve quadratic equations.

As you watch each of these videos, pay attention to the following:

1. List down the steps involved in each method.

Methods	Steps
Extracting the Roots	
Factoring	
Completing the Square	
Quadratic Formula	

2. Highlight differences in the various methods.

3. List down the algebraic skills that are needed in each method.

Methods	Steps
Extracting the Roots	
Factoring	
Completing the Square	
Quadratic Formula	

You are not yet expected to master these methods. You are only expected to look at each method and make comparisons. Click on the different links below. You may key in your notes on the table provided below.

A. Extracting the roots http://www.youtube.com/watch?v=-0dQ6yRXI38
B. Factoring http://www.youtube.com/watch?v=SDe-1IGeS0U
C. Completing the square http://www.youtube.com/watch?v=xGOQYTo9AKY
D. Quadratic formula http://www.youtube.com/watch?v=3ayhvAI3IeY

PROCESS QUESTIONS:

1. What is involved in each method?

2. Name the algebraic skills needed in each method.

3. Does each method have limitations?

4. Why are there numerous methods needed to solve quadratic equations?

ACTIVITY 8. EVERNOTE

Go to www.evernote.com. Download the Evernote program. You will be writing your notes on the four videos that you have viewed through this program. Take note that you will not be submitting the notes to the teacher for you will use these as you proceed with the other activities.

When you are writing your notes, pay attention to the following:

1. Describe each method of solving for the roots.
2. List down the algebraic skills needed in each method.
3. Write down the steps involved.

Be sure to unlock the meaning of different concepts like Square Root Principle and Principle of Zero Products Property. You can search for the definitions of these properties by using the search engine www.google.com.ph. Type inside the search box these properties and click enter. Look for websites that provide a definition of these properties.

ACTIVITY 9. EXTRACTING THE ROOTS

Let us look at the first method of solving for the roots of a quadratic equation. Before you practice, let us first look at several illustrative examples that will show the use of this method.

1. $x^2 + 4 = 102$	
$x^2 = 98$	Subtract 4 from both sides.
$x = \pm\sqrt{98}$	Use the square root principle.
$x = \pm\sqrt{49 \bullet 2}$	Factor the radicand.
$x = \pm 7\sqrt{2}$	Simplify the radical.
2. $(x + 2)^2 = 25$	
$\sqrt{(x+2)^2} = \sqrt{25}$	Use the square root principle.
$x + 2 = \pm 5$	Simplify the radical.
$x = \pm 5 - 2$	Subtract 2 from both sides.
3. $(4x - 7)^2 = -20$	
$\sqrt{(4x-7)^2} = \sqrt{-20}$	Use the square root principle.

PROCESS QUESTIONS:

1. Define the Square Root Principle.

2. Describe the kinds of roots that were derived.

3. Try to look at the following possibilities as you consider the equation $x^2 = c$.

   a. What can you say about c if the equation has no real solution?

b. What can you say about c if the equation has exactly one solution?

c. What can you say about c if the equation has two solutions?

d. If c is a prime number, what type of solutions does the equation have?

e. If a is a positive perfect square, what type of solutions does the equation have?

ACTIVITY 10. SKILL BOOSTER

After looking at several examples that show how to solve quadratic equations by extracting the roots, let us now find out whether you can do the solving. Answer the worksheet found below.

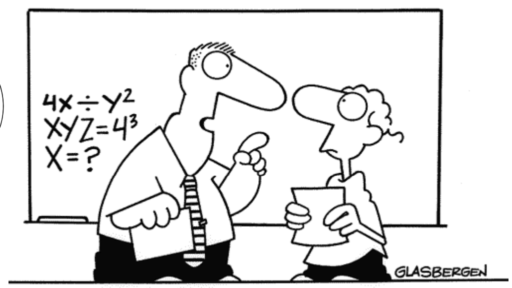
Solve each equation by using the square root property.

PRACTICE		
1. $4t^2 = 108$	2. $(p + 6)^2 = 9$	3. $(x + 5)(x - 5) = 11$
MORE PRACTICE		

4. $3x^2 = 600$	5. $3(4x - 1)^2 = 27$	6. $(x + 3)(x - 3) = 18$

ACTIVITY 11. When the linear term is present (SOLVING BY FACTORING)

After learning how to solve quadratic equations by extracting the roots, let us now look at the second method of solving. Look at each example and try practicing how to solve a similar example.



"Algebra class will be important to you later in life because there's going to be a test six weeks from now."

Solving Quadratic Equations by Factoring

Objective: To use the zero factor property to solve equations.

Zero Factor Property

- If $ab = 0$, then $a = 0$ or $b = 0$ or *both a and b* $= 0$.
- In other words, if a product is zero, then one or all of the factors are zero.

Steps for Solving Quadratic Equations w/ Factoring

1. Put equation in standard form. ($ax^2 + bx + c = 0$)
2. Factor completely.
3. Set each factor = 0.
4. Solve each equation.
5. Check. (VERY IMPORTANT!)

Examples – Solve each equation.

- **a) $(2x+3)(x-5)=0$ b) $x^2 - 36 = 0$**

Examples – SOLUTIONS

- **a) $(2x+3)(x-5)=0$**
- (already factored and set = 0, so...)

$2x+3=0$ or $x-5=0$

$2x=-3$ or $x=5$

$x=\frac{-3}{2}$

$x=\left\{\frac{-3}{2},-5\right\}$

- **b) $x^2 - 36 = 0$**

$(x+6)(x-6)=0$

$x+6=0$ or $x-6=0$

$x=-6$ $x=6$

$x=\{-6,6\}$

Examples

c) $a^2 - 24a = -144$ d) $-3m^2 + 27m = 0$

Examples -- SOLUTIONS

c) $a^2 - 24a = -144$

$$a^2 - 24a + 144 = 0$$
$$(a-12)(a-12) = 0$$
$$a = 12$$
$$a = \{12\}$$

d) $-3m^2 + 27m = 0$

$$-3m^2 + 27m = 0$$
$$-3m(m-9) = 0$$
$$-3m = 0 \quad \text{or} \quad m - 9 = 0$$
$$m = 0 \quad \text{or} \quad m = 9$$
$$m = \{0, 9\}$$

More Examples

e) $m^2 - 25 = 24m$ f) $x^3 + 2x^2 = 15x$

SOLUTIONS

e) $m^2 - 25 = 24m$

$$m^2 - 24m - 25 = 0$$
$$(m-25)(m+1) = 0$$
$$m - 25 = 0 \quad \text{or} \quad m + 1 = 0$$
$$m = \{25, -1\}$$

f) $x^3 + 2x^2 = 15x$

$$x^3 + 2x^2 - 15x = 0$$
$$x(x^2 + 2x - 15) = 0$$
$$x(x+5)(x-3) = 0$$
$$x = 0 \text{ or } x + 5 = 0 \text{ or } x - 3 = 0$$
$$x = \{0, -5, 3\}$$

Here's a real doozy

g) $(3x+2)(x-3) = 7x - 1$

SOLUTION

g) $(3x+2)(x-3) = 7x - 1$

$$(3x+2)(x-3) = 7x - 1$$
$$3x^2 - 9x + 2x - 6 - 7x + 1 = 0$$
$$3x^2 - 14x - 5 = 0$$
$$(3x+1)(x-5) = 0$$
$$3x + 1 = 0 \text{ or } x - 5 = 0$$
$$3x = -1 \quad \text{or} \quad x = 5$$
$$x = \frac{-1}{3}$$
$$x = \left\{\frac{-1}{3}, 5\right\}$$

PROCESS QUESTIONS:

1. How different is the new method from the previous one?

2. Why call the method solving quadratic equations by factoring?

3. List down the factoring techniques that you used for solving.

ACTIVITY 12. PRACTICE MAKES PERFECT

It is now time to practice using the new method that you learned. Answer the worksheet you find below. Try to check you answers at the end of the activity

Solve for the roots mentally.

Quadratic Equation	Root 1	Root 2
1. $(x + 5)(x - 2) = 0$		
2. $t(6t + 5) = 0$		
3. $(2m - 7)(m - 3) = 0$		
4. $6y(4y + 9) = 0$		
5. $(x + \frac{1}{2})(2x - \frac{1}{3}) = 0$		

Solve each equation by factoring and check your answers.

6. $2x^2 - x = 15$	8. $5m^2 - 22m + 8 = 0$
7. $3m^2 - 6m = 24$	9. $x^2 - 36 = 0$

ACTIVITY 13. YOU COMPLETE ME

Find out why mathematicians made a third method to solve for the roots of quadratic equations. Click on this link:

http://www.glencoe.com/sec/math/algebra/algebra1/algebra1_03/study_guide/pdfs/alg1_pssg_G077.pdf

When you open the reading material, look at the illustrative examples then do the succeeding exercises.

PROCESS QUESTIONS:

1. Why is completing the square necessary in solving for the roots of a quadratic equation?

2. Compare the new method that you learned with the previous methods.

3. List down the steps involved in the method.

4. Why is it important to follow the steps carefully?

ACTIVITY 14. IN GENERAL

Watch the video again by clicking this link: http://www.youtube.com/watch?v=3ayhvAI3IeY. As you review the method, try to fill out the worksheet below.

	Write the quadratic equation in general form.
	Divide both sides by a.
	Isolate the terms containing the variable.
	Complete the square by adding $\left(\frac{b}{2a}\right)^2$ to both sides.
	Simplify.
	Write the left side as a perfect square trinomial.
	Apply the square root property.
	Simplify. Isolate x on one side of the equation.

PROCESS QUESTIONS:

1. When deriving the quadratic formula, what other methods were done?

2. Describe the process of solving quadratic equations using quadratic formula.

3. Compare quadratic formula from factoring and completing the square.

4. Why is it important to learn this method?

ACTIVITY 15. 3-2-1 Chart

In this activity, you will be asked to complete the 3-2-1 Chart regarding the special products that you have discovered.

3-2-1 Chart

Three things you found out:

1. ____________________
2. ____________________
3. ____________________

Two interesting things:

1. ____________________
2. ____________________

One question I still have:

1. ____________________

Please email your question to your teacher so the needed clarification will be addressed.

ACTIVITY 16. THE CHOICE IS YOURS

After learning the different methods of solving quadratic equations, let us now find out whether you know how to determine which method would be appropriate in solving each given equation.

1. $49x^2 - 25 = 0$	4. $\frac{1}{12}x^2 - \frac{1}{2}x + \frac{1}{3} = 0$
2. $2x^2 + 5 = 7x$	5. $\left(\frac{1}{3}x + 1\right)^2 = 49$
3. $2x^2 + 8x = 3$	6. $(x - 1)(x + 2) = 10$

PROCESS QUESTIONS:

1. How many items did you solve correctly?

2. Which method did you use the most? Why?

3. What realizations have you made for yourself after doing the exercise?

ACTIVITY 17. IN TABULAR FORM

Let us now make a summary of the four methods which you just learned. Complete the table below with the advantages and disadvantages of each method of solving for the roots of quadratic equations.

Methods	**Advantages**	**Disadvantages**
Square Root Property		
Factoring		
Methods	**Advantages**	**Disadvantages**
Completing the Square		

	________________________________ ___ ________________________________ ___ ________________________________ ___ ________________________________ ___	________________________________ ___ ________________________________ ___ ________________________________ ___ ________________________________ ___
Quadratic Formula	________________________________ ___ ________________________________ ___ ________________________________ ___ ________________________________ ___ ________________________________ ___ ________________________________ ___ ________________________________ ___	________________________________ ___ ________________________________ ___ ________________________________ ___ ________________________________ ___ ________________________________ ___ ________________________________ ___ ________________________________ ___

ACTIVITY 18. MEMORY LANE

Can you still recall rational algebraic equations? Visit this website: http://www.virtualnerd.com/algebra-1/rational-expressions-functions/solve/example-solutions/equation-definition to watch the video. After watching, answer the process questions that follow.

1. What is a rational equation?

2. How is a rational equation different from a rational expression?

3. How do you think will you solve a rational equation?

ACTIVITY 19. MAKING CONNECTIONS

Quadratic equations and rational algebraic equations are related to each other. Many rational algebraic equations when solved will lead to quadratic equations. Let us look at the example below.

$$(y+1)(y-1)\left(\frac{2}{y+1}+\frac{1}{y-1}\right)=(1)(y+1)(y-1)$$

$$2(y-1)+(y+1)=(y+1)(y-1)$$

$$2y-2+y+1=y^2-1$$

$$y^2-3y=0$$

$$y(y-3)=0$$

$$y=0,\ 3$$

tarted with a rational equation (which has a limited domain) we mus

PROCESS QUESTIONS:

1. Describe the given equation. What kind of equation is it?

2. How did the equation lead to a quadratic equation?

3. What generalizations can you make about the two?

Now that you have learned from the given illustrative examples how rational algebraic equations can lead to quadratic equations, it is now time to make more connections by watching videos on applications of rational algebraic equations in real life.

VIDEO 1: http://www.youtube.com/watch?v=r6N8mDRNktw
PROCESS QUESTIONS:

1. In which aspect of life can the concept of rational algebraic equations be used?

2. How can the concept facilitate the solution to the real life problem?

VIDEO 2: http://www.youtube.com/watch?v=fG-zjUR9mM8
PROCESS QUESTIONS:

1. What did you see in the video?

2. In which aspect of life can the concept of rational algebraic equations be used?

ACTIVITY 20. TRY IT!

Let us see if you know how to solve rational algebraic equations that lead to quadratic equations. Try to answer the two problems below.

1. $$3\left(\frac{x}{x+1}\right)^2+7\left(\frac{x}{x+1}\right)-6=0$$

2. $$9\left(\frac{x+2}{x+1}\right)^2-6\left(\frac{x+2}{x+1}\right)+1=0$$

ACTIVITY 21. MUDDIEST POINT

In this activity, you will complete the journal below.

The part of the lesson that I still find confusing is ____________________________ because ____.

ACTIVITY 22. ROOT INVESTIGATION

Fill out the table with the needed information.

EQUATIONS	$b^2 - 4ac$	ROOTS	
		ROOT 1	ROOT 2
1. $x^2 + 4x - 21 = 0$			
2. $x^2 - 3x - 18 = 0$			
3. $x^2 + x - 1 = 0$			
4. $x^2 + 4x + 4 = 0$			
5. $x^2 - 2x + 1 = 0$			

PROCESS QUESTIONS:

1. Describe the values that you got under the column $b^2 - 4ac$.

2. For each kind of number you got under the second column, observe the roots. What generalizations can you make?

ACTIVITY 23. IT'S THE DISCRIMINANT

Do you ever wonder what $b^2 - 4ac$ stands for? Why don't you watch a video by clicking on this link: http://www.youtube.com/watch?v=SkUATohNR78.

PROCESS QUESTIONS:

1. What is a discriminant?

2. Why is getting the discriminant important?

3. What are the different values that a discriminant could have?

ACTIVITY 24. SHOWCASE YOUR THOUGHTS

Showcase the concepts that you have gained by using present.me. Present.me is a video online presentation software. In doing this activity, focus on the new insights that you have discovered with regards to discriminants.

ACTIVITY 25. SHOWCASE YOUR SKILLS

After learning what discriminants are, let us find out if you can answer the worksheet below.

Solve for the discriminant of the following equations and describe the nature of the roots.

Equation	Discriminant	Nature of Roots
1. $3x^2 + 5x - 9 = 0$		
2. $6a^2 + 17a - 14 = 0$		
3. $(3b - 1)(2b + 5) = 3$		
4. $4z(z - 1) = 19$		
5. $12p^2 - 4p + 3 = 0$		

Find the value(s) of k for each given condition.

6. $x^2 - 6x + k = 0$ for which the equation has two distinct real solutions.
7. $x^2 - kx + 2k = 0$ for which the equation has two equal real roots.

ACTIVITY 26. SOUL MATES: ROOTS AND COEFFICIENTS

In this activity, let us find out whether the roots of quadratic equation are related with the coefficients. Click on this link to watch the video: http://www.youtube.com/watch?v=IGhrjwE-QoU

PROCESS QUESTIONS:

1. What new concept did you learn from watching the video?

2. How are the roots of the quadratic equation related to the coefficients?

3. How can you use these concepts to master your skills in quadratic equations?

4. Outline the steps presented in the video.

ACTIVITY 27. LEARNING LOG

After learning the preliminary concepts on quadratic equations, write a one paragraph essay on your progress in this module. What are the things that you need more assistance on?

End of FIRM UP:

In this section, the discussion was about the different methods that you can use to solve quadratic equations. You were also given the opportunity to learn how the different coefficients and roots of a quadratic equation are related.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision? What new learning goal should you now try to achieve?

Now that you know the important ideas about this topic, let us go deeper by moving on to the next section.

DEEPEN

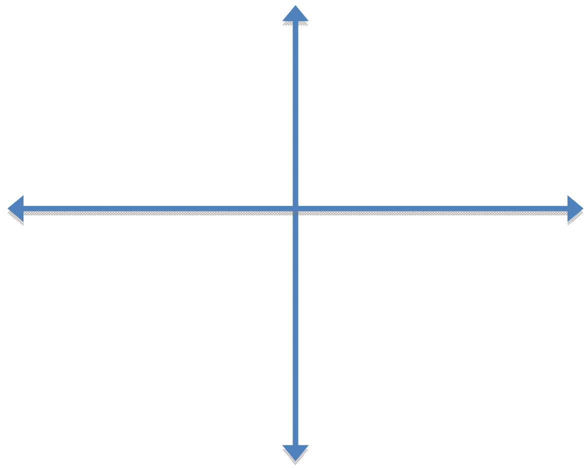
Now that you have learned the important skills needed in solving quadratic equations, your goals in this section is to look at some real-life situations where we can apply the concepts that you have learned.

ACTIVITY 28. THE SCIENCE OF ALGEBRA

In this activity, you will do a simple experiment. Click on this link: http://stargazers.gsfc.nasa.gov/pdf/activities/math_activities/math_teacher/math_act_09_t.pdf. Read and understand very well the experiment. Make sure to prepare the needed materials. Once you have the materials ready, you may start with the experiment. Record your findings in the table below.

Time (s)	0										
Water height (mm)											

Make a graph using the values in the table. Label the x-axis with time and the y-axis with water height.



PROCESS QUESTIONS:

1. What do you notice about the relationship of the two variables?

2. Is there constant change? Why? Why not?

3. Describe the graph. Compare this with the graph of a linear equation.

4. What is its lowest point? highest point?

5. Why are the negative values not considered?

ACTIVITY 29. NO MONKEY BUSINESS

Imagine that you inherited ₱500,000 from your grandmother. Think of a business that you would like to run. What kind of products or services are you willing to offer your customers? What are the different costs associated with running a business?

Watch a video about the different costs that businesses incur by by clicking this link: http://www.youtube.com/watch?v=nQ5APwtB-ig.

PROCESS QUESTIONS:

1. What are the costs incurred in a business?

2. How is a fixed cost different from a variable cost?

3. Why do businesses incur losses?

4. What might lower the profits of businesses?

ACTIVITY 30. THE ALGEBRA IN BUSINESS

Do you ever wonder where quadratic equations are used is business? Read and understand the problem below. After which, answer the process questions.

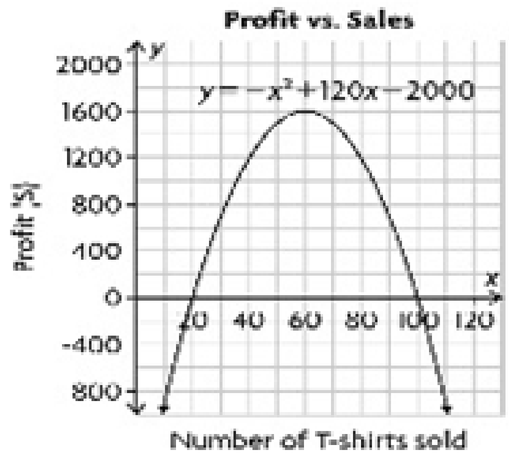
The profit P of the business is given by the equation $P = -x^2+120x-2000$ where x is the number of t-shirts sold.

Look at the table of values below.

x	0	20	40	60	80	100
P	-2000	0	1200	1600	1200	0

How were the values of P derived?

Now, look at the graph of the different ordered pairs.



PROCESS QUESTI

1. Describe the graph.

2. How does it look like?

3. Recall the graph of a linear function. How is it different from the graph that you see above?

4. When would profits be equal to zero?

5. How much profit could you gain for selling 60 shirts?

6. Where the does the graph cross the x-axis? What do these points represent?

7. Why does the graph reach the fourth quadrant? What does this imply?

8. How could you explain why selling 60 shirts would give you more profit than selling 80 shirts?

9. **How can various real-life situations involving maximum and minimum values be solved and analyzed?**

ACTIVITY 31. SAILING THE SEA

It is time to look at another way in which quadratic equations can be used to model real-life problems. You will first view a video that explains how a boat sails in apparent wind. Click on the link: http://www.youtube.com/watch?v=8xo0ySZ98oc.

After watching the video, read and understand the problem below. After which, answer the questions that follow.

