PEAC
PRIVATE EDUCATION ASSISTANCE COMMITTEE
Trustee of the Fund for Assistance to Private Education

DepED
DEPARTMENT OF EDUCATION

2018

LEARNING MODULE

Mathematics G10 | Q3

Statistics and Probability: Outcomes-Based Decisions

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NOTICE TO THE SCHOOLS

This learning module (LM) was developed by the Private Education Assistance Committee under the GASTPE Program of the Department of Education. The learning modules were written by the PEAC Junior High School (JHS) Trainers and were used as exemplars either as a sample for presentation or for workshop purposes in the JHS In-Service Training (INSET) program for teachers in private schools.

The LM is designed for online learning and can also be used for blended learning and remote learning modalities. The year indicated on the cover of this LM refers to the year when the LM was used as an exemplar in the JHS INSET and the year it was written or revised. For instance, 2017 means the LM was written in SY 2016-2017 and was used in the 2017 Summer JHS INSET. The quarter indicated on the cover refers to the quarter of the current curriculum guide at the time the LM was written. The most recently revised LMs were in 2018 and 2019.

The LM is also designed such that it encourages independent and self-regulated learning among the students and develops their 21st century skills. It is written in such a way that the teacher is communicating directly to the learner. Participants in the JHS INSET are trained how to unpack the standards and competencies from the K-12 curriculum guides to identify desired results and design standards-based assessment and instruction. Hence, the teachers are trained how to write their own standards-based learning plan.

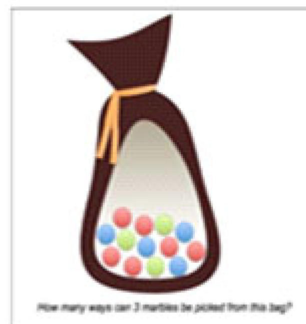
The parts or stages of this LM include Explore, Firm Up, Deepen and Transfer. It is possible that some links or online resources in some parts of this LM may no longer be available, thus, teachers are urged to provide alternative learning resources or reading materials they deem fit for their students which are aligned with the standards and competencies. Teachers are encouraged to write their own standards-based learning plan or learning module with respect to attainment of their school's vision and mission.

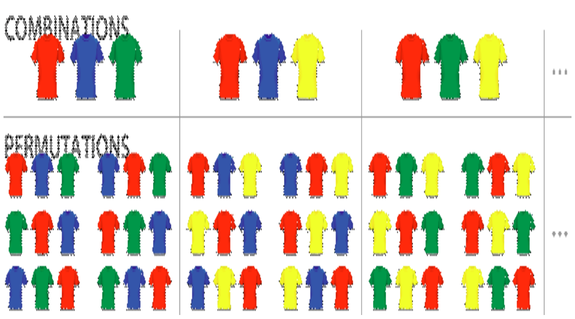
The learning modules developed by PEAC are aligned with the K to 12 Basic Education Curriculum of the Department of Education. Public school teachers may also download and use the learning modules.

MATHEMATICS 10

Module 3: Statistics And Probability: Outcomes-Based Decisions

☑ INTRODUCTION AND FOCUS QUESTION(S):





Have you ever wondered how counting of arrangements, groupings which involve very large numbers in complex situations and chances of events to happen are to be done? This module focuses on how they should be accomplished. The fundamental counting techniques, permutations, combinations and probability are to be explored and covered. These concepts and skills will equip you to communicate, formulate, investigate, analyze and solve real-life problems in order to come up with conclusions which will help in making sound decisions.

This module seeks to find the answers of the questions,

1. How can you predict outcomes accurately?

2. How can you count without counting?
   What is the best method in predicting the chances of favorable outcomes? (topical)

☑ **LESSONS AND COVERAGE:**

In this module, you will examine this question when you take the following lessons:

Lesson 1 – PERMUTATIONS

In these lessons, you will learn the following:

Lesson 1	✓ Illustrates the permutation of objects (K) ✓ Derives the formula for finding the number of permutations of *n* objects taken *r* at a time (P) ✓ Solves problems involving permutations (P)

☑ **MODULE MAP:**

☑ **EXPECTED SKILLS:**

To do well in this module, you need to remember and do the following:

1. Illustrate the fundamental counting principle and evaluate factorial notations.
2. Explore certain websites indicated in the module that would be of great help for your better understanding of the lessons on permutations and work on the interactive activities.
3. Take down notes of the important concepts and follow the formulae in answering FCP and permutations exercises.
4. Perform the specific activities or tasks and complete the exercises and assessments provided.
5. Collaborate with the teacher and peers.

