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PEAC
PRIVATE EDUCATION ASSISTANCE COMMITTEE
Trustee of the Fund for Assistance to Private Education

DepED
DEPARTMENT OF EDUCATION

LEARNING MODULE

Mathematics G9 | Q2

Variations and Radicals

PEAC
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NOTICE TO THE SCHOOLS

This learning module (LM) was developed by the Private Education Assistance Committee under the GASTPE Program of the Department of Education. The learning modules were written by the PEAC Junior High School (JHS) Trainers and were used as exemplars either as a sample for presentation or for workshop purposes in the JHS In-Service Training (INSET) program for teachers in private schools.

The LM is designed for online learning and can also be used for blended learning and remote learning modalities. The year indicated on the cover of this LM refers to the year when the LM was used as an exemplar in the JHS INSET and the year it was written or revised. For instance, 2017 means the LM was written in SY 2016-2017 and was used in the 2017 Summer JHS INSET. The quarter indicated on the cover refers to the quarter of the current curriculum guide at the time the LM was written. The most recently revised LMs were in 2018 and 2019.

The LM is also designed such that it encourages independent and self-regulated learning among the students and develops their 21st century skills. It is written in such a way that the teacher is communicating directly to the learner. Participants in the JHS INSET are trained how to unpack the standards and competencies from the K-12 curriculum guides to identify desired results and design standards-based assessment and instruction. Hence, the teachers are trained how to write their own standards-based learning plan.

The parts or stages of this LM include Explore, Firm Up, Deepen and Transfer. It is possible that some links or online resources in some parts of this LM may no longer be available, thus, teachers are urged to provide alternative learning resources or reading materials they deem fit for their students which are aligned with the standards and competencies. Teachers are encouraged to write their own standards-based learning plan or learning module with respect to attainment of their school's vision and mission.

The learning modules developed by PEAC are aligned with the K to 12 Basic Education Curriculum of the Department of Education. Public school teachers may also download and use the learning modules.

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MATHEMATICS 9

Module 2: Variations And Radicals

☑ INTRODUCTION AND FOCUS QUESTION(S):

Do you know why humans cannot dive to a depth that submarines can? Pressure increases as one dives deeper and there is only much pressure that our body, especially our eardrums, can endure. When the pressure is too much, the eardrums break, which leads to hearing problems.

Have you always wanted to reach your destination in the shortest time? To do so, you either take the short cut or increase your speed. However, remember that the faster you drive, the more time and distance you need to stop safely, and the less time you have to react. Once brakes are applied, the braking distance varies directly as the square of the car's speed. These problems of depth and distance involve quantities that need to be estimated or computed. These problems also show relationship of certain quantities such as pressure with depth and speed with distance.

In this module, you will identify other similar related quantities in real life and how they increase or decrease with respect to another quantity. Remember to search for the answer to the following questions:**How can mathematical formulas in scientific investigations be determined?How can a quantity be influenced by another?**

☑ LESSONS AND COVERAGE:

In this module, you will examine this question when you take the following lessons:

Lesson 1 – Variations
Lesson 2 –Radicals

In these lessons, you will learn the following:

Lesson 1	• Illustrates situations that involve the following variations (a) direct; (b) inverse; (c) joint; (d) combined. • Translates into variation statement a relationship between two quantities given by: (a) a table of values; (b) a mathematical equation; (c) a graph and vice versa. • Solves problems involving variation.
Lesson 2	• Applies the laws involving positive integral exponents to zero and negative integral exponents. • Illustrates expressions with rational exponents • Simplifies expressions with rational exponents • Writes expressions with rational exponents as radicals and vice versa • Derives the laws of radicals • Simplifies radical expressions using the laws of radicals • Performs operations on radical expressions • Solves equations involving radical expressions • Solves problems involving radicals

☑ **MODULE MAP:**

Here is a simple map of the above lessons you will cover:

SOLVING REAL LIFE PROBLEMS

VARIATIONS

Can be translated in
- Table of values
- Mathematical equation
- graph

Types
- Direct variation
- Inverse variation
- Joint variation
- Combined variation

RADICALS

Operations on radicals
- Addition
- Subtraction
- Multiplication
- Division

☑ **EXPECTED SKILLS:**

To do well in this module, you need to remember and do the following:

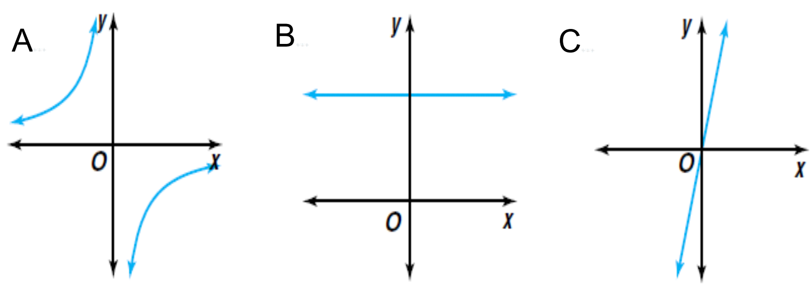
1. Follow the instructions provided for each activity.
2. Review and evaluate your work using the rubric provided before submission.
3. Complete all activities.
4. Be mindful of the meaning of unfamiliar words you encounter in this module. A glossary of terms is provided in the last part of this module.
5. Maximize the use of online resources in each lesson. Online resources can be accessed multiple times. The summary of online resources is provided in the end of the module.

PRE-ASSESSMENT:

Let's find out how much you already know about this module. Answer the pre-test below.

Click on the letter that you think best answers the question. Please answer all items. After taking this short test, you will see your score. Take note of the items that you were not able to correctly answer and look for the right answer as you go through this module.

1. Which of the following graphs illustrates a direct relationship?



2. What formula represents the relationship shown in the table?

m	2	5	8	3	7
P	12	30	48	18	42

A. P = 6 + m
B. P = 6 – m
C. P = 6m
D. P = $\frac{m}{6}$

3. If y varies jointly as x and the cube of z and y =16 when x = 4 and z =2, find an equation that represents this relationship.
   A. $y = xz^3$
   B. $y = 2xz^3$
   C. $y = \frac{2kx}{z^3}$
   D. $y = \frac{xz^3}{2}$

4. What is the simplified form of $4^{3/2} = 8$?
   A. $(\sqrt{4})^3 = 8$
   B. $(\sqrt{4})^2 = 8$
   C. $(\sqrt[3]{4})^2 = 8$
   D. $(\sqrt{2})^3 = 8$

5. Which of the following is the simplified form of $\sqrt{4} + \sqrt{8} + 2\sqrt{4}$?
   A. $8\sqrt{2}$
   B. $6 + 2\sqrt{2}$
   C. $\sqrt{8}$
   D. $2\sqrt{16}$

6. The monthly salary ***S*** of Nestor is directly proportional to the number of days ***d*** he worked. His daily wage is ₱ 320. Which table represents the relationship of the Nestor's monthly salary and the number of days he worked?
   A.

d	1	5	10	22
S	320	320	320	320

   B.

d	1	5	10	22
S	320	1600	3200	7040

   C.

d	1	5	10	22
S	320	1920	4800	10240

D.

d	1	5	10	22
S	320	1600	4800	7040

7. Which of the following is the shows the appropriate procedure in simplifying $\sqrt{\sqrt[3]{729}}$?

A. $\sqrt[(3)(2)]{729} = \sqrt[6]{729} = 3$
B. $\sqrt{729} = 27 = \sqrt[3]{27} = 3$
C. $\sqrt[3]{729} = 9 = \sqrt{9} = 3$
D. $\frac{\sqrt{729}}{\sqrt[3]{729}} = \frac{27}{9} = 3$

8. Find the solution of the equation $\sqrt{x+4} - 2 = 3$?

A. 21
B. 22
C. 23
D. 24

9. The number of calories C burned while rowing is directly proportional to the time spent rowing. During a 10 – minute rowing competition, a 130 – pound woman will burn 118 calories while 150 – pound woman will burn 124 calories. Write a model for the calories c burned from rowing in this competition for m minutes. How many minutes will they have to row together to burn at least 200 calories?
   A. 14 minutes
   B. 15 minutes
   C. 16 minutes
   D. 17 minutes

10. The total cost for a hotel room varies directly with the numbers of nights you stay. At Hotel A, three nights costs a total of PhP 6,000. At Hotel B, five nights stay

costs a total of Php 9,000. If you will stay for four nights, which hotel has the cheaper cost?

A. Hotel A
B. Hotel B
C. Both offers the same
D. It cannot be determined

11. Which of the following expressions violates the condition in simplifying radicals?

A. $\sqrt[6]{x^4}$
B. $\sqrt{x}$
C. $\sqrt[5]{x}$
D. $\sqrt[5]{3x}$

12. Three times the square root of 2 greater than a number is equal to the square root of 4 more than 11 times that number. Find the number.

A. 4
B. 5
C. 6
D. 7

13. Which of the following is true about the laws of radicals?

A. The quotient of two radicals having the same index **n** is not equal to the **n**th root of the quotient of their radicands.
B. The quotient of two radicals having the same index **n** is equal to the **n**th root of the quotient of their radicands.
C. The product of two radicals having different index **n** is equal to the **n**th root of the product of their radicands.
D. The **n**th root of a number raised to **n** is not equal to the number.

14. If *y* varies jointly as *x* and *z* and *y* = 16 when *x* = 4 and *z* =2, what will happen to the value of z if the value of x increases and the value of y remained.

A. The value of z will increase.
B. The value of z will decrease.
C. The value of z will remain.
D. The value of z cannot be determined.

15. A man in a tower estimated that the distance he can see on the horizon is about 18 mi. How high above the ground is the man in the tower? (The approximate distance, ***d***, in miles that a person can see to the horizon from a height of ***h*** feet is given by the equation $d = \sqrt{\frac{3h}{2}}$.)

a. 210 ft.
b. 220 ft.
c. 216 ft.
d. 218 ft.

16. SPO3 Jesu Erin, investigated a car collision and found out that one of the drivers is under the influence of alcohol. He wanted to investigate how fast the driver was going due to the damaged vehicle during the collision, where the skid marks are the only remaining evidence. Where do you think SPO3 Jesu Erin should start to look into?
    A. The speed of the car
    B. The length of the skid marks
    C. The speed of the car and the length of the skid marks
    D. The alcohol content and the weather.

17. Based on your answer in #16, if the length of the skid marks 1220 ft, how fast is the drunk driver going?
    A. 121 miles per hour
    B. 120 miles per hour
    C. 125 miles per hour
    D. 200 miles per hour

18. Isometric growth, where the various parts of an organism grow in direct proportion to each other, is rare in living organisms. In contrast, most organisms grow non-isometrically; the various parts and organisms do not increase in size in a one-to-one ratio. One of the best known examples of non-isometric growth is human growth. Children have proportionately larger heads and shorter legs than adults. The relative proportions of a human body change dramatically as the human grows. What will happen to humans if they grow isometrically?
    A. Young children would not grow old.
    B. Young children would grow taller by 6 inches every year.
    C. Young children would look just like adults.
    D. Young children would look just like adults, only smaller.

19. Andy's Canteen serves budget meals. The monthly number of orders varies directly with the number of menu, and varies inversely with the cost of each budget meal. Last month's total order is 2000 budget meals when itserves 6 menu and the cost per meal was PhP 50. If the number of menu does not change, what would be the effect on the monthly total number of orders for increasing the cost per meal to PhP 60? Will it be beneficial for the store?
    A. The number of orders will remain, therefore the canteen will gain more.
    B. The number of orders will increase, therefore the canteen will gain more.
    C. The number of orders will decrease, thus the canteen will lose a lot of money.
    D. The number of orders will decrease but the canteen will gain as much as when the price was lower.

20. As a weather expert, you are requested by the PAGASA to make a brochure to be distributed to the residents near the coastal areas for their information campaign on tsunamis. The brochure is to be presented to the board of directors. In order for the board of the directors to approve the brochure for distribution, what are the standards to be considered?
    A. Organization of the presentation, mathematical concept, clarity of the graphics and representations accuracy of data and fluency of the presentation.

B. Practicality, mathematical concept, clarity of the graphics and representations accuracy of data and fluency of the presentation.
C. Organization of the presentation, mathematical concept, accuracy of data and fluency of the presentation.
D. Practicality, Content, Graphics and Representation

LESSON 1: VARIATION

Let's begin by determining the type of relationship in given quantities.

EXPLORE

:

ACTIVITY 1. RELATED QUANTITIES

DESCRIPTION: Name two quantities and describe the relationship. Click "SUBMIT" when you're finished.

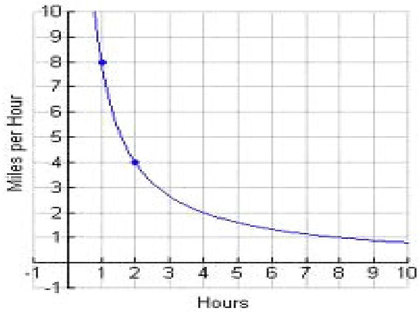
Quantity 1	Quantity 2	Relationship of quantities
Depth of dive	Pressure	As depth of dive increases, pressure also increases
Speed of a car	Time to reach the destination	As the speed of the car increases, the time to reach the destination decreases.
Slice of pizza	Number of people eating	
Number of dogs		
	Fare	

PROCESS QUESTIONS:

1. Can you think of more quantities that are related? Give at least 2 more.
2. What did you observe about the relationship of the quantities? Explain.
3. How do you think can a quantity be influenced by another? Discuss.

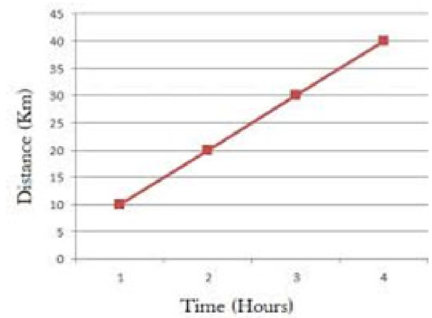
ACTIVITY 2. GRAPH ANALYSIS

DESCRIPTION: Determine the type of relationship shown in the graph. Put a check mark on your answer.



____ Directly related

____ Inversely related



____ Directly related

____ Inversely related

ACTIVITY 3. IRF WORKSHEET

DESCRIPTION: Complete the first part of the Initial Answer section of the worksheet below. You will revisit this worksheet as you progress in this topic. Read aloud what you've written. Click "SAVE" when you're done.

Initial Answer
If two quantities are directly related, then ____________________ ____ ____. If two quantities are inversely related, then __________________ ____ ____.
Revised Answer
Final Answer

PROCESS QUESTIONS:

1. What did you observe from the graphs?
2. What makes them different?
3. How can a quantity be influenced by another?
4. How can mathematical formulas in scientific investigations be determined?

End of EXPLORE:

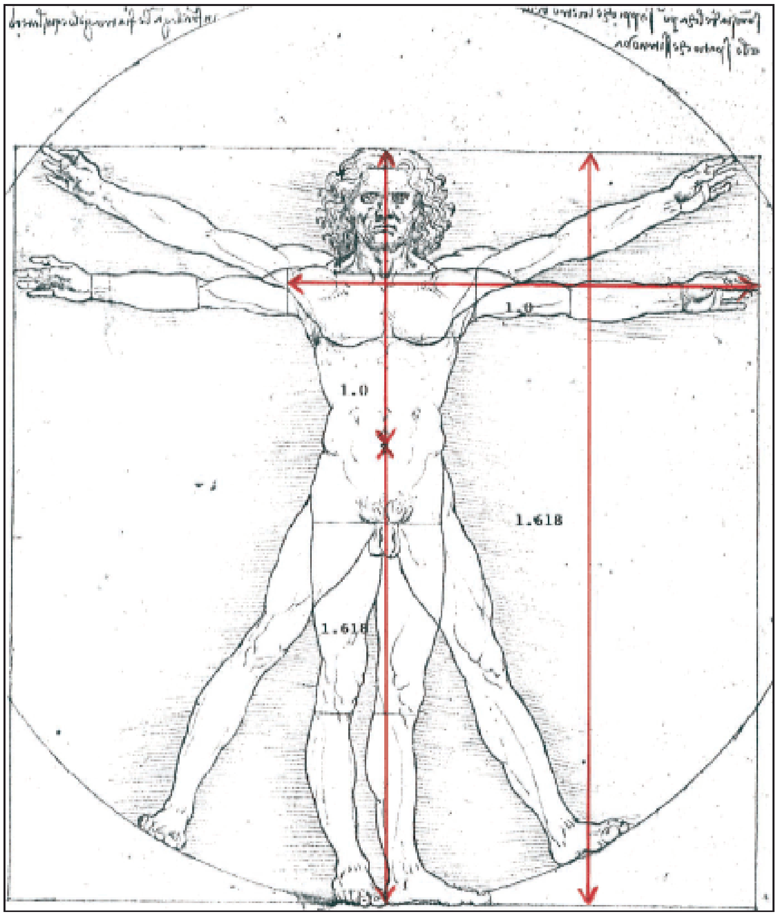
You have just given your ideas about variation. Find out in the next section if your initial ideas are correct. Keep a record of the important concepts that might aid you in affirming or revising your initial ideas in preparation for your final project for the lesson, which is a presentation of a case analysis regarding the factors that lead to vehicular accidents. Let's start by doing the next activity.

FIRM-UP

Your goal in this section is to learn and understand key concepts about direct and inverse variation. To see the competencies for variation, click this link to go back to list of lessons and competencies. You are expected to complete all activities before moving to the next section.