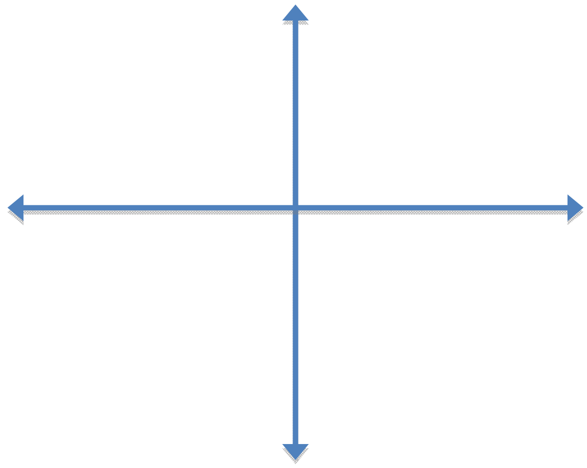
Sailors need to consider the speed of the wind when adjusting the sails on their boat. The force *F* (in pounds per square foot) on a sail when the wind is blowing perpendicular to the sail can be modeled by the equation $F = 0.004v^2$ where *v* is the wind speed (in knots).

PROCESS QUESTIONS:

1. Complete the table below.

v	0	10	20	30	40	50
F						

2. Graph the ordered pairs.



3. Describe the shape of the graph.

4. Is there a limit to the wind speed?

5. What is the lowest point the graph?

6. Are negative values considered? Why?

7. **How can various real-life situations involving maximum and minimum values be solved and analyzed?**

ACTIVITY 32. DO NOT GET WET

Here is the last problem that we are going to talk about. Read and understand the problem below and answer the process questions that follow.

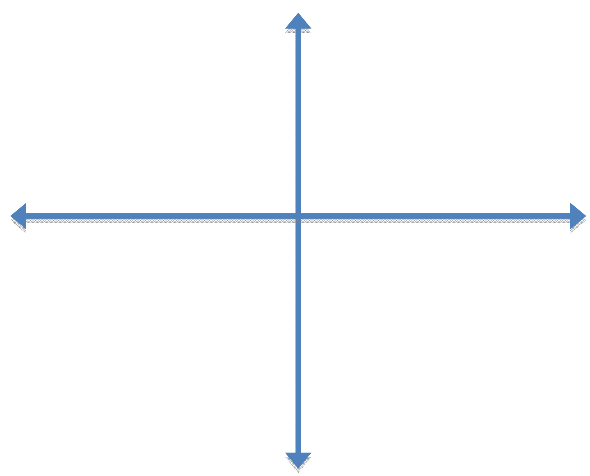
A jet of water is spraying from the center of a circular fountain. The height, h, in meters above the ground, of the jet of water is modeled by the relation $h = -0.5x^2 + 1.8x + 1.2$, where x represents the distance that the water travels horizontally, in meters.

PROCESS QUESTIONS:

1. Complete the table below.

x	0	1.5	2	2.82	3.5	4
h						

2. Graph the ordered pairs.



3. Describe the shape of the graph.

4. What is the maximum height of the jet of water?

5. Why will water eventually go down?

6. Are negative values considered? Why?

7. **How can various real-life situations involving maximum and minimum values be solved and analyzed?**

ACTIVITY 33. SUMMING EVERYTHING UP

You will now revisit the different situations that you have used to apply the concepts of quadratic equations. What are the common things that you notice among the various situations presented?

From the various real-life connections presented in the different activities, I noticed that the situations have the following things in common:

1. ____________________
2. ____________________
3. ____________________

After summing up everything, you will now watch another video by clicking this link: http://www.youtube.com/watch?v=UcZhGkU6sIM. Try to see if the generalizations that you have made can also be used in the new situation.

PROCESS QUESTIONS:

1. What real-life situation is used in the video?

2. Look at the commonalities that you listed down, can you also find them in the new situation?

3. What generalization can you make after looking at the new situation?

ACTIVITY 34. ANTICIPATION REACTION GUIDE

Let us now revisit the Anticipation-Reaction Guide. After learning the different concepts on quadratic equations and their application to real-life situations, your task is to fill in the reaction column by writing the letter A if you agree with the statement and letter D if you disagree with the statement. If you disagree with the statement, correct it to make it a true statement and explain your correction afterwards.

Anticipation	Quadratic Equations	Reaction	Explanation
	1. A equation of the form 0 = ax + by + c is a quadratic equation.		
	2. The standard form of a quadratic equation is $ax^2 + bx + c = 0$, were a, b, and c can be any real number.		
	3. All quadratic equations can be solved through factoring method.		
	4. In solving quadratic equations using completing the square method, there is a need to write the equation in standard form.		
	5. The roots of a quadratic equation will always be a real numbers.		
	6. The completing the square method and the quadratic formula are used to solve quadratic equations which are not factorable.		
	7. The discriminant tells us the nature of the roots of a certain quadratic equation.		
	8. Imaginary numbers are possible roots of a quadratic equation.		
	9. The roots of a quadratic equation will always be distinct numbers.		
	10. In solving problems involving quadratic equations, it is possible to have one of the roots to be considered as extraneous.		

End of DEEPEN:

In this section, the discussion was about using the concepts of quadratic equations to be able to solve real-life problems. Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.

TRANSFER

Your goal in this section is to apply your learning to real life situations by doing scaffold activities which will help you in the making of final project. You will be given a practical task which will demonstrate your understanding.

ACTIVITY 35. SCAFFOLD FOR TRANSFER 1

In this activity, you will observe the following pictures below. Identify a real-life experience that you can associate with the given pictures. After which, answer the process questions that follow.

http://baseballrebellion.com/cpippitt/the-myths-of-hitting-down-and-through/

http://free-workout-routines.net/image-files/shooting-arc.gif

http://l1.yimg.com/bt/api/res/1.2/j4VgbIO0qdup4Ca6msyoaQ--/YXBwaWQ9eW5ld3M7Zmk9aW5zZXQ7aD00MDU7cT03NTt3PTYzMA--/http://media.zenfs.com/en_us/News/Reuters/2013-11-18T190718Z_1030131390_GM1E9BJ08KP01_RTRMADP_3_SPACE-MARS-LAUNCH.JPG

http://www.jdtravelservices.co.uk/index%2520images/

Airports/Other/Airplane%2520Landing.jpg

http://www.isnetworks.com/wp-content/plugins/blindall-slideshow/Slideshow/business-money.jpg

PROCESS QUESTIONS:

1. Name real life situations that are represented by each picture.

2. What is common among the pictures?

3. How does each picture show the use of quadratic equations?

4. Can you name other real life experiences that use the concepts of quadratic equations like the ones shown in the pictures?

ACTIVITY 36. SCAFFOLD FOR TRANSFER 2

After learning how quadratic equations can be used to model real-life situations, you will now extend your learning by interviewing any person in any field where quadratic equations can be used. You will be asking your interviewee questions that pertain to the importance of parabolic paths in the given field.

You may use the following questions in your interview:

1. Name some math skills that you utilize in your work. Why are these skills very important?
2. How do you use quadratic equations in doing your work?
3. Why do you use parabolic paths in doing your work?

After the interview, present what you have learned by creating customized avatars through voki.com.

Now, that you have completed the first lesson of this module, you will now complete the synthesis journal found below.

ACTIVITY 37. SYNTHESIS JOURNAL

The unit's lesson was on ____.
One key idea was
____. This is important because
____.
Another key idea was ____.
This is also important because
____.
In summary, the unit's lesson
____

____
_____.

In this section, your task was to look at how the skills needed to accomplish the performance task were done.

End of TRANSFER:

You have completed this lesson. Congratulations!

Lesson 2: Quadratic Inequalities

EXPLORE

In grade 8, you studied linear inequalities and were able to identify linear inequalities, illustrate and model linear inequalities, and solve problems involving linear inequalities. In the previous lesson you studied quadratic equations. Now, we study quadratic inequalities and I will challenge you to answer the problem below in any ways. Record your answer and justification in the box provided.

CHALLENGE PROBLEM

Vertical Leap Record. ***Guinness Book of World Records reports that German shepherds can make vertical leaps of over 10 feet when scaling walls. If the distance*** s ***(in feet) off the ground after t seconds is given by the equation*** $s = -16t^2 + 24t + 1$***, for how many seconds is the dog more than 9 feet off the ground? Justify/Explain your answer.***

(Note: Highlight the given and the phrase which illustrates inequality)

Answer	***Justification/Explanation***

PROCESS QUESTIONS:

1. How do you find the activity?

2. How did you come up with your decision?

3. What supports you with your decision?

4. How can we use equations and inequalities to solve real-life problems where certain quantities are unknown?

5. How do quadratic inequalities help us solve real life problems?

6. **How can various real-life situations involving maximum and minimum values be solved and analyzed?**

With your answer, justification and explanation above, it is obvious that you have initial knowledge about quadratic inequalities. Take note of what you have written because we will go back to your answer, justification and explanation after we study and fully grasp quadratic inequalities. To check and highlight how far you know about quadratic inequalities, perform the activity below.

ACTIVITY 1. KWL CHART

Answer the question in the first and second columns only.

What I *know* about quadratic inequalities?	**What I *want* to know about quadratic inequalities?**	What I *learn* about quadratic inequalities?

Hold on to what you know about quadratic inequalities. For a couple of weeks, we will study quadratic inequalities and you will verify and enrich what you initially know about this topic. We will explore its nature and its usefulness to our lives. There are relatively more inequalities in life than equalities. This is evident in the different real-life situations below. Now you will identify real-life situations related to inequalities and illustrate the inequalities involve by doing the next activity.

ACTIVITY 2. CLASSIFY ME!

Study and analyze each situation in the first column then write the inequality model illustrated in the situation and classify this as linear or quadratic inequality.

Situations	Inequality model	Classification: Linear or Quadratic
1. You are on a treasure-diving ship that is hunting for gold and silver coins. Objects collected by the divers are placed in a wire basket. One of the divers signals you to reel in the basket. It feels as if it contains no more than 50 pounds of material. If each gold coin weighs about 0.5 ounce and each silver coin weighs about 0.25 ounce, what are the different amounts of coins that could be in the basket?		
2. For a drug to have a beneficial effect, its concentration in the bloodstream must exceed a certain value, which is called the minimum therapeutic level. Suppose that the concentration c (in mg/L) of a particular drug t hours after it is taken orally is given by $c = 20t/(t^2 + 4)$. If the minimum therapeutic level is 4 mg/L,		

determine when this level is exceeded.		
3. You are a car dealer. You have Php63,360,000 available to purchase compact cars and sport utility vehicles for your lot. The compact car costs Php495 000 and the sport utility vehicle costs Php990 000. If x represents the number of compact cars and y represents the number of sport utility vehicles you purchase.		
4. The braking distance d (n feet) of a certain car travelling v mi/hr is given by the equation d = v + (v^2/20). Determine the velocities that result in braking distance of less than 75 feet?		

PROCESS QUESTIONS:

1. How do you find the activity?

2. Which of the situations/problems illustrate linear inequality? Quadratic inequality?

3. How does linear inequality differ from quadratic inequality?

4. How do you identify them?

5. How do quadratic inequalities help us solve real life problems?

6. **How can various real-life situations involving maximum and minimum values be solved and analyzed?**

End of EXPLORE:

You will try finding out and verify what the answer is by the doing the next part. What you learn in the next sections will also enable you to do an activity in preparation to the final task which involve decision making.

Let's now find out what the answer is by doing the next part.

FIRM-UP

Your goal in this section is to learn and understand key concepts about illustrating and solving quadratic inequalities.

Illustrating and solving quadratic inequality

An inequality involving a quadratic polynomial is a quadratic inequality. The solution of a quadratic inequality may be obtained using the principles of real numbers and the properties of inequality.

Quadratic inequalities can be of the following forms:

$$ax^2 + bx + c > 0$$
$$ax^2 + bx + c \geq 0$$
$$ax^2 + bx + c < 0$$
$$ax^2 + bx + c \leq 0$$

To solve a quadratic inequality, we must determine which part of the graph of a quadratic function lies above or below the x-axis. An inequality can therefore be solved graphically using a graph or algebraically using a table of signs to determine where the function is positive or negative.

Example 1: Solving quadratic inequalities

Question:
Solve for x: $x^2 - 5x + 6 \geq 0$

Illustrative Answer:
Factorize the quadratic
$(x - 3)(x - 2) \geq 0$

Determine the critical values of x
From the factorized quadratic we see that the values for which the inequality is equal to zero are $x = 3$ and $x = 2$. These are called the critical values of the inequality and they are used to complete a table of signs.

Complete a table of signs
We must determine where each factor of the inequality is positive and negative on the number line:

- to the left (in the negative direction) of the critical value
- equal to the critical value
- to the right (in the positive direction) of the critical value

In the final row of the table we determine where the inequality is positive and negative by finding the product of the factors and their respective signs.

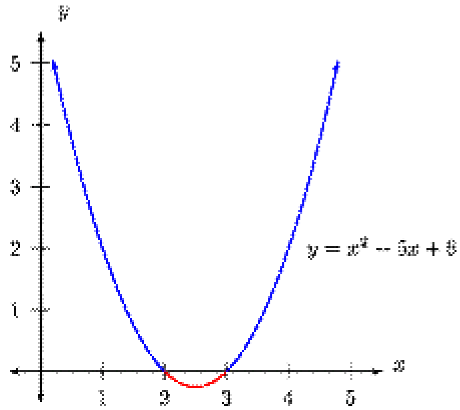
Critical values		$x = 2$		$x = 3$	
$x - 3$	−	−	−	0	+
$x - 2$	−	0	+	+	+
$f(x) = (x - 3)(x - 2)$	+	0	−	0	+

Table 1

From the table we see that $f(x)$ is greater than or equal to zero for $x \leq 2$ or $x \geq 3$.

A rough sketch of the graph
The graph below does not form part of the answer and is included for illustration purposes only. A graph of the quadratic helps us determine the answer to the inequality. We can find the answer graphically by seeing where the graph lies above or below the x-axis.

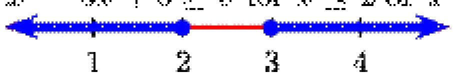
- From the standard form, $x^2 - 5x + 6$, $a > 0$ and therefore the graph is a “smile” and has a minimum turning point.
- From the factorized form, $(x - 3)(x - 2)$, we know the x-intercepts are $(2; 0)$ and $(3; 0)$.



The graph is above or on the x-axis for $x \leq 2$ or $x \geq 3$.

Write the final answer and represent on a number line

$x^2 - 5x + 6 \geq 0$ for $x \leq 2$ or $x \geq 3$



Example 2: Solving quadratic inequalities

Question:
Solve for x: $4x^2 - 4x + 1 \leq 0$

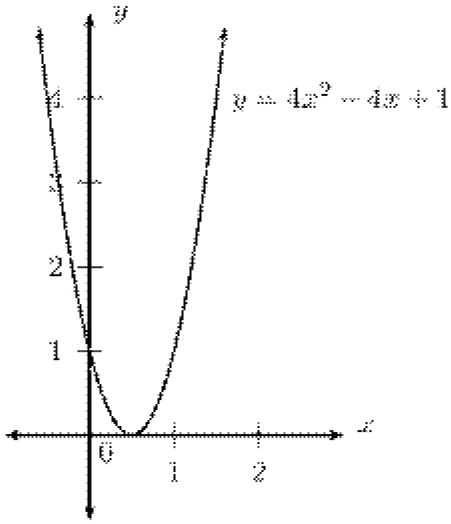
Illustrative Answer:
Factorized the quadratic
$(2x - 1)(2x - 1) \leq 0$

$$(2x - 1)^2 \leq 0$$

Determine the critical values of x
From the factorized quadratic we see that the value for which the inequality is equal to zero is $x = \frac{1}{2}$. We know that $a^2 > 0$ for any real number $a, a \neq 0$, so then $(2x - 1)^2$ will never be negative.

A rough sketch of the graph
The graph below does not form part of the answer and is included for illustration purposes only.

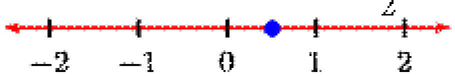
- From the standard form, $4x^2 - 4x + 1$, $a > 0$ and therefore the graph is a "smile" and has a minimum turning point.
- From the factorized form, $(2x - 1)(2x - 1)$, we know there is only one x-intercept at $\left(\frac{1}{2}; 0\right)$.



Notice that no part of the graph lies below the x-axis.

Write the final answer and represent on a number line

$4x^2 - 4x + 1 \leq 0$ for $x = \frac{1}{2}$



Example 3: Solving quadratic inequalities
Question:
Solve for x: $-x^2 - 3x + 5 > 0$

Illustrative Answer:
Examine the form of the inequality
Notice that the coefficient of the x^2 term is −1. **Remember** that if we multiply or divide an inequality by a negative number, then the inequality sign changes direction. So we can write the same inequality in different ways and still get the same answer, as shown below.
$-x^2 - 3x + 5 > 0$

Multiply by −1 and change direction of the inequality sign
$x^2 + 3x - 5 < 0$

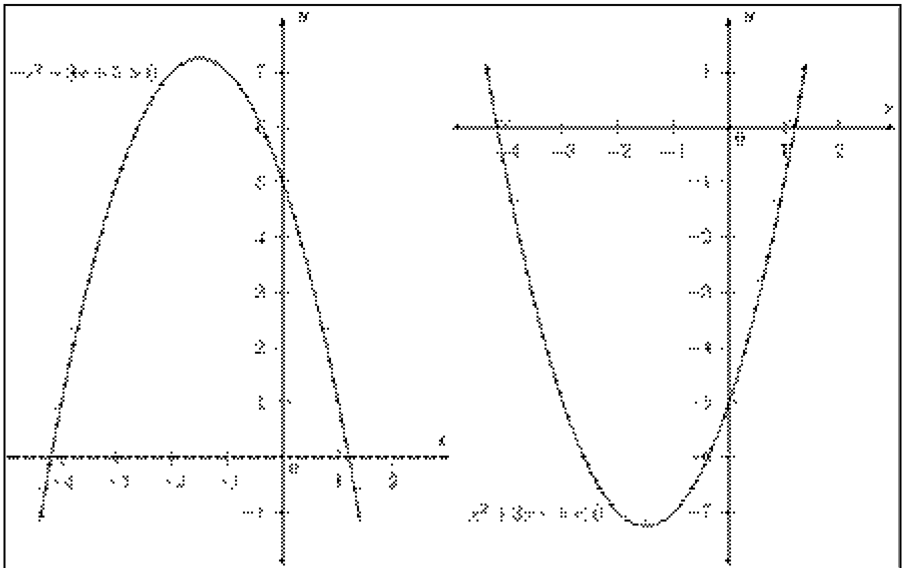


Table 2

From this rough sketch, we can see that both inequalities give the same solution; the values of x that lie between the two x-intercepts.

Factorized the quadratic
We notice that $-x^2 - 3x + 5 > 0$ cannot be easily factorized. So we let $-x^2 - 3x + 5 = 0$ and use the quadratic formula to determine the roots of the equation.

$$
\begin{aligned}
-x^2 - 3x + 5 &= 0 \\
x^2 + 3x - 5 &= 0 \\
\therefore x &= \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-5)}}{2(1)} \\
&= \frac{-3 \pm \sqrt{29}}{2} \\
x_1 &= \frac{-3 - \sqrt{29}}{2} \approx -4,2 \\
x_2 &= \frac{-3 + \sqrt{29}}{2} \approx 1.2
\end{aligned}
$$

Therefore we can write, correct to one decimal place,

$$
\begin{aligned}
x^2 + 3x - 5 &< 0 \\
\text{as } (x - 1,2)(x + 4,2) &< 0
\end{aligned}
$$

Determine the critical values of x
From the factorized quadratic we see that the critical values are $x = 1,2$ and $x = -4,2$.

Complete a table of signs

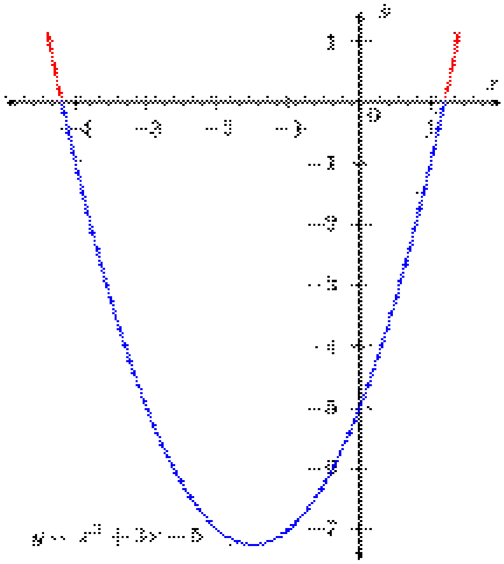
Critical values		$x=-4,2$		$x=1,2$	
$x+4,2$	$-$	0	$+$	$+$	$+$
$x-1,2$	$-$	$-$	$-$	0	$+$
$f(x)=(x+4,2)(x-1,2)$	$+$	0	$-$	0	$+$

Table 3

From the table we see that the function is negative for $-4,2<x<1,2$.

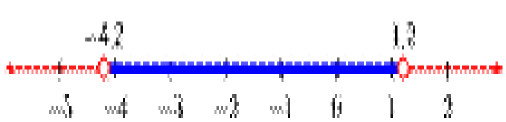
A sketch of the graph

1. From the standard form, x^2+3x-5, $a>0$ and therefore the graph is a "smile" and has a minimum turning point.
2. From the factorized form, $(x-1,2)(x+4,2)$, we know the x-intercepts are $(-4,2;0)$ and $(1,2;0)$.



From the graph we see that the function lies below the x-axis between −4, 2 and 1,2.