PRE-ASSESSMENT:

Let's find out how much you already know about this module. Click on the letter that you think best answers the question. Please answer all items. After taking this short test, you will see your score. Take note of the items that you were not able to correctly answer and look for the right answer as you go through this module.

1. 0! is equal to
   a. -1 b. 0 c. 1 d. 2

2. ${}_7P_7$ equals
   a. 7.6.5.4.3.2.1 b. 7! c. $\frac{7!}{0!}$ d a, b, & c

3. ${}_5P_8$ means
   a. $\frac{8!}{5!}$ b. $\frac{8!}{3!}$ c. $\frac{8!}{5!3!}$ d. $\frac{8!5!}{3!}$

4. The arrangement of 3 books in English, Math and Science by two's on a shelf are
   a. (EM, ES, SM)
   b. (EM, ES, ME, MS, SM, SE)
   c. (EMS, SEM, MES)
   d. (EMS, ESM, SEM, SME, MES, MSE)

5. To determine the number of varieties of ice-cream with 3 flavors from 8 different flavors with no 2 flavors alike will be done by
   a. 8•8•8 b. 8•7•6 c. 8•3 d. 8•7•6•5•4

6. How many plate numbers composed of 3 letters followed by three digits can be made from 26 letters of the English alphabets excluding I and O and the 3 digits does not start with zero?
   a. 24 24 24 10 10 10
   b. 26 25 24 10 9 8
   c. 26 26 26 10 10 10
   d. 24 24 24 9 10 10

7. From the ground floor to the second floor, there are 3 staircases, to the third floor there are also 3 staircases and each classroom has 2 doors, how many choices of passage ways are there in entering the classroom?
   a. 8 b. 9 c. 11 d. 18

8. Which of the following is the correct meaning of combination?

a. It is a counting technique where order of elements/members is counted individually.
b. It is a counting technique where order of elements/members matters.
c. It is a counting technique that considers all possible outcomes where order of elements is necessary.
d. It is a counting technique that considers all possible outcomes where order of elements is not important.

9. Which of the following is the correct way in getting the combination of 10 objects taken 3 at a time?

a) ${}_{3}C_{10}$ b. ${}_{10}C_{3}$ c. $\frac{{}_{3}C_{10}}{10}$ d. $\frac{{}_{10}C_{3}}{3}$

10. What is the number of outcomes if you want to form a 6-member committee composed of 4 boys and 2 girls from a group of 6 boys and 4 girls?

a) ${}_{6}C_{4}+{}_{4}C_{2}$ b. ${}_{4}C_{6}\bullet{}_{2}C_{4}$ c. ${}_{6}C_{4}\bullet{}_{4}C_{2}$ d. ${}_{4}C_{6}+{}_{2}C_{4}$

1. 11. Which of the following is the correct process that relates combination and permutation of 4 objects taken 2 at time?

$a)$

$$
\begin{aligned}
{}_{4}P_{2} &= 2!({}_{4}C_{2}) \\
\frac{4!}{(4-2)!} &= 2!({}_{4}C_{2}) \\
\frac{\frac{4!}{(4-2)!}}{2!} &= {}_{4}C_{2} \\
{}_{4}C_{2} &= \frac{\frac{4!}{(4-2)!}}{2!} \\
{}_{4}C_{2} &= \frac{4!}{(4-2)!}\bullet\frac{1}{2!} \\
{}_{4}C_{2} &= \frac{4!}{2!(4-2)!}
\end{aligned}
$$