As you progress through the lesson, you will learn **how can mathematical formulas in scientific investigations be determined?**

One of the foremost Renaissance artists, Leonardo da Vinci, believed that in the perfect body, the parts should be related by certain ratios. Below is an illustration of his Vitruvian man.



http://gurneyjourney.blogspot.com/2013/01/part-2-golden-mean-and-leonardo.html

Da Vinci believes that the perfect length of the arm should be 3 times the length of the hand, and the perfect length of the foot should be 6 times the length of the big toe. These relationships can be expressed by the linear equations
a = 3h, where ***a*** is arm length and ***h*** is hand length, and
f = 6t, where ***f*** is foot length and ***t*** is the length of the big toe.

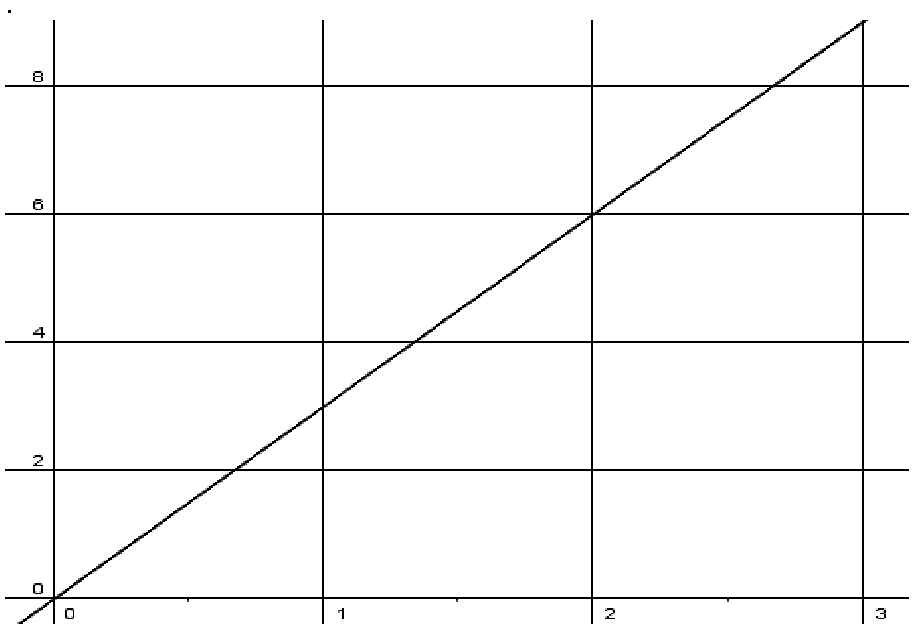
From the mathematical equation ***a = 3h***; if ***h = 1***, then ***a = 3***; if ***h = 2***, then ***a*** = 6. ***h*** is the independent variable, ***a*** is the dependent variable and 3 is the constant.

So using a = 3h and presenting in a table of values:

h (in feet)	1.0	1.25	1.5	1.75	2.0	2.25	2.50
a (in feet)	3	3.75	4.5	5.25	6	6.75	7.5

What can you observe? What happens to ***a*** as increases? The table of values can also be translated in a graph

h (length of hand)



***a* (length of arm)**

Notice that as □ increases, ***a*** also increases. Likewise, as ***h*** decreases, ***a*** also decreases. This type of relationship is called direct variation.

ACTIVITY 4. MORE ON DIRECT VARIATION

DESCRIPTION: Access the website below then read about direct variation. Then answer the questions that follow. Click "SAVE" when you're done.

http://www.regentsprep.org/regents/math/algebra/ao4/ldirect.htm
This is an article about direct variation.
http://www.mathwords.com/d/direct_variation.htm
This is an article about direct variation.
http://learner.org/workshops/algebra/workshop7/index.html#1
This is an article about direct variation.

PROCESS QUESTIONS:

1. What is direct variation?

2. If quantity ***a*** is directly related to quantity ***b***, what happens to ***a*** as ***b*** increases?

3. If quantity ***a*** is directly related to quantity ***b***, what happens to ***a*** as ***b*** decreases?

4. What mathematical equations model direct variation?

5. Complete the statement.

If ***y* = *kx***, then the quantity ***y*** varies ________________

with ***x;*** or ***y*** is ________________ with ***x***. ***k*** is the called

the____________ of variation or constant of proportionality.

6. How can a quantity be influenced by another?

As illustrated in the case of the length of arms and the length of the hand, the direct relationship of two quantities can be translated into a mathematical equation, table of values and graph. In the next activity, you will do all methods to translate situations involving direct variation.

ACTIVITY 5. TRANSLATING DIRECT VARIATIONS

DESCRIPTION: Access these websites then observe how the relationship of the directly related quantities were translated in different forms. Then do the same for the situation below. Click "SAVE" when you're done.

http://www.regentsprep.org/regents/math/algebra/ao4/ldirect.htm
This is an article is about translating direct variations.
http://www.virtualnerd.com/algebra-1/linear-equation-analysis/direct-variation/direct-variation-examples/direct-variation-equation-word-problem
This is an article is about translating direct variations.
http://www.youtube.com/watch?v=o31s1daJaWw
This is a video about translating direct variations.
http://www.youtube.com/watch?v=NX-K3KasEDU
This is a video about translating direct variations.

Situation:
The number of pages printed by a printer varies directly with the amount of time the printer is printing. A printer prints 4 colored pages per minute.

1. Write the mathematical equation

Let:

y = number of colored pages printed

x = number of minutes the printer is printing

k(constant of variation) = ______

equation: __________

2. Complete the table of values.

x (in minutes)	0	1	2	3	4
y (pages printed)					

3. Upload the graph the relationship of the two quantities by using the software Geogebra.

Downloading and installing

a. Download the software in http://www.geogebra.org/cms/en/download/
b. Install in your computer by clicking the installation file. Follow the instructions to install Geogebra.

Using Geogebra

c. Open Geogebra. To explore tutorials, click “Help” then choose “tutorials”. You will be directed to a website for the tutorials.
d. To graph, open Geogebra. Under “View”, choose “Input bar” then click “Show”.
e. In the input box located at the bottom of your screen, write the equation you made in number 1.
f. Press “Enter” and you should be able to see the graph.
g. To save the graph as a picture, click “File”, then choose “Export”, then click “Graphics view as picture (png,eps).
h. Click “Save” in the dialog box.
i. Write the filename then click “Save” to save the graph in your computer.

Uploading the graph

j. Click “Upload File”below.
k. Locate where you saved your graph in your computer.
l. Choose your file then upload. Your graph should be visible in the box below.

PROCESS QUESTIONS:

1. What is the relationship of the number of colored pages printed and the time the printer is printing?

2. What are the different ways of translating direct variations?

3. What type of equation was formed?

4. What type of graph was formed?

5. Did the different ways of translating help in identifying the relationship of the two quantities? Which is the most effective? Take note that you only translated **ONE** situation, so the relationship of the two quantities is the same even if it was translated in different ways.

6. Which is the main reason of translating, illustrating the relationship in different forms or understanding the relationship of the quantities?

7. Can you reverse the process? Meaning, describe the relationship of the quantities given the a) equation, b) table of values and c) graph?

Many commonplace situations are examples of direct relationships like the total cost for some number of CDs when the price is ₱150 per CD and the number of miles traveled if one drives at 40 miles per hour. When these situations are translated in equation form, table of values or graph, ***making estimates and predictions becomes easier***.

ACTIVITY 6. HOW TO SOLVE PROBLEMS INVOLVING DIRECT VARIATIONS

DESCRIPTION: Access the websites below then take notes on how to solve problems involving direct variation. Use your Evernote account in www.evernote.com to take notes on how to solve problems involving direct variation. Then, summarize the steps in the box below. **Read aloud** what you've written in the box. Click "SAVE" when you're finished.

http://voices.yahoo.com/how-set-solve-direct-variation-word-problems-1762543.html
This website describes how to solve problems involving direct variation
http://www.youtube.com/watch?v=wKXXrBV_RcA
This website shows a video about solving problems involving direct variation
http://math.info/Algebra/Word_Probs_Direct_Variation/
This website describes how to solve word problems involving direct variation

Steps in Solving a Direct Variation Problem

After translating direct relationships in different ways and listing steps in solving direct variation problems, are you ready to solve problems related to direct variation?

Click “Like” if you are ready then proceed with Activity 7.Otherwise, click “dislike” then visit these additional websites before you proceed to Activity 7.

☐     ☐

http://www.youtube.com/watch?v=q6IlRl6Bh8s
This website shows a video about solving problems involving direct variation
http://math.info/Algebra/Word_Probs_Direct_Variation/
This website describes how to solve word problems involving direct variation

ACTIVITY 7. SOLVING PROBLEMS INVOLVING DIRECT VARIATIONS

DESCRIPTION: Answer the following problems. Email your complete solution to your teacher.

1. Decide if the second quantity increases or decreases.
   a. As the distance a taxi travels increases, the fare ___________.
   b. The circumference of a circle decreases as its radius ________.
   c. The area of a square decreases as its side ________________.
   d. The total cost increases as the price ____________________.

2. In these situations, y varies directly as x:
   a. When $x = 2$, $y = 14$. Find y when $x = 10$.
   b. When $x = \frac{3}{4}$, $y = \frac{1}{8}$. Find y when $x = 4$.
   c.

x	4	7
y	24	?

d.

x	8	?
y	14	16

3. The weight of an object on the moon varies directly as its weight on Earth. An astronaut who weighs 80 kg on Earth weighs 12.8 kg on the moon. How much would a 100 – kg person weigh on the moon?

4. The pressure of water on an object varies directly with its distance from the surface. A submarine experiences pressure of 26 lbs per square inch at 60 feet below the surface. By how much will the pressure increase if the submarine is at 150 feet below the surface then moves to 208 feet below the surface?

5. The number of kilograms of water in a human body varies directly as the total weight. A person weighing 63 kg contains 42 kg of water. How many more kilograms of water are in a 120 – kg person than a 90– kg person?

6. The area of a square varies directly as the square of its side. What is the area of the square if the side measures 1 meter? 2 meters? 3 meters? 4 meters? 5 meters? Show it in a table and graph.

7. The stopping distance *d* of a car after the brakes have been applied varies directly as the square of the speed of the car *s*.
   a. If a car traveling 60 mph can stop in 200 ft, how far will it take a car to stop if it is traveling at 50 mph?
   b. Suppose the car was able to stop in 50 ft, at what speed was it traveling?

PROCESS QUESTIONS:

1. What is the common with the relationship of the two quantities in each item?

2. Did you translate the situation into an equation? Into table of values? Into graph?

3. How can a quantity be influenced by another?

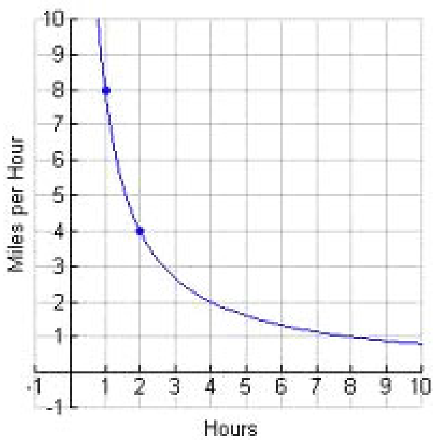
4. How can mathematical formulas in scientific investigations be determined?

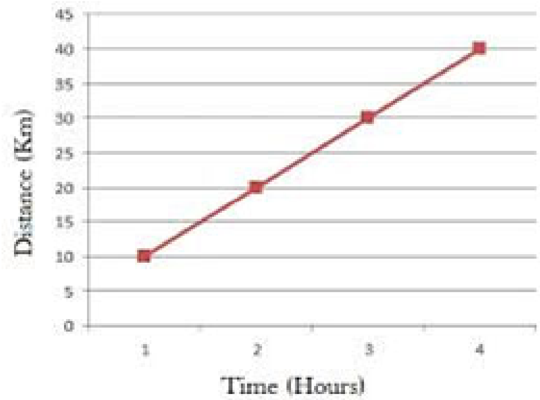
5. In the various situations above (# 3 – 6), what was common in the process of representing the situation in a mathematical way? Discuss and support your answer with specific examples.

Before starting with the next topic, revisit Activity 2 and 3. Are there answers you need to revise?

ACTIVITY 8. GRAPH ANALYSIS REVISITED

DESCRIPTION: Determine the type of relationship shown in the graph. Put a check mark on your answer.





____ Directly related
____ Inversely related

____ Directly related
____ Inversely related

ACTIVITY 9. IRF WORKSHEET REVISITED

DESCRIPTION: You completed the Initial Answer section of the worksheet below. This time, revise your ideas under the "Revised answer". Read aloud what you've written and when satisfied, click "SAVE".

Initial Answer
To the programmer, please show the answer of the student here from Activity 2
Revised Answer If two quantities are directly related, then ________________ ____ ____. If two quantities are inversely related, then ________________ ____ ____.
Final Answer

Now that you revised your ideas, let's see about inverse variation.

During sports competitions like the Palarong Pambansa, many groups of athletes have to hire buses. If the amount for hiring the bus is ₱30000, and the athletes will divide the amount to be paid equally, how much will they pay if only 10 athletes will ride in the bus? Only 20 athletes? Only 30 athletes? Only 50 athletes?60 athletes?

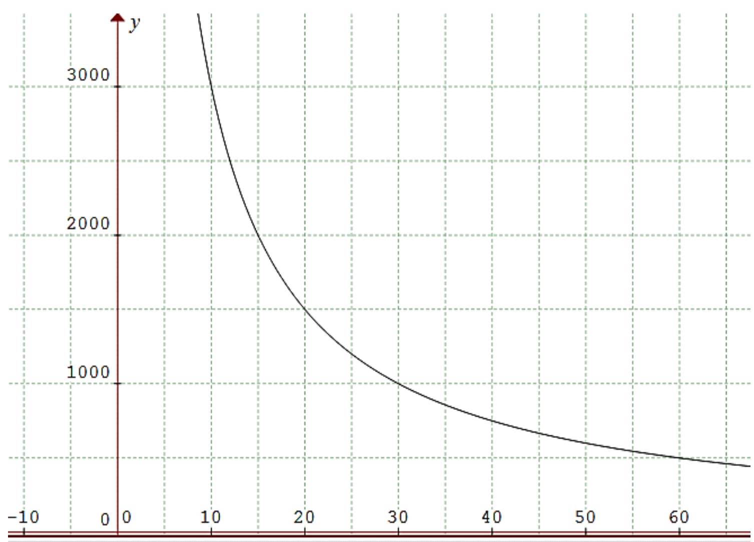
If the amount to be paid is *A* and the number of athletes is *n*, then the equation A = $\frac{30000}{\square}$ models how to compute the amount each athlete will pay. If only 10 athletes will ride in the bus, A = $\frac{30000}{10} = 3000$; if only 20 athletes, A = $\frac{30000}{20} = 1500$; if only 30 athletes, A = $\frac{30000}{30} = 1000$; and if there are 60 athletes, A = $\frac{30000}{60} = 500$.

So using A = $\frac{30000}{\square}$ and presenting in a table of values:

n (number of athletes)	10	20	30	40	50	60
A (amount in ₱)	3000	1500	1000	750	600	500

What can you observe? What happens to the amount *A* to be paid as the number of n increases? The table of values can also be translated in a graph.

A (Amount in ₱)



n(number of athletes)

Notice that as *n* increases, *A* decreases. Likewise, as *n* decreases, *A* increases. This type of relationship is called inverse variation.

ACTIVITY 10. MORE ON INVERSE VARIATION

DESCRIPTION: Access the websites below then read about inverse variation. Then answer the questions that follow.

http://www.regentsprep.org/Regents/math/algtrig/ATE7/Inverse%20Variation.htm
This article is about inverse variation.

http://www.mathwords.com/i/inverse_variation.htm
This article is about inverse variation.

http://learner.org/workshops/algebra/workshop7/index2.html#1
This article is about solving inverse variation.
http://www.shelovesmath.com/algebra/beginning-algebra/direct-inverse-and-joint-variation/
This article is about solving direct and inverse variation.

PROCESS QUESTIONS:

1. What is inverse variation?

2. If quantity *y* is inversely related to quantity *x*, what happens to *y* as *x* increases?

3. If quantity *y* is inversely related to quantity *x*, what happens to *y* as *x* decreases?

4. What mathematical formulas model inverse variation?

5. Complete the statement.

6.

If $y = \frac{k}{x}$, then the quantity y varies ________________ with x; or y is ________________ with x. k is the called the____________ of variation or constant of proportionality.

As illustrated in the case of the number of athletes and the amount to be paid by each, inverse relationships can be translated in a mathematical equation, table of values and graph.

ACTIVITY 11. TRANSLATING INVERSE VARIATIONS

DESCRIPTION: Access these websites then observe how the relationship of the inversely related quantities were translated in different forms. Then do the same for the situation below. Click "SAVE" when you're done.

http://www.sparknotes.com/math/algebra1/variation/section2.rhtml
This article is about translating quantities having inverse relationships.

http://www.shelovesmath.com/algebra/beginning-algebra/direct-inverse-and-joint-variation/
This article is about translating quantities having inverse and direct relationships.

Situation: The estimated distance from Baguio to Manila is 240 km. How long will it take Andy to travel from Manila to Baguio if he travels at an average speed of 40 kph?50 kph?60 kph?70 kph?80 kph?