Write the final answer and represent on a number line
$x^2+3x-5<0$ for $-4,2<x<1,2$



Important: When working with an inequality in which the variable is in the denominator, a different approach is needed. Always remember to check for restrictions. *For more examples, please access this site and click ok and enlarge the page.*

http://www.teaching.martahidegkuti.com/shared/lnotes/4_collegealgebra/inequalities/inequality2.pdf
This site contains sample problems with solutions on solving quadratic inequalities.

http://www.regentsprep.org/Regents/math/algtrig/ATE6/Quadinequal.htm
Contains examples on how to solve quadratic inequalities

ACTIVITY 3. FIND MR. X

You have studied and explored illustrative examples on how to illustrate and solve quadratic inequalities. At this point, you will access a site and answer the activity. Be sure to answer the given problem first before clicking on the answer provided. http://www.regentsprep.org/Regents/math/algtrig/ATE6/quadinequalpractice.htm This site contains activity on solving quadratic inequalities. Each activity is provided with answer for you to check and verify your answer.

PROCESS QUESTIONS:

1. How do you find the activity?

2. How are the solutions of quadratic inequalities obtained?

3. How do you know that an interval is a solution to the given inequalities?

4. How are quadratic inequalities in two variables solved and graphed?

ACTIVITY 4. 3-2-1 PAUSE

Answer this formative assessment:

Three things I learned about illustrating and solving quadratic inequalities	Two things I want to know more	One thing I don't understand

End of FIRM UP:

In this section, the discussion was about the illustrating and solving quadratic inequalities by different methods. You also learned how to sketch the graph of quadratic inequalities.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

Now that you know the important ideas about quadratic inequalities, let's go deeper by moving on to the next section.

DEEPEN

Your goal in this section is to take a closer look at some aspects of quadratic inequalities. Solving real-life problems involving quadratic inequalities will be illustrated and shown in this section. Also, you will be challenged to answer problems involving quadratic inequalities in an interactive site.

D. Solving Real-Life Problems Involving Quadratic Inequalities

ACTIVITY 5. WATCH AND SOLVE ME!

You already understand how to solve quadratic inequalities; now, you will deepen your understanding by analyzing the example on how to apply quadratic inequalities to real- life problems illustrated in this site. Please click the link below.
http://www.dailymotion.com/video/xgeavf_how-to-apply-quadratic-inequalities-to-real-life-problems_tech
This contains video on how to apply quadratic inequality to real-life problems.

PROCESS QUESTIONS:

Answer the following:

- How did you find the problem?
- How was the problem solved?
- How can quadratic inequality help us solve real life problems?
- How can various real-life situations involving maximum and minimum values be solved and analyzed?

ACTIVITY 6. TRY ME!

Enrich your understanding on how to solve real –life problems involving quadratic inequalities by performing the problems in the site below. Access the link and click **practice icon** to show the problem, solve the problem and enter your answer and click **try icon** to know whether your answer is correct or not.
http://braingenie.ck12.org/skills/106856
Interactive site on solving real life problems involving quadratic inequalities

PROCESS QUESTIONS:

1. How did you find the activity in the site?

2. Were you able to solve the problems correctly? Why?

3. How do you solve real-life problems involving quadratic inequalities?

4. **How can various real-life situations involving maximum and minimum values be solved and analyzed?**

ACTIVITY 7. POSE IT!

Given the situations below, pose a problem and solve.

How is problem posing done?

Given a problem/situation, pose a problem by doing any or all of the following:

Asking questions (such as: "what – if-not" questions
Change the context
Change the numbers
Change the numbers of conditions
Reverse the given and the wanted information
Change some combination of context, numbers' condition, and given/wanted information

Kindly click this link: http://repository.nie.edu.sg/jspui/bitstream/10497/391/1/TL-18-1-64.pdf to see a sample of how problem posing is done.

(Note: Underline what you have posed)

Situations	Problem Posed	Solutions and Answer
Situation 1 **Weight in space.** After an astronaut is launched into space, the astronaut's weight decreases until a state of weightlessness is achieved. The weight of a 125-pound astronaut at an altitude of x kilometers above sea level is given by W = 125 (6400/6400 + x)^2. At what altitude is the astronaut's weight less than 5 pounds? .		
Situation 2 **Drug concentration.** For a drug to have a beneficial effect, its concentration in the bloodstream		

must exceed a certain value, which is called the minimum therapeutic level. Suppose that the concentration c (in mg/L) of a particular drug t hours after it is taken orally is given by $c = 20t/(t^2 + 4)$. If the minimum therapeutic level is 4 mg/L, determine when this level is exceeded.		
Situation 3 **Aircraft landing speed.** In the design of certain small turbo prop aircraft, the landing speed V(in ft/sec) is determined by the formula $W = 0.0033V^2S$, where W is the gross weight (in pounds) of the aircraft and S is the surface are (in ft^2) of the wings. If the gross weight of the aircraft is between 7500 pounds and 10,000 pounds and $S = 210ft^2$, determine the range of the landing speeds in miles per hour.		

PROCESS QUESTIONS:

1. How do you find the activity?

2. Did you experience difficulty in posing a problem? Why?

3. What generalization can you make about quadratic inequalities as shown in the three situations above? Support your statement with details from the above situations.

4. How do you pose and solve real-life problems involving quadratic inequalities?

5. **How can various real-life situations involving maximum and minimum values be solved and analyzed?**

End of DEEPEN:

In this section, the discussion was about how real-life situations involving quadratic inequalities are solved. Let us revisit the challenge problem we had earlier. Would there be changes in your answer now? Would you like to revise your answer and justification? Revise your answer and justification and report it through this link.
http://www.voki.com/create.php

Present your revised answer and justification here by creating your own video.

What new realizations do you have about the topic? What new connections have you made for yourself? Go back to your **KWL Chart**, accomplish the third column.

ACTIVITY 8. KWL CHART (REVISIT)

Answer the question in the third column only.

What I *know* about quadratic inequalities?	What I *want* to know about quadratic inequalities?	**What I *learn* about quadratic inequalities?**
		Put together what you've learned with a graphic organizer. Click the link below to make your graphic organizer. ***http://www.com/index-f.php***

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.

TRANSFER

Your goal in this section is apply your learning to real life situations. You will be given a practical task which will demonstrate your understanding and decision making ability.

ACTIVITY 9. DO THE TASK!

Prompt

You are interviewing an applicant for a sales agent position in your company. Based on the company's records, almost every company sales agent makes a total revenue of $150x - x^2$ pesos for selling x products in a week. If the applicant confidently assures you that he/she can easily make at least P 10,000 weekly revenues, how many products should he/she sell to meet this quota? Will you or will you not hire this applicant? Present your answer, justification and explanation through a power point presentation. Your answer and the data used should be accurate, while the justification and explanation should be valid and organized. Power point presentation must also be creative and clear.

Use the link below to publish your power point.

https://www.present.me/content/?utm_expid=39514825-9.RmSjXrfCTbOTiqm2SPHDWQ.0
Used in making a slide show of the power point

Use the rubric below as your guide in accomplishing the task and in rating your output. Your work should show the traits listed under Good or 3. If your work has these traits, you are ready to submit your work.

If you want to do more, your work should show the traits listed under Excellent or 4. If your work does not qualify to the traits under 3 or 4, revise your work before submitting it.
`

	4 *Excellent*	*3* Good	*2* Developing	*1* *Beginning*
Mathematical Concept and Justification	The explanation and justification are valid and show sophisticated	The explanation and justification are valid and show solid understanding	The explanation and justification are not valid and show	The explanation and justification are not valid

	understanding of the relevant ideas and processes. Main concept is accurately presented in an in-depth way that makes connections between each information. All sub concepts are logically organized which branch out appropriately from the main idea	of the relevant ideas and processes. All sub concepts are organized and consistently branch out from the main idea	somewhat limited understanding of the relevant ideas and processes and sub-concepts don't consistently branch out from the main idea. It shows some misunder-standing of key ideas.	and show little apparent understan-ding of the relevant ideas and issues and sub concepts don't consis-tently branch out from the main idea. It shows major misunder-standings of the key ideas.
Clarity and creativity of the power point presentation	The presentation is original, creative and attractive. Used striking graphics which enhanced the presentation and aid in comprehension; very clear and well-situated.	The presentation is creative and attractive. Used appropriate graphics which enhanced the presentation and aid in comprehension; clear and well-situated.	The presentation is creative but not attractive. Used some graphics which distract the meaning of the presenta-tion.	The presenta-tion is not clear. Many graphics and representa-tions used are inappro-priate and poorly selec-ted and do not enhance the topic; some graphics are ill-placed.
Accuracy of data	The data are credibly accurate and precise. Math concepts and procedures are applied appropriately. Use of efficient strategy that leads directly to a correct solution are highly evident.	The data are correct. Math concepts and procedures are applied correctly. Use of strategy that leads to a solution is evident.	The data contain minor errors. Some math concepts are used but not all of the necessary ones. Some strategies used are inappro-priate	The data contain major errors. Inappropriate math concepts or procedures are used. No evidence of a strategy or the strategy

				shown is inappropriate

End of TRANSFER:

In this section, your task was to present your decision together with justification and explanation through a power point presentation according to the criteria or rubric given. Now, you will summarize what you have learned by writing a synthesis journal.

ACTIVITY 10. SYNTHESIS JOURNAL

Write a synthesis journal by answering and reflecting on the following questions.

How did you find the task?

____

____

____

____

How did the task help you see the real world use of the quadratic inequalities?

____

____

____

How can various real-life situations involving maximum and minimum values be solved and analyzed?

You have completed this lesson. You've learned enough about quadratic inequality and its usefulness to real-life situation. Congratulations and be ready for the next lesson. END!

Lesson 3: Quadratic Functions

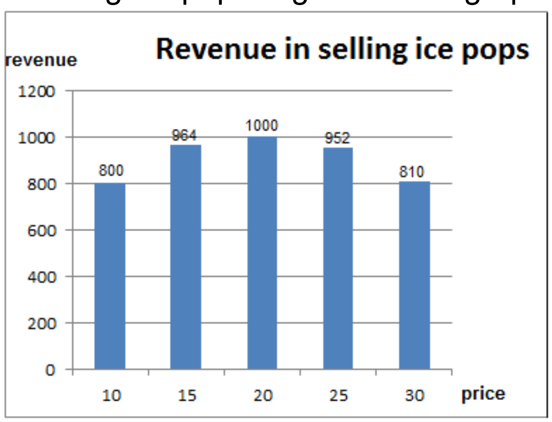
EXPLORE

In the previous lesson, you learned how quadratic equations and inequalities are solved. These skills are essential to fully understand quadratic functions.

Let's start the module by answering the problem below.

ACTIVITY 1. Data Analysis

The total revenue from selling ice pops is given in the graph below.



Process Questions:

A. What kind of model best fits the data above? Is it linear or quadractic? Why?

B. Find an equation to model these data.

C. At what price did the revenue of selling ice pops in the maximum level?

D. Using your model, predict the revenue if the ice pops is sold for 18 and 27 pesos?

E. How well does your model predict these values?

F. Give a reasoble domain and range for your model?

G. If you are to choose among the prices given, what price will you choose to maximize your revenue? Why?

H. How can various real-life situations involving maximum and minimum values be solved and analyzed?

ACTIVITY 2. Anticipation/ Reaction Guide

Respond to each statement twice: once before the lesson and again after the lesson.
Write **A** if you agree with the statement.
Write **B** if you disagree with the statement.

Before Lesson	*Statement*	*After Lesson*
	1. Quadratic function is used to model situations which follow a parabolic path or trend.	
	2. A quadratic model is the same as a linear model.	
	3. The maximum projection of sales and revenue can be determined by the vertex of the quadratic graph.	

	4. Models of quadratic functions such as graph, equations, and table of values are used to analyze, solve, and predict different situations/problems.	
	5. Quadratic function makes life harder since in the real life, it does not help at all.	
	1. One of the applications of quadratic functions has something to do with freely falling bodies.	
	2. If a maximum or minimum problem can be expressed as quadratic function, it can be solved by finding the vertex of the corresponding parabola.	
	3. There is one and only one method of determining the minimum or maximum values of a quadratic function.	
	4. Different methods can be used to solve math problems.	
	5. The speed of an object increases as it falls towards the ground. The speed d and the time t it will take for the object to reach the ground from where it came from are related by the linear function y = mx + b, where m is its slope and b is its y intercepts.	
	6. Quadratic functions has no real-world applications and parabolas does not exist in our everyday life.	
	7. Quadratic equation is a tool that you can use to find the roots to a quadratic function.	
	8. The x- intercepts of the parabola may imply break-even points where total revenue equals total costs of production.	
	9. If the parabola opens upward, the vertex is the maximum point and if the parabola opens downward, the vertex is the minimum point.	
	10. Revenue is the total payment received by business and usually calculated by multiplying the unit price by the number of units sold while profit is calculated by subtracting the total cost from revenue.	

End of EXPLORE:

You gave your initial ideas on the question given above by answering the first column of the ARG sheet.

Let's find out how others would answer the above and compare their ideas to our own. As you compare, you will find out if your ideas are in line with the standard. You will also learn other concepts which will help you complete a required project found at the end. This project is about making a s making a study and come up with a proposal or recommendations for better operation of a school canteen.

FIRM-UP

Your goal in this section is to learn and understand key concepts of graphing quadratic functions, writing the equation of quadratic function, and use the characteristics of its graphs and equation to analyze, solve and predict real life situations.

LESSON 3.1. GRAPHING QUADRATIC FUNCTIONS

In this lesson, be guided by the questions: **"What are the important characteristics of the graphs of quadratic functions? How are these used to analyze, solve, and predict real-life situations?"**

Recognizing Graphs of Quadratic Functions

KEY CONCEPTS

Quadratic function: A quadratic function can be written in the standard form or vertex form of a quadratic.

The greatest exponent is 2.

Standard form: $f(x) = ax^2 + bx + \boxed{c}$

Quadratic term

Linear term

Constant term

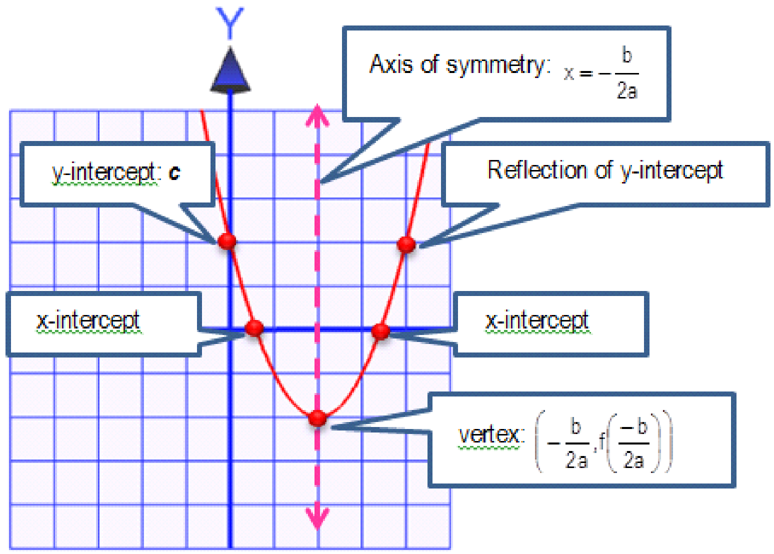
where a, b, and c are real numbers and a $\neq$ 0.

Example: $f(x) = 3x^2 - 2x + 1$

Vertex form: $f(x) = a\,(x - h)^2 + k$

Where a, h, and k are real numbers and a $\neq$ 0 and the vertex has coordinates (h, k).

Parabola : The graph of a quadratic function. A parabola opens either upward or downward.
Vertex. The point on a parabola where the graph changes direction. The maximum or minimum function value occurs at the vertex of a parabola.
Axis of Symmetry: It is an imaginary line through the graph of a parabola that divides the graph into two congruent halves. Each side of the parabola is a reflection of the other side.



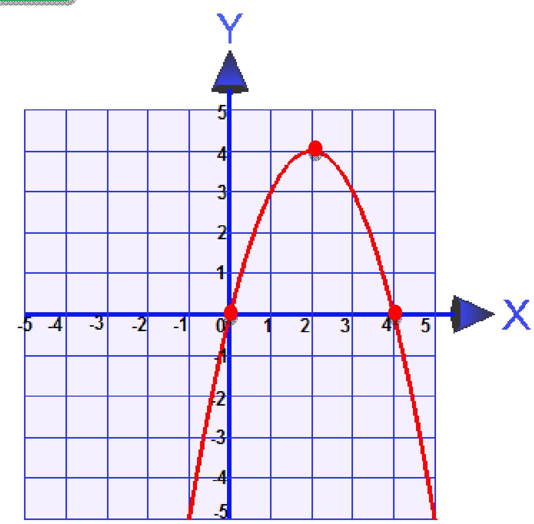
x-intercepts: These are points on the parabola that crosses the x-axis where y= 0.
y-intercept: Is a point on the parabola that crosses the y-axis where x = 0.
Domain: The set of all possible values for x.
The domain of all quadratic functions are ***all real numbers*** $(-\infty,+\infty)$.
Range: The set of all possible values for y.
If the vertex of the quadratic equation is (h,k) then the range is
$y \geq k$**,** if the graph opens upward
$y \leq k$**,** If the graph opens downward

Take down notes on the important terms in Evernote.

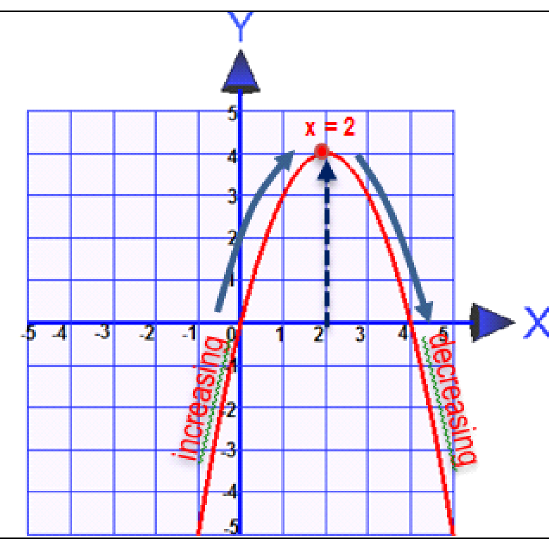
Example 1: Analyzing a Quadratic Graph

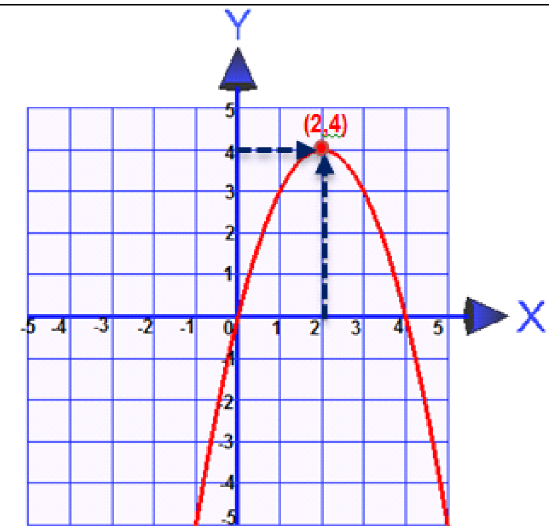
Use the graph of f(x) to estimate the following.

1. For what x-values is this curve increasing? Decreasing? Write your answer as inequalities.
2. Vertex
3. x-intercepts
4. y-intercept
5. axis of symmetry
6. reflection of the y-intercept
7. f(5)
8. What x-values will make f(x) = 3
9. Domain
10. Range



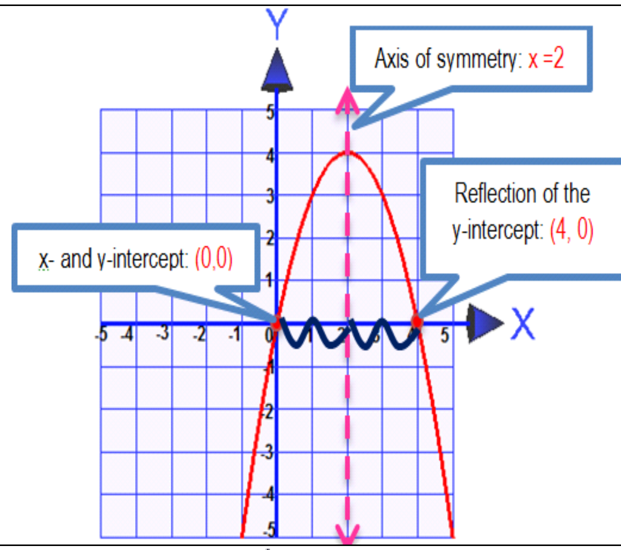
Solution

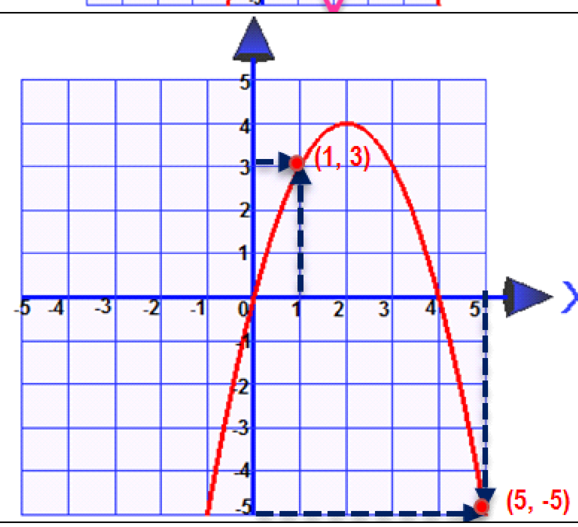
1. Reading the graph from left to right, we see that the curve is increasing for **x < 2** and decreasing for **x > 2.**	

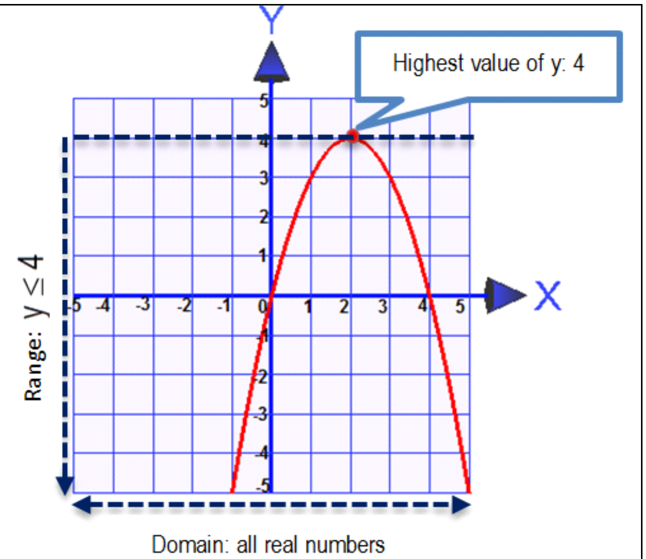
<table>
<tr>
<td>2. The curve changes from increasing to increasing when x = 2, so the vertex is(2, 4).</td>
<td>
</td>
</tr>
<tr>
<td>3. The curve crosses the x-axis at x = 0 and x = 4, so the x-intercepts are (0,0) and (4, 0).