$b)$

$$
\begin{aligned}
{}_{2}P_{4} &= 2!({}_{4}C_{2}) \\
\frac{2!}{(2-4)!} &= 2!({}_{4}C_{2}) \\
\frac{\frac{2!}{(2-4)!}}{2!} &= {}_{4}C_{2} \\
{}_{4}C_{2} &= \frac{\frac{2!}{(2-4)!}}{2!} \\
{}_{4}C_{2} &= \frac{2!}{(2-4)!}\bullet\frac{1}{2!} \\
{}_{4}C_{2} &= \frac{1}{(2-4)!}
\end{aligned}
$$

c) ${}_nP_r = n!({}_nC_r)$

$$\frac{n!}{(n-r)!} = n!({}_nC_r)$$

$$\frac{\frac{n!}{(n-r)!}}{n!} = {}_nC_r$$

$${}_nC_r = \frac{\frac{n!}{(n-r)!}}{n!}$$

$${}_nC_r = \frac{n!}{(n-r)!} \bullet \frac{1}{n!}$$

$${}_nC_r = \frac{1}{(n-r)!}$$

d) ${}_nP_r = r!({}_nC_r)$

$$\frac{n!}{(n+r)!} = r!({}_nC_r)$$

$$\frac{\frac{n!}{(n+r)!}}{r!} = {}_nC_r$$

$${}_nC_r = \frac{\frac{n!}{(n+r)!}}{r!}$$

$${}_nC_r = \frac{n!}{(n+r)!} \bullet \frac{1}{r!}$$

$${}_nC_r = \frac{n!}{r!(n+r)!}$$

12. The school will be celebrating its Family Day soon. You are a member of the games and amusement committee and you are assigned to make some alphanumeric codes for your raffle tickets. Each code is composed of three letters and two digits. How many possible unordered outcomes are possible?
    a. 26,000 b. 88,500 c. 100,000 d. 117,000

13. Barangay Sampalok Zone 1 is organizing a Hearts' Week, an event for a cause. The proceeds will be donated to HeartSmile, a foundation that funds for the medication of children with heart conditions. The barangay chairman is taking the lead in organizing various committees. The first committee is the planning committee. From a group of 7 men and 6 women, five persons are to be selected to form the committee so that at least 3 men are there on the committee. How can the barangay chairman determine the possible number of committees where each member is equal in position?
    a. He can just randomly select from the qualified persons to lead the committee and count all the possibilities.
    b. He can use permutation to find the number of outcomes since every position in the committee is important.
    c. He can use combination to find the number of outcomes because every member is equal in rank and ordering their position does not matter.
    d. He can just multiply the number of men by the number of women to determine the number of possible outcomes since the position of every member is not important.

14. A coin is tossed three times. Let E be the event that exactly one head comes up and let F be the event that at least two tails come up. What is $P(E \cup F)$?

a. 3/8 b. 1/8 c. 0/8 or 0 d. 4/8 or 1/2

15. A deck of 52 cards are shuffled and a single card is drawn. Find the probability that the card drawn is a diamond or a queen.

a. 1/26d. b. 4/13 c. 17/52 d. 1/52

16. Carlo bought 4 batteries of which one is defective. He selects two batteries at random to be used for the TVs remote control. What is the probability that the second battery is not defective given that the first battery is not defective?

a. 2/3 b. 1/4 c. 1/3 1/2

17. A class has 20 boys and 15 girls. The class has to elect a President, Vice President, Treasurer, Secretary, and PIO. What is the probability that a boy will be elected as president and all girls for the other 4 positions?

a. $\frac{325}{5797}$ b. $\frac{1}{75}$ c. $\frac{195}{11594}$ d. $\frac{19}{60}$

18. As the number of electronic devices increases, so does the use of rechargeable batteries. A particular manufacturer produces batteries in lots of 100. In each lot, two of the batteries will be defective. The batteries are randomly packaged in groups of four batteries. You work in the Quality Control department of that company. You want to know the probability that all of the batteries in a package will not be defective. The goal is to make this probability as low as possible to minimize the warranty costs. What is the probability that all of the batteries in a package will not be defective?

a. $\frac{{}_{98}C_4}{{}_{100}C_4}$ b. $\frac{{}_{98}P_4}{{}_{100}P_4}$ c. $\frac{{}_{4}C_2}{{}_{98}C_4}$ d. $\frac{{}_{4}P_2}{{}_{98}P_4}$

19. You are the organizer of a basketball tournament in your barangay. You are to submit to the tournament committee head the budget for the referees fees. There are 12 teams who registered. You need to decide what format of the game to follow so that the budget will be minimal. Which of the following formats will you recommend to have minimal budget for referees fees based on the number of games played?