1. Write the mathematical equation

Let:

s = average speed of the car in kph

t = duration of travel (time in hours)

k (constant of variation) = ________

equation:

2. Complete the table of values

speed (in kph)	40	50	60	70	80
time (in hrs)	7.5				

3. Upload the graph the relationship of the two quantities by using the software Geogebra.

Downloading and installing

a. Download the software in http://www.geogebra.org/cms/en/download/
b. Install in your computer by clicking the installation file. Follow the instructions to install Geogebra.

Using Geogebra

c. Open Geogebra. To explore tutorials, click “Help” then choose “tutorials”. You will be directed to a website for the tutorials.
d. To graph, open Geogebra. Under “View”, choose “Input bar” then click “Show”.
e. In the input box located at the bottom of your screen, write the equation you made in number 1. Use x for s and y for t.
f. Press “Enter” and you should be able to see the graph.
g. To save the graph as a picture, click “File”, then choose “Export”, then click “Graphics view as picture (png,eps).
h. Click “Save” in the dialog box.
i. Write the filename then click “Save” to save the graph in your computer.

Uploading the graph

j. Click “Upload File” below.
k. Locate where you saved your graph in your computer.
l. Choose your file then upload. Your graph should be visible in the box below.

PROCESS QUESTIONS:

1. What is the relationship of the speed and duration of travel?

2. What are the different ways of translating inverse variations?

3. What type of equation was formed?

4. What type of graph was formed?

5. Did the different ways of translating help in identifying the relationship of the two quantities? Which is the most effective? Take note that you only translated **ONE** situation, so the relationship of the two quantities is the same even if it was translated in different ways.

6. Which is the main reason of translating, illustrating the relationship in different forms or understanding the relationship of the quantities?

7. Can you reverse the process, meaning, describe the relationship of the quantities given the a) equation, b) table of values and c) graph?

8. How can mathematical formulas in scientific investigations be determined?

Many commonplace situations are examples of inverse relationships like the number of days a drum of water lasts varies inversely as the number of people who consume it; and the time required to finish a job varies inversely as the number of people working on the job. When these situations are translated in equation form, table of values or graph, ***making estimates and predictions becomes easier***.

ACTIVITY 12. HOW TO SOLVE PROBLEMS INVOLVING INVERSE VARIATION

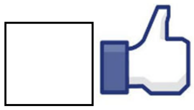
DESCRIPTION: Access the websites below then take notes on how to solve problems involving inverse variation. Use your Evernote account in www.evernote.com to take notes on how to solve problems involving inverse variation. Then, summarize the steps in the box below. **Read aloud** what you've written in the box. Click "SAVE" when you're finished.

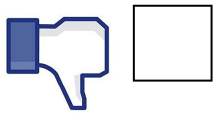
http://www.mesacc.edu/~scotz47781/mat120/notes/variation/inverse/inverse.html
This article is about solving problems involving inverse variation.
http://math.info/Algebra/Word_Probs_Inverse_Variation/
This article is about solving problems involving inverse variation.
http://www.youtube.com/watch?v=NMztX6yG1Dc
This video is about solving problems involving inverse variation.

Steps in Solving an Inverse Variation Problem

After translating inverse relationships in different ways and listing steps in solving inverse variation problems, are you ready to solve problems related to inverse variation?

Click "Like" if you are ready then proceed with Activity 13. Otherwise, click "dislike" then visit these additional websites before you proceed to Activity 13.





http://www.shelovesmath.com/algebra/beginning-algebra/direct-inverse-and-joint-variation/
This article is about solving problems involving inverse variation.

ACTIVITY 13. SOLVING PROBLEMS INVOLVING INVERSE RELATIONSHIPS

DESCRIPTION: Answer the following problems. Email your complete solution to your teacher.

1. Decide if the second quantity increases or decreases.
   a. As the distance a bus travels increases, the fare ________.
   b. The quantity of an item you can buy with ₱500 decreases as the price of the item _______________.
   c. The amount of remaining ink in a pen decreases as the number of hours it has been used _______________.
   d. The time required to empty a tank increases as the rate of pumping _________________.
2. In these situations, y varies inversely as x:
   a. When $x = 3$, $y = 2$. Find y when $x = 8$.
   b. When $x = \frac{1}{3}$, $y = 6$. Find y when $x = 10$.
   c.

x	12	3
y	3	?

   d.

x	6	?
y	$\frac{1}{3}$	1

3. The time required to do a certain job varies inversely as the number of people working on the job. It takes 2 hours for 6 people to scrub the corridors of a building. How long would it take 4 people to do the same job?

4. The length of time a bag of dog food lasts varies inversely with the number of dogs. If a bag of dog food will feed 3 dogs for 10 days, how long will it feed 10 dogs?

5. According to Boyle's Law, when the temperature is held constant the volume of the gas varies inversely with the pressure on the gas. If the gas occupies 36 cm^3 under a pressure of 10 atmospheres, find the volume when the pressure is changed to 8 atmospheres.

6. Kai decides to empty her pool for the winter. She knows that the time ***t*** required to empty a pool varies inversely as the rate ***r*** of pumping.
   a. Write an equation that represents this situation. Let □be theconstant of variation.

   b. In the past, Kai was able to empty her pool in 45 minutes at a rate of 800 liters per minute. She now owns a new pump that can empty the pool at a rate of 1 kiloliters per minute. How long will it take Kai to empty the pool using this new pump? Is it faster or slower? By how many minutes?

7. The intensity of light ***I***, measured in lux, is inversely proportional to the square of the distance d between the light source and the object illuminated.
   a. Write an equation that represents this situation.

   b. Using a light meter, a lighting director measures the intensity of the light from a bulb hanging 6 feet overhead a circular table at 16 lux. If the table has a5-foot diameter, what illumination reading will the director find at the edge of the table where the actors will sit? Round to the nearest tenth.

PROCESS QUESTIONS:

1. What is common with the relationship of the two quantities in each item?

2. Did you translate the situation into an equation? Into table of values? Into graph?

3. How can a quantity be influenced by another?

4. How can mathematical formulas in scientific investigations be determined?

5. In the various situations above (# 3 – 7), what was common in the process of representing the situation in a mathematical way? Discuss and support your answer with specific examples.

ACTIVITY 14. IRF WORKSHEET FINALIZED

DESCRIPTION: You completed the Initial and Revised Answer sections of the worksheet below. This time, finalize your ideas under the "Final Answer". **Read aloud** what you've written and you're satisfied, click "SAVE".

Initial Answer
Revised Answer
Final Answer If two quantities are directly related, then _________________ ____ ____. If two quantities are inversely related, then ________________ ____ ____.

PROCESS QUESTIONS:

1. What did you change in your previous answer?

2. What is the new concept that you learned?

Now that you have finalized your thoughts about direct and inverse variations, are you confident to answer problems related to variations? Click **this link**to revisit the required competencies for variation. When ready, proceed to the next activity.

ACTIVITY 15. DIRECT vs INVERSE VARIATION

DESCRIPTION: In this activity, you will summarize and share what you have learned about direct and inverse variations.

1. Use www.gliffy.comto help you develop your mind map.
2. Save your map as an image/picture.
3. Post the image/picture in your Facebook account and solicit at least 10 comments.
4. Message your teacher through Facebook to check your post.

To check on your understanding about direct and inverse variation, the following activity is a quiz. You can use your notes in www.evernote.com and the summary

you uploaded in www.facebook.com as your guide. Be sure to review your answers before you click "SUBMIT".

ACTIVITY 16. HOW FAR / HOW FAINT

DESCRIPTION: To complete this activity, follow the steps below. Write your complete solution and answers the email to your teacher.

1. Read the article about the Inverse Square Law in the website below.
http://www.if.ufrgs.br/ast/solar/eng/edu/invsquar.htm

2. State the Inverse Square Law in sentence form.

3. Translate the Inverse Square Law in equation form.
Let s = amount of sunlight,
k = constant of variation
d = distance as compared to Earth's distance from the sun

4. Based from the website, what equation represents the amount of sunlight received by the satellite? Compare it with the amount of sunlight received by Earth.

5. Based from the website, what equation represents the amount of sunlight received by Neptune? Compare it with the amount of sunlight received by Earth.

6. Based from the website, what equation represents the amount of sunlight received by Mercury? Compare it with the amount of sunlight received by Earth.

7. Show the relationship of the amount of sunlight (s) and distance (d) in a table.

	Satellite	Neptune	Mercury
d			
s			

8. Answer the question below. Give supporting evidences by using the equations and table that you had written.
a. What will happen if the distance of Earth to the Sun is as close as Mercury?
b. What will happen if the distance of Earth to the sun is as far as Neptune?

End of FIRM UP:

In this section, the discussion was about direct and inverse variations.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision? What new learning goal should you now try to achieve?

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.

DEEPEN:

Your goal in this section is to take a closer look at some aspects of joint and combined variations. Click this link to check on the expected competencies for joint and combined variations. As you go on, be focused on how can mathematical formulas in scientific investigations be determined?

You already know relationships where one quantity is directly or inversely related with another quantity. However, is it possible that a quantity can vary directly or inversely with two or more variables? Let's start by doing this activity.

ACTIVITY 17. AREAS, VOLUMES AND VARIATIONS

DESCRIPTION: Complete the table by computing the area and volume using the values provided. Click "SAVE" when you're finished.

Area of a Rectangle A = lw		
length	width	Area
5	6	
5	12	
5	18	
10	6	
15	6	

Volume of Cylinder V = $\pi r^2 h$		
radius	height	Volume
3	8	
3	16	
3	24	
6	8	
12	8	

PROCESS QUESTIONS:

1. As the width increases and the length remains constant, what happens to the area?

2. The length increases and the width remains constant, what happens to the area?

3. As the height increases and the radius remains constant, what happens to the volume?

4. As the radius increases and the height remains constant, what happens to the volume?

5. How many variables affect the area of a rectangle?

6. How many variables affect the volume of a cylinder?

7. How can a quantity be influenced by another?

As you have observed, as the length increases and the width remain constant, the area increases. Also, as the width increases and the length remain constant, the area also increase. Clearly, the area of a rectangle is proportional to the product of its length and width. Thus, area varies jointly with its length and width.

Similarly, as the radius increases and the height remain constant, the volume of the cylinder increases. As the height increases and the radius remain constant, the volume increases. Clearly, the volume of a cylinder is affected by its radius and height. Thus, volume varies jointly with the square of its radius and height.

To learn more about joint variation, do the next activity.

ACTIVITY 18. MORE ON JOINT VARIATION

DESCRIPTION: Access the websites below then read about joint variation. Then answer the questions that follow.

http://www.shelovesmath.com/algebra/beginning-algebra/direct-inverse-and-joint-variation/
This article is about joint variation.
http://www.youtube.com/watch?v=QIjpoEE65v4
This video is about joint variation.

PROCESS QUESTIONS:

1. What is joint variation?

2. If quantity ***y*** is jointly related to quantity ***x*** and ***z***, what happens to ***y*** as ***x*** and ***z*** increases?

3. If quantity ***y*** is jointly related to quantity ***x*** and ***z***, what happens to ***y*** as ***x*** and ***z*** decreases?

4. What mathematical formulas model joint variation?

5. Complete the statement.

If $y = kxz$ then the quantity ***y*** varies ________________

________________________. k is the called

the____________ of variation.

6. How can a quantity be influenced by another?

ACTIVITY 19. HOW TO SOLVE PROBLEMS INVOLVING JOINT VARIATION

DESCRIPTION: Access the websites below then take notes on how to solve problems involving joint variation. Use your Evernote account in www.evernote.com to take notes on how to solve problems involving joint variation. Then, summarize the steps in the box below. **Read aloud** what you've written in the box. Click "SAVE" when you're finished.

http://www.mesacc.edu/~scotz47781/mat120/notes/variation/joint/joint.html
This article is about solving problems involving joint variation.

http://www.shelovesmath.com/algebra/beginning-algebra/direct-inverse-and-joint-variation/
This article is about solving problems involving joint variation.

Steps in Solving a Joint Variation Problem

Now that you've written the steps in solving joint variation, open the website below then try to solve the problems. The answer and solution are also provided for you to compare. Take note of the problem on interest rate because you encountered this in the pretest and soon on the posttest.

http://www.mesacc.edu/~scotz47781/mat120/notes/variation/joint/joint_practice.html
This article contains problems involving joint variation.

After reading articles, watching videos and solving some problems, are you ready to solve problems related to joint variation?

Click “Like” if you are ready then proceed with Activity 20. Otherwise, click “dislike” then visit these additional websites before you proceed to Activity 20.

☐   ☐

http://www.youtube.com/watch?v=QIjpoEE65v4
This video is about solving problems involving joint variation.

http://www.granby.k12.ct.us/file.cfm?resourceid=14602&filename=VariationReview_with_answers_extra_practice.pdf
This article contains problems involving joint variation.

ACTIVITY 20. SOLVING PROBLEMS INVOLVING JOINTVARIATIONS

DESCRIPTION: Answer the following problems. Email your complete solution to your teacher.

1. Find an equation in which y varies jointly as x and z and the following conditions exist.
   ______________a. $y = 4$ when $x = 2$ and $z = 1$
   ______________b. $y = 35$ when $x = 7$ and $z = 15$
   ______________c. $y = 9$ when $x = \frac{1}{4}$ and $z = 12$
2. Find an equation of joint variation. Then solve for the missing value.
   a. p varies directly as q and t, and $p = 60$ when $q = 24$ and $t = 5$. Find p when $q = 12$ and $t = 4$.
   b. v varies directly as w and u, and $v = 28$ when $w = 8$ and $u = 7$. Find v when $w = 14$ and $u = 9$.
   c. If y varies jointly as x^2 and z, and $y = 56$ when $x = 4$ and $z = 7$. Find y when $x = 9$ and $z = 3$.
3. The area of a triangle varies jointly with the base and height of the triangle. The area of the triangle is 16 m^2 when the base is 4 meters and the height is 8 meters. What is the area if the base is 24 meters and the height is 10 meters?
4. A lumber company needs to estimate the volume of wood a load of timber will produce. The supervisor knows that the volume of wood in a tree varies jointly as the height □ and the square of the tree’s girth (distance around the tree)□. The

supervisor observes that a tree 40 meters tall with a girth of 1.5 meters produces 288 cubic meters of wood.

a. Write an equation that represents this situation.
b. What volume of wood can the supervisor expect to obtain from 50 trees averaging 75 meters in height and 2 meters in girth?

5. The power (P) in watts generated by a windmill varies jointly with its efficiency (E), the square of the diameter (D) of its blades in feet and the cube of the wind velocity (V) in feet per second. At a wind velocity of 9.3 ft/sec, a windmill whose blades have a diameter of 10 feet and whose efficiency is 0.4 generates a power of 10,000 watts.

   a. Solve for the variation constant. Write a variation equation.
   b. How much power does the windmill generate when the wind velocity is 10 ft/sec?
   c. Suppose the diameter of the blades is increased to 12 feet, and the other quantities remain constant, how much power would the windmill generate?

PROCESS QUESTIONS:

1. What is common with the relationship of the quantities in each item?

2. Did you translate the situation into an equation? Into table of values? Into graph?

3. How can a quantity be influenced by another?

4. How can mathematical formulas in scientific investigations be determined?

5. In the various situations above (# 3 – 5), what was common in the process of representing the situation in a mathematical way? Discuss and support your answer with specific examples.

Now that you finished solving problems about joint variation, consider this situation.

Why do tires vary from one kind of vehicle to another? The size of a tire considers the weight of the vehicle and the terrain where the vehicle will primarily be used, among others. Tire pressure also varies. The formula $P = \frac{0.25W}{A}$ gives the recommended tire pressure for each tire for the total weight (W) of a car and the area (A) of the ground covered by each tire. Notice that pressure varies directly as the weight of the car and inversely as the area of the ground. The formula $P = \frac{0.25W}{A}$ is an example of combined variation.

Do the next activity to learn more about combined variation.

ACTIVITY 21. MORE ON COMBINED VARIATION

DESCRIPTION: Access the websites below then read about combined variation. Then answer the questions that follow.

http://www.shelovesmath.com/algebra/beginning-algebra/direct-inverse-and-joint-variation/
This article is about combined variation.

http://www.youtube.com/watch?v=QIjpoEE65v4
This video is about combined variation.

PROCESS QUESTIONS:

1. What is combined variation?

2. If quantity y is directly related to quantity x and inversely related to z, what happens to y as x increases?

i

3. If quantity y is directly related to quantity x and inversely related to z, what happens to y as z increases?

4. What mathematical equations model combined variation?

5. Complete the statement.

If $y = \frac{kx}{z}$ then the quantity y varies ____________________

______________________________. k is the called

the ______________ of variation.

6. How can a quantity be influenced by another?

Now that you gave your thoughts of combined variation, let's see how problems involving combined variations are solved.

ACTIVITY 22. HOW TO SOLVE PROBLEMS INVOLVING COMBINED VARIATION

DESCRIPTION: Access the websites below then take notes on how to solve problems involving combined variation. Use your Evernote account in www.evernote.com to take notes on how to solve problems involving combined variation. Then, summarize the steps in the box below. **Read aloud** what you've written in the box. Click "SAVE" when you're finished.

http://www.mesacc.edu/~scotz47781/mat120/notes/variation/combined/combined.html
This article shows how to solve combined variation.

http://www.shelovesmath.com/algebra/beginning-algebra/direct-inverse-and-joint-variation/
This article shows how to solve combined variation.

Steps in Solving Combined Variation Problem

After reading articles and watching videos about combined variation, are you ready to solve problems related to combined variation?