4. The curve crosses the y-axis at y = 0, so the y-intercept is (0,0).

5. Axis of symmetry: x = 2

6. Reflection of the y-intercept: (4, 0)</td>
<td>
</td>
</tr>
<tr>
<td>7. When x = 5, the curve has an output of y = -5, so f(5) = -5.

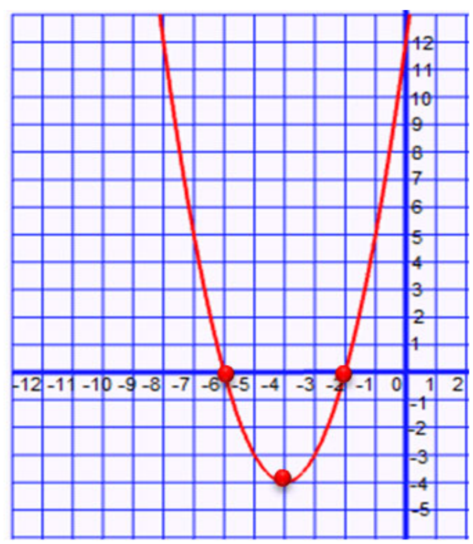
8. The output of the function is y = 3 when x= 1.</td>
<td>
</td>
</tr>
</table>

9. Domain: **all real numbers** since the parabola gets wider and wider and it will continue up to infinity. **10.** Range: $y \leq 4$ since the graph opens downward	

Example 2: Analyzing a Quadratic Graph

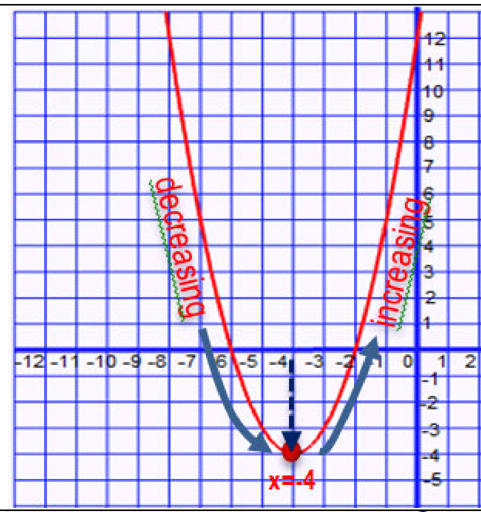
Use the graph of f(x) to estimate the following.

- For what x-values is this curve increasing? Decreasing? Write your answer as inequalities.
- Vertex
- x-intercept
- y-intercept
- axis of symmetry
- Reflection of the y-intercept
- f(-8)
- What x-values will make f(x) = 5
- Domain
- Range

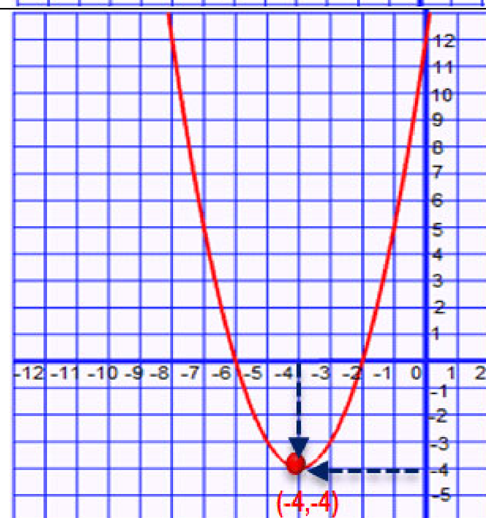


Solution

a. Reading the graph from left to right, we see that the curve is decreasing for **x < -4**and increasing for **x > -4.**



b. The curve changes from decreasing to increasing when x = -4, so the vertex is**(-4, -4).**

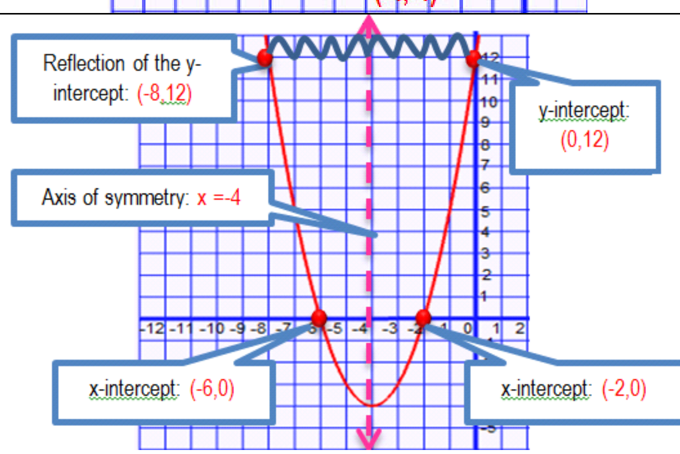


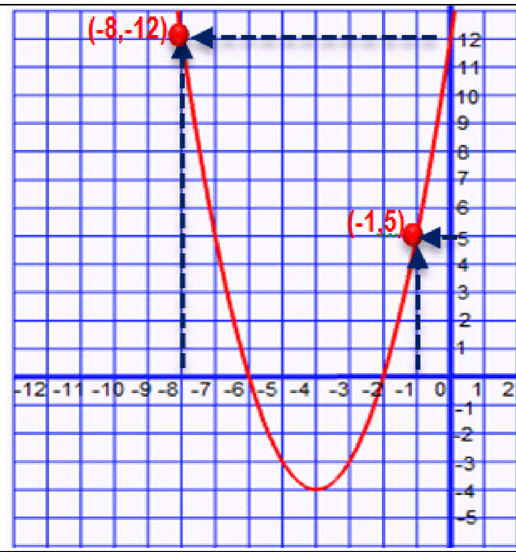
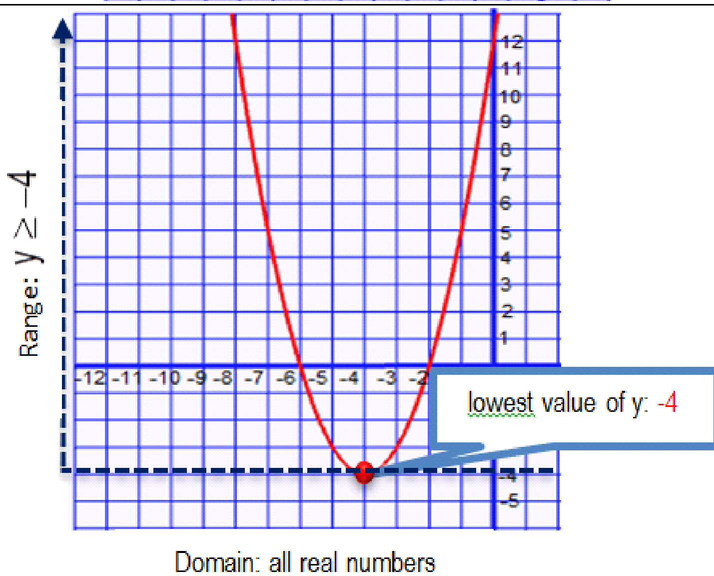
c. The curve crosses the x-axis at x = -6 and x = -2, so the x-intercepts are **(-6,0)** and **(-2, 0).**

d. The curve crosses the y-axis at y =12, so the y-intercept is **(0,12).**

e. Axis of symmetry: **x = -4**

f. Reflection of the y-intercept: **(-8, 12)**



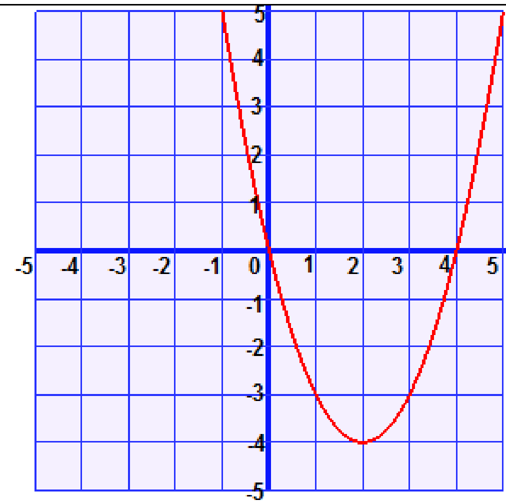
a. When x = -8, the curve has an output of about y = 12, so **f(-8) = 12**. **g.** The output of the function is y = 5 when x = -1.	
h. Domain: All real numbers **i.** Range: $y \geq -4$ since the graph opens downward	

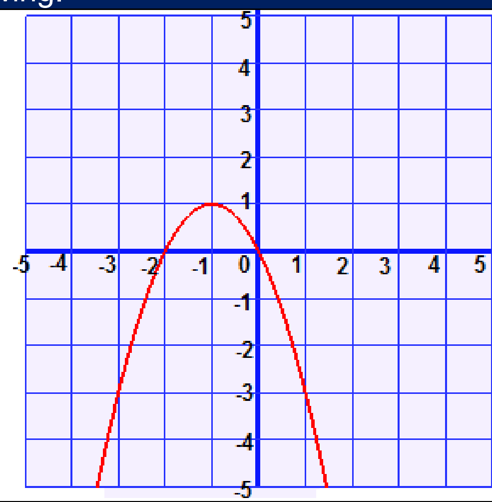
ACTIVITY 3. Skills Practice: Analyzing a Quadratic Graph

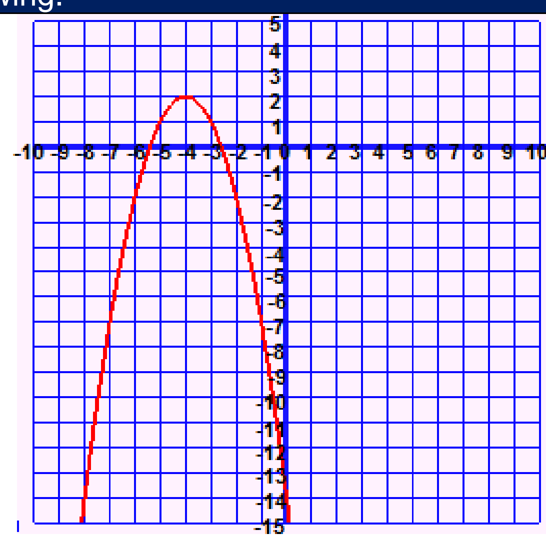
Analyze the components of each graph of a quadratic function.

SET A

Use the graph of f(x) to estimate the following.

a. For what x-values is this curve increasing? Decreasing? b. Vertex: c. x-intercepts: d. y-intercept: e. axis of symmetry: f. reflection of the y-intercept: g. f(1): h. What x-values will make f(x) = -3 : i. Range:	

Use the graph of f(x) to estimate the following.	
a. For what x-values is this curve increasing? Decreasing? **b.** Vertex: **c.** x-intercepts: **d.** y-intercept: **e.** axis of symmetry: **f.** f(-3): **g.** What x-values will make f(x) = -3: **h.** Range:	

Use the graph of f(x) to estimate the following.	
1. For what x-values is this curve increasing? Decreasing? 2. Vertex: 3. x-intercepts: 4. y-intercept: 5. axis of symmetry: 6. reflection of the y-intercept: 7. f(-7): 8. What x-values will make f(x) = 1: 9. Range:	

Skills Readiness Check: Reflect on the level of your performance today. Check the first column if you **need more practice** or you are now **ready to move on to the next activity**.

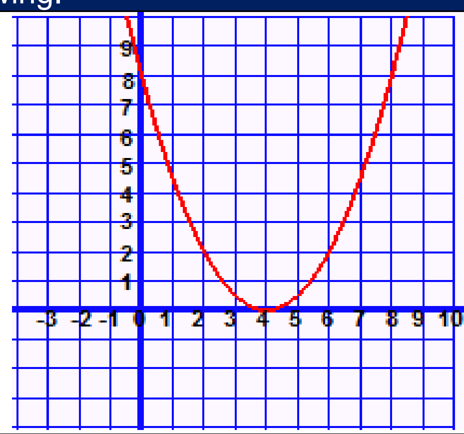
I Need more practice (if some of your answers are incorrect)	I am ready to move on to the next activity (if you incur 1-2 errors in the SET A practice problems)
Continue answering the next practice problems.	You may proceed to the next activity

SET B

Analyze the components of each graph of a quadratic function.

Use the graph of f(x) to estimate the following.

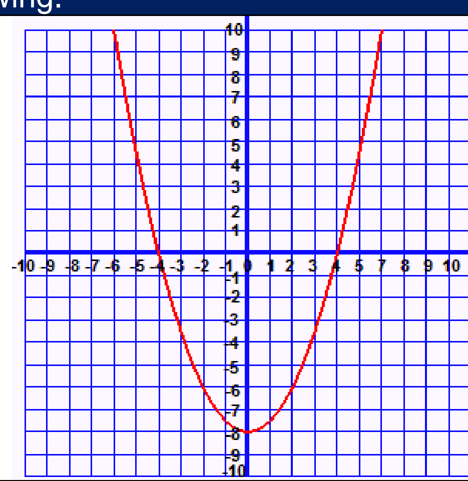
a. For what x-values is this curve increasing? Decreasing?
b. Vertex:
c. x-intercept:
d. y-intercept:
e. axis od symmetry:
f. reflection of the y-intercept:
g. f(2):
h. What x-values will make f(x) = 8
i. Range

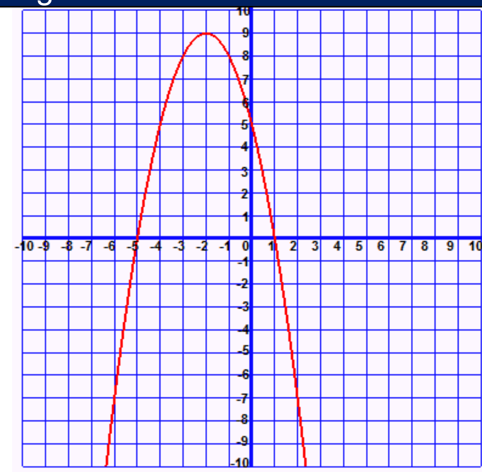


Use the graph of f(x) to estimate the following.

2.
a. For what x-values is this curve increasing? Decreasing?
b. Vertex:
c. x-intercepts:
d. y-intercept:
e. axis of symmetry:
f. reflection of the y-intercept:
g. f(-2):

h. What x-values will make f(x) = 5:
i. Range



Use the graph of f(x) to estimate the following.	
a. For what x-values is this curve increasing? Decreasing? b. Vertex: c. x-intercepts: d. y-intercept: e. reflection of the y-intercept: f. f(2): g. What x-values will make f(x) = 8 h. Range:	

*Click on **SUBMIT** if you have answered all the items before clicking **Answer Key** to check your answers.*

Example 3: Using a Table to find intercepts

Remember that the x-intercepts are the points on the parabola that crosses the x-axis where y= 0. It is represented by the ordered pair: (x, 0)

The y-intercept on the other hand is a point on the parabola that crosses the y-axis where x = 0. It is represented by the ordered pair: (0, y)

Use the table to find the y- and x-intercepts.

a.

Input (x)	Output (y)	
-10	114	
-7	0	x-intercept
-4	-78	
0	-126	y-intercept
3	-120	
9	0	x-intercept
12	114	

b.

Input (x)	Output (y)	
-8	-72	
-6	0	x-intercept
-4	4.8	
-2	7.2	
0	7.2	y-intercept
2	4.8	
4	0	x-intercept

ACTIVITY 4. Skills Practice: Using a Table to Find Intercepts

Use the table to find the x- and y-intercepts.

a.

Input (x)	Output (y)
-6	-9
-4	0
-2	5
0	6
1	5
3	0

x-intercepts: (___, ___) and (___, ___)
y-intercept: (___, ___)

b.

Input (x)	Output (y)
-15	15
-10	0
-5	-10
0	-15
5	-15
10	-10
15	0

x-intercepts: (___, ___) and (___, ___)
y-intercept: (___, ___)

c.

Input (x)	Output (y)
-2	20
0	9
2	2
3	0
4	-1
6	0
7	2

x-intercepts: (___, ___) and (___, ___)
y-intercept: (___, ___)

d.

Input (x)	Output (y)
-2	180
0	80
2	20
4	0
6	20
8	80

x-intercept: (___, ___)
y-intercept: (___, ___)

e.

Input (x)	Output (y)
-1	48
0	24
1	8
2	0
3	0
4	8
5	24

x-intercepts: (___, ___) and (___, ___)
y-intercept: (___, ___)

f.

Input (x)	Output (y)
-2	36
-1	25
0	16
1	9
2	4
3	1
4	0
5	1

x-intercept: (___, ___)
y-intercept: (___, ___)

Example 4: Using an Equation to find the y-intercept, the Axis of symmetry and the Vertex

1. Consider $f(x) = x^2 + 4x - 12$ where **a = 1, b = 4 and c= -12** 1. **Find the y-intercept** (0,***c***): **(0, -12)** 2. **Find the axis of symmetry:** $x = -\frac{b}{2a}$ $= -\frac{4}{2(1)}$ $= -2$ The equation of the axis of symmetry is **x = -2.**	3. **Coordinates of the vertex**: Substitute x = -2 in the function $f(x) = x^2 + 4x - 12$. $F(-2) = x^2 + 4x - 12$ $= (-2)^2 + 4(-2) - 12$ $= 4 - 8 - 12$ $= -16$ The vertex has coordinates **(-2, -16)**
2. Consider $f(x) = 3x^2 - 2x + 5$ where **a = 3, b = -2 , and c = 5**. 1. **Find the y-intercept** (0,***c***): **(0, 5)** 2. **Find the axis of symmetry:** $x = -\frac{b}{2a}$ $= -\frac{-2}{2(3)}$ $= \frac{2}{6} \text{ or } \frac{1}{3}$ The equation of the axis of symmetry is **x = $\frac{1}{3}$.**	3. **Coordinates of the vertex**: Substitute **x =** $\frac{1}{3}$ in the function f (x)= $3x^2 - 2x + 5$. $f\left(\frac{1}{3}\right) = 3x^2 - 2x + 5$ $= 3\left(\frac{1}{3}\right)^2 - 2\left(\frac{1}{3}\right) + 5$ $= 3\left(\frac{1}{9}\right) - \frac{2}{3} + 5$ $= \frac{1}{3} - \frac{2}{3} + 5$ $= -\frac{1}{3} + 5$ $= \frac{14}{3} \text{ or } 4.67$ The vertex has coordinates $\left(\frac{1}{3}, \frac{14}{3}\right)$**.**

ACTIVITY 5. Skills Practice: Finding the y-Intercept, the Axis of Symmetry and Coordinates of the Vertex Using an Equation

Consider the functions given below.	
i. $f(x) = x^2 - 2x - 8$ **a. Determine the values of a,** a = ____, b = ____ , c = ____ b. **Find the y-intercept** (0,***c***): **c. Find the axis of** d. **Coordinates of the vertex**:	ii. $f(x) = 2x^2 - 3x + 3$ **a. Determine the values of a, b,** a = ____, b = ____ , c = ____ b. **Find the y-intercept** (0,***c***): **c. Find the axis of symmetry:** d. **Coordinates of the vertex**: 1.