A. Follow the single round robin system and the team with the most number of wins becomes the champion.
B. Separate the 12 teams into two groups. Each group of 6 teams will play single round robin. The most number of wins per group will play for the championship in a best of 3.

a. format A
b. format B
c. both formats yield same number of games
d. the number of games cannot be determined

20. In a small assembly plant with 50 employees, each worker is expected to complete work assignments on time and in such a way that the assembled product will pass a final inspection. On occasion, some of the workers fail to meet the performance standards by completing work late or assembling a defective product. At the end of a performance evaluation period, the production manager found that 5 of the 50 workers completed work late, 6 of the 50 workers assembled a defective product, and 2 of the 50 workers both completed work late and assembled a defective product.

After reviewing the performance data, the production manager decided to assign a poor performance rating to any employee whose work was either late or defective. What is the probability that the production manager assign an employee a poor performance rating?

The problem involves probability of
a. mutually exclusive events
b. not mutually exclusive events
c. dependent events
d. independent events

EXPLORE

You begin by engaging in an activity which will explore the concepts of the fundamental counting principle and permutations and how they are applied in the real-world.

ACTIVITY 1. Situational Analysis: How Safe Is My Treasure?

In groups of 4 students, you read the given situation and come up with the answers of the questions which follow.
When opening an account in a bank, every client is required to submit a PIN which he will use as a password in accessing the account. This PIN should be kept confidential for security reasons.

1. If the PIN is composed of 4 numbers, how many possible choices do a client has?
2. How do you count the choices?
3. What is the chance of having duplication?
4. Is counting the choices easy to do? Why or why not?
5. *How can you count without counting?*
6. What influences choices or decisions?
7. What happen to a decision/choice that is not based on the possible outcomes?
8. *How can outcomes be predicted?*

ACTIVITY 2. Eliciting Prior Knowledge

ANTICIPATION-REACTION GUIDE

Read all the statements in the chart and put a ☆ if you agree and X mark if you disagree with the statement in the before column.

BEFORE	STATEMENTS	AFTER
	1. Fundamental counting principle is used in finding the number of possible outcomes.	
	2. Listing, the use of tables and tree diagram are methods of counting.	
	3. Permutation is used to determine the number of committees of 4 members that can be formed out of 10 members.	

	4. The notation 5! means 5x1.	
	5. P (10,4) =10.9.8.7	

Take turns in sharing your answers to the class.

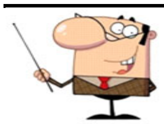
End of EXPLORE:

You have just given your initial ideas and heard how others answered the questions, so you will now start finding out which answers are valid or not by doing the next part.

FIRM-UP

Your goal in this section is to learn and understand key concepts of combinatorics, so let's start by knowing the fundamental counting principle and permutations.

ACTIVITY 3. Let's Count



In a person's life, almost every action of the day has corresponding outcomes or every action has a corresponding reaction as the famous physicist Sir Isaac Newton puts it. Each of these happenings requires him to make choices not just one but many. That is why there is a need to count.

Do you know how to count?

Let's check if you really know how.

1. If you throw a die, how many possible outcomes will there be?
   Answer: ________
2. If you throw a die and coin, how many outcomes are there?
   Answer: __________

PROCESS QUESTIONS:

1. How did you know the answer?
2. How did you go about counting?
3. What method did you use in counting?
4. Illustrate the method that you use.

5. Are there other ways to count?
6. What are the different methods of counting?

For every event that occurs there are always outcomes and these can be counted in a variety of ways. In order not to miss a single outcome a certain system has to be followed; one way to do it is through organized listing, tables or grids could also be used. Another systematic way of counting is to come up with a tree diagram especially for cases where there is a sequence of events.