Click "Like" if you are ready then proceed with Activity 23. Otherwise, click "dislike" then visit these additional websites before you proceed to Activity 23.

☐   ☐

http://www.purplemath.com/modules/variatn3.htm

This article shows how to solve combined variation.

ACTIVITY 23. SOLVING PROBLEMS INVOLVING COMBINED VARIATIONS

DESCRIPTION: Answer the following problems. Email your complete solution to your teacher.

Find an equation of combined variation. Then solve for the missing value.

1. y varies directly as x and inversely as z, and $y = 6$ when $x = 8$ and $z = 4$. Find y when $x = 12$ and $z = 9$.

2. y varies directly as x and inversely as z, and $y = 4$ when $x = 3$ and $z = 6$. Find y when $x = 5$ and $z = 20$.

3. p varies directly as q and inversely as r, and $p = 6$ when $q = 12$ and $r = 0.5$. Find p when $q = 8$ and $r = \frac{1}{3}$.

PROCESS QUESTIONS:

1. What is common with the relationship of the quantities in each item?

2. Did you translate the situation into an equation? Into table of values? Into graph?

3. How can a quantity be influenced by another?

4. How can mathematical formulas in scientific investigations be determined?

ACTIVITY 24. JOINT vs COMBINED VARIATION

DESCRIPTION: In this activity, you will summarize and share what you have learned about joint and combined variations.

1. Use www.gliffy.com to help you develop your mind map.
2. Save your map as an image/picture.
3. Post the image/picture in your Facebook account and solicit at least 10 comments.

4. Message your teacher through Facebook to check your post.

End OF DEEPEN:

In this section, the discussion was about joint and combined variation. You have learned about joint and combined variation through reading and summarizing articles, watching videos while taking notes, constructing and posting your own mind map, revisiting your prior knowledge and solving lots of problems.

What new realizations do you have about the topic? What new connections have you made for yourself? What helped you make these connections?

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.

TRANSFER

Your goal in this section is apply your learning to real life situations. You will be given a practical task which will demonstrate your understanding on variations.

ACTIVITY 25. DVD FOR RENT

DESCRIPTION: Read and analyze the situation below. Answer completely using the skills you've learned in this lesson.

Zander's Digital World rents out DVDs. The manager observed that the weekly total number of rentals varies directly with the total inventory, and varies inversely with the cost of each rental. Last week's total rental is 300 DVDs when its inventory was 960 and the cost per rental was ₱25. If its inventory does not change, what would be the effect on the weekly total number rentals if the cost per rental will be increased to ₱30? Will it be beneficial for the store? Justify your answer.

While doing this performance task you may have questions or you may need help from your teacher or peers. You may click on the email button to send your concern to your teacher or you can post your concern to our Discussion Forum to communicate with your peers. *(SRL-Resource Management seeking help from others)*

After doing the task, evaluate your work using the rubric below.

CRITERIA	Outstanding 4	Satisfactory 3	Developing 2	Beginning 1	RATING
Mathematical reasoning	Explanation shows thorough reasoning and insightful justifications.	Explanation shows substantial reasoning.	Explanation shows gaps in reasoning.	Explanation shows illogical reasoning.	
Accuracy	All computations are correct and shown in detail.	All computations are correct.	Most of the computations are correct.	Some the computations are correct.	
Presentation	The presentation uses appropriate and creative visual materials. It is delivered in a very convincing manner.	The presentation uses appropriate visual materials. It is delivered in a clear manner.	The presentation uses some visual materials. It is delivered in a disorganized manner.	The presentation does not use any visual materials. It is delivered in a clear manner.	
				OVERALL RATING	

End of TRANSFER:

In this section, your task was the application of variations.

How did you find the performance task? How did the task help you see the real world use of the topic?

You are almost finished with this lesson. Click NEXT to answer the last activity.

ACTIVITY 26. LAC CARD

DESCRIPTION: Fill-in the Learned, Affirmed, Challenged cards given below to evaluate what you've learned about variations. When you're finished, click "SUBMIT".

Learned	Affirmed	Challenged
• What new realizations and learning do you have about the topic? • 1. • 2. • 3.	• What new connections have you made? Which of your old ideas have been affirmed/confirmed? • 1. • 2. • 3.	• What questions do you still have? Which areas seem difficult for you? Which do you want to explore? • 1. • 2. • 3.

Congratulations! You can now proceed to the next lesson.

GLOSSARY OF TERMS USED IN THIS LESSON:

direct variation – describes a situation where a variable increases as another variable increases such that as x increases, y increases and when x decreases, y also decreases.

inverse variation – describes a situation where a variable increases as another variable increases such that as x increases, y decreases and when x decreases, y increases.

joint variation – describes a situation where a variable depends on the product of other variables

combined variation - describes a situation where a variable depends on two (or more) other variables, and varies directly with some of them and varies inversely with others

REFERENCES AND WEBSITE LINKS USED IN THIS LESSON:

Oronce, Orlando and Marilyn Mendoza.(2014). E-Math. Rex Book Store. Manila, Philippines

Nivera, Gladys. (2011). Intermediate algebra: Patterns and Practicalities. Salesiana Books. Makati City, Philippines

Direct Variation
http://www.youtube.com/watch?v=o31s1daJaWw
The video in this website is about solving problems involving direct variation.

Direct and inverse variation
http://www.youtube.com/watch?v=RBLjdzqyPl4
The video in this website is about solving problems involving direct and inverse variation.

Direct and other forms of variation
http://www.onlinemathlearning.com/direct-variation.html
This website gives a detailed explanation of direct and other forms of variation. Examples and videos are also included.

Inverse variation
http://www.youtube.com/watch?v=YkGBiyZgEIM
The video in this website is about solving problems involving inverse variation.

Joint ad combined variation
http://www.onlinemathlearning.com/joint-variation.html
This website contains the explanation of the joint and combined variation. Examples and videos are also included.

Joint and combined variation
http://www.youtube.com/watch?v=h45fqKZfg0E
The video in this website is about solving problems involving joint and combined variation.

Joint and combined variation
http://www.youtube.com/watch?v=QIjpoEE65v4
The video in this website is about solving problems involving joint and combined variation.

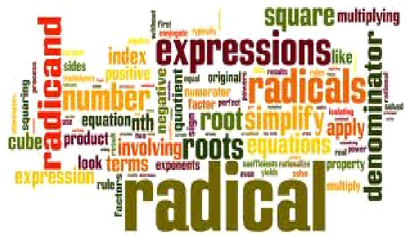
Practice problems about variation
http://hotmath.com/help/gt/genericalg1/section_9_9.html
This website contains several problem sets about variation. It also shows the solution for your reference.

The different forms of variation
http://www.shelovesmath.com/algebra/beginning-algebra/direct-inverse-and-joint-variation/
This article in this website presented the different forms of variation complete with explanation, examples and graphs.

Vitruvian Man
http://gurneyjourney.blogspot.com/2013/01/part-2-golden-mean-and-leonardo.html

LESSON 2: RADICALS

☑ INTRODUCTION AND FOCUS QUESTION(S):



Have you thought of why Algebra is difficult to understand and often perceived to have a confusing set of letters, numbers and symbols? Have you at a certain time asked yourself how Algebra can help you in choosing the best option if there is a variety of choices?

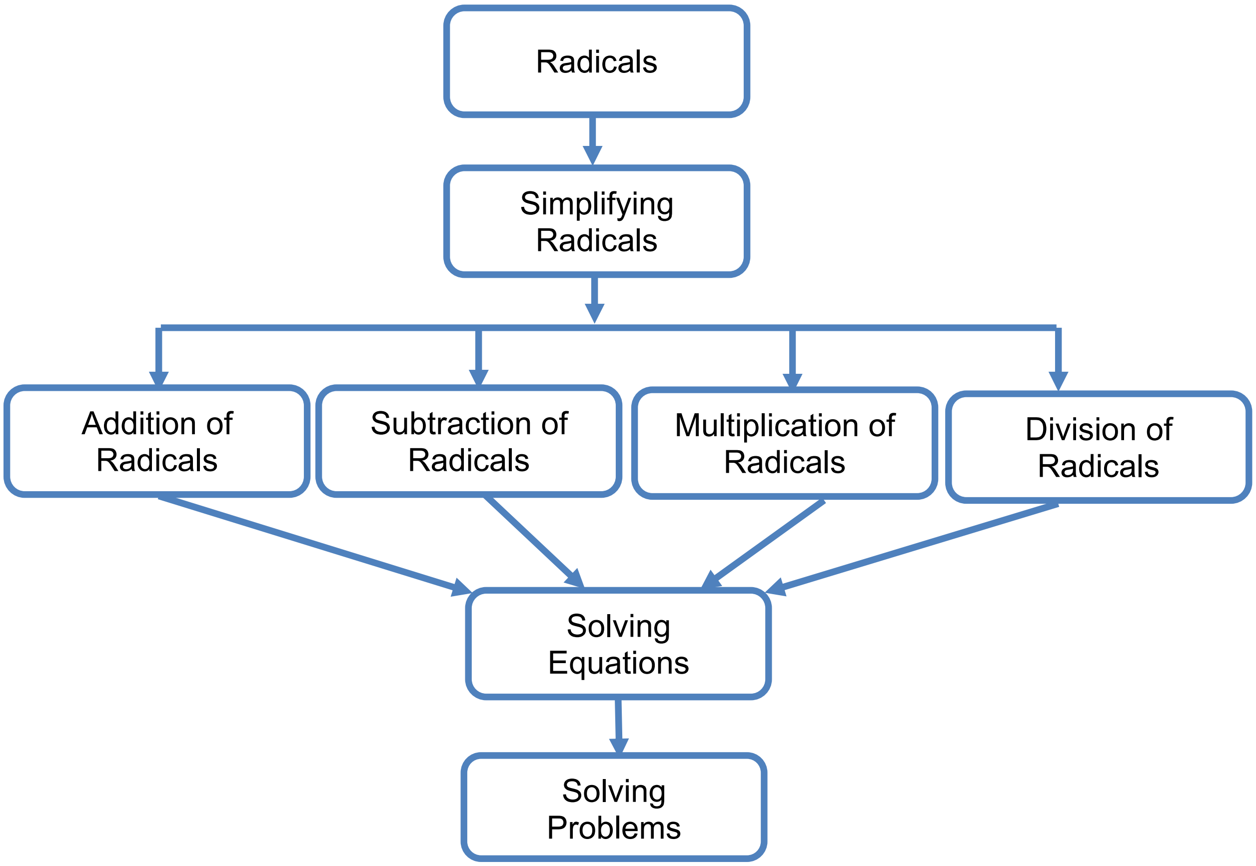
In this module, you will pay attention to radical expressions where the knowledge and skills learned in the previous lessons can be applied. You will find out the different concepts involved in Radicals. As you go over the exercises, you will be able to answer the questions: How can mathematical formulas in scientific investigations be determined? How can a quantity be influenced by another?

In these lesson, you will learn the following:

Lesson 2	• **Applies the laws involving positive integral exponents to zero and negative integral exponents.** • **Illustrates expressions with rational exponents** • **Simplifies expressions with rational exponents** • **Writes expressions with rational exponents as radicals and vice versa** • **Derives the laws of radicals** • **Simplifies radical expressions using the laws of radicals** • **Performs operations on radical expressions** • **Solves equations involving radical expressions** • **Solves problems involving radicals**

☑ LESSON MAP:

Here is a simple map of the topics you will cover:



☑ EXPECTED SKILLS:

To do well in this lesson, you need to remember and do the following:

1. Look up the meaning of words you do not know.
2. Complete all activities and exercises.
3. Take note on the proper modeling of situations using oral, written, graphical and algebraic methods to solve problems.
4. Use the checklist and rubric provided to evaluate your work before submission.
5. Be mindful of the meaning of unfamiliar words you encounter in this lesson. A glossary of terms is provided in the last part of this lesson.

6. Maximize the use of online resources in each lesson. Online resources can be accessed multiple times. The summary of online resources is provided in the end of the lesson.

EXPLORE

Have you thought of why Algebra is difficult to understand and often perceived to have a confusing set of letters, numbers and symbols? In this section, we will try to conquer our confusions by understanding what these symbols are all about.

ACTIVITY 1. Situation Analysis – CAUGHT IN THE CAMERA

Description: this video shows an excerpt from different amazing videos caught in the camera.

http://www.youtube.com/watch?v=2EW5hauGXv4

Process Questions:

1. What is the video all about?

2. In what way did the Don Mariano accident affect you?

3. Why do you think such accidents happen?

4. What do you think are the best ways to solve these problems?

5. How are accidents properly investigated?

ACTIVITY 2. Generalization Table

Fill in the first column of the generalization table below by stating your initial thoughts on the question, then save your answer.

How can mathematical formulas in scientific investigations be determined?
How can a quantity be influenced by another?

My Initial Thoughts	*My Findings and Corrections*	*Supporting Evidence*	*Qualifying Conditions*	*My Generalizations*

End of Explore

What you wrote showed your thoughts and ideas about some of the questions that were asked. Let's take a closer look at our ideas by exploring the important key concepts about radicals..

FIRM-UP

Your goal in this section is to learn and understand key concepts about radicals.

ACTIVITY 3. THE SEEKER

Directions: Consider the situation below. Open the website below and follow the given instructions.


Situation
You are on your way to Baguio City to attend the Panagbenga Festival. A few hours later, you experienced a car accident on the expressway. You then went to the police station for the investigation. During the investigation, the police officer provided the following information about the accident: the length of the skid mark, the number of skid marks and the damage of your car.

To check those, click on the website **http://www.harristechnical.com/skid33.htm.** Click on the site and a vehicle speed estimator will be displayed. Below are spaces where you can type in the information. Through these, the breaking and the speed of your car will be determined. Vary the length of the skid marks such as 10, 20, 30, 40 and 50 ft. observe what happens to the speed.

Length of Skid Marks	Speed of the Car	Damage to the Car
10 ft.		
20 ft.		
30 ft.		
40 ft.		
50 ft.		

Process Questions:

1. What have you observed about the speed upon increasing the length of the skid mark?

2. How do you describe the difference of the speed upon increasing the length of the skid mark?

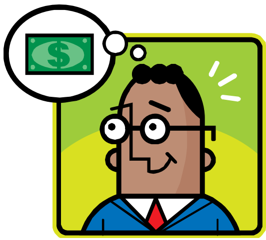
3. By how much do you think is the change in the speed once the length is changed?

4. How is the knowledge of radical expressions and equations used to solve real-life problems?

ACTIVITY 4. Read and Think Aloud.

Read the information given below. You may highlight important part of the discussion which can be useful in the next activities. You may click the websites below for video clips regarding the application of radicals and additional reading materials.

Given below are different applications of Radical Expressions in real life:



For instance in, FINANCIAL PLANNING

- A financial planner has been asked to determine the inflation rate for homes.
- To calculate the inflation of homes that increases from p1 to p2 over n years, the annual rate of inflation (expressed as a decimal) can be modeled as:

$$i = \left(\frac{p2}{p1}\right)^{\left(\frac{1}{n}\right)} - 1$$

In MEDICINE,

Doctors can approximate the Body Surface Area of an adult (in square meters) using an index called BSA where H is height in centimeters and W is weight in kilograms:

$$BSA = \sqrt{\frac{H * W}{3600}}$$

In Geology/Meteorology/Oceanography

- The Pacific Tsunami Warning Center is responsible for monitoring earthquakes that could potentially cause tsunamis in the Pacific Ocean. Through measuring the water level and calculating the speed of a tsunami, scientists can predict arrival times of tsunamis.
- The speed (in meters per second) at which a tsunami moves is determined by the depth d (in meters) of the ocean:

$s = \sqrt{g * d}$ in which g is acceleration due to gravity, which is 9.8 meters per square second

In Supply Chain Management/Business

- You are a purchasing manager for a medical device company. You are responsible for directing the way the company buys, stores, and sells supplies to other companies.
- You want to reduce the company's warehousing costs by ordering the supplies needed to produce the medical devices just in time to use them.
- In order to determine the most economic order quantity E for parts used in production of the medical devices, you need to use the following formula:

$E = \sqrt{\frac{2 * A * S}{I}}$ Where A is the quantity the plant will use in one year, S is the cost of setup for making the device and I is the cost of holding one unit in stock for one year

- http://www.youtube.com/watch?v=tvvTaFaaMjc
  this site involves concepts of radical expressions and how they are applied in real life.

- http://www.youtube.com/watch?v=OgfnHDQwxVU
  site showing an example of solving real life problems involving radical expressions

- http://www.youtube.com/watch?v=N6CUxqQY0KU&list=PLDkXP6pQoNb4QJdSw0azDlQ3nVlzYgNj6
  in this 6 – part video, you will see the application of radical expressions in solving architectural problems, specifically involving the Pythagorean theorem

- http://www.youtube.com/watch?v=5fmnKCOnwpE
  This video provides an example of how to use a radical equation to determine the speed of a vehicle based upon the length of its skid mark.

Something to Ponder.
Integer Exponents.

Definition 1: a^n, *n* an integer and *a*, a real number

1. For n a positive integer:

   $a^n = a \cdot a \cdots a$

   Example: $3^6 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

2. For n = 0

   $a^0 = 1$

   $135^0 = 1$

3. For n, a negative number

$a^n = \frac{1}{a^{-n}}$ and $a \neq 0$

Example 1

Using the Definition of Integer Exponents
Write each part as a decimal fraction or using positive exponents. Assume all variables represent nonzero real numbers.