3. $f(x) = -x^2 - 2x + 3$ a. **Find the y-intercept**: **b. Find the axis of** c. **Coordinates of the vertex**:	4. $f(x) = 3x^2 + 6x + 7$ 1. **Find the y-intercept**: **2. Find the axis of symmetry:** 3. **Coordinates of the vertex**:

Analyzing The Effects of the Changing Values of a, h, and k in the Equation y= a(x – h)^2 + k of a Quadratic Function on its Graph

In the previous section, we looked at some graphical characteristics of quadratic functions such as the general shape and vertex and used several quadratic functions as model of real-world data. In this lesson, you will take a close look at the vertex form and see how the constants ***a, h*** and ***k*** affect the graph of the function. Still keep in mind the following questions: **"What are the important characteristics of the graphs of quadratic functions? How are these used to analyze, solve, and predict real-life situations?"**

ACTIVITY 6. Investigation on the Effects of h and k to the Graph of a Quadratic $f(x) = a(x-h)^2 + k$

INVESTIGATION

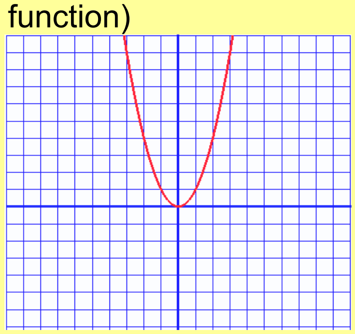
What do h and k do to the graph?

Study the followings graphs of quadratic functions.

1. Focus on ***k*** in ***f(x)= x^2 + k or f(x) = (x^2 + 0) + k***

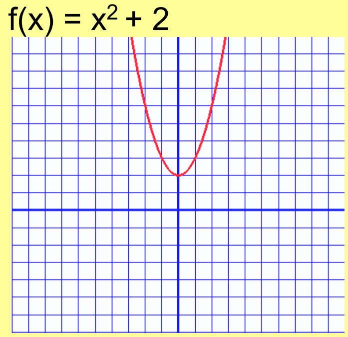
I. In these functions, we are considering how a positive ***k-value*** changes the graph of a basic quadratic function. Explain what a **positive k-value** does to the graph?

1. $f(x) = x^2$ (basic quadratic function)



Vertex: (0, 0)

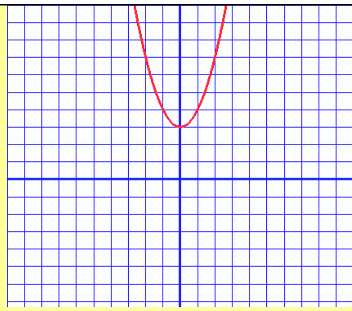
2. $f(x) = x^2 + 2$



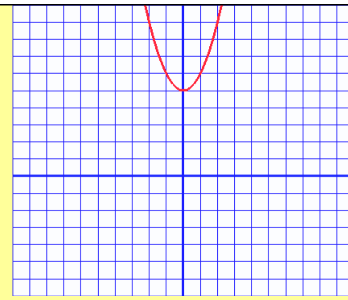
Vertex: (0, 2)

1. c. $f(x) = x^2 + 3$

2. $f(x) = x^2 + 5$



Vertex: (0, 3)

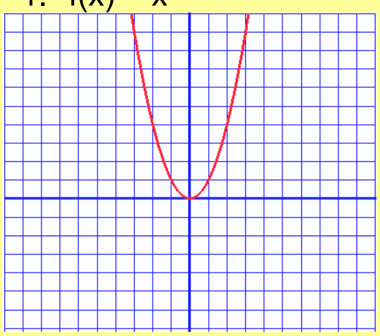


Vertex: (0, 5)

Answer: The positive k-value shifts each graph ___________(up or down).

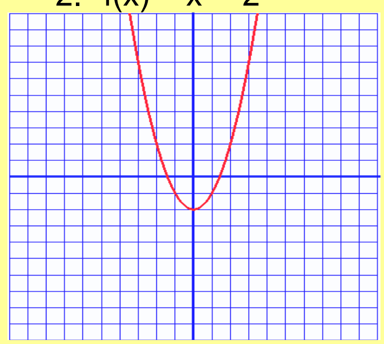
II. In these functions, we are considering how a negative ***k-value*** changes the graph of a basic quadratic function. Explain what a **negative k-value** does to the graph?

1. $f(x) = x^2$



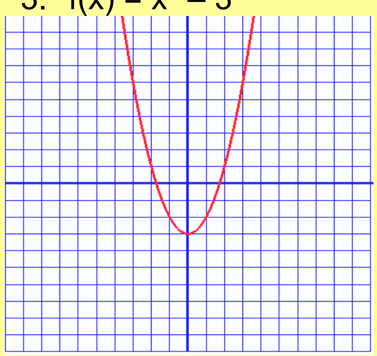
Vertex: (0, 0)

2. $f(x) = x^2 - 2$



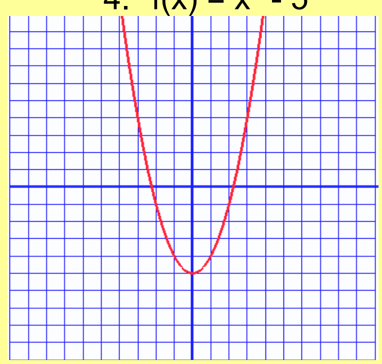
Vertex: (0, -2)

3. $f(x) = x^2 – 3$



Vertex: (0, -3)

4. $f(x) = x^2 - 5$



Vertex: (0, -5)

Answer: The negative k-value shifts each graph ___________(up or down).

2. Focus on ***h*** in ***$f(x)= (x – h)^2$***

In the vertex form $f(x) = a(x - h)^2 + k$, the value of h is confusing.

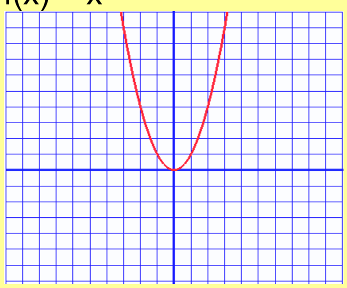
Consider the expression **x – h** and how the value of *h* appears once the expression has been simplified.

If h = 1, we have x – 1.
If h = -1, we have x – (-1) resulting to x + 1

Therefore, *h* appears to have the opposite sign

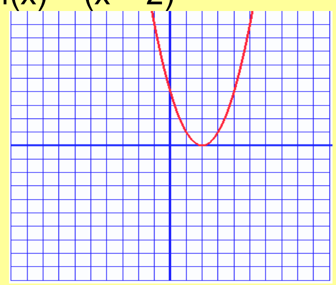
1. In these functions, we are considering how a positive h-value changes the graph of a basic quadratic function. Explain what a positive h-value does to the graph.

1. $f(x) = x^2$



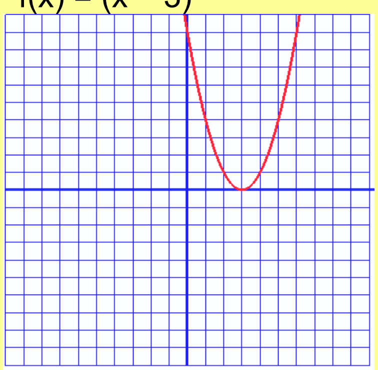
Vertex: (0, 0)

2. $f(x) = (x - 2)^2$



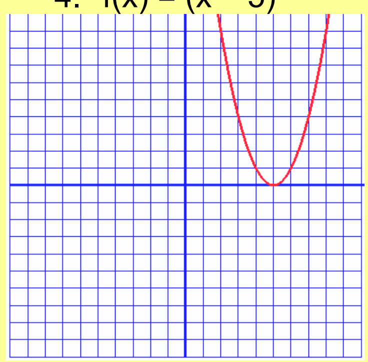
Vertex: (2, 0)

3. $f(x) = (x - 3)^2$



Vertex: (3, 0)

4. $f(x) = (x - 5)^2$



Vertex: (5, 0)

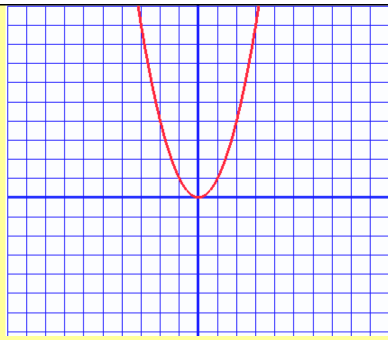
Answer:

The positive h-value shifts each graph to the___________(left or right).

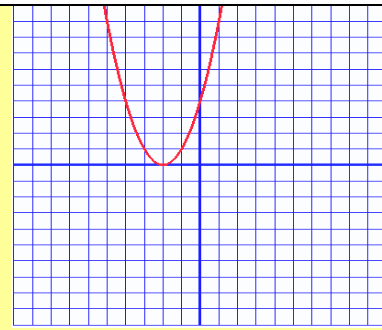
2. In these functions, we are considering how a negative h-value changes the graph of a basic quadratic function. Explain what a positive h-value does to the graph.

a. $f(x) = x^2$

b. $f(x) = (x + 2)^2$

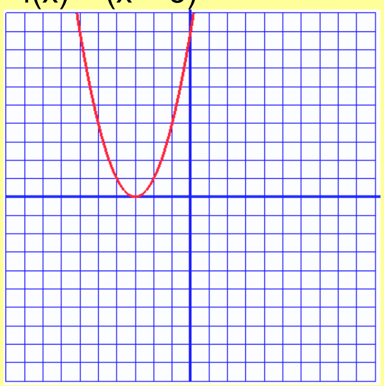


Vertex: (0, 0)



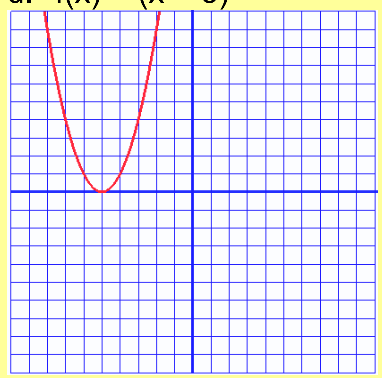
Vertex: (-2, 0)

c. $f(x) = (x + 3)^2$



Vertex: (-3, 0)

d. $f(x) = (x + 5)^2$



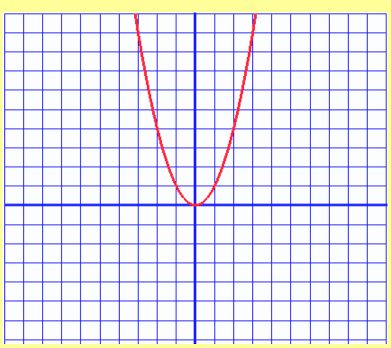
Vertex: (-5, 0)

Answer: The negative h-value shifts each graph to the____________(left or right).

3. Focus on ***h*** in ***f(x)= a(x – h)² + k, where a = 1***

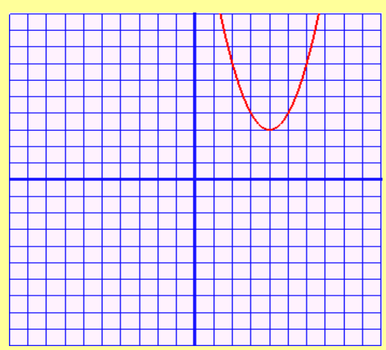
Given the graphs of the following functions below, find the vertex of the parabola.

1. $f(x) = x^2$



h = 0 , k = 0 , vertex: (0,0)

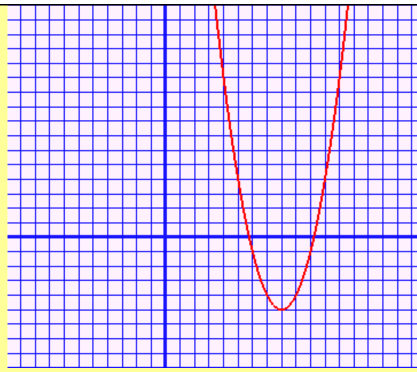
2. $f(x) = (x – 4)^2 + 3$



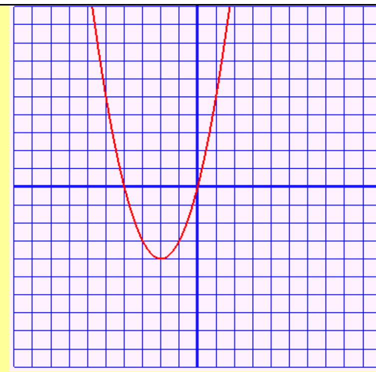
h =_____ , k = _____ , vertex: (____, _____)

3. $f(x) = (x – 8)^2 – 5$

4. $f(x) = (x + 2)^2 – 4$

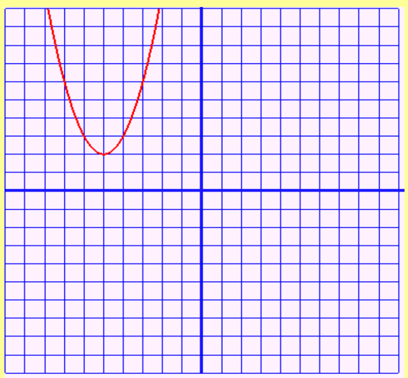


h =_____ , k = _____ , vertex: (____, _____)



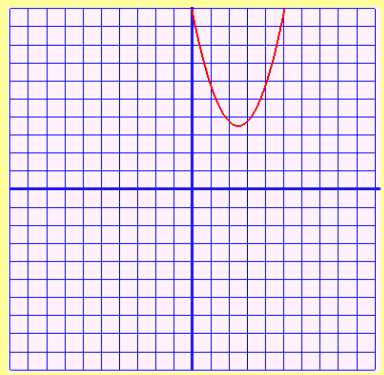
h =_____ , k = _____ , vertex: (____, _____)

5. $f(x) = (x + 5)^2 + 2$



h =_____ , k = _____ , vertex: (____, _____)

6. $f(x) = (x – 2.5)^2 + 3.5$



h =_____ , k = _____ , vertex: (____, _____)

How do we determine the vertex of a parabola from the equation $f(x)= a(x – h)^2 + k$?

Answer: The vertex of the parabola is (____, ____).

ACTIVITY 7. Concept Map

Sum-up what you have learned in the investigation you have just performed by writing on the blank boxes the effects of h and k to the graph of a quadratic function.

Effects of *h* and *k*In the graph of a quadratic *f(x) =a(x-h)²+ k)*

- **Effects of *h* to the graph**
  - If *h* is greater than 0 (*positive)* → ______
  - If *h* is lesser than 0 (*negative*) → ______
- **Effects of *k* to the graph**
  - If *k* is greater than 0 ((*positive*) → ______
  - If *k* is lesser than 0 (*negative*) → QUADRATIC FUNCTIONS

CONCLUSION:

In the graph of the quadratic function in the vertex form $f(x) = a(x-h)^2 + k)$,
the horizontal shifting of the parabola to either left or right depends on the value of _____;

the vertical shifting of the parabola to either up or down depends on the value of _____.

The vertex of the parabola can be read directly from the vertex form of the quadratic $f(x) = a(x - h)^2 + k$ where the vertex = (____, _____).

CONCLUSION:

In the graph of the quadratic function in the vertex form ***f(x) =a(x-h)²+ k),***

the horizontal shifting of the parabola to either left or right depends on the value of ***h***; the vertical shifting of the parabola to either up or down depends on the value of ***k***.

The vertex of the parabola can be read directly from the vertex form of the quadratic $f(x) = a(x - h)^2 + k$ where the vertex = ***(h, k)***.

To complete our examination of the vertex form, we still have to investigate what the value of ***a*** does to the graph of a parabola.

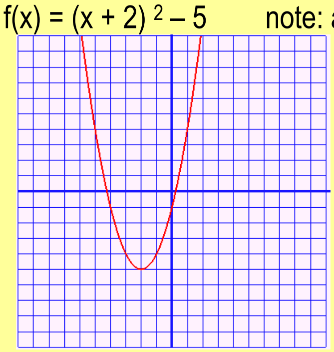
ACTIVITY 8. Investigation on the Effects of *a* to the Graph of a Quadratic $f(x) = a(x-h)^2 + k$

INVESTIGATION

What does *a* do to the graph?

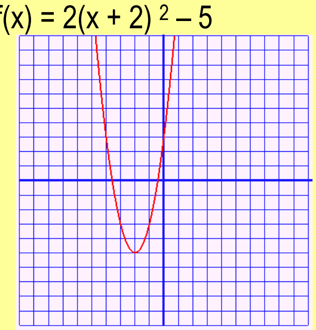
a. Examine the each parabola below where a > 1. Then determine the vertex of each.

a. $f(x) = (x + 2)^2 - 5$ note: a = 1



Vertex : (___, ___)

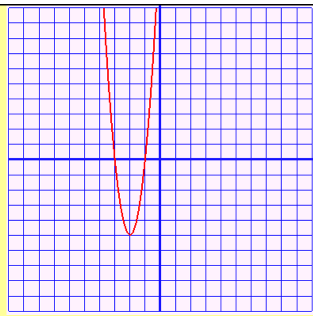
b. $f(x) = 2(x + 2)^2 - 5$



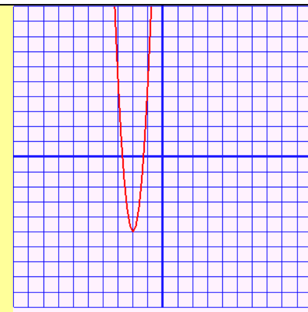
Vertex : (___, ___)

c. $f(x) = 5(x + 2)^2 - 5$

d. $f(x) = 10(x + 2)^2 - 5$



Vertex : (___, ___)



Vertex : (___, ___)

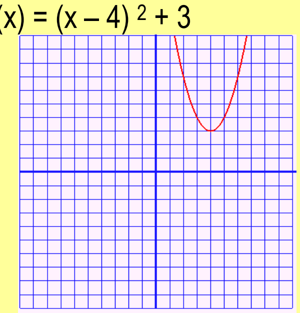
1. Does the value of ***a*** affect the vertex of the graph? (Yes or No)

2. What happened to the width of the parabola as the $|a|$ increases (absolute value of a)? (narrower or wider)

3. Explain what a positive ***a***-value does to the graph? (opens upward or downward)

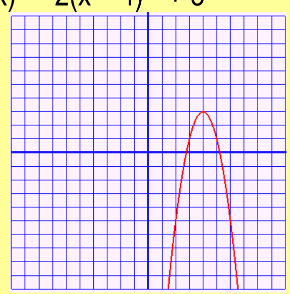
b. Examine the each parabola below where a < 1. Then determine the vertex of each.

a. $f(x) = (x - 4)^2 + 3$



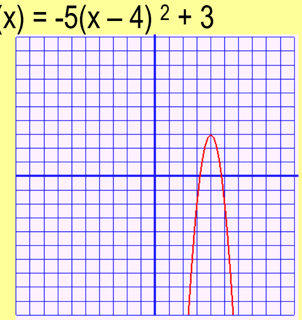
Vertex : (___, ___)

b. $f(x) = -2(x - 4)^2 + 3$



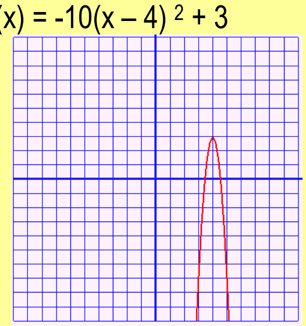
Vertex : (___, ___)

c. $f(x) = -5(x - 4)^2 + 3$



Vertex : (___, ___)

d. $f(x) = -10(x - 4)^2 + 3$



Vertex : (___, ___)

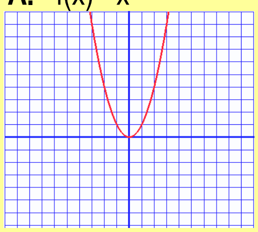
1. Does the value of ***a*** affect the vertex of the graph? (Yes or No)

2. What happened to the width of the parabola as the $|a|$ (absolute value of a) increases? (narrower or wider)

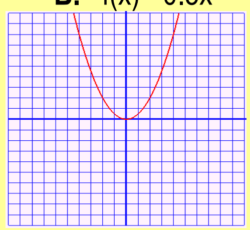
3. Explain what a negative ***a***-value does to the graph? (opens upward or downward)

c. Some other values of a and its effect to the graph.

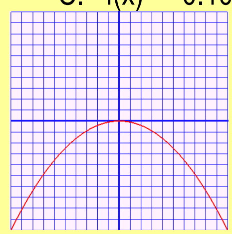
A. $f(x) = x^2$



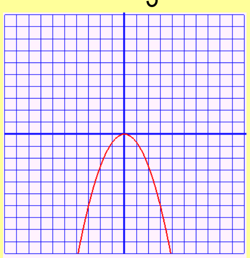
B. $f(x) = 0.5x^2$



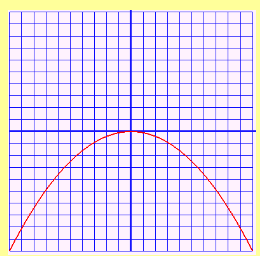
C. $f(x) = -0.10x^2$



D. $f(x) = -\frac{2}{3}x^2$



E. $f(x) = -\frac{1}{10}x^2$



What happened to the width of the parabola as the $|a|$ (absolute value of a) decreases? (narrower or wider)

ACTIVITY 9. Concept Map

Sum-up what you have learned in the investigation you have just performed by writing on the blank boxes the effects of ***a*** to the graph of a quadratic function.

Effects of *a* in the Graph of the Quadratic Function in the Vertex Form $f(x) = a(x-h)^2 + k$

If $a > 0$ (positive)	If $a < 0$ (negative)	If $\|a\| > 1$	If $\|a\| < 1$

CONCLUSION:

a will affect the __________of the graph as well as determine whether the parabola opens ___________ or ____________.