In general;

These are the 3 different methods of counting:
1. Listing
2. Use of grid or table
3. Tree Diagram

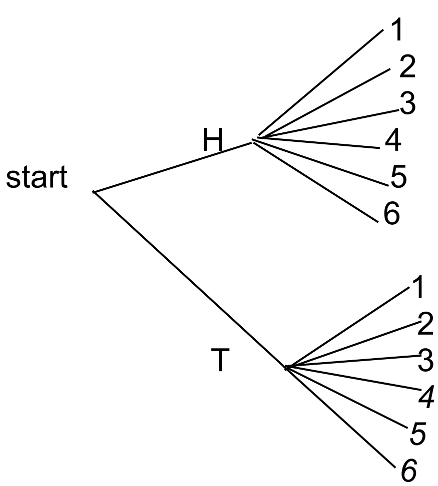
1. LISTING
A die has 6 faces, so if it is thrown it will reveal either 1, 2, 3, 4, 5 or 6 Therefore, there are 6 possible outcomes.

2. USE OF GRID OR TABLE
The die has 6 sides and a coin has 2 sides

Die	1	2	3	4	5	6
coin						
Head	H1	H2	H3	H4	H5	H6
Tail	T1	T2	T3	T4	T5	T6

When throwing a die and a coin simultaneously, there are 12 possible outcomes.

3. TREE DIAGRAM



There are 12 possible outcomes when throwing a die and a coin.

DO THESE:

Find the number of possible outcomes for each given situation using the specified method.

A. Listing
   1. The genders of 3 kids (use B for boy and G for girl)

B. Tree Diagram
   2. Choosing a cell phone plan whether prepaid or postpaid, 3G or 4G phone that comes in black, white or gold.

C. Tabular
   3. PE shirts in white, green, yellow, blue and red that comes in small, medium or large

Process Questions:
1. Using the specified methods, how many possible outcomes are there in #1?
   In #2? In #3?
2. Based on the answers obtained, is there a common solution of how they are derived?
3. What is the process of obtaining the answer?
4. Does the process work in all situations? Justify the answer by showing the solutions.

ACTIVITY 4. Investigate

You have done a lot of counting using the different methods, now verify if such method will work out well with these situations.

1. If you throw 2 dice, how many outcomes are there?
   Answer: ________

2. How many choices of snacks are there if you get to pick a sandwich from 5 varieties (chicken, ham, egg, tuna, bacon) and 1 drink from 8 flavors of fruit juices (orange. Lemon, mango, apple, pineapple, strawberry, dalandan, melon).
   Answer: _________

PROCESS QUESTIONS:

1. How was the counting done?
2. Do the methods of counting used work out well?
3. Is there another way to arrive at the answer without going through a very tedious way of counting?
4. What is it?
5. How is this done? Illustrate the solutions.
6. What are these numbers being multiplied represent?

This process where the number of choices/outcomes for every action is multiplied is referred to as the **fundamental counting principle.**

Solutions:

1. Since a die has 6 faces and there are 2 dice, then there are 6x6=36 outcomes.
2. There are 5 sandwiches to choose from and 8 flavors of juices, so there are 5x8=40 choices of snacks.

ACTIVITY 5. #MULTIPLYTOCOUNT

Use the Fundamental Counting Principle to find the total number of outcomes for each given situation.

1. selecting an outfit from 8 choices of shirt and 4 choices of pants

2. answering a 5-item True or False test

3. picking up 3 kinds of fruits from a choice of 9 kinds

4. choosing a PIN of 3 digits

5. picking a name with 3 letters of the English Alphabet composed of 2 vowels and a consonant.

When you are done answering, look for a partner, exchange your work, discuss the answers and show the solutions on the board.

Compare your answers with the answers below and explanations are provided.