A. $(u^3v^2)^0 = 1$
B. $10^{-3} = \frac{1}{10^3} = \frac{1}{1000} = 0.001$
C. $x^{-8} = \frac{1}{x^8}$
D. $\frac{x^{-8}}{y^{-5}} = \frac{x^{-8}}{1} \cdot \frac{1}{y^{-5}} = \frac{1}{x^8} \cdot \frac{y^5}{1} = \frac{y^5}{x^8}$

Matched Problem 1

Write each part as a decimal fraction or using positive exponents. Assume all variables represent nonzero real numbers.

1. $\mathbf{636^0}$

2. $\frac{1}{10^{-3}}$

3. $\frac{u^{-7}}{v^{-4}}$

4. $(x^2)^0$

5. 10^{-5}

For Your Info.
The French philosopher/mathematician Rene Descartes (1596-1650) is generally credited with the introduction of the very useful exponent notation x^n. The notation as well as the other improvements in algebra may be found in his *Geometr,* published in 1637. If n is a natural number, x^n denotes the product of n factors, each equal to x. In this section, the meaning of x^n will be expanded to allow the exponent n to be any rational number. Each of the following expressions will then represent a unique real number:

8^5 4^{-3} 3.1416^0 $7^{1/2}$ $16^{-5/3}$

ACTIVITY 5. Let's Talk About Rules!

First, read the text and, using your notepad, take note of important rules being discussed. Then you may click the link below to add more information. Use your notepad to take important notes about the previous discussion. You may also highlight important parts of the discussion that will guide you in the next activities.

The basic properties of integer exponents are summarized in the table below:

Properties of Integral Exponents

For *n* and *m* integers and *a* and*b* real numbers:

1. $a^m a^n = a^{m+n}$
2. $(a^n)^m = a^{mn}$
3. $(ab)^m = a^m b^m$
4. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ $\qquad b \neq 0$
5. $\frac{a^m}{b^m} = \begin{cases} a^{m-n} \\ \frac{1}{a^{n-m}} \end{cases}$ $\qquad a \neq 0$

Examples. Simplify using exponent properties, and express answers using positive exponents.

1. $(3a^5)(2a^{-3}) = (3 \cdot 2)(a^5 \cdot a^{-3}) \quad = 6a^2$

2. $\frac{6x^{-2}}{8x^{-5}} = \frac{3x^{-2-(-5)}}{4} = \frac{3x^3}{4}$

3. $-4y^3 - (-4y)^3 = -4y^3 - (-4)^3 \cdot y^3$
$= -4y^3 + 64y^3 = 60y^3$

By "simplify" we mean eliminating common factors from numerators and denominators and reducing the number of times a given constant or variable appears in an expression. We ask that answers be expressed using positive exponents only in order to have a definite form for an answer. Later, we will encounter situations where we will want negative exponents in a final answer.

Caution

Be Careful when using the relationship $a^{-n} = \frac{1}{a^n}$:

$ab^{-1} \neq \frac{1}{ab}$ $ab^{-1} = \frac{a}{b}$ $and\ (ab)^{-1} = \frac{1}{ab}$

$\frac{1}{a+b} \neq a^{-1} + b^{-1}$ $\frac{1}{a+b} = (a+b)^{-1} and\ \frac{1}{a} + \frac{1}{b} = a^{-1} + b^{-1}$

Do not be confused at properties 1 and 2

$a^3 a^4 \neq a^{3 \cdot 4}$ $a^3 a^4 = a^{3+4} = a^7$ property 1

$(a^3)^4 \neq a^{3+4}$ $(a^3)^4 = a^{3 \cdot 4} = a^{12}$ property 2

If all exponent properties are to continue to hold even if some of the exponents are rational numbers then,

$$(5^{\frac{1}{3}})^3 = 5^{3/3} = 5 \text{and} (7^{1/2})^2 = 7^{2/2} = 7$$

Since, the number of Real nth roots of a real number b states that:

	n even	*n* odd
b positive	Two real nth roots	One real nth root
	-3 and +3 are both fourth roots of 81	2 is the only real cube root of 8
b negative	No real nth root	One real nth root
	-9 has no real roots	-2 is the only real cube root of -8

Then the number 5 has one real cube root, it seems reasonable to use the symbol

$5^{\frac{1}{3}}$ to represent this root. On the other hand, it also states that 7 has two real square roots. *Which real square root of 7 does* $7^{1/2}$ *represent?* We answer this question in the following definition:

Definition $b^{1/n}$, *Principal nth Root*

For *n* a natural number and *b* a real number, $b^{1/n}$ is the **principal nth root of *b*** defined as follows:

1. If n is even and b is positive, then $b^{1/n}$ represents the positive nth root of b.
2. If n is even and b is negative, then $b^{1/n}$ does not represent a real number.
3. If n is odd, then $b^{1/n}$ represents the real nth root of b.
4. $\mathbf{0^{1/n} = 0}$

A number or an expression is a **perfect square** if it can be expressed as product of two same rational numbers or expressions.

Examples:

1. **4** is a perfect square because it can be expressed as product of two same rational numbers $2 \cdot 2 = 4$ or $(2)(2) = 4$

2. The number $\frac{16}{36}$ is a perfect square because

$$\left(\frac{4}{6}\right)\left(\frac{4}{6}\right) = \frac{16}{36} \text{ or } \left(-\frac{4}{6}\right)\left(-\frac{4}{6}\right) = \frac{16}{36}$$

3. The expression **25x²** is a perfect square because it can be expressed as product of two same rational expressions.

$$5x \cdot 5x = 25x^2 \text{ or } (-5x)(-5x) = 25x^2$$

A number or an expression is a **perfect cube** if it can be expressed as product of three same rational numbers or expressions.

Examples:

1. **8** is a perfect cube because it can be expressed as product of three same rational numbers.

$$2 \cdot 2 \cdot 2 = 8$$

2. The number $-\frac{27}{64}$ is a perfect cube because

$$\left(-\frac{3}{4}\right)\left(-\frac{3}{4}\right)\left(-\frac{3}{4}\right) = -\frac{27}{64}.$$

3. The expression $\mathbf{8x^3y^3}$ is a perfect cube because it can be expressed as product of three same rational expressions.

$$2xy \cdot 2xy \cdot 2xy = 8x^3y^3$$

ACTIVITY 6. Let's Find Out!

Which of the following numbers or expressions are perfect squares or perfect cubes? Explain or justify your answer. Put a check mark on the space provided.

Given	Perfect Square	Perfect Cube	Explanation
1. 0.09			
2. ¼			
3. $\frac{16}{100}$			
4. $\frac{27}{8}$			
5. 256			
6. -125			
7. $-\frac{1}{64}$			

8. 169			
9. -27			
10. 1000			

PROCESS QUESTIONS:

1. How did you find the activity?
2. Did you encounter any difficulty in answering the exercises? Why?
3. How did you overcome these difficulties?
4. How can radicals help us solve real life problems?

Exercise: Find each of the following:

1. $4^{1/2}$

2. $64^{1/2}$

3. $(81)^{1/2}$

4. $8^{1/3}$

5. $0^{1/8}$

How should a symbol such as $5^{2/3}$ be defined? If the properties of exponents are to hold for a rational exponents, then $5^{2/3} = (5^{1/3})^2$; that is $5^{2/3}$ must represent the square of the cube root of 5. That will lead to the definition:

Definition: $b^{m/n}$ and $b^{-m/n}$, Rational Number Exponent

For m and n natural numbers and b any real number (except b cannot be negative when n is even):

$$b^{m/n} = (b^{1/n})^m \; and \; b^{-m/n} = \frac{1}{b^{m/n}}$$

We have discussed $b^{m/n}$ for all numbers m/n and real number b. it can be shown, though we will not do so, that all five properties of exponents listed continue to hold for rational exponents as long as we avoid even roots of negative numbers.

ACTIVITY 7. My Property.

Simplify Problems 1 – 5 and express answers using positive exponents.

1. $(a^2b^3)^5$

$$a^{10}b^{15}$$

3. $\left(\frac{x^4y^{-1}}{x^{-2}y^8}\right)^2$

$$=\frac{x^{12}}{y^{18}}$$

4. $\left(\frac{6mn^{-2}}{3m^{-1}n^2}\right)^{-3}$

$$=\frac{n^{12}}{8m^6}$$

5. $\left(\frac{m^{-2}n^8}{m^4n^{-1}}\right)^2$

PROCESS QUESTIONS:

1. How did you find the activity?
2. Did you encounter any difficulty in answering the exercises? Why?
3. How did you overcome these difficulties?

4. How can radicals help us solve real life problems?

Let's Classify.

What do the following algebraic expressions have in common?

$2^{1/2}$	$2x^{2/3}$	$\frac{1}{x^{1/2}+y^{1/2}}$
$\sqrt{2}$	$2\sqrt[3]{x^2}$	$\frac{1}{\sqrt{x}+\sqrt{y}}$

Each vertical pair represents the same quantity, one in rational exponent from and the other in radical form. There are occasions when it is more convenient to work with radicals than with rational exponents, or vice versa. In this section we will see how two forms are related and investigate some basic operations on radicals.

We start this discussion by defining an nth – root radical:

Definition: $\sqrt[n]{b}$, nth – root radical

For *n* a natural number greater than 1 and *b* a real number, we define $\sqrt[n]{b}$ to be the principal nth root of b that is $\sqrt[n]{b} = b^{1/n}$. The symbol $\sqrt{\ }$ is called a **radical**, *n* is called the **index** and *b* is called the **radicand**.

Example

= $25^{1/2}$

= $-25^{1/2}$

If *n* = 2, we write $\sqrt{b}$ in place of $\sqrt[2]{b}$.

$\sqrt{25}$ = 5

$-\sqrt{25}$ = - 5

$\sqrt{-25}$ *is not real*

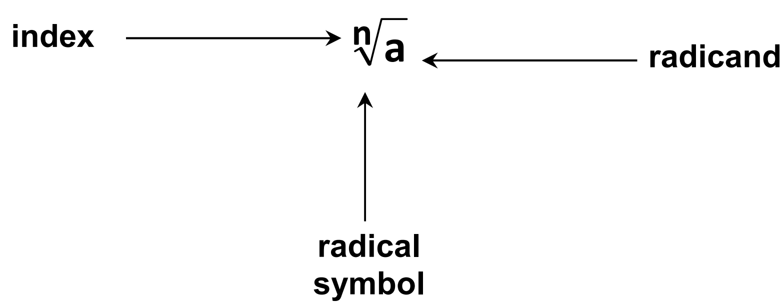
As already stated, it is often an advantage to be able to shift back and forth between rational exponent forms and radical forms. The following relationships, which are direct consequences of the stated definitions, are useful in this regard:

Expressions such as $\sqrt{12}$, $\sqrt{2x}$, $\sqrt[3]{20}$, or $\sqrt[3]{4x}$ are radical expressions. Radical expression is any term that contains both a radical symbol $\sqrt{\;}$ and a radicand.

The idea of square root and cube root can be extended to fourth root, fifth root, and so on. In general, the principal **n**th root of a number **a** is denoted by $\sqrt[n]{a}$, where **n** is an integer greater than 1. If $\sqrt[n]{a} = b$, then **b** is one of the **n** equal factors of **a**.

$$\overbrace{b \cdot b \cdot b \cdot \ldots \cdot b}^{\textbf{n factors}} = b^n = a$$

In the expression $\sqrt[n]{a}$, **n** is the index and **a** is the radicand. The whole is expression is called radical.



Examples:

Given	Index	Radicand
$\sqrt[3]{81}$	3	81
$\sqrt[7]{2x-3}$	7	2x-3
$15\sqrt[3]{4xy}$	3	4xy
$\sqrt[8]{256}$	8	256

Simplifying Radical Expressions

If **n** is a positive even integer, we say that $\sqrt[n]{a} = b$ if $b^n = a$ and $-\sqrt[n]{a} = -b$ if $(-b)^n = a$. This means that a number greater than 0 has two real nth roots if the index is a positive even integer.

Examples:

1. $\sqrt[4]{16} = 2$ because $2^4 = 16$.
2. $-\sqrt[4]{16} = -2$ because $(-2)^4 = 16$.
3. $\sqrt[6]{729} = 3$ because $3^6 = 729$.
4. $-\sqrt[6]{729} = -3$ because $(-3)^6 = 729$.
5. $\sqrt[4]{81x^4} = 3x$ because $(3x)^4 = 81x^4$.
6. $-\sqrt[4]{81x^4} = -3x$ because $(-3x)^4 = 81x^4$.

If **n** is a positive odd integer, we say that $\sqrt[n]{a} = b$ if $b^n = a$ and $\sqrt[n]{-a} = -b$ if $(-b)^n = -a$.

Examples:

1. $\sqrt[3]{8} = 2$ because $2^3 = 8$.
2. $\sqrt[3]{-8} = -2$ because $(-2)^3 = -8$.
3. $\sqrt[5]{32x^5} = 2x$ because $(2x)^5 = 32x^5$.

ACTIVITY 8. : CLASSIFIED Radicals.

Determine the index and the radicand of each of the following radical expressions. Then to tell whether the given is rational or not.

	Given	Index	Radicand	Rational or Irrational
1	$\sqrt[3]{343}$			
2	$\sqrt{72}$			
3	$\sqrt{81}$			
4	$\sqrt{15}$			
5	$\sqrt[3]{343}$			
6	$\sqrt[3]{\frac{27}{1000}}$			
7	$\sqrt[3]{12}$			

8	$\sqrt[5]{243}$			
9	$\sqrt[5]{1024}$			
10	$\sqrt[5]{-\frac{1}{32}}$			

PROCESS QUESTIONS:

1. Based on your answers above, how did you the index and the radicand?
2. Do you have any basis to easily classify whether the given radical expression is rational or not?
3. What makes radical expression different from exponential expression?

ACTIVITY 9. Generalization Table

Fill in the unshaded columns of the generalization table below by stating your findings and corrections, supporting evidence and qualifying conditions on the question. Then save your answer.

How can mathematical formulas in scientific investigations be determined? How can a quantity be influenced by another?

My Initial Thoughts	*My Findings and Corrections*	*Supporting Evidence*	*Qualifying Conditions*	*My Generalizations*

Let's find out how others would answer the question and compare their ideas to our own. As you compare, you will also learn other concepts which will help you complete the required project. The project is about making a Police Report and a PowerPoint presentation as an application of Radicals.

We will start by doing the next activity.

In the previous activity you were able to determine the important parts of an expression in radical form. The next activity will test you on how you can represent the given exponent form to radical form and vice versa. You will also know the properties of radicals.

Rational Exponent/Radical Conversions

For *m* and*n* positive integers $(n > 1)$, and *b* not negative when *n* is even,

$$b^{m/n}\begin{cases}(b^m)^{1/n} = \sqrt[n]{b^m}\\(b^m)^{1/n} = (\sqrt[n]{b})^m\end{cases}$$

Examples:

Exponential Form	Radical Form
$x^{1/7}$	$\sqrt[7]{x}$
$(3u^2v^3)^{3/5}$	$\sqrt[5]{(3u^2v^3)^3}$
$y^{-2/3}$	$\sqrt[3]{\dfrac{1}{y^2}}$
$u^{1/5}$	$\sqrt[5]{u}$

In the previous examples you were able to see how the properties of exponents are applied. The next activities will show you how the exponent of properties considered earlier imply the following properties of radicals.

ACTIVITY 10. Check Point

A. Write the following expressions in radical form in as many ways as you can.

1. $16^{\frac{1}{4}}$

2. $5^{-\frac{1}{3}}$

3. $(8b)^{\frac{1}{6}}$

4. $(m-1)^{-\frac{1}{5}}$

5. $\left(\frac{4}{5}\right)^{\frac{1}{2}}$

B. Write the following radicals as expressions with rational exponents.

6. $(\sqrt[5]{3})^4$

7. $\sqrt[4]{(9a)^3}$

8. $\left(\sqrt[3]{\left(\frac{5}{12}\right)}\right)^8$

9. $\sqrt[4]{\frac{3}{10}}$

10. $\left(\sqrt[3]{15}\right)^2$

C. Write the following radicals as expressions with rational exponents.

6. $(\sqrt[5]{3})^4$

$=3^{\frac{4}{3}}$

7. $\sqrt[4]{(9a)^3}$

$=(729a^3)^{\frac{1}{4}}$

8. $(\sqrt[3]{\left(\frac{5}{12}\right)}\,)^{8}$

$$=\frac{5}{12}^{\frac{3}{8}}$$

9. $\sqrt[4]{\frac{3}{10}}$

$$=\frac{3}{10}^{\frac{1}{4}}$$

10. $\left(\sqrt[3]{15}\right)^{2}$

$$=15^{\frac{2}{3}}$$

The following laws of radicals can be applied when simplifying or evaluating radicals. The laws of radicals will provide us with the means of changing algebraic expressions containing radicals to a variety of equivalent forms.