CONCLUSION: ***a*** will affect the **width** of the graph as well as determine whether the parabola opens **upward** or **downward.**

ACTIVITY 10. Concept Map: Summing up the Results in the investigation

Effects of *a, h, and k* in the Graph of the Quadratic Function in the Vertex Form $f(x) = a(x-h)^2 + k$

$$f(x) = a(x-h)^2 + k$$

Click on ***SUBMIT*** *if you have completed the graphic organizer before clicking* ***Answer Key*** *to check your answers.*

SUBMIT

In the activities above, you have found out the effects of a, h, k in the graph of a quadratic function. Click on http://www.mathxtc.com/Downloads/NumberAlg/files/Quad%20Functions%20and%20Models.ppt for input and examples.

In the next lesson, we will apply what you learned in the previous lessons to analyze the equation of the quadratic function in the vertex form **$f(x) = a(x-h)^2 + k$.**

ACTIVITY 11. Skills Practice: Analyzing Graphs of Quadratic $f(x) = a(x-h)^2 + k$ by Determining the a, h, k, Axis of Symmetry, Direction, and Width

SET A

		My Answer	Remarks (right or wrong)
1. $f(x) = (x - 3)^2$	**In vertex form $f(x) = a(x-h)^2 + k$**		
	a		
	h		
	k		
	vertex		
	Axis of symmetry (x=h)		
	Direction		
	Width		
	Shifting h/k		
	Total number of correct answers		

Click Answer Key to the view the correct answers.

		My Answer	Remarks (right or wrong)
2. $f(x) = 0.35(x - 2)^2 + 1$	In vertex form $f(x) = a(x-h)^2 + k$		
	a		
	h		
	k		
	vertex		
	Axis of symmetry (x=h)		
	Direction		

	Width		
	Shifting h/k		
	Total number of correct answers		

Click on the answer key to check whether your answers are correct or wrong.

		My Answer	Remarks (right or wrong)
3. $f(x)= x^2 – 2x – 5$	In vertex form $f(x) =a(x-h)^2 + k$		
	a		
	h		
	k		
	vertex		
	Axis of symmetry (x=h)		
	Direction		
	Width		
	Shifting h/k		
	Total number of correct answers		

Skills Readiness Check: Reflect on the level of your performance for this lesson. Check the first column if you **need more practice** or you are now **ready to move on to the next activity**.

I Need more practice (if some of your answers are incorrect)	I am ready to move on to the next activity (if you incur only few errors in the first three practice problems)
Go back to the PowerPoint you have viewed to review the input and examples. Then continue answering the next set of practice problems.	You may proceed to the next activity

SET B

1. $f(x) = -3(x + 5)^2$		My Answer	Remarks (right or wrong)
	In vertex form $f(x) = a(x-h)^2 + k$		
	a		
	h		
	k		
	vertex		
	Axis of symmetry (x=h)		
	Direction		
	Width		
	Shiftingh/k		
	Total number of correct answers		

2. $f(x) = -4(x + 3)^2 - 1$		My Answer	Remarks (right or wrong)
	In vertex form $f(x) = a(x-h)^2 + k$		
	a		
	h		
	k		
	vertex		
	Axis of symmetry (x=h)		
	Direction		
	Width		
	Shiftingh/k		
	Total number of correct answers		

3. f(x)=		My Answer	Remarks (right or wrong)
	In vertex form $f(x) = a(x-h)^2 + k$		
	a		
	h		
	k		
	vertex		
	Axis of symmetry (x=h)		
	Direction		

$-3x^2 + 24x$	Width		
	Shifting h/k		
	Total number of correct answers		

Skills Evaluation:

1. How would you compare your performance in the first set of problems to the second set of problems?

2. What did you do in order to overcome the difficulties you have encountered in solving the set of problems?

3. What positive attitude have you learned while doing the activity above?

Click on *SAVE* if you are through answering all the items. Post your answers in the **Skills Evaluation Check** *Discussion Forum.*

Graphing Quadraticsin Vertex , Standard, and Intercept Forms

In the previous activities, you have learned how to determine the vertex, axis of symmetry, intercepts, and the effects of a, h, and k to the graph.
You will use these in graphing quadratic equations in the vertex, standard, and intercept forms.

ACTIVITY 12. Graphing quadratic functions in vertex, standard and intercept forms.

Do the following:

1. Watch a video from the site http://www.youtube.com/watch?v=RPOkbkL2fkk. This video will teach you how to graph a quadratic function in vertex form.
2. List down the steps in Evernote.
3. Watch a video from the site http://www.youtube.com/watch?v=LoscdQJyZPo. This video will teach you how to graph a quadratic function in standard form.
4. List down the steps in Evernote.

5. Watch a video from the site http://www.youtube.com/watch?v=pme-ZrnCl3o.This video will teach you how to graph a quadratic function in intercept form.
6. List down the steps in Evernote.

A. View a PowerPoint presentation from http://www.quia.com/files/quia/users/pamm1409/HALG2/4.1_PowerPoint_(formerly_5.1).ppt (this site provides examples and practice problems)

Process Questions:

a. What are the things you need to know and have inorder for you to graph quadratic functions in
a. vertex form
b. standard form
c. intercept form

ACTIVITY 13. Quiz: Graphing quadratic functions and writing the equation of quadratic function using its graph

Download thispdf file from the site http://frontenacss.limestone.on.ca/teachers/dcasey/0E67293F-00870BC8.84/3_Graphing_Quadratic_Functions_Worksheet.pdf.

Instruction:

1. Print all 9 pages
2. Graph items 1-32.
3. Write the quadratic equation for items 33- 41.
4. Email your output to your teacher or hand-it in to him/her during your face-to-face meeting.

LESSON 3.3. QUADRATIC MODELS

ACTIVITY 14. Wrting the equation of quadratic function given the table of values, graph, and zeros.

Do the following:

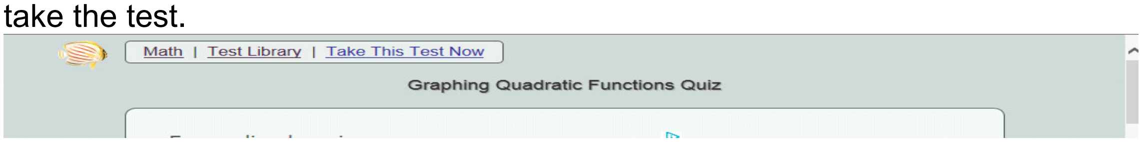
1. http://www.youtube.com/watch?v=Sje8fgdBY2Y .This video will teach you how to write a quadratic equation given the table of values.
2. Write the steps in Evernote.
3. http://www.youtube.com/watch?v=vAPPYoBV2Ow. This video will teach you how to write a quadratic equation given the graph of the function.
4. Write the steps in Evernote.
   http://www.youtube.com/watch?v=89UYFIRkgP8 and http://www.youtube.com/watch?v=OXViZtD2BTE&list=PLaIUqRPLCu47yy0WoM1NzA6-No4Jw1I3A.These video will teach you how to write a quadratic equation given the zeroes of the function.
5. Write the steps in Evernote.

ACTIVITY 15. Interactive Quiz.

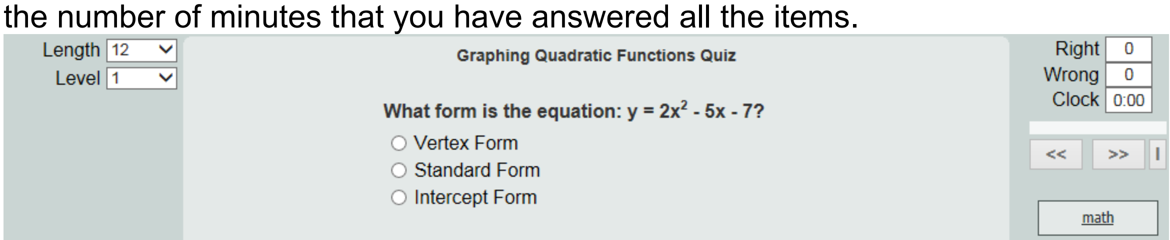
Take an interactive quiz from the site http://www.thatquiz.org/tq/previewtest?M/N/Q/5/8S8C1353086320 . This is a 10-item interactive quiz about quadratic function.

Procedure:

1. Click on the ***Take This Test Now*** icon on the upper part of the page to take the test.



2. Click on the correct answer. On the right part of the page you will view how many items you have correctly answered and wrongly answered and the number of minutes that you have answered all the items.



3. After taking all the items, you will view the items that you have wrongly answered with its corresponding answer.

4. If you want to take the test again, just click on **Reset.**

MAXIMUM AND MINIMUM VALUES OF QUADRATIC FUNCTION

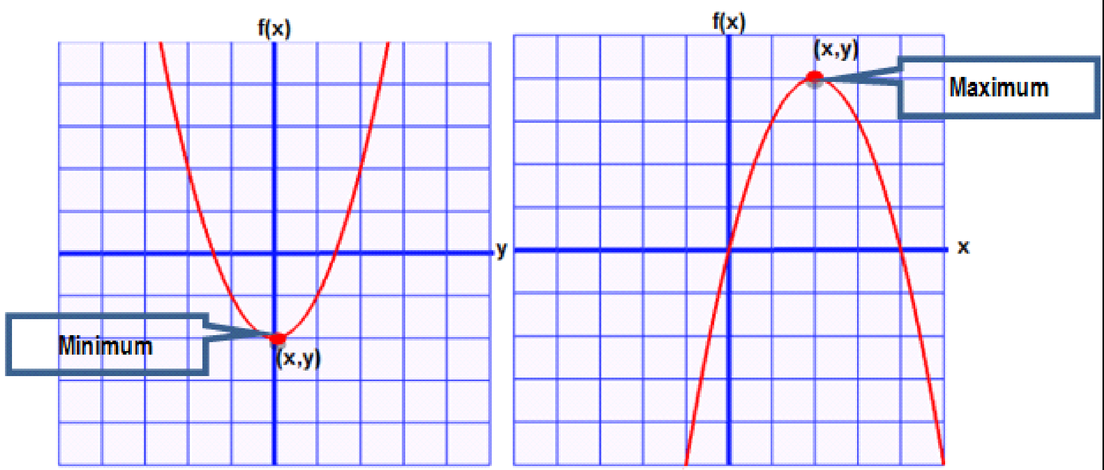
KEY CONCEPTS

Maximum and Minimum Values: These are the y-coordinate of the vertex of a quadratic function.These values represent the greatest or lowest possible value the function can reach.

The graph of $f(x) = ax^2 + bx + c$, where $a \neq 0$,

1. opens up and has a minimum value when a > 0 (positive real numbers), and
2. opens down and has maximum value when a < 0 (negative real numbers.

Model:



The y-coordinate is the minimum value. The y-coordinate is the maximum value.

The terms minimum point and minimum value are not interchangeable. The **minimum point**on the graph of a quadratic function is the **ordered pair** that describes the location of the vertex. The**minimum value**of the function is the **y-coordinate**of the minimum point. It is the smallest value obtained when f(x) is evaluated for all values of x.

Example 4: Maximum and Minimum Values Using an Equation

Consider $f(x) = -x^2 - 10x + 16$

1. **Determine the values of a, b, and c.**
   a = -1, b = -5, and c = 6
2. **Determine whether the function has a maximum or minimum value.**
   The value of a = -1, meaning the function has a maximum value. This also means that the graph opens downward.

3. **State the maximum or minimum value of the function.**

Solve for the x-coordinate by using the equation of the axis of symmetry	Solve for the y-coordinate by substituting the x value to the given equation.
$x = -\frac{b}{2a}$ $= -\frac{-10}{2(-1)}$ $= -5$	$f(x) = -x^2 - 10x + 16$ $= -(-5)^2 - 10(-5) + 16$ $= -25 + 50 + 16$ $= 41$

The maximum value of the function is **41.**

ACTIVITY 16. Skills Practice: Determining the Maximum or Minimum Values of a Quadratic Function

Consider the following functions:

$f(x) = x^2 - 8x + 64$

1. **Determine the values of a, b, and c.**

a =___, b =___, and c = ___

2. **Determine whether the function has a maximum or minimum value.**

The value of a =___, meaning the function has a ___________ value. This also means that the graph opens ______________.

3. **State the maximum or minimum value of the function.**

Solve for the x-coordinate by using the equation of the axis of symmetry $x = -\frac{b}{2a}$	Solve for the y-coordinate by substituting the x value to the given equation $f(x) = x^2 - 8x + 64$

The ___________value of the function is _____.

$f(x) = -2x^2 + 8x + 3$

1. **Determine the values of a, b, and c.**

   a =___, b =___, and c = ___

2. **Determine whether the function has a maximum or minimum value.**

   The value of a =___, meaning the function has a ___________ value. This also means that the graph opens ___________.

3. **State the maximum or minimum value of the function.**

Solve for the x-coordinate by using the equation of the axis of symmetry $x = -\frac{b}{2a}$	Solve for the y-coordinate by substituting the x value to the given equation $f(x) = -2x^2 + 8x + 3$.

The ___________value of the function is _____.

PROCESS QUESTIONS:

1. How are the maximum and minimum values related to the coordinates of the vertex of the graph of the function?

2. Are there other ways of determining the y-coordinate of the vertex aside from computing a function of x or f(x)? Discuss.

Example Application Problems of Finding the Maximum and Minimum Values of a Quadratic Function

1. Elmer is organizing a fund raising basketball tournament in their barangay. He plans to charge Php 20 entry fee for each of the 80 players. He recently decided to raise the entry fee by Php 5 and 5 fewer players entered with the increase. How much would Elmer charge in order to maximize the income?

Solution:

Define the variables

Let x = the number of price increases
Let f(x) = the total fund raised as a function of x

Words to Function form

fund raised equals fee times the number of entrants
$F(x) = (20+5x) \bullet (80 - 5x)$

3. Solve for the value of x which is the x-coordinate of the vertex/axis of symmetry

Write the equation in standard form

$F(x) = (20 + 5x) \bullet (80 - 5x)$	Distribute
$= (20 \bullet 80) + (20 \bullet -5x) + (5x \bullet 80) + (5x \bullet -5x)$	Multiply
$= 1600 - 100x + 400x - 25x^2$	Simplify
$= \mathbf{-25x^2 + 300x + 1600}$	Write in $ax^2 + bx + c$ form

Use the formula for the axis of symmetry, $x = -\frac{b}{2a}$ to find the x-coordinate which is the number of increases

$$x = -\frac{b}{2a} = -\frac{300}{-25} = 6$$

Elmer needs to have 6 increases. So the maximum entry fee is 20 + 5(6) = 50 pesos.

What will be the maximum value of the fund raised?

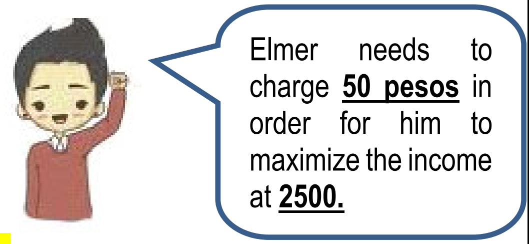
Evaluate F(6) by substituting x = 6 to the equation

$F(x) = -25x^2 + 300x + 1600$

$= -25(6)^2 + 300(6) + 1600$

$= -900 + 1800 + 1600$

$= 2500$



The maximum fund raised will be 2500.

Application in Fireworks.

A skyrocket is shot into the air. It's altitude ***h*** in feet after ***t*** seconds is given by the function $h = -16t^2 + 128t$.

1. In how many seconds does the skyrocket reach its maximum altitude?

Solve for *t* (represents the x-coordinate of the maximum point) where a = -16, b = 128

$$t = -\frac{b}{2a}$$

$$t = -\frac{128}{2(-16)}$$

$$t = \frac{128}{32}$$

$$t = 4$$

The skyrocket can reach its maximum height in **4 seconds.**

1. What is the skyrocket's maximum altitude?

Solve for *h* (represents the y-coordinate of the maximum point) by substituting t= 4 in the function $h = -16t^2 + 128t$.

$h = -16t^2 + 128t$
$h = -16(\mathbf{4})^2 + 128(\mathbf{4})$
$h = -16(16) + 512$
$h = -256 + 512$
$h = 256$

The skyrocket's maximum height is **256 ft.**

ACTIVITY 17.Refresh Online!

DESCRIPTION: Visit the suggested sites below to reinforce your understanding of the concepts. Make an outline in your notes and share your insights to your talk group by signing up in voxopop.com.

1. http://www.purplemath.com/modules/quadprob.htm
http://www.youtube.com/watch?v=qS3OpIAAG6Y
These websites show illustrative examples of the applications of quadratic functions.

2. http://www.youtube.com/watch?v=ipBoVSMVzp0
This website is a video on how a problem regarding maximum area is solved.

3. http://www.youtube.com/watch?v=3ShdbCN7i_Y
This website is a video on how real-life problems involving time and vertical height are solved?

4. http://www.youtube.com/watch?v=ssb0yxpNmiI
This video illustrates how we determine break- even points in business related problems that are modelled by quadratic functions.

5. http://www.youtube.com/watch?v=KIGmg4yWRKE
This video showcases how typically quadratic function models are used in order to find either maximum or minimum values of something.

PROCESS QUESTIONS:

1. How did the sites help you enhance your understanding of the concepts? Did they make sense?
2. What is the essence of quadratic functions in relation to real-life experiences?
3. Why do we use different methods to solve math problems?
4. Why do we need to use appropriate method in solving math problems?
5. What are the types of problems that require minimum values?

6. What are the types of problems that require maximum values?

7. How this activity enhances your self- confidence in solving problems?

ACTIVITY 18. Skills Readiness Check

Reflect if you already have the skill in solving real-life problems involving quadratic functions. Kindly check your readiness on the table below.

I do not have a full grasp of the information and skills needed in solving real-life problems related to quadratic functions	I have a full grasp of the information and skills needed in solving real-life problems related to quadratic functions
Here is what you are going to do: Go back to the examples and videos given above. You may seek help from anybody to assist you in understanding the examples and videos given. Then go back to this activity and check the next column if you are ready to solve the next set of problems.	*Here is what you are going to do:* You may proceed to the next activity

ACTIVITY 19. Apply it !

Answer the quiz on the site below and the items that follows. Write your answers in the box provided and click submit.

A. http://math.about.com/od/Exercises/a/Quadratic-Quiz.htm
This site contains a 6-item quiz on application of quadratic functions.

B. Answer the following problems.

1. A baseball player hits the ball. Its motion is modeled by the graph above.
   1. Estimate the maximum height that the ball can reach.
   2. Estimate the height of the ball after 1 second and 3 seconds.
   3. Estimate the time it reaches the ground.
   4. Estimate the time the ball reaches its maximum height.

2. Last year, 500 people attended the NDC High School Drama Club's Begin Again show. The ticket price was 100 pesos. The finance committee estimates that 18 fewer people would attend for each 20 pesos increase in ticket price.
   1. What ticket price would give the greatest income for the drama club?
   2. If the drama club raised the ticket to this price, how much income should it expect to bring in?

   The finance committee should charge_____in order to maximize the profit at ___________.

3. A farmer has 1000 ft of fencing material and a big field. He can enclose a rectangular area with dimensions x feet and 500 – x feeet. What is the largest area he can create?

4. Last year, 300 people attended the Stage Play for a Cause. The ticket price was 8 peso. The advisor estimates that 20 fewer people would attend for each 1 peso increase in ticket price.
   a. What ticket price would give the greatest income to the organizer?
   b. If the organizer raised its tickets to this price, how much income should it expect to bring in?

5. A ball is thrown from the top of a 144-foot bell tower. The ball follows a trajectory. The height (h) reached by the ball after t seconds is given by the quadratic function $h(t) = 64t - 16t^2$.

   1. How high did the ball reach?
   2. How long did it take to reach that height?
   3. Where was the ball after 5 seconds?
   4. How long did it take for the ball to hit the ground?

PROCESS QUESTIONS

1. How are the maximum or minimum values asked in different situations solved?

2. What difficulties did you experience in solving word problems that made use of quadratic functions?

3. What solutions can you think of which will minimize or eliminate these problems?

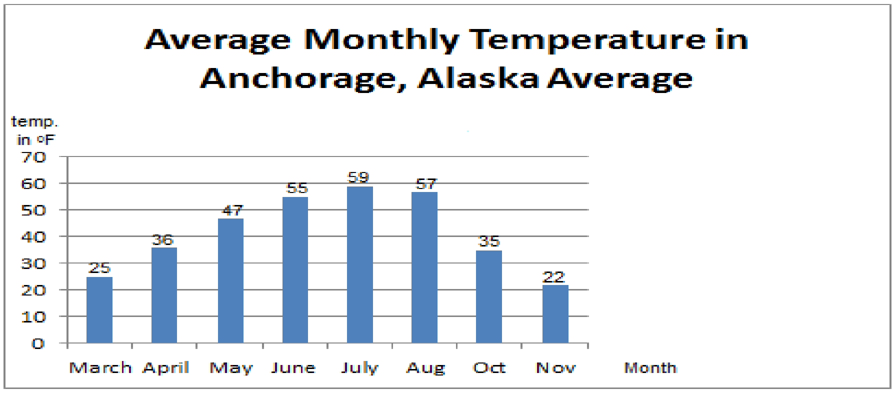
FINDING A QUADRATIC MODEL FOR A REAL WORLD DATA

Steps in Finding a Quadratic Model
1. Define the variables and adjust the data (if needed). 2. Create a scatter plot. 3. Select a model type. 4. Quadratic model: Pick a vertex and substitute it for ***h*** and ***k*** in the vertex form $f(x) = a\,(x - h)^2 + k$. 5. Pick another point and use it to find ***a.*** 6. Write the equation of the model using function notation. Note: a. In the **context of an application**, the **domain** of a quadratic model expands beyond the data. Be sure to avoid inputs that will cause model breakdown to occur. b. In the **context of an application**, the **range** of a quadratic model is the lowest to highest points on the graph within the domain.