1. There are 8 choices of shirt and 4 choices of pants, so

   Solution:

   8 x 4 = 32 different outfits to select

2. Item 1 can be answered in 2 ways, item 2 can be answered in 2 ways also and so is #3, #4 and #5, thus there are

Solution:
2.2.2.2.2 = 32 ways to answer a 5-item T or F test

3. Because there are 9 kinds of fruits, there are 9 choices of the first fruit to pick up, leaving 8 choices for the second, and 7 choices for the third, which means that there are

Solution:
9.8.7 = 504 ways to pick up 3 kinds of fruits from a choice of 9 kinds

4. There are actually 10 numbers from 0-9 which means that for the first digit there are 10 choices, for the second digit there are again 10 choices and for the third digit there are still 10 choices, therefore there are

Solution:
10.10.10 = 1,000 PINs to choose from

5. The English alphabet has 26 letters; 5 vowels and 21 consonants, so if a 3-letter name is composed of 1 consonant and 2 vowels, then there are

Solution:
5.21.5 = 525 names to pick or 21.5.5, or 5.5.21

Why do you think no. 5 has 3 possible solutions? Explain each solution.

ACTIVITY 6. Drill Exercise

ACTIVITY 6: **DRILL EXERCISE**
Using FCP, find the number of possible ways/outcomes:
1. Six coins are tossed.
2. A coin is tossed 4 times.
3. A die is rolled thrice.

4. A 4-item multiple choice test which can be answered with a, b, c or d.
5. A meal is composed of rice or bread, 1 dish which is either a chicken, pork or beef, 1 fruit which can be a banana, apple or orange and a beverage which can be a cola, tea, coffee or juice

ACTIVITY 7. Arrange ME

As you have experienced, there are so many ways to count, and you can even count without actual counting but you only need to do multiplication. At times, when determining the number of outcomes, it will involve certain arrangement of objects or events which follow a definite order. This arrangement is called permutation.

Consider this:

CASE 1:

1. How many 4-digit numbers can be formed using the digits 1, 3, 5 & 7 where repetition is not allowed?

   The numbers are as follows;

1357	3157	5137	7135
1375	3175	5173	7153
1537	3517	5317	7315
1573	3571	5371	7351
1735	3713	5713	7513
1753	3731	5731	7531

Getting the answer 24 through listing method is very taxing.

Is there a way you can get the number of arrangements at the shortest possible time?

Yes. Using FCP, there are 4 choices for the first, 3 choices for the second, 2 choices for the third and 1 for the fourth digit; that is

$4 \times 3 \times 2 \times 1 = 24$ 4-digit numbers that can be formed
or $4!$ (This is the numerical way.)

Look at the list of the numbers above, the first digit can be either 1,3,5 &7 as seen in columns 1,2,3 & 4 respectively, but the second digit has only 3 choices left, leaving only 2 choices for the third and only 1 for the fourth digit. (if the number starts with a 1, the choices for the second are either 3, 5 or 7. If the second digit is a 3, then the choices for the third are 5 & 7 and if the third digit is 5, it will leave 7 as the only choice for the fourth digit.)

The exclamation point after a number is factorial symbol, which is read as "4 factorial" which is equal to the product of 4 and all other integers lesser than 4.

In general,
n! is the product of all integers less than or equal to n.
n! = n(n-1)(n-2)(n-3). . . 2.1

2. In how many ways can the letters a, b & c be arranged without repetition?
   Through listing we have the following arrangements:

abc	bac	cab
acb	bca	cba

   Using FCP, there are 3 choices for the first letter, because no repetition is allowed only 2 choices is left for the second letter and one choice for the last letter. Thus, there are

   3.2.1 = 6 ways to arrange the 3 letters
   Or 3!

Therefore in #1, permutations of 4 objects taken 4 at a time denoted by $_4P_4$ = 4! = 24

And in # 2, permutations of 3 objects taken 3 at a time that is $_3P_3$ = 3! = 6

Examples:

1. $_5P_5$ = 5!
   = 5.4.3.2.1
   = 120

2. $_{10}P_{10}$ =10!
   = 10.9.8.7.6.5.4.3.2.1
   =3,628,800

3. $_7P_7$ = 7!
   = 7.6.5.4.3.2.1
   = 5040

ACTIVITY 8. Group and Arrange

CONSIDER THIS:

CASE 2
Using the digits 1, 2, 3, 4 & 5, without repetition

1. How many single digit numbers are there?
   Answer: 1, 2, 3, 4, 5
   There are 5 single digit numbers that can be formed, since there are 5 choices.
   So, permutation of 5 objects taken 1 at a time = 5

2. How many 2-digit numbers can be formed?
   Answer:

12	21	31	41	51
13	23	32	42	52
14	24	34	43	53
15	25	35	45	54

   There are 20 2-digit numbers that can be formed.