The **n**th root of a number raised to **n** is equal to the number.

$$\sqrt[n]{x^{n}} = x \quad \text{or} \quad \left(\sqrt[n]{\mathbf{x}}\right)^{n} = \mathbf{x}$$

Examples

$\sqrt[3]{16^3}$	16
$(\sqrt[6]{21})^6$	21
$\sqrt[5]{(3x-4)^5}$	3x – 4
$\left(\sqrt{\frac{3x}{y}}\right)^2$	$\frac{3x}{y}$
$\sqrt{(3x-4)^2}$	3x – 4
$(\sqrt[7]{8-4y})^7$	8 – 4y

2

The product of two radicals having the same index **n** is equal to the **n**th root of the product of their radicands.

$$\sqrt[n]{xy} = \sqrt[n]{x} \cdot \sqrt[n]{y}$$

Examples

$\sqrt{2} \cdot \sqrt{18}$	$= \sqrt{36}$ $= 6$
$\sqrt[3]{4} \cdot \sqrt[3]{16}$	$= \sqrt[3]{64}$ $= 4$
$\sqrt[5]{2y} \cdot \sqrt[5]{16y^4}$	$= \sqrt[5]{32y^5}$ $= 2x$
$\sqrt[5]{8a^3} \cdot \sqrt[5]{8a^3}$	$= \sqrt[5]{64a^6}$ $\sqrt[5]{32 \cdot 2 \cdot a^5 \cdot a}$ $2a\sqrt[5]{2a}$

3

The quotient of two radicals having the same index **n** is equal to the **n**th root of the quotient of their radicands.

$$\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$$

Examples

$\frac{\sqrt{32}}{\sqrt{2}}$	$=\sqrt{\frac{32}{2}}=\sqrt{16}=4$
$\frac{\sqrt[3]{54}}{\sqrt[3]{2}}$	$=\sqrt[3]{\frac{54}{2}}=\sqrt[3]{27}=3$
$\sqrt{\frac{64}{9}}$	$=\frac{\sqrt{64}}{\sqrt{9}}=\frac{8}{3}$
$\sqrt[3]{\frac{8}{27}}$	$\frac{\sqrt[3]{8}}{\sqrt[3]{27}}=\frac{2}{3}$

4

The **n**th root of the **m**th root of a number **x** is equal to a radical whose radicand is **x** and whose index is the product of **m** and **n**.

$$\sqrt[mn]{x}=\sqrt[n]{\sqrt[m]{x}}$$

Examples

$\sqrt{\sqrt[3]{729}}$	$=\sqrt[(2)(3)]{729}$ $=\sqrt[6]{729}$ = 3
$\sqrt[4]{\sqrt[3]{a^{12}}}$	$=\sqrt[(3)(4)]{a^{12}}$ $=\sqrt[12]{a^{12}}$ = a

ACTIVITY 11. Tell Me!

Tell whether each expression is in its simplest form, if not, identify which condition it violated.

Given	Simplest Form or Not	Condition Violated if Not in Simplest Form
$\sqrt{24}$		
$\dfrac{4}{\sqrt{x-3}}$		
$\dfrac{\sqrt[4]{3x-5}}{3x-5}$		
$\sqrt[5]{10xy}$		
$\sqrt[3]{8}$		
$\sqrt{12x^3y^6}$		
$\sqrt[3]{6}$		
$\dfrac{\sqrt{10}}{\sqrt{2}}$		
$\sqrt{2x^7}$		

The laws of radicals provide us with the means of changing algebraic expressions containing radicals to a variety of equivalent forms. One form that is often useful is a *simplified form*. An algebraic expression is said to be in simplified form if all four of the conditions listed in the following definitions are satisfied.

Radical Form Conditions

1. No radicand (the expression within the radical sign) contains a factor to a power greater than or equal to the index of the radicand.

For example, $\sqrt{x^5}$ is a violation to the condition

2. No power of the radicand and the index of the radical have a common factor other than 1.

For example, $\sqrt[6]{x^4}$ violates this condition

3. No radical appears in the denominator

For example, $\frac{1}{\sqrt{x}}$ is a violation to this condition

4. No fraction appears within a radical

ACTIVITY 12. Make IT simple.

Use the laws of radicals to simplify each of the following expressions. Explain how you arrived at your answer.

1. $\sqrt[5]{32m^{10}n^{15}}$

2. $\sqrt{25b^2}$

3. $\sqrt{36t^2}$

4. $\sqrt{50x^3}$

5. $\dfrac{\sqrt{24t^5}}{\sqrt{3t^3}}$

6. $\dfrac{\sqrt[6]{256x^7}}{\sqrt[6]{2x}}$

7. $\sqrt{\dfrac{144m^6}{n^8}}$

8. $\sqrt[5]{\dfrac{486a^{10}}{2b^5}}$

9. $\sqrt[3]{\sqrt[4]{12x^{12}}}$

10. $\sqrt[3]{\frac{a^3b^9}{125}}$

End of Firm up

In this section, the discussion focused on transforming radical form to exponential form and vice versa and how they are easily converted through the use of different properties. Laws of Radicals were also discussed in this section which can be applied in simplifying radical expressions.

You may go back to the previous section to check and compare your ideas with the discussed topic.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

For additional information on how to determine solutions of linear inequalities in two variables, you may visit http://www.youtube.com/watch?v=IMTudaflcik&playnext=1&list=PLD76A144881E89E96&feature=results_video .

Before proceeding to the next activity, let's check first if what level are you now in terms of the previous concepts and activities given. You may go back to the list of skills, and check whether which of the skills you consider yourself as an expert and which among the skills you are having difficulty.

If you have questions or activities that still confuses you, just send an email to your teacher.

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.

DEEPEN

In the previous activities you were able to: apply the laws involving positive integral exponents to zero and negative integral exponents, illustrate expressions with rational exponents, simplify expressions with rational exponents, write expressions with rational exponents as radicals and vice versa, derive the laws of radicals and simplify radical expressions using the laws of radicals.

The next activity will test you on how you can apply the previous concepts in the operations of radicals, how to solve radical equations, how to solve real life problems involving radicals. As you move on, think of these questions asked in the previous activity: How can mathematical formulas in scientific investigations be determined? How can a quantity be influenced by another?

Radicals may be combined into simple radical expressions by adding similar terms. The sum of two radicals cannot be simplified if the radicals have different indices or different radicands.

Adding and subtracting radical expressions is very much similar to adding and subtracting similar terms of polynomials. Similar terms or like terms are those with the same literal parts or literal factors. Radicals are similar if they have the same index and the same radicands.

Examples:

$\sqrt{5}+\sqrt[3]{5}$ the given cannot be combined because of the difference in their indices.

$\sqrt{5}+\sqrt{2}$ the indices may be the same. However, this cannot be combined also because the radicands are different. There are also some situations where there is a need to simplify first to the simplest radical form before adding and subtracting. For example in adding $\sqrt{5}$and $\sqrt{125}$, $\sqrt{125}$ can further be simplified to $5\sqrt{5}$ because $\sqrt{125}=\sqrt{5\cdot5\cdot5}$, so the equation will now become

$$\sqrt{5}+\sqrt{125}=\sqrt{5}+\sqrt{5\cdot5\cdot5}$$
$$\sqrt{5}+\sqrt{125}=\sqrt{5}+5\sqrt{5}$$

$\sqrt{5}+\sqrt{125}=6\sqrt{5}$

The same concept in terms of the rules for addition and subtraction of polynomials applies in addition and subtraction of radicals. In adding and subtracting radicals, you should make sure of the following: make sure that they are like or similar; the coefficients of the radicals then affix the common radical expression and it should be simplified.

Examples
Study each item. Simplify by collecting like terms. Show complete solutions.

1. $\sqrt{2}+\sqrt{18}$ $= \sqrt{2}+\sqrt{2\cdot2\cdot2\cdot3}$
   $= \sqrt{2}+2\sqrt{2\cdot3}$
   $= \sqrt{2}+2\sqrt{6}$
   $= \sqrt{2}+2\sqrt{6}$

You can not add the coefficients of different radicand

2. $\sqrt{25}+\sqrt{64}$ $= 5+8$
$= 13$

3. $\sqrt[3]{128}+\sqrt[3]{54}$ $= \sqrt[3]{4\cdot4\cdot4\cdot2}+\sqrt[3]{3\cdot3\cdot3\cdot2}$
$= 4\sqrt[3]{2}+3\sqrt[3]{2}$
$= 7\sqrt[3]{2}$

4. $-5\sqrt{24}-2\sqrt{54}$ $= -5\sqrt{2\cdot2\cdot2\cdot3}-2\sqrt{3\cdot3\cdot3\cdot2}$
$= (2)(-5)\sqrt{2\cdot3}+(3)(-2)\sqrt{2\cdot3}$
$= -10\sqrt{6}-6\sqrt{6}$
$= -16\sqrt{6}$

5. $5\sqrt{75x^2}-2\sqrt{12x^2}$ $= 5\sqrt{5\cdot5\cdot2\cdot x\cdot x}-2\sqrt{3\cdot2\cdot2\cdot x\cdot x}$
$= (5)(5)(x)\sqrt{2}+(2)(x)(-2)\sqrt{2}$
$= 25x\sqrt{2}-4x\sqrt{2}$
$= 21x\sqrt{2}$

6. $\sqrt{72xy}+2\sqrt{2xy}+\sqrt{128xy}$ $= \sqrt{6\cdot6\cdot2\cdot xy}+2\sqrt{2xy}+\sqrt{8\cdot8\cdot2\cdot xy}$
$= 6\sqrt{2xy}+2\sqrt{2xy}+8\sqrt{2xy}$
$= 6+2+8\sqrt{2xy}$
$= 16\sqrt{2xy}$

ACTIVITY 13. MATCHED Problems

Perform the indicated operations.

1. $4\sqrt{2}\ +5\sqrt{2}$

2. $5\sqrt{3} + 2\sqrt{3} + 10\sqrt{3}$

3. $16\sqrt{7} + 8\sqrt{7} + \sqrt{7}$

4. $15\sqrt{11} + 3\sqrt{11} + 5\sqrt{11} + 6\sqrt{11}$

5. $\frac{1}{2}\sqrt{13} + \frac{2}{3}\sqrt{13}$

6. $2\sqrt{27} + 3\sqrt{16}$

7. $\sqrt{180} + \sqrt{125} + 10\sqrt{45}$

8. $3\sqrt{36a} + \sqrt{100a}$

9. $\sqrt{75} + 2\sqrt{27} + \sqrt{12}$

10. $5\sqrt{b} + \sqrt{4b}$

Process Questions:

1. How did you go about the activity?
2. How did you plan your solution?
3. What did you consider? What has helped you in answering each item?
4. Were there difficulties or confusions? What were they?
5. *How is the knowledge of radical expressions and equations used to solve real-life problems?*

To have more information in addition and subtraction of radical expressions, click on the website below:

http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=112

You may also check this website for additional exercises in the four fundamental operations of radicals.
http://www.kutasoftware.com/FreeWorksheets/Alg1Worksheets/Adding+Subtracting%20Radical%20Expressions.pdf

What you are going to see is the web page www.explorelearning.com, particularly on the operations on radical expressions. This internet exploration activity is timed. At most, you will be given five minutes to do the activity. To have more time doing the activity at home, you may subscribe by registering. You are entitled for a 30 – day trial. After doing the activity, you may answer the following questions:

1. How was the experience working with the explore learning website?

2. Where there difficulties? How did you manage to resolve them?

ACTIVITY 14. 3-2-1

3. In each of the items you answered, what were your frequent mistakes as determined by online feedback given to you? How are you able to correct your mistakes in the process?

We already have come up with the rules with the rules on how to add subtract radical expressions? What if our radical expressions involve multiplication? What set of rules should we formulate to help us simplify these expressions?

In the process of multiplying radicals, there are three cases to be considered.

a. when indices are the same. Multiply radicals and apply $\sqrt[n]{x} \cdot \sqrt[n]{y} = \sqrt[n]{xy}$ Simplify the resulting radicand when necessary.

b. when indices are different but radicands are the same. Apply the following steps:

1. Transform the radical to fractional exponents.
2. Multiply the powers by applying: $x^m \cdot x^n = x^{m+n}$ (law of exponents)
3. Rewrite the product as a single radical.
4. Simplify the resulting radicand if necessary.

c. when indices and radicands are different. Do the following steps:

1. Transform the radicals to powers with fractional exponents.
2. Change the fractional exponents into similar fractions.
3. Rewrite the product as a single radical
4. Simplify the resulting radicand if necessary.

Examples:

1. $\sqrt{2} \cdot \sqrt{7} \cdot \sqrt{5} = \sqrt{2 \cdot 7 \cdot 5}$ Write product as single expression

$= \sqrt{70}$

2. $\sqrt{12} \cdot \sqrt{18} = \sqrt{12 \cdot 18}$

$= \sqrt{2 \cdot 2 \cdot 3 \cdot 2 \cdot 3 \cdot 3}$

$= (2)(3)\sqrt{3 \cdot 2}$

$$= \quad 6\sqrt{6}$$

3. $3\sqrt{15x} \cdot \sqrt{6x^3y}$

$$\begin{aligned} &= 3\sqrt{(15)(6)(x)(x^3)(y)} \\ &= 3\sqrt{90(x^4)(y)} \\ &= 3\sqrt{(3)(3)(2)(5)(x^2)(x^2)(y)} \\ &= (3)(3)(x^2)\sqrt{(2)(5)(y)} \\ &= 9x^2\sqrt{10y} \end{aligned}$$

4. $x\sqrt{5} \cdot y\sqrt{7} \quad = \quad xy\sqrt{7}$

5. $(-5\sqrt{15a})(-2\sqrt{5ab})(9\sqrt{3a}) = (-5)(-2)(9)\sqrt{(15)(5)(3)(a)(ab)(a)}$

$$\begin{aligned} &= 90\sqrt{(3)(5)(5)(3)(a)(a)(b)(a)} \\ &= 90(3)(5)(a)\sqrt{ab} \\ &= 720a\sqrt{ab} \end{aligned}$$

ACTIVITY 15. Go and Multiply

Perform the indicated operations. Simplify all answers as completely as possible.

1. $\sqrt{3} \cdot \sqrt{12}$	
2. $\sqrt{3} \cdot \sqrt{27}$	
3. $\sqrt{3} \cdot \sqrt{4} \cdot \sqrt{5}$	
4. $(\sqrt[3]{20b^2})(\sqrt[3]{50b^2})$	
5. $\sqrt{12x^3y} \cdot \sqrt{9x^2y^3}$	
6. $(\sqrt{5xy})(\sqrt{10xy})(15\sqrt{xy})$	
7. $5\sqrt{9xyz^4} \cdot 3\sqrt{12xz} \cdot 6\sqrt{x^2y^3}$	
8. $\sqrt{4x} \cdot \sqrt{4x} \cdot 4x$	

In some cases if the radicals have different indices but same radicands, transform the radical form to exponential form. Then, multiply the powers by applying the law of exponents and then rewriting the product as a single radical.

1. $\sqrt{3} \cdot \sqrt[4]{3}$ $= 3^{\frac{1}{2}} \cdot 3^{\frac{1}{4}}$
$= 3^{\frac{1}{2}+\frac{1}{4}}$
$= 3^{3/4}$
$= \sqrt[4]{3^3}$

2. $(\sqrt[4]{3x-5})(\sqrt[3]{3x-5})$ $= (3x-5)^{\frac{1}{4}}(3x-5)^{1/3}$
$= (3x-5)^{\frac{1}{4}+\frac{1}{3}}$
$= (3x-5)^{\frac{7}{12}}$
$= \sqrt[12]{(3x-5)^7}$

In case where the indices and the radicands are different, transform the radicals into exponential form. Change exponents as similar fractions and rewrite the product as a single radical. Simplify the answer if possible.

For Example:

1. $(\sqrt{2})(\sqrt[3]{3})$ $= 2^{\frac{1}{2}} \cdot 3^{\frac{1}{3}}$
$= 2^{\frac{3}{6}} \cdot 3^{\frac{2}{6}}$

$= \sqrt[6]{2^3} \cdot \sqrt[6]{3^2}$

$= \sqrt[6]{8} \cdot \sqrt[6]{9}$

$= \sqrt[6]{72}$

2. $\sqrt[3]{9} \cdot \sqrt[4]{5}$ $= 9^{\frac{1}{3}} \cdot 5^{\frac{1}{4}}$
$= 9^{\frac{4}{12}} \cdot 5^{\frac{3}{12}}$

$= \sqrt[12]{9^4} \cdot \sqrt[12]{5^3}$

$= \sqrt[12]{6561} \cdot \sqrt[12]{125}$

$= \sqrt[12]{820{,}125}$

3. $\sqrt[3]{x^2y^2} \cdot \sqrt[4]{x^3y^7}$ $= (x^2y^2)^{\frac{1}{3}}(x^3y^7)^{\frac{1}{4}}$
$= (x^2y^2)^{\frac{4}{12}}(x^3y^7)^{\frac{3}{12}}$

$= \sqrt[12]{(x^2y^2)^4} \cdot \sqrt[12]{(x^3y^7)^3}$

$$= \sqrt[12]{x^8y^8} \cdot \sqrt[12]{x^9y^{21}}$$

$$= \sqrt[12]{x^{17}y^{29}}$$

$$= xy^2\sqrt[12]{x^5y^5}$$

Exercise: Determine the product of the following:

1. $\sqrt[3]{6} \cdot \sqrt[4]{5}$

2. $\sqrt{5} \cdot \sqrt[3]{7}$

3. $\sqrt[3]{2x^3y^2z} \cdot \sqrt{5x^3yz}$

4. $\sqrt[8]{5} \cdot \sqrt{2}$

5. $\left(\sqrt[8]{7} \cdot \sqrt{5}\right)^2$

ACTIVITY 16. Let's Get it Online!

Follow the process which can be observed in the website below. Focus on the multiplication of radical expressions only.