Write these steps in your Evernote.

Example

The average monthly temperature in Anchorage, Alaska, is given in the chart.



Source: Weatherbase.com

I. Will a linear or quadratic model fit these data better? Why?

Answer: A quadratic model will fit these data best because the distribution is shaped like a downward facing parabola.

II. Find an equation for a model of these data.

Step 1: Define the variables and adjust the data (if needed).

T(m) = Average temperature in Anchorage, Alaska in ^{0}F (as replacement of f(x)

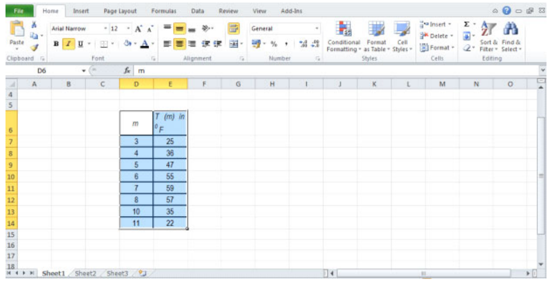
m = Month of the year; eg.,*m* = 3 represents March (as replacement of x)

The months given in the bar chart must be translated into numerical values as shown in the table below.

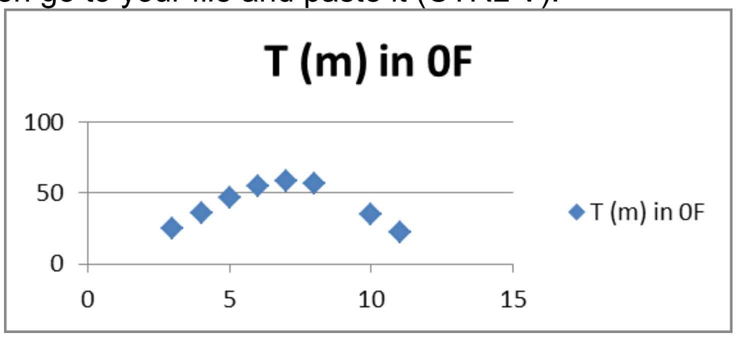
m	*T (m) in* ^{0}*F*	
3	25	
4	36	
5	47	
6	55	
7	59	→ Highest point
8	57	
10	35	
11	22	

Step 2: Create a Scatter Plot. (Use Excel in the absence of a graphing calculator)

1. Enter data in Excel.



2. Highlight the data then click in **Insert** icon and then click **Scatter** icon so that the scatter plot will appear. Click the scatter plot, right click, click copy then go to your file and paste it (CTRL V).



Step 3: Select a model type. Choose between linear and quadratic model. Since the data have the shape of downward-facing parabola, we choose a quadratic model.

Step 4: **Quadratic Model**. Pick a vertex and substitute it for ***h*** and ***k*** in the vertex form.
The highest point on this scatter plot looks like a reasonable vertex, so we will use it as a vertex.

Vertex = (7, 59) where $h = 7$ and $k = 59$
Change $f(x) = a(x - h)^2 + k$ to $T(m) = a(m - h)^2 + k$
$T(m) = a(m - 7)^2 + 59$

Step 5: Pick another point and use it to find ***a.***
We can choose the last point in the data set because it is farther away from the vertex and seems to follow a smooth curve.

Other point: (11, 22) where $T(m) = 22$ and $m = 11.$
Substitute thesein $T(m) = a(m - 7)^2 + 59$
$22 = a(11 - 7)^2 + 59$

$$22 = a(4)2 + 59$$
$$22 = 16a + 59$$
$$22 - 59 = 16a + 59 - 59$$
$$\frac{-37}{16} = \frac{16a}{16}$$
$$-2.31 = a$$

Step 6: Write the equation of the model using function notation.
We now have h = 7, k= 59, and a = 2.31, so the equation that will model the given data is written as

$T(m) = -2.31(m - 7)^2 + 59$ *in vertex form*
$T(m) = -2.31(m^2 - 14m + 49) + 59$
$T(m) = -2.31m^2 + 32.34m - 113.19 + 59$
$T(m) = -2.31m^2 + 32.34m - 54.19$ *in standard form*

III. Using your model, estimate the average temperatures during September and February. Would either of these months be a good time to travel to Alaska if you wanted to visit when temperatures were mild?

Solution:
September is the 9th month of the year and February is the 2nd month of the year, so

September	February
$T(m) = -2.31(m - 7)^2 + 59$ $T(9) = -2.31(9 - 7)^2 + 59$ $T(9) = -2.31(2)^2 + 59$ $T(9) = 49.8$	$T(m) = -2.31(m - 7)^2 + 59$ $T(2) = -2.31(2 - 7)^2 + 59$ $T(2) = -2.31(-5)^2 + 59$ $T(2) = 1.5$

The average temperature in Anchorage Alaska during September is 49.8 ^{0}F, and during February is 1.5 ^{0}F or (-16.94^{0}C). February would not be a good month to travel because it is too cold. Sept would be better month to visit Alaska.

IV. The actual average temperature September is 48^{0}F, and during February is 18^{0}F. How well does your model predict these values?

Answer:
The estimate for September is fairly accurate, but the estimate for February is not close to the actual value. Since our estimate for February is 16.5 ^{0}F less than the actual temperature, it probably represents a model breakdown in this situation.

V. Give a reasonable domain and range for your model.

Answer:
Domain: $[3,11]$ or $3 \le m \le 11$. The months included in the graph starts form March (3rd month) and ends with November (11th month).

Range: $[22,59]$ or $22 \le m \le 59$. The lowest average temperature is at 22^{0}F and the highest average temperature is at 59 ^{0}F.

Process Questions:

A. What do you think is the reason why the quadratic model in the given data was not able to give the correct estimation of the temperature for the month of February?

B. Can man control nature? Explain.

C. How can various real-life situations involving maximum and minimum values be solved and analyzed?

Post your answers in the ***Discussion forum*** *and discuss with your other classmates.*

ACTIVITY 20. Skills Check

Check the skills that you have already learned and mastered

I know how to solve quadratic equations and inequalities.	
I know how to analyze the graphs of quadratic function (identifying the vertex, intercepts, axis of symmetry, domain, range, and the direction of the parabola)	
I know how to graph quadratic functions in the vertex form, in the standard form, and in the intercept form.	
I know how to determine the maximum and minimum values of a quadratic function.	
I know how to solve problems involving the maximum and minimum values of a quadratic function.	
I know how model real-life situations using any quadratic model such as graph, table of values and equation and make predictions out of it.	

End of FIRM UP:

In this section, the discussion was about; (a) analyzing graphs of quadratic function and its effect as the values of a, h, and k are changing representing; and (b) representing a quadratic data through the use of any quadratic models such graphs, table of values, and equations and use these in making predictions.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision? What new learning goal should you now try to achieve?

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.

DEEPEN

Your goal in this section is to take a closer look at some aspects of the topic. Using the knowledge and skills that you have acquired in the explore and firm up part of this lesson, let us now look at how the key concepts on quadratic functions can be used to look at other aspects of the subject and real life problems as a whole.

ACTIVITY 21. Quiz

III. Misconception Check.

5. Monica and Maxine are asked to find the maximum of $f(x) = -4x^2 + 8x - 6$. Is either of them correct? Explain your reasoning.

Monica

$$x = -\frac{8}{2(-4)} = 1$$

$$f(1) = -4(1)^2 + 8(1) - 6$$

$$= -2$$

The maximum value is -2

Maxine

$$x = \frac{8}{2(-4)} = -1$$

$$f(1) = -4(1)^2 + 8(1) - 6$$

$$= \boldsymbol{-18}$$

The maximum value is -18

6. Sven and Bryan are asked to solve the problem below.
   A model for a company's revenue is R = - 15p^2 + 300p + 12 000, where p is the price in peso of the company's product. What price will maximize revenue?
   Each of them is asked to show their solution. Who among the two is correct?

Sven

$$p = \frac{-300 \pm \sqrt{300^2 - 4(-15)(12000)}}{2(-15)}$$

$$p = \frac{-300+90}{-30} = -20 \quad \text{or} \quad p = \frac{-300-90}{-30} = 40$$

The price that will maximize the revenue is 40.

Bryan

$$p = -\frac{300}{2(-15)} = 10$$

The price that will maximize the revenue is 10.

IV. Applying Quadratics in the Real Life

I. SPORTS

Solve the following problems.

1. A hit at a major baseball game flew along the following path from home plate.

Horizontal Distance (feet)	Height (feet)
10	15
75	78
110	103
190	136
230	140
310	123
400	65

a. Find an equation for a model of these data. Find the vertex and use (310, 123) as the other point.

b. Give a reasonable domain and range for this model.

c. Find the vertex of this model and explain its meaning.

2. A fly ball hit at the same major league baseball game had the following vertical heights.

Time (seconds)	Height (feet)
1	132
3	292
5	324
6	290
7	228
9	4

a. Find an equation for the model of these data. Find the vertex and use (7, 228) as the other point.

b. Give a reasonable domain and range for this model.

c. Estimate the height of the ball at 8 seconds.

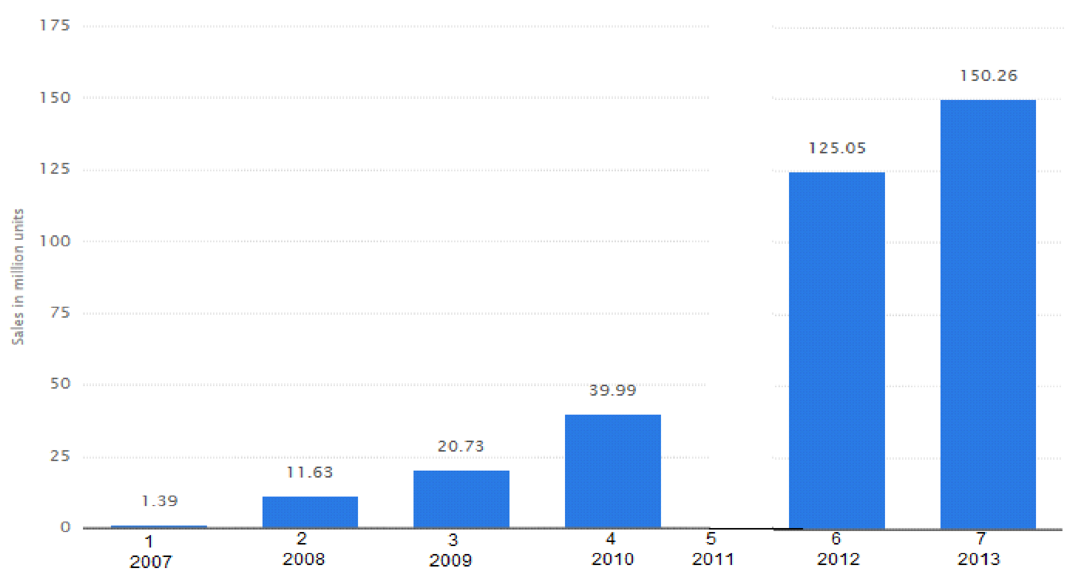
d. Use your graph to estimate how long it took for the ball to hit the ground.

e. Find the maximum height of the ball and the time when it reached that height.

J. BUSINESS

Analyze the given graph.

Global Apple iPhone sales in the fiscal years 2007 to 2013 (in million units)



1. What trend can you see? Is it quadratic or linear? Why?

2. Write an equation for a model of these data. Find the vertex and use (3, 20.73) as the other point.

3. Using your equation, estimate the number of sales in 2011.

4. The actual sales in 2011 is 72.29 million units. Was your equation able to give you a good estimate?

5. Give a reasonable domain and range for this model.

6. Predict Apple's sales in 2014.

7. Will Apple's iPhone sales continue to increase in the next 5 years? Why?

3 – The answer was correct with a clear explanation and contains accurate computations.
2 – The answer was correct with a clear explanation and contains accurate computations.
1 – The answer was correct but no explanation was given.
0 – No answer and explanation was given.

Process Question:

How can various real-life situations involving maximum and minimum values be solved and analyzed?

Pass the answers and solutions to these problems to your teacher on your next face-to-face meeting.

ACTIVITY 22. Is it Linear or Quadratic?

Study the three situations below.
Note: **Profit = Sales – Capital**

<table>
<tr>
<td>Situation A

Andrea sells cellphone casings at Php 150 each which she bought originally for Php 125 only. Below is a table of Andrea's sales for one month.
<table>
<tr><td>Number of casings sold per week at Php 150 each</td><td>140</td><td>145</td><td>160</td><td>148</td></tr>
</table>
</td>
<td>Situation B

Maxine sells cellphone casings at Php 150 each which she originally bought for Php 125 only. She thought of maximizing the number of casings sold by decreasing the price every week by 5 pesos for one month. The number of casings sold per week as follows:
3. 140 casings for first week
4. 160 casing for the second week</td>
</tr>
</table>

Profit for selling cellphone casing	3500	3 625	4 000	3 700

1. What is the maximum profit for the month?
2. What is her total profit for one month?
3. Using the table of values, draw a scatter plot of Andrea's sales for the whole month?
4. What trend can you see? Is it quadratic or linear?
5. What quantity is represented in the y-axis?
6. What quantity is represented in the x-axis?
7. Write an equation to model the data given above.
8. Give a reasonable domain and range of your model.

5. 185 for the third week
6. 210 for the fourth week.

Below is the table of Maxine's sales for one month.

Price per week	150	145	140	135
Profit for selling cellphone casing	3500	3200	2775	2100

1. What is the maximum profit for the month?
2. What is her total profit for one month?
3. Using the table of values, draw a scatter plot of Maxine's sales for the whole month?
4. What trend can you see? Is it quadratic or linear?
5. What quantity is represented in the y-axis?
6. What quantity is represented in the x-axis?
7. Write an equation to model the data given above.
8. Give a reasonable domain and range of your model.

Situation C
Margaret owns an accessory store in the City. Her most saleable item is the cellphone casing. The store sells an average of 140 pieces per week where each item is sold at Php 150 which she originally bought for Php 125. She plans to raise the price to maximize the sales by Php 15 per week starting on the second week of the month.After the second week's increase, she observed that fewer casings were not sold. The number of items sold for per week are as follows:

11. 135 casings for the first week
12. 130 casings for the second week
13. 117 casing for the third week
14. 85 casings for the fourth week

Her one–month sale is shown in the table below.

Price per week	150	165	180	195
Profit for selling cellphone casing	3375	5200	6435	5950

1. What is the maximum profit for the one-month sales? At what price?
2. What is her total profit for one month?
3. Using the table of values you have created, create a scatter plot of Margaret's sales for the whole month?
4. What trend can you see? Is it quadratic or linear?
5. What quantity is represented in the y-axis?
6. What quantity is represented in the x-axis?
7. How will compare the quantities used in the x- axis and in the y-axis of each graph?
8. How will you compare the graph in this situation to the graph of the previous situations?
9. Write an equation to model the data given above.
10. Give a reasonable domain and range of your model.
11. Which of the two trends (quadratic or linear) will help maximize your profit?
12. Can Margaret continue on increasing the price per week? What will happen if she keeps on increasing the price? Why?

Conclusion: What determines a maximum profit? How does one ensure a maximum profit?
Pass your answers to your teacher on your next face-to-face meeting.

Conclusion: What determines a maximum profit? How does one ensure a maximum profit?
Based from the three situations, the *price increases* greatly determines a maximum profit than *the increase in the number of items sold.* To ensure *a* maximum profit, we should consider the number of possible customers who will buy and will not buy the products as we increase the price of the item.

ACTIVITY 23. Revisiting Activity No. 1: Data Analysis

Revisit your answers in Activity 2. Revise answers that need to be revised. Click on **Submit** after having done with the revision. Post your answer for item 8 in the **Discussion Forum.**

ACTIVITY 24. Tourism Watch

Study the problem below.

The manager of a domestic travel and tour advertises a price for as low as Php 20 000 per person for a package of 10 tourists. The price per person decreases by Php 500 for every additional person in the package. What is the number of tourists maximizes the income?

Price per person or tourist (Php)	Number of tourists	Total income or Revenue (Php)
20 000	10	200 000
20 000 - 1 (500)	10 + 1 (1)	214 500
20 000 – 2 (500)	10 + 2(1)	228 000
20 000 – 3 (500)	10 + 3(1)	240 500
20 000 – x(500)	10 + x (1)	(20 000-500x)(10 + x)

$R(x) = (20\,000 - 500x)(10 + x)$
$R(x) = -500x^2 + 20\,000x - 5000x + 200\,000$
$R(x) = -500x^2 + 15\,000x + 200\,000$
$R(x) = -x^2 + 30x + 400$ (simplified form)
$h = -b / 2a = -30/ - 2(-1) = 15$
$k = R(15) = 312\,500$
The vertex is at (15, 312 500). This means that 15 is the number of tourists that maximizes the income since the parabola of the quadratic function $R(x) = -500x^2 + 15\,000x + 200\,000$ pens downward.

ACTIVITY 25. Manager's Challenge! (Scaffold 3)

DESCRIPTION: Analyze and investigate the problem below and use the link www.helloslide.com in the presentation of your output.

Apple Company has released its new iphone in the market with a suggested retail price of Php 35, 500. Many consumers have been waiting for this model because of its unique and more advanced features. You are working as a marketing manager of the i-store, one of Apple Company's dealer stores. As the marketing manager, it is your responsibility to maximize the sales and profit for four months, making sound recommendations regarding the pricing of the product. It has been observed in the 3rd week of the first month that for every Php 300 increase in the suggested retail price, 3 fewer customers will not buy the product. The proposal must be accurate in computations, and represented using a model.

Scoring Rubric

Criteria	4 Excellent	3 Proficient	2 Progressing	1 Beginning
Representation	The quadratic equation created is correct with	The quadratic equation created is correct.	The quadratic equation created does not use the	The quadratic equation used is incorrect.

	additional model to support it.		defined variables.	
Accuracy of the Computations	Computations are accurate and supported with correct and clear interpretation.	Computations are accurate and supported with correct interpretation	Computations are correct but interpretation is incorrect.	Most of the computations and interpretations are erroneous.

PROCESS QUESTIONS:
1. What realizations did you have while doing the task?
2. What are some difficulties or problems you have encountered while performing the task?
3. Did the concepts learned in earlier part of the module help you in doing the task? Explain.

Post your output and answers to these questions in the Discussion Forum.

ACTIVITY 26. Internet Trends

You are tasked to conduct a research of the trends in the Internet (eg., the number of photos, data, videos, and music uploaded and shared) for 2013 worlwide. You will present the result in PowerPoint. The research must contain authentic data, it must be represented by different models, computations are accurate, and interpretation is correct.

Scoring Rubric

Criteria	4 Excellent	3 Proficient	2 Progressing	1 Beginning
Authenticity	The data used are authentic and updated. Data are taken from reliable resources.	The data used are authentic. Data are taken from reliable resources.	Some of the data used are not authentic.	Data are not useful and relevant.
Representation	The quadratic equation created is correct with additional model to support it.	The quadratic equation created is correct.	The quadratic equation created does not use the defined variables.	The quadratic equation used is incorrect.

Accuracy and Interpretation of the Computations	Computations are accurate and supported with correct and clear interpretation.	Computations are accurate and supported with correct interpretation	Computations are correct but interpretation is incorrect.	Most of the computations and interpretations are erroneous.

Process Questions:

11. How can internet trends be analyzed and predicted?

12. What does one need to know in order to come up with a good interpretation and prediction?

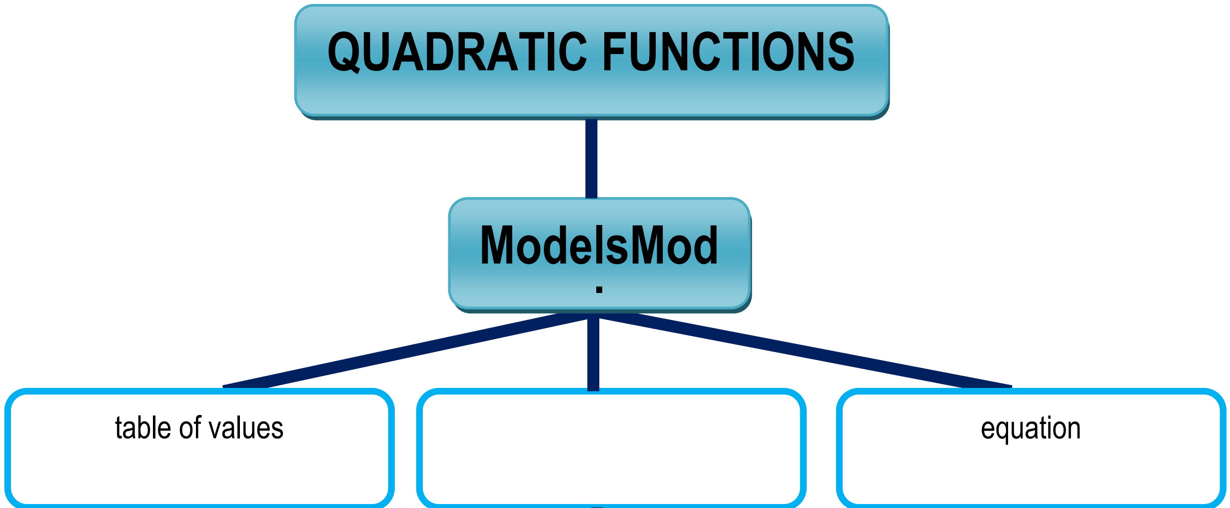
13. How can various real-life situations involving maximum and minimum values be solved and analyzed?