   Using FCP, there are 5 choices for the first digit and 4 choices for the second digit thus, 5.4 = 20.

   So, the permutations of 5 objects taken 2 at a time = 20

3. How many 3-digit numbers can be formed?

123	142	213	241	312	341	412	431	512	531
124	143	214	243	314	342	413	432	513	532
125	145	215	245	315	345	415	435	514	534
132	152	231	251	321	351	421	451	521	541
134	153	234	253	324	352	423	452	523	542
135	154	235	254	325	354	425	453	524	543

   There are 60 3-digit numbers that can be formed.

   Using FCP, there are 5 choices for the 1st digit, 4 choices for the 2nd digit and 3 choices for the 3rd digit thus, 5.4.3 = 60.

   So, the permutations of 5 objects taken 3 at a time = 60.

4. How many 4-digit numbers can be formed?

Using FCP, there are 5.4.3.2 = 120 4-digit numbers that can be formed. So, the permutations of 5 objects taken 4 at a time = 120.

In summary,

1. ${}_5P_1 = 5$ = 5!/4!
2. ${}_5P_2 = 5.4$ = 5!/3!
3. ${}_5P_3 = 5.4.3$ = 5!/2!
4. ${}_5P_4 = 5.4.3.2$ = 5!/1!
5. ${}_5P_5 = 5.4.3.2.1$ = 5!

Observe the pattern above and give the expanded form & equivalent expression of the following permutations;

1. ${}_{10}P_3$ = ________________ = ________________
2. ${}_9P_4$ = ________________ = ________________
3. ${}_{12}P_5$ = ________________ = _________________
4. ${}_6P_6$ = ________________ = _________________
5. ${}_nP_n$ = ________________ = _________________
6. ${}_nP_r$ = ________________ = _________________

Process questions:
1. What have you noticed about the numbers in the expanded form?
2. What are the missing numbers in the expanded form?
3. What is another way of expressing these numbers?
4. Where do you see these numbers in the equivalent expression in the last column?
5. What relation exists between this number in the divisor and the given numbers beside P?
6. What will happen if these numbers beside P are equal?
7. What is the value of 0!?
8. Is it necessary to write 0!, even if you already know its value? Why or why not?

Permutation is an ordered arrangement of objects or events.

$$_{n}P_{n} = n!$$

$$_{n}P_{r} = \frac{n!}{(n-r)!}$$

ACTIVITY 9. Many Doesn't Make It More

CONSIDER THIS:

CASE 3

In the previous activity, you have seen that the 3 letters a, b, & c can be arranged in 6 different ways. Each letter is distinguishable from the others.
Will counting be affected if the letters are non-distinguishable?

Examples:

1. How many ways can we arrange the letters of the word ANA?
   The arrangement of any 3 different letters can be expressed as 3! = 6, but in this case 2 letters of the name ANA are non- distinguishable, so for illustration purposes mark the 1st A as A_1 and the 2nd A as A_2

$A_1N A_2$	NA_1A_2	$A_1A_2 N$
$A_2N A_1$	NA_2A_1	$A_2A_1 N$

   There are 6 different arrangements, but 2 arrangements are exactly the same. Whether A_1 and A_2 are being interchanged the arrangement is still the same therefore it is counted as one. Thus, the letters of the word ANA can be arranged in 3 different ways $= \frac{3!}{2!}$. Why is it divided by 2!? It is because any 2 arrangements are counted as one.