1 http://regentsprep.org/Regents/Math/math-topic.cfm?TopicCode=radicals

You may answer each radical expression, choose the correct answer by clicking on the small circle corresponding to the right answer found on the right side of the page.

2 http://home.xnet.com/~fidler/triton/math/review/mat110/exprad/rad/prod/prod1.htm

Process Questions:

1. How did you go about the online activities?

2. Which part of the solution process you committed mistakes? How did you correct them?

Like the multiplication of radicals, Division of radicals follows several rules, one is simplifying by rationalization of denominators. See the examples and apply them in the exercise that follows.

Examples:

1. $\frac{\sqrt{243}}{\sqrt{5}} = \frac{\sqrt{9\cdot 9\cdot 3}}{\sqrt{5}}$ Simplify $\sqrt{243}$

$= \frac{9\sqrt{3}}{\sqrt{5}}$

$= \frac{9\sqrt{3}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$ Rationalize

$= \frac{9\sqrt{3\cdot 5}}{\sqrt{5\cdot 5}}$

$= \frac{9\sqrt{15}}{5}$

2. $\frac{\sqrt{6x^5}}{\sqrt{30xy}} = \sqrt{\frac{6x^5}{30xy}}$ make one radical expression, then reduce to the lowest term

$= \sqrt{\frac{x^4}{5y}}$

$= \frac{\sqrt{x^4}}{\sqrt{5y}}$

$= \frac{x^2}{\sqrt{5y}}$

$= \frac{x^2}{\sqrt{5y}} \cdot \frac{\sqrt{5y}}{\sqrt{5y}}$ rationalize

$= \frac{x^2\sqrt{5y}}{5y}$

3. $\frac{\sqrt{2}}{\sqrt{5}-\sqrt{3}} =$ solution to this problem may involve conjugates of denominators.

When the denominator of a fraction contains two terms where one or both terms contain a square root, the denominator can be rationalized by multiplying it by its conjugate.

Here are examples on getting the conjugate of one binomial.

Given	Conjugate
$\sqrt{7}+\sqrt{2}$	$\sqrt{7}-\sqrt{2}$
$\sqrt{3xy}-2$	$\sqrt{3xy}+2$
$-2-\sqrt{3}$	$-2+\sqrt{3}$
$2x+\sqrt{3}$	$2x-\sqrt{3}$

Process Questions:

1. Given the examples of binomials and its conjugate, what do you think is process of getting the conjugate of one binomial?
2. What prior knowledge can be used to help us simplify the expression?
3. What new knowledge did you gain?
4. Did your idea change because of this?

Let's continue:

3. $\frac{\sqrt{2}}{\sqrt{5}-\sqrt{3}}$ = $\frac{\sqrt{2}}{\sqrt{5}-\sqrt{3}}\cdot\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ conjugate the denominator

= $\frac{\sqrt{2}(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})}$ determine the product of the given binomials

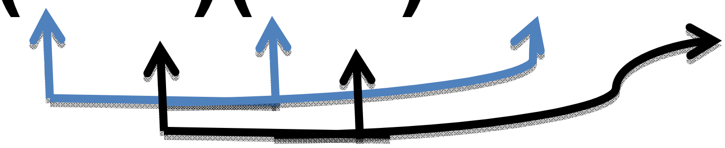
A **conjugate** is a binomial formed by negating the second term of a binomial. The conjugate of *x* + *y* is *x* − *y*, where *x* and *y* are real numbers.

The product of conjugates are always rational numbers. The product of a pair of conjugates is always a difference of two squares ($a^2 - b^2$). Multiplication of a radical expression by its conjugate results in an expression that is free of radicals.

Example:

In getting the product of $(a+b)(a-b)$ only the first and the last terms are multiplied. So,

$$(a+b)(a-b) = a^2 - b^2$$



Now, in getting for example the product of $\sqrt{a}-\sqrt{b}$ and its conjugate, we follow the solution below:

$$(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b}) = [(\sqrt{a})(\sqrt{a})] - [(\sqrt{b})(\sqrt{b})]$$
$$= \sqrt{a^2} - \sqrt{b^2}$$
$$= a - b$$

So, let's continue the previous example:

3. $\frac{\sqrt{2}}{\sqrt{5}-\sqrt{3}} \quad = \quad \frac{\sqrt{2}}{\sqrt{5}-\sqrt{3}} \cdot \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$= \frac{\sqrt{2}(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})}$

$= \frac{\sqrt{2\cdot 5}+\sqrt{2\cdot 3}}{\sqrt{5\cdot 5}-\sqrt{3\cdot 3}}$

$= \frac{\sqrt{10}+\sqrt{6}}{5-3}$

$= \frac{\sqrt{10}+\sqrt{6}}{2}$ final answer!

4. $\frac{\sqrt{5}}{\sqrt{7}-3} \quad = \quad \frac{\sqrt{5}}{\sqrt{7}-3} \cdot \frac{\sqrt{5}(\sqrt{7}+3)}{(\sqrt{7}-3)(\sqrt{7}+3)}$

$= \frac{\sqrt{35}+\sqrt{15}}{7-9}$

$= \frac{\sqrt{35}+\sqrt{15}}{-2}$

$= \frac{-\sqrt{35}-\sqrt{15}}{2}$

5. $\frac{\sqrt{3}-\sqrt{5}}{\sqrt{3}-3\sqrt{5}} \quad = \quad \frac{\sqrt{3}-\sqrt{5}}{\sqrt{3}-3\sqrt{5}} \cdot \frac{(\sqrt{3}+3\sqrt{5})}{(\sqrt{3}-3\sqrt{5})}$

$$= \frac{(\sqrt{3}-\sqrt{5})(\sqrt{3}+3\sqrt{5})}{(\sqrt{3}-3\sqrt{5})(\sqrt{3}-3\sqrt{5})}$$ Apply FOIL Method

$$= \frac{(\sqrt{3})(\sqrt{3})+(\sqrt{3})(3\sqrt{5})+(-\sqrt{5})(\sqrt{3})+(-\sqrt{5})(3\sqrt{5})}{(\sqrt{3})(\sqrt{3})-(3\sqrt{5})(3\sqrt{5})}$$

$$= \frac{\sqrt{9}+3\sqrt{15}-\sqrt{15}-3\sqrt{15}}{3-9(5)}$$ Combine Similar Terms

$$= \frac{3-\sqrt{15}}{3-45}$$ Simplify

$$= \frac{3-\sqrt{15}}{-42}$$ Final Answer

ACTIVITY 17. Divide and Conquer

Perform the indicated operations.

1. $\sqrt{4xy^2z^2} \div \sqrt[6]{16xy^2z^4}$

2. $\sqrt{2} \div (2 + \sqrt{3})$

3. $\frac{1}{2+\sqrt{5}}$

4. $\frac{1}{3-\sqrt{11}}$

5. $\dfrac{1}{\sqrt{3}-1}$

"click for the first step" to see the first part of the solution which can be used as your guide. Please remember that you have to solve first the problem before clicking the correct step. After solving the given problems answer the question: *Which part of the solution you committed mistakes? What did you do to correct them?*

http://home.xnet.com/~fidler/triton/math/review/mat110/exprad/rad/quot/quot1.htm

ACTIVITY 18. Generalization Table

Fill in the unshaded columns of the generalization table below by stating your generalization on the question, then save your answer.

How is the knowledge of radical expressions and equations used to solve real-life problems?

My Initial Thoughts	*My Findings and Corrections*	*Supporting Evidence*	*Qualifying Conditions*	*My Generalizations*

In the previous activities, we learned to transform exponential form to radical forms and vice versa. We were also able to simplify and apply the four fundamental operations involving radicals. In the proceeding activities you will be given more information on how to apply radicals involving real life problems through radical equations.

Now let's learn about radical equations:

A radical equation is an equation in which an unknown letter/variable occurs under a radical sign, or with a fractional exponent.

lve an equation that contains a square root, cube root, or nth root radical, the following procedures could be followed.

1. Isolate the radical from the other terms of the equation.
2. Raise both sides of the resulting equation to a number equal to the index of the radical.
3. Solve the equation by applying the different properties of equality and other related mathematics concepts.

For Example:

Example 1:

$\sqrt{x} - 2 \quad = \quad 5$

To solve the equation, isolate the radical by adding 2 on both sides of the equation.

$\sqrt{x} - 2 + 2 \quad = \quad 5 + 2$

$\sqrt{x} \quad = \quad 7$

Square both sides of the resulting equation then solve.

$(\sqrt{x})^2 \quad = \quad 7^2$

$x \quad = \quad 49$

Therefore, x = 49 is the solution of $\sqrt{x} - 2 = 5$

Example 2:

$\sqrt{x+4}-2$	=	5
$\sqrt{x+4}-2+2$	=	5 + 2
$\sqrt{x+4}$	=	7
$\left(\sqrt{x+4}\right)^2$	=	7^2
$x+4$	=	49
$x+4-4$	=	49 - 4
x	=	45

Therefore, x = 45 is the solution of $\sqrt{x+4}-2 = 5$

Example 3:

$4\sqrt[3]{3x-1}+5$	=	13
$4\sqrt[3]{3x-1}+5-5$	=	13 – 5
$4\sqrt[3]{3x-1}$	=	8
$\frac{1}{4}\left(4\sqrt[3]{3x-1}\right)$	=	$\frac{1}{4}(8)$
$\sqrt[3]{3x-1}$	=	2
$\left(\sqrt[3]{3x-1}\right)^3$	=	2^3
$3x-1$	=	8
$3x-1+1$	=	8 + 1
$3x$	=	9
$\frac{3x}{3}$	=	$\frac{9}{3}$
x	=	3

Therefore, x = 3 is the solution of $4\sqrt[3]{3x-1}+5 = 13$

Example 4:

$\frac{\sqrt{3x+4}}{2}$	=	5
$2\left(\frac{\sqrt{3x+4}}{2}\right)$	=	2(5)
$\sqrt{3x+4}$	=	10
$\left(\sqrt{3x+4}\right)^2$	=	10^2
$3x+4$	=	100
$3x+4-4$	=	100 – 4
$3x$	=	96
$\frac{3x}{3}$	=	$\frac{96}{3}$
x	=	32

Therefore, x = 32 is the solution of $\frac{\sqrt{3x+4}}{2} = 5$

Example 5:

$\frac{\sqrt{8x+3}}{\sqrt{x}}$	=	3
$\left(\sqrt{\frac{8x+3}{x}}\right)^2$	=	3^2
$\frac{8x+3}{x}$	=	9
$x\left(\frac{8x+3}{x}\right)$	=	9(x)
$8x+3$	=	9x
$8x-9x$	=	-3
-x	=	-3
(-1)(-x)	=	-1(-3)
x	=	3

Therefore, x = 3 is the solution of $\frac{\sqrt{8x+3}}{\sqrt{x}}$ = 3

Example 6:

When a radical equation requires squaring a binomial on one side of the equation, be careful not to square only the binomial in the product but also the middle term which is twice the product of the first and last terms of the binomial.

$\sqrt{x}+1$	=	$\sqrt{x-3}$	square both sides
$\left(\sqrt{x}+1\right)^2$	=	$\left(\sqrt{x-3}\right)^2$	
$\sqrt{x^2}+2\sqrt{x}+1$	=	$x-3$	
$x+2\sqrt{x}+1$	=	$x-3$	
$2\sqrt{x}$	=	$x-3-x-1$	
$2\sqrt{x}$	=	-4	
$\left(2\sqrt{x}\right)^2$	=	-4^2	
4x	=	16	
$\frac{4x}{4}$	=	$\frac{16}{4}$	
x	=	4	

ACTIVITY 19. THE EXPERT

Now that you are familiar with the rules in simplifying radicals and also solving radical equations. Do the activity below. Choose one among the two situations which you want to answer. You may use any educational sites to assist you in doing the activity. Your product will be evaluated using the rubric below.

Situation 1

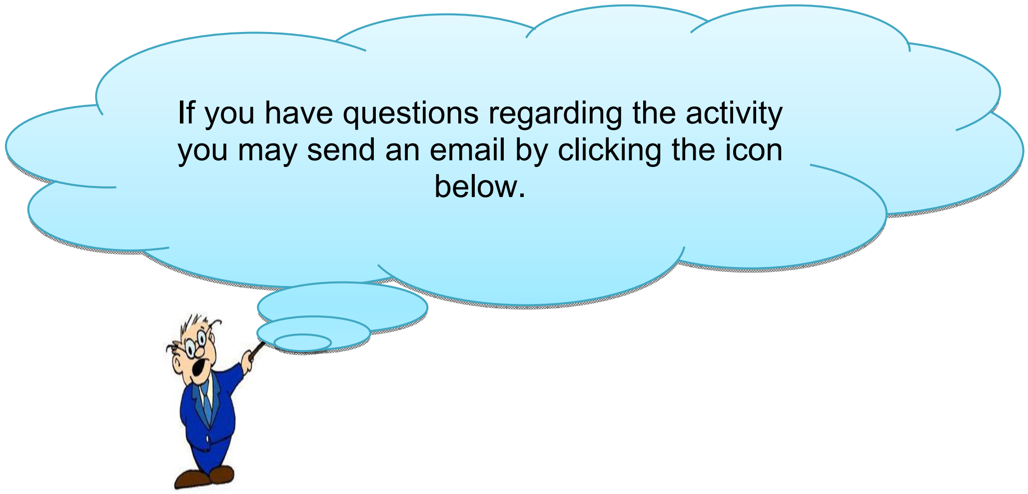
As one of the leading investigators in the country, you are requested by the head of the MMDA to prepare a leaflet to be distributed to the criminology interns as their guide in the preservation of evidence during car accidents. It is expected that you will provide the following information: the speed of the car, the length of the skid marks and the possible damage to the vehicle. Your brochure will be evaluated according to the following standards: use of mathematical concepts, clarity of the graphics and representations, and accuracy of data.

Situation 2

As one of the respected weather experts in the country, you are requested by the NDRRMC to prepare a leaflet to be distributed to the PAGASA interns to be used as an information campaign about the damages and speed of tsunamis. It is expected that you will provide the following information: speed of the wind, height of the tsunami and the tsunami's equivalent damage to properties. Your brochure will be evaluated according to the following standards: use of mathematical concepts, clarity of the graphics and representations, and accuracy of data.

	4 Excellent	3 Satisfactory	2 Developing	1 Beginning
Use of Mathematical Concepts	The presentation showsdeep understanding of the relevant ideas and processes. Main concepts are accurately presented in an in-depth way	The presentation shows adequate understanding of the relevant ideas and processes. All sub concepts are organized and	The presentation shows limited understanding of the relevant ideas and processes and sub-concepts don't	The presentation shows little understanding of the relevant ideas and issues and sub concepts don't consistently branch out

	that makes connections between each information. All sub concepts are logically organized.	consistently branch out from the main idea.	consistently branch out from the main idea.	from the main idea.
Clarity of the graphics and representations	All graphics and representations used are original and appropriate appropriate and attractive which enhanced the topic and aid in comprehension; properly and situated in a striking and original way	All graphics and representations used are appropriate which enhanced the topic and aid in comprehension; clear and well-situated.	Few graphics and representations used are partially appropriate which enhanced the topic and give a little aid in comprehension; confusing and wrongly placed in some parts.	Many graphics and representations used are inappropriate and poorly selected and don't enhance the topic; some graphics are ill-placed.
Accuracy of data	The data are credibly accurate and precise. Math concepts and procedures are detailed and applied appropriately. Use of efficient strategy that leads directly to a correct solution is original and evident.	The data are correct. Math concepts and procedures are applied correctly. Use of strategy that leads to a solution is evident.	The data contain minor errors. Some math concepts are used but not all of the necessary ones. Some strategies used are inappropriate	The data contain major errors. Inappropriate math concepts or procedures are used. No evidence of a strategy or the strategy shown is inappropriate



ACTIVITY 20. EQUATE Me.

Solve each of the following equations. Explain how you arrived at the solution.

1. $\sqrt{3x+8}=\sqrt{2x+11}$

2. $\dfrac{\sqrt{2x-1}}{3}=3$

3. $\dfrac{4\sqrt[4]{x}}{\sqrt[4]{x}+1} = 3$

4. $6\sqrt[3]{x-2} = -18$

5. $\dfrac{\sqrt{3x-1}}{5} = \dfrac{2}{\sqrt{3x-1}}$

To add more information in solving radical equations. You may click the website below. The website contains additional discussions on radical equations. It is suggested that you take down important ideas and concepts. After reviewing the website you may do the next activity.

http://www.wtamu.edu/academic/anns/mps/mathlab/col_algebra/col_alg_tut19_radeq.htm

After reading the above website, you may click the website below for an interactive exercise involving radical equations.

http://regentsprep.org/regents/mathb/7D3/rationalprac.htm

ACTIVITY 21. COMPARE AND CONTRAST CHART

To see how well you understand the process solving radical equations, accomplish the Compare and Contrast Chart based on what you saw in the solution.

Compare and Contrast Chart Graphic Organizer

Solving Radical Equations	Solving Linear Equations

How are they alike?

How are they different?

ACTIVITY 22. FLOW – Tion

Now that you already know how to solve radical equations. Make a summary of the steps on how to solve radical equations, write your answers on the given flow chart.

Solving Radical Equations

↓

STEP 1:

↓

STEP 2:

↓

STEP 3:

↓

STEP 4:

END OF DEEPEN

In this section, the discussion was about applying what you learned in real life. You will now be given a practical task which will demonstrate your understanding of the topic.