Post your output and answers to these questions in the Discussion Forum.

ACTIVITY 27. Revisiting Activity No. 2

In this activity, you will revisit the A-R Guide found on the explore part of this module. Fill in the Reaction Column by ticking the box of the corresponding statement which you agree in. Explain the corrections that you need to make.

ACTIVITY 28. Concept Map.



Conclusion:
These models are used to ____________and_____________ real-world data. But remember that sometimes a model breakdown happens when a quadratic ___________ is used as model of the data.

ACTIVITY 29. Writing a Synthesis Journal

Complete the table below.

Synthesis Journal			
Lesson	**What I did.**	**What I learned.**	**How I can use it**
Models of Quadratic Equations			
Graph of a Quadratic Function			
Making a Quadratic Model for a Real- World Data			

End of DEEPEN:

In this section, the discussion was about the real world use of the quadratic functions and its models and its advantage in maximizing sales.

What new realizations do you have about the topic? What new connections have you made for yourself? What helped you make these connections?

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.

TRANSFER

Your goal in this section is apply your learning to real life situations. You will be given a practical task which will demonstrate your understanding.

ACTIVITY 30. The Final Task

ASEAN 2015 brings life to a country's tourism. You are an ARGF travel and tour agent. You are tasked to make promo packages for the different Asian tourists. You are to present a written report of your proposal to your manager. The proposal should demonstrate practicality, accuracy, authenticity and application of concepts on quadratic functions.

Scoring Rubric

Criteria	4 Excellent	3 Proficient	2 Progressing	1 Beginning
Authenticity of Data	The data used are authentic and updated. Data are taken from reliable resources.	The data used are authentic. Data are taken from reliable resources.	Some of the data used are not authentic.	Data are not useful and relevant.
Accuracy of the Computations	Computations are accurate and supported with correct and clear interpretation.	Computations are accurate and supported with correct interpretation	Computations are correct but interpretation is incorrect.	Most of the computations ad interpretations are erroneous.

Practicality of Proposal	The amount of the price increase is realistic and yields maximum profit.	The amount of the price increase is realistic and yields profit.	The amount of the price increase is realistic but yields minimal profit.	The amount of the price increase is unrealistic and does not yield profit.
Application of the Concepts of Quadratic Equations	The quadratic equation created is correct with additional model to support it.	The quadratic equation created is correct.	The quadratic equation created does not use the defined variables.	The quadratic equation used is incorrect.

PROCESS QUESTIONS:

5. What are the important factors did you consider which contributed to the success of this task?
6. To what extent is your knowledge, skills and understanding of quadratic functions have helped you perform the task?
7. How can various real-life situations involving maximum and minimum values be solved and analyzed?

Reflection Log!
1. What have I learned?
2. How do I feel with what I've learned?
3. What can I do with what I've learned?

Should you have any clarifications on this lesson, type your question and email this to the teacher or post it to the discussion forum.

End of TRANSFER:

In this section, your task was to research on the trends in the Internet.

How did you find the performance task? How did the task help you see the real world use of the topic?

You have completed this lesson. Before you go to the next lesson, you have to answer the following post-assessment.

MODULE: POST-ASSESSMENT

It's now time to evaluate your learning. Click on the letter of the appear after you answer all items. If you do well, you may move on to the answer that you think best answers the question. Your score will only next module. If your score is not at the expected level, you have to go back and take the module again.

1. Which equation has unreal roots?
   A. $-3x^2 + 2x – 15 = 0$
   B. $4x^2 – 40x + 100 = 0$
   C. $2x^2 + 10x + 5 = 0$
   D. $10x^2 – 3x – 1 = 0$

2. A quadratic equation in one variable is an equation of the form $ax^2 + bx + c = 0$. Which could <u>NOT</u> have a value of 0?
   A. a
   B. b
   C. c
   D. a and b

3. A quadratic equation has one real solution if $b^2 – 4ac$ is ______.
   A. less than zero
   B. equal to zero
   C. greater than zero but not a perfect square
   D. greater than zero and a perfect square

4. In using the method of completing the square to solve $2x^2 – 10x = –8$, a student began by adding the square of half the coefficient of x to both sides of the equation. He then encountered difficulty in his later steps. What was the error?
   A. The student should have divided both sides of the equation by the coefficient of x^2.
   B. The student should have written the equation in general form prior to the process he performed.
   C. The student should have factored out the variable x first from the left side of the equation.
   D. The student should have taken half the coefficient of x^2 prior to the process he performed.

5. Which are the roots of the quadratic equation $2x^2 - 7x + 3 = 0$?
   A. $\frac{1}{2}$ and 3
   B. $-\frac{3}{2}$ and -1
   C. $-\frac{1}{2}$ and -3
   D. $\frac{3}{2}$ and 1

6. Which of the following illustrates a quadratic inequality?
   A. $2x(x + 3) > 2x^2$
   B. $(x + 2)(x - 3) < 0$
   C. $x(x - 5)(x - 4) \leq 4$
   D. $x^3 + x^2 \leq 2x^2 + x^3 - 4$

7. Solve the quadratic inequality $x^2 - x - 12 > 0$. Express your answer in interval notation.
   A. $(-\infty, 3)$ $(-4, \infty)$
   B. $(-\infty, -3)$ $(4, \infty)$
   C. $(-\infty, 4)$ $(-3, \infty)$
   D. $(-\infty, -4)$ $(3, \infty)$

8. A stuntman will jump off a 20 m building. A high-speed camera is ready to film him between 15 m and 10 m above the ground. When should the camera film him? The camera should film him from
   A. 1.0 to 1.4 seconds after jumping.
   B. 1.2 to 1.6 seconds after jumping.
   C. 1.4 to 1.8 seconds after jumping.
   D. 1. 6 to 2.0 seconds after jumping.

9. The unit cost in dollars for manufacturing n starters is given by $C = 0.004n^2 - 3.2n + 660$. For what number of starters is the unit cost at a minimum?
   A. 100
   B. 200
   C. 300
   D. 400

10. The table below shows the records of sales profit (P) from selling x units of commodity. How many units of the commodity must be sold to have a break even?

x	500	1000	3000	5000
P	87500	150 000	150 000	-250 000

A. 1500
B. 3000
C. 3500
D. 4000

X	20	40	60	70
y	440	200	280	440

A. 30 years of age
B. 40 years of age
C. 45 years of age
D. 50 years of age

12. The profit (P) in dollars on each cellphone manufactured by Clear Lines Inc., is related to the number of cell phones produced each day according to the equation $P = 0.82x^2 + 4.25x + 45$ where x is the number of cellphones produced each day (in hundreds) and $2 \le x \le 8$. About how many cell phones should be produced each day to make a profit of \$70 per phone?
    A. 4
    B. 3
    C. 5
    D. 7

13. If the total costs are $C(x) = 500 + 90x$, and total revenues are $R(x) = 150x – x^2$.
    Find the break- even point(s). At break-even, there is no profit, the costs equal the revenue, so *R(x) = C(x).*
    A. 20 and 25
    B. 50 and 10
    C. 5 and 100
    D. 4 and 125

14. At what value of x is the profit at a maximum?
    A. 15
    B. 20
    C. 25
    D. 30

15. You are hired one of the consultants of a fast food chain in your city. The owner asked you to come up with a business plan proposal to increase its sales and maximize its profits. Which should not be the possible characteristics of your proposal?
    A. presentable, accurate and organized
    B. practical, authentic and accurate

C. authentic, grammatically correct and organized
D. presentable, realistic and neat

16. You are a crew from an entertainment who launches fireworks at angle of 60° from the horizontal. The height of one particular type of display can be approximated by the function, $h(t) = -16t^2 + 160\ t$ where h(t) is measured in feet and t is measured in seconds. How long will it take the fireworks reach the maximum height?
    A. 4 sec
    B. 5 sec
    C. 6 sec
    D. 7 sec

17. You are engaged in a business that sells personalized pens. The number of pens sold (x), and the daily profit (y), are shown in the table below. What is the vertex of the profit function and its meaning in the context of this problem?

x	1	2	4	6
y	5	3	11	35

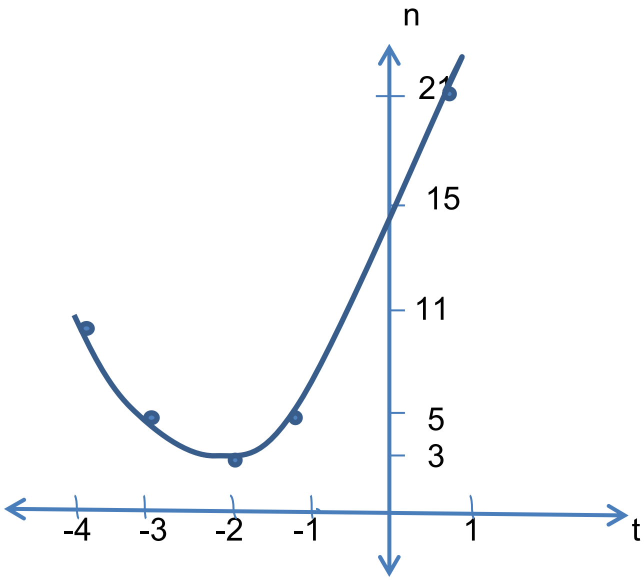
    A. The vertex (2, 3) means a daily profit of 3 peso is obtained when 2 pens are sold.
    B. The vertex (2, 3) means a daily loss of 3 peso when 2 pens are sold.
    C. The vertex (3, 2) means a daily profit of 2 peso is obtained when 3 pens are sold.
    D. The vertex (3, 2) means a daily loss of 2 peso when 3 pens are sold.

18. Rex has a 150 m fencing wire to set up around a rectangular lot beside a river for his ducks and geese. If a 10 m opening on the side of the river is left, what will be the length and width for the maximum area?

    A. 76 m and 37 m
    B. 75 m and 37.5 m
    C. 80 m and 35 m
    D. 82 m and 34 m

19. The number of bacteria in a refrigerated food is given by $n(t) = 5t^2 – 40t + 100$, where t is the temperature of the food in Celsius. As a microbiologist, you are tasked to answer the question: At what temperature will the number of bacteria be minimal?
    A. 4°C
    B. 5°C
    C. 6°C
    D. 50°C

20. You are a medical technologist doing an experiment regarding the number of
bacteria (n) in a refrigerated food where t is the temperature of the food in Celsius. The result of an experiment is illustrated by the graph below. Which of the following functions best describes the graph?



A. $n(t) = 2t^2 + 4t + 11$
B. $n(t)\ \ 2t^2 - 4t + 5$
C. $n(t) = 2t^2 + 8t + 11$
D. $n(t) = 2t^2 + 8t + 5$

GLOSSARY OF TERMS USED IN THIS LESSON:

AXIS OF SYMMETRY **-** It is an imaginary line through the graph of a parabola that divides the graph into two congruent halves. Each side of the parabola is a reflection of the other side.

DISCRIMINANT – refers to the radicand in the quadratic formula.

DOMAIN- The set of all possible values for x. The domain of all quadratic functions are ***all real numbers*** $(-\infty,+\infty)$.

IRRATIONAL NUMBERS – are numbers that cannot be expressed as a quotient of two integers.

MAXIMUM AND MINIMUM VALUES- These are the y-coordinate of the vertex of a quadratic function.These values represent the greatest or lowest possible value the function can reach.

PARABOLA- The graph of a quadratic function. A parabola opens either upward or downward.

PRINCIPLE OF ZERO PROPERTY PRODUCTS – states that if ab = 0, the a = 0 or b = 0.

QUADRATIC EQUATION – is any equation in the form $ax^2 + bx + c = 0$ where a, b and c are real numbers and $a \neq 0$.

QUADRATIC INEQUALITY - An inequality involving a quadratic polynomial.

QUADRATIC FUNCTION - A quadratic function can be written in the standard form or vertex form of a quadratic.The greatest exponent is 2.

RANGE **-** The set of all possible values for y. If the vertex of the quadratic equation is (h,k) then the range is
$y \geq k$, if the graph opens upward
$y \leq k$, If the graph opens downward

RATIONAL EQUATION – an equations that contains one or more rational expressions.

ROOTS – the solutions to an equation.

STANDARD FORM **-** $f(x) = ax^2 + bx + c$

VERTEX **-** The point on a parabola where the graph changes direction. The maximum or minimum function value occurs at the vertex of a parabola.

VERTEX FORM - $f(x) = a\,(x - h)^2 + k$

X-INTERCEPTS - These are points on the parabola that crosses the x-axis where $y = 0$.

Y-INTERCEPT - Is a point on the parabola that crosses the y-axis where $x = 0$.

REFERENCES AND WEBSITE LINKS USED IN THIS LESSON:

LESSON 1: QUADRATIC EQUATIONS

VIDEOS

http://www.youtube.com/watch?v=D87sogoU_68.
This video shows different real-life scenes that make use of parabolic paths.

http://www.youtube.com/watch?v=-0dQ6yRXI38
This video will explain the steps needed in solving quadratic equations using extraction of roots.

http://www.youtube.com/watch?v=SDe-1IGeS0U
This video will explain the steps needed in solving quadratic equations using factoring.

http://www.youtube.com/watch?v=xGOQYTo9AKY
This video will explain the steps needed in solving quadratic equations using completing the square.

http://www.youtube.com/watch?v=3ayhvAI3IeY
This video will explain the steps needed in solving quadratic equations using quadratic formula.

http://www.virtualnerd.com/algebra-1/rational-expressions-functions/solve/example-solutions/equation-definition
This video will help the student recall the lesson on rational algebraic equations.

http://www.youtube.com/watch?v=r6N8mDRNktw
This video will allow the student to see how distance problems could be solved using rational algebraic equations.

http://www.youtube.com/watch?v=fG-zjUR9mM8
This video will show other real life problems that could solved using rational algebraic equations.

http://www.youtube.com/watch?v=SkUATohNR78
This video shows how the discriminant is used.

http://www.youtube.com/watch?v=IGhrjwE-QoU
This video will let the students see the relationship between the roots and coefficients of quadratic equations.

http://www.youtube.com/watch?v=nQ5APwtB-ig
This video shows the costs incurred by businesses.

http://www.youtube.com/watch?v=8xo0ySZ98oc
This video will explain how boats sail on apparent wind speeds.

http://www.youtube.com/watch?v=UcZhGkU6sIM
This video will allow students to see a new situation on constructing bridges as a way to verify the generalizations that they had made.

PDF FILES
http://www.mth.msu.edu/~kadyrova/lectures/Lecture_04.pdf
http://www.glencoe.com/sec/math/algebra/algebra1/algebra1_03/study_guide/pdfs/alg1_pssg_G077.pdf
http://stargazers.gsfc.nasa.gov/pdf/activities/math_activities/math_teacher/math_act_09_t.pdf

LESSON 2: QUADRATIC INEQUALITY

Textbooks:

1. Real-World Mathematics
   By: Melanie M. Covar& Rita May L. Fetalvero on pp. 286- 297

2. Algebra and Trigonometry with Analytic Geometry
   By: Swokowsli and Cole on pp. 122 - 132

Websites:

1. http://www.algebra.com/algebra/homework/Systems-of-equations/change-this-name32481.lesson
   Solving quadratic inequalities

2. http://www.wikihow.com/Solve-Quadratic-Inequalities
   Three ways of solving quadratic inequalities

3. http://www.mathwarehouse.com/quadratic-inequality/how-to-solve-and-graph-quadratic-inequality.php
   Solving by graphing

4. http://www.mathwarehouse.com/quadratic-inequality/how-to-solve-and-graph-quadratic-inequality.php
   Best method in solving quadratic inequalities

5. http://www.purplemath.com/modules/ineqquad.htm

Solving quadratic inequalities

6. http://www.purplemath.com/modules/quadprob.htm
   Module on quadratic inequalities- application

7. http://braingenie.ck12.org/skills/106856
   Interactive site on solving real life problems involving quadratic inequalities

8. http://moodle.anoka.k12.mn.us/pluginfile.php/113155/mod_page/content/67/4.6.2. pdf
   Solving Quadratic Inequalities and Application

9. http://www.regentsprep.org/Regents/math/algtrig/ATE6/Quadinequal.htm
   Contains examples on how to solve quadratic inequalities

10. *https://www.present.me/content/?utm_expid=39514825-9.RmSjXrfCTbOTiqm2SPHDWQ.0*
    Used in making a slide show of the power point

11. ***http://www.gliffy.com/index-f.php***
    This contains different graphic organizers

12. *http://www.voki.com/create.php*
    This site can be used to create your power point

13. http://repository.nie.edu.sg/jspui/bitstream/10497/391/1/TL-18-1-64.pdf
    This site will allow students to look at a sample problem posing activity in math.

LESSON 3: QUADRATIC FUNCTIONS

Videos:
http://www.youtube.com/watch?v=RPOkbkL2fkk. This link provides a video that will teach students how to graph a quadratic function in vertex form.

http://www.youtube.com/watch?v=LoscdQJyZPo. This link provides a video that will teach students how to graph a quadratic function in standard form.

http://www.youtube.com/watch?v=pme-ZrnCl3o. This link provides a video that will teach students how to graph a quadratic function in intercept form.

http://www.youtube.com/watch?v=Sje8fgdBY2Y . This link provides a video that will teach students how to write a quadratic equation given the table of values.

http://www.youtube.com/watch?v=vAPPYoBV2Ow. This link provides a video that will teach students how to write a quadratic equation given the graph of the function.

http://www.youtube.com/watch?v=89UYFIRkgP8. This link provides a video that will teach students how to write a quadratic equation given the zeroes of the function.

http://www.youtube.com/watch?v=OXViZtD2BTE&list=PLaIUqRPLCu47yy0WoM1NzA6-No4Jw1I3A. This link provides a video that will teach students how to write a quadratic equation given the zeroes of the function.

http://www.youtube.com/watch?v=qS3OplAAG6Y These websites show illustrative examples of the applications of quadratic functions.

http://www.youtube.com/watch?v=ipBoVSMVzp0 This website is a video on how a problem regarding maximum area is solved.

http://www.youtube.com/watch?v=3ShdbCN7i_Y This website is a video on how real-life problems involving time and vertical height are solved?

http://www.youtube.com/watch?v=ssb0yxpNmiI This video illustrates how we determine break- even points in business related problems that are modeled by quadratic functions.

http://www.youtube.com/watch?v=KIGmg4yWRKEThis video showcases how typically quadratic function models are used in order to find either maximum orminimum values of something.

PDF Files :
http://www.quia.com/files/quia/users/pamm1409/HALG2/4.1_PowerPoint_(formerly_5.1).ppt
This site provides a PowerPoint presentation that contains examples and practice problems in graphing quadratic functions in vertex, standard, and intercept forms.

http://frontenacss.limestone.on.ca/teachers/dcasey/0E67293F-00870BC8.84/3_Graphing_Quadratic_Functions_Worksheet.pdf.
This site provides a pdf file that contains problems in graphing quadratic functions.

http://www.mcgrawhill.ca/web_resources/sch/Functions11_sec1_6.pdf. This site provides information about the zeros of a quadratic function and how to use it in solving problems in the real world.

PowerPoint Presentation:
http://www.mathxtc.com/Downloads/NumberAlg/files/Quad%20Functions%20and%20Models.ppt This site provides a PowerPoint presentation file that gives input and examples for the effects of a, h, k in the graph of a quadratic function .

http://www.quia.com/files/quia/users/pamm1409/HALG2/4.1_PowerPoint_(formerly_5.1).ppt This site provides examples and practice problems about graphing quadratic functions in vertex, standard, and intercept forms.

Interactive Quiz:
http://www.thatquiz.org/tq/previewtest?M/N/Q/5/8S8C1353086320 . This is a 1-item interactive quiz about quadratic function.

http://math.about.com/od/Exercises/a/Quadratic-Quiz.htm
This site contains a 6-item quiz on application of quadratic functions.

http://math.about.com/od/Exercises/a/Quadratic-Quiz.htm
This site contains a 6-item quiz on application of quadratic functions.

Textbooks:

Aufmann, R., and Lockwood, J. 2011. Intermediate Algebra: An Applied Approach. Brooks/Cole, Cengage Learning. United States of America.

Bernabe, Julieta G., et al. Our World of Math. Vibal Publishing House: Quezon City, 2013.

Clark, M., and Anfinson. 2012. Intermediate Algebra: Connecting Concepts through Applications. Brooks/Cole, Cengage Learning. United States of America.

Lim, Yvette, et al. Math for Engaged Learning. Sibs Publishig House: Quezon City 2014.

Ogena, Ester, et al. Our Math. Mc Graw Hill and Vibal Publishing House: Quezon City, 2013.

Orines, Fernando B., et al. Next Century Mathematics.Phoenix Pblishing House: Quezon City, 2014.

Oronce, Orlando A and Mendoza, Marilyn O, e- math. Rex Book Store: Manila, 2014.

Nivera, Gladys C and Lapinid, Minie Rose C. Grade 9 Mathematics Patterns and Practicalities. Salesiana Books by Don Bosco Press, Inc.: Makati, 2013.