2. How many permutations can be done of the letters of the word MAMA?
   $_{4}P_{4} = 4! = 24.$

$M_1\ A_1\ M_2\ A_2$	$A_1\ M_1\ A_2\ M_2$	$M_1\ M_2\ A_1\ A_2$
$M_1\ A_2\ M_2\ A_1$	$A_2\ M_1\ A_1\ M_2$	$M_2\ M_1\ A_1\ A_2$
$M_2\ A_1\ M_1\ A_2$	$A_1\ M_2\ A_2\ M_1$	$M_1\ M_2\ A_2\ A_1$
$M_2\ A_2\ M_1\ A_1$	$A_2\ M_2\ A_1\ M_1$	$M_2\ M_1\ A_2\ A_1$
$A_1\ A_2\ M_1\ M_2$	$M_1\ A_1\ A_2\ M_2$	$A_1\ M_1\ M_2\ A_2$

$A_1\,A_2\,M_2\,M_1$	$M_2\,A_1\,A_2\,M_1$	$A_2\,M_1\,M_2\,A_1$
$A_2\,A_1\,M_1\,M_2$	$M_1\,A_2\,A_1\,M_2$	$A_1\,M_2\,M_1\,A_2$
$A_2\,A_1\,M_2\,M_1$	$M_2\,A_2\,A_1\,M1$	$A_2\,M_2\,M_1\,A_1$

Take note that for every four arrangements it is counted as one, because even if the 2A's or the 2M's are interchanged the arrangements are still the same. Therefore, the letters of the word MAMA can be arranged in 6 different ways, that is $\frac{24}{4}=6$ or $\frac{4!}{2!2!}$.

2!, since there are 2 A's and the other **2!** is attributed to the 2 M's.

3. How many 4-digit numbers can be formed using 2, 2, 2 & 5?
   If you apply the concept of permutations of n objects taken n at a time and there are 4 objects, then it would be ${}_4P_4$= 4! = 24. Remember the 3 objects are non-distinguishable. Nevertheless, the following numbers are listed;

   2225, 5222, 2522, 2252

   Only 4 numbers can be formed whenever you arranged the numbers 2, 2, 2 & 5, which means that out of 24 there are 6 arrangements that are counted as one. See the illustration below.

   $2_1\,2_2\,2_3\,5$
   $2_1\,2_3\,2_2\,5$
   $2_2\,2_1\,2_3\,5$
   $2_2\,2_3\,2_1\,5$
   $2_3\,2_1\,2_2\,5$
   $2_3\,2_2\,2_1\,5$

   Therefore ,4 is obtained by dividing 24 by 6 which is equal to

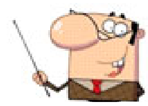
   $$\frac{4.3.2.1}{3.2.1}=\frac{4!}{3!}$$

PROCESS QUESTIONS:

1. As illustrated, how is the number of arrangements of objects which are non-distinguishable determined?
2. Where do the divisors based from?

Hence, to find the permutation of n objects where some objects are non-distinguishable, one object appears ***a*** times, the second object appears
b times and a third object appears ***c*** times and so on, then

$$P = \frac{n!}{a!b!c!}$$



ACTIVITY 10. Practice Exercise

Perform this exercise with a "learning buddy"

A. Evaluate the following:
   1. ${}_{13}P_6 =$
   2. ${}_{8}P_8 =$
   3. ${}_{15}P_{13} =$
   4. $\frac{10!}{2!3!2!} =$
   5. $\frac{6!}{3!} =$

B. Find the permutations of the letters of the ff.
   1. TOMATO =
   2. GEOMETRY =
   3. ROSE =
   4. BANANA =
   5. HAPPY =

ACTIVITY 11. Interactive Activity

Visit this site and answer the given exercise to check how much you have grasp about the lesson.
http://www.regentsprep.org/regents/math/algtrig/ats5/pcprac.htm
This site contains interactive exercise on permutations.

ACTIVITY 12. Round & Round in Circles

Arrangements of objects in a line do not have uniform solutions. They vary according to the conditions set for a particular purpose. Now, arrangements in circular form, as the seating arrangements in round tables must also be investigated and verify whether the formulae for linear permutations work in the same way in circular permutations.

CASE 4
1. How many ways can 3 persons be