TRANSFER

All throughout the lesson, the question about how radicals can help in your daily life was asked repeatedly. The Transfer section of the lesson will guide you in determining the best answer to the question. **How can problems were two quantities bounded by conditions are solved? How related quantities affect each other?** How is the knowledge of radical expressions and equations used to solve real-life problems?

Your goal in this section is apply your learning to real life situations. You will be given a practical task which will demonstrate your understanding of the topic..

Now that you are already acquainted with solving radical equations and have even summarized your thoughts and learning in a flowchart, you will be given some activities showing how radicals can be applied in real life.

Many real life situations or problems involve the concept of radicals. The use of radical equations facilitates finding of the solutions to these problems.

Examples:

1. The formula V = $3.5\sqrt{h}$ is used to approximate how far V, in km, a person can see to the horizon from a given height, *h*, in meters. How far can you see to the horizon through an airplane window at a height of 9000 m?

(convert first 10000m to km)

Solution:

V= $3.5\sqrt{h}$
$V = 3.5\sqrt{10000}$
$V = 3.5(100)$
$V = 350km$

You can see the about 350 km horizon at a height of 10000m.

2. If person can see 35 km to the horizon from the top of a building. How high is the building?

Solution:

$V = 3.5\sqrt{h}$

$35 = 3.5\sqrt{h}$

$\frac{35}{3.5} = \frac{3.5\sqrt{h}}{3.5}$

$10 = \sqrt{h}$

$10^2 = (\sqrt{h})^2$

$100 = h$

The building is 100 meters high

3. The formula $\boldsymbol{V} = \sqrt{\boldsymbol{12S}}$ is used to approximate the velocity ***V*** in miles per hour of a car based on the length of its skid marks ***S*** (in feet) on wet pavement. How long would the skid marks of a car that travels 36 miles an hour?

Solution:

$V = \sqrt{12S}$

$36 = \sqrt{12S}$

$36^2 = (\sqrt{12S})^2$

$1296 = 12S$

$\frac{1296}{12} = \frac{12S}{12}$

$108 = S$

The length of the skid marks of the car would be 108 miles

4. The approximate distance, ***d***, in miles that a person can see to the horizon from a height of ***h*** feet is given by the equation $\boldsymbol{d} = \sqrt{\frac{\boldsymbol{3h}}{\boldsymbol{2}}}$. A man in a tower estimated that the distance he can see on the horizon is about 24 mi. How high above the ground is the man in the tower?

Solution:

$d = \sqrt{\frac{3h}{2}}$

$24 = \sqrt{\frac{3h}{2}}$

$24^2 = \left(\sqrt{\frac{3h}{2}}\right)^2$

$2(576) = (\frac{3h}{2})(2)$

$1152 = 3h$

$\frac{1152}{3} = \frac{3h}{3}$

$384 = h$

Therefore, the man is 384 feet above the ground.

ACTIVITY 23. Your Turn!

Show different ways of representing and solving each of the following problems.

1. Use the equation $T = 2\pi\sqrt{\frac{L}{384}}$, where T is the period of a pendulum and L is its length in inches, to answer the following questions.

   a. If the period of the pendulum is given, how would you represent its length?

   b. If the length of the pendulum is 12 inches, what is its period?

2. A car that was involved in a mishap left a skid marks 120 ft. long. A witness told that the car was travelling at about 50 miles an hour. Was the witness right? Justify your answer by showing a solution.

3. The equation $v = 8\sqrt{h}$ can be used to find the approach velocity v (in feet per second) of a pole-vaulter. h represents the height a pole-vaulter can reach.
   a. Erwin and Elmer are pole-vaulters. In one of the events they participated, Erwin's approach velocity was 40 feet per second while Elmer's approach velocity was 35 feet per second. Who reached greater height? Justify your answer.

   b. Suppose the height Erwin reaches is 25 ft., what must be the approach velocity of Eric for him to beat Elmer? Justify and explain your answer.

4. Isaac Newton established the formula $V_{esc} = \sqrt{\frac{2GM}{R}}$ to calculate the escape velocity from a planet or star. Where, V is the escape velocity, G is the Gravitational Constant, M is the mass of the planet star and R is the radius of the planet star. What will the equation be if you are going to solve for the Radius of the planet star.

Process Questions:

5. The speed that a tsunami can travel is modeled by the equation $S = 356\sqrt{d}$ where S is the speed in kilometers per hour and *d* is the average depth of the water in kilometers. What is the speed of the tsunami when the average water depth is 0.645 kilometers? (*round to the nearest tenth)*

ACTIVITY 24. PERFORMANCE TASK: THE EXPERT

The Disaster Reduction Management Council will be having its conference about disasters and how to measure its effect. As an expert in this field, you are invited to present a case analysis regarding the factors that leads to natural disasters. Your presentation will be evaluated according to the following standards: mathematical concept, clarity of the graphics and representations, accuracy of data, fluency of presentation, and organization of the presentation.

Now that you are done with your work, use the rubric below to evaluate your work. Your work should show the traits listed as Good or 3. If your work has these traits, you are ready to submit your work. Click on “Submit”.

If you want to do more, your work should show the traits listed as Excellent or 4. If your work does not have the traits for 3 or 4, revise your work before submitting it.

PERFORMANCE TASK RUBRIC

	4 Excellent	3 Satisfactory	2 Developing	1 Beginning
Use of Mathematical Concepts	The presentation shows deep understanding of the relevant ideas and processes. Main concepts are accurately presented in an in-depth way that makes connections between each information. All sub concepts are logically organized.	The presentation shows adequate understanding of the relevant ideas and processes. All sub concepts are organized and consistently branch out from the main idea.	The presentation shows limited understanding of the relevant ideas and processes and sub-concepts don't consistently branch out from the main idea.	Use of Mathematical Concepts
Clarity of the graphics and representations	All graphics and representations used are original and appropriate appropriate and attractive which enhanced the topic and aid in comprehension; properly and well-situated.	All graphics and represent-tations used are appropriate which enhanced the topic and aid in comprehen-sion; clear and well-situated.	Few graphics and represent-tations used are partially appropriate which enhanced the topic and give a little aid in comprehend-sion; confusing and wrongly placed in some parts.	Many graphics and represent-tations used are inappropriate and poorly selected and don't enhance the topic; some graphics are ill-placed.
Accuracy of data	The data are credibly accurate and precise. Math concepts and procedures are detailed and applied appropriately. Use of efficient	The data are correct. Math concepts and procedures are applied correctly. Use of strategy that leads to a solution is evident.	The data contain minor errors. Some math concepts are used but not all of the necessary ones. Some strategies used	The data contain major errors. Inappropriate math concepts or procedures are used. No evidence of a strategy or the

	strategy that leads directly to a correct solution is original and evident.		are inappropriate	strategy shown is inappropriate
Fluency of presentation	Fluent, confident and thoroughly explained each point by providing support that contains rich, vivid and powerful detail.	Generally fluent, confident and clearly explained the proposal.	Somewhat hesitant, less confident and failed to explain significant number of points	Hesitant, not confident. Explanation is missing.
Organization of the presentation	Highly organized and done in an interesting way. Flows smoothly. Observes logical connections of points.	Satisfactorily organized. Sentence flow is generally smooth and logical.	Somewhat disorderly. Flow is not consistently smooth, appears disjointed.	Illogical and unclear. No logical connections of ideas. Difficult to determine the meaning.

Now that you have completed your project, you may do the next activity.

ACTIVITY 25. CONNECT me!!!

Answer the following by reflecting on the following questions.

How did you find the performance task? → How did the task help you see the real world use of the topic? → How can radicals help us solve real life problems?

End of TRANSFER:

In this section, your task was to create a power point presentation and a written proposal. At this point, it is expected that the question about confusions regarding letters, numbers and symbols are being dealt with. Now that we already learned about the topic of radicals, its definition and its uses. We are now able to apply this in a different context.

ACTIVITY 26. The Way I see It

Complete the table below.

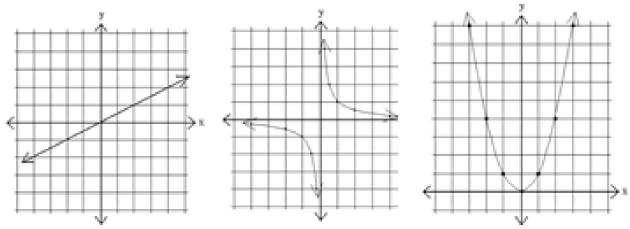
What did I Learn About Radicals?	How Will I apply it as a student?	How will I apply it as a member of a community?

You have completed this lesson. Before you go to the next lesson, you have to answer the following post-assessment.

POST-ASSESSMENT:

Click on the letter of the answer that you think best answers the question. Your score will only appear after you answer all items. If you do well, you may move on to the next module. If your score is not at the expected level, you have to go back and take the module again.

1. Which of the following graphs illustrates a direct relationship?



A B C

2. What formula represents the relationship shown in the table?

m	1	2	3	5	10
P	5	9	13	21	41

A. P = 4m
B. P = 4+m
C. P = 4m
D. P = $\frac{4}{m}$

3. If y varies jointly as x and the square of z and y =72 when x =3 and z =2, find choose the equation that represents this relationship.

A. $y = xz^2$
B. $y = 4xz^2$
C. $y = 6xz^2$
D. $y = 8xz^2$

4. What is the simplified form of $16^{3/2}$?

a. 64
b. 8
c. 12
d. 16

5. Which of the following radicals gives $5+2\sqrt{3}$ when combined?

A. $\sqrt{16} - \sqrt{3} + 1 + 3\sqrt{3}$
B. $\sqrt{16} - 2\sqrt{3} + 1 + 3\sqrt{3}$
C. $\sqrt{16} - \sqrt{3} + 5 + 3\sqrt{3}$
D. $\sqrt{16} - \sqrt{3} + 1 + 3\sqrt{4}$

6. The monthly salary ***S*** of Andy is directly proportional to the number of days ***d*** he worked. His daily wage is PhP 405. Which table represents the relationship of the Andy's monthly salary and the number of days he worked?

A.

D	1	6	14	20
S	405	1800	5670	8000

B.

D	1	6	14	20
S	405	2000	5000	8000

C.

D	1	6	14	20
S	405	2025	5670	8100

D.

D	1	6	14	20
S	405	2025	5670	9100

7. Which of the following is true about $\sqrt[mn]{x} = \sqrt[n]{\sqrt[m]{x}}$?

A. The nth root of the mth root of a number x is equal to a radical whose radicand is x and whose index is the product of m and n

B. The nth root of the mth root of a number x is equal to a radical whose radicand is x and whose index is the quotient of m and n

C. The nth root of the mth root of a number x is equal to a radical whose radicand is the product of m and n

D. The root of a number x is not equal to a radical whose radicand is x and whose index is the product of m and n

8. Which of the following radical equation will give a solution of 64?

A. $\sqrt{x+4} = 2$

B. $\sqrt[3]{x+2} = 8$

C. $\sqrt[3]{x+2} = 4$

D. $\sqrt[3]{x+2} = 64$

9. The number of calories **C** burned while rowing is directly proportional to the time ***t*** spent rowing. During a 5 – minute rowing competition, a 130 – pound woman will burn 59 calories while 75 – pound woman will burn 124 calories. Write a model for the calories *c* burned from rowing in this competition for *m* minutes. What is the minimum number of minutes they have to row together to burn at least 300 calories?

A. 10 minutes
B. 12 minutes
C. 26 minutes
D. 33 minutes

10. The rental of a beach cottage varies directly with the numbers of hours you stay. At cottage A, three hours costs a total of PhP 250. At cottage B, five hours stay costs a total of Php 420. If you will stay for 13 hours and the cost will be computed per hour, which cottage has the cheaper rental?

A. Cottage A
B. Cottage B
C. Both offers the same
D. It cannot be determined

11. Which of the following expressions violates the condition in simplifying radicals?

A. $\sqrt{2x^7}$
B. $x\sqrt{2}$
C . $3\sqrt{2}$
D. $x^7\sqrt{2}$

12. Which of the following problems would result to 1?

A. Twice the square root of four more than three times a number is equal to five more than three times the same number.
B. Twice the square root of four more than three times a number is equal to twenty five more than three times the same number.
C. Three times the square root of four more than three times a number is equal to twenty five more than three times the same number.
D. Twice a number is equal to twenty five more than three times the same number.

13. Which of the following is not true about the laws of radicals?

A. The quotient of two radicals having the same index **n** is equal to the **n**th root of the quotient of their radicands.
B. The product of two radicals having the same index **n** is equal to the **n**th root of the product of their radicands.
C. The **n**th root of the **m**th root of a number **x** is equal to a radical whose radicand is **x** and whose index is the product of **m** and **n**.

D. The **n**th root of a number raised to **n** is not equal to the number.

14. If *y* varies jointly as *x* and *z* and *y* = 60 when *x* = 3 and *z* =4, what will happen to the value of x if the value of z increases and the value of y remained?

A. The value of x will decrease.
B. The value of x will increase.
C. The value of x will remain.
D. The value of x cannot be determined.

15. What is the height of a person with a Body Surface Area of approximately 0.2109?

A. 2 meters
B. 3 meters
C. 3.5 meters
D. 4 meters

16. As a police officer, you found out that The formula $\boldsymbol{V} = \sqrt{\boldsymbol{12S}}$ is used to approximate the velocity ***V*** in miles per hour of a car based on the length of its skid marks ***S*** (in feet) on wet pavement. How long would the skid marks of a car that travels 40 miles an hour?

A. Approximately 133.33 miles
B. Approximately 233 miles
C. 200 miles
D. 100 miles

17. The Pacific Tsunami Warning Center is responsible for monitoring earthquakes that could potentially cause tsunamis in the Pacific Ocean. Through measuring the water level and calculating the speed of a tsunami, scientists can predict arrival times of tsunamis.
If you are a weather forecaster, do you think it is important to determine the depth of the ocean in measuring tsunamis?

A. No, because tsunamis are measured according to speed.
B. No, because tsunami is difficult to measure.
C. Yes, because The speed (in meters per second) at which a tsunami moves is determined by the depth d (in meters) of the ocean.
D. Yes, because tsunamis is from under the ocean.

18. The number of hours *h* that it takes *m* men to assemble *x* machines varies directly as the number of machines and inversely as the number of men. If 6 men can assemble 36 machines in 8 hours, can 4 men be able to assemble 12 machines in 3 hours?

A. Yes, they can finish it in less than 3 hours.
B. Yes, they can finish it in exactly 3 hours.
C. No, they need four hours to finish it.

D. No, they need five hours to finish it.

19. Nestor's Eatery serves budget meals. The monthly number of orders varies directly with the number of menu, and varies inversely with the cost of each budget meal. Last month's total order is 1800 budget meals when it serves 6 menu and the cost per meal was PhP 40. If the number of menu does not change, what would be the effect on the monthly total number of orders for increasing the cost per meal to PhP 50? Will it be beneficial for the store?

A. The number of orders will remain, therefore the eatery will gain more.
B. The number of orders will increase, therefore the eatery will gain more.
C. The number of orders will decrease but the canteen will gain as much as when the price was lower.
D. The number of orders will decrease, thus the eatery will lose a lot of money.

20. As a weather expert, you are requested by the PAGASA to provide residents of coastal areas information about possible effects of tsunamis. You are provided by the board of directors a set of criteria in doing this tasked which are Organization of the presentation, mathematical concept, clarity of the graphics and representations accuracy of data and fluency of the presentation. What do you think is the best product to provide?

A. Comics
B. Brochure
C. Windmill Model
D. Rain Gauge

GLOSSARY OF TERMS USED IN THIS LESSON:

Conjugates - is a binomial formed by negating the second term of a binomial. The conjugate of x + y is x − y, where x and y are real numbers. If y is imaginary, the process is termedcomplex conjugation: the complex conjugate of a + bi is a − bi, where a and b are real.

Radicals - an expression that has a square root, cube root, etc.

Radicand - the value inside the radical symbol. The value you want to take the root of.

Skid Marks - a long black mark left on a road surface by the tires of a skidding vehicle.

REFERENCES AND WEBSITE LINKS USED IN THIS LESSON:

http://wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/col_alg_tut19_radeq.htm
This site presents a radical equation tutorial featuring the steps in solving radical equations.

http://regentsprep.org/regents/mathb/7D3/radlesson.htm
This site provides the steps on how to solve radical equations. Several examples are given for clarification on the steps.

http://regentsprep.org/regents/mathb/7D3/rationalprac.htm
This site provides good exercises on solving radical equations.

http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=112
This site contains interactive practice exercises for students. To be able to maximize the resources it contains, users are encouraged to subscribe to www.explorelearning.com in which a free trial is given.

http://regentsprep.org/Regents/Math/math-topic.cfm?TopicCode=radicals
This site contains 5 sub-topics on operations of radicals. It represents students with steps on how to simplify radicals and steps on how to add, subtract, multiply, and divide radical expressions. The teacher can utilize this site to enhance his/her discussion on the operations of radicals. It also contains interactive games and activities related to operations on radicals like online multiple choice items which can provide the learners an immediate feedback.

http://home.xnet.com/~fidler/triton/math/review/mat110/exprad/rad/prod/prod1.htm http://home.xnet.com/~fidler/triton/math/review/mat110/exprad/rad/quot/quot1.htm

Intermediate Algebra by Zenaida E. Diaz and Maharlika P. Mojica on pp.83-88
Worktext in Intermediate Algebra II by Ferdinand C. Pascual, el.al. on pp.51-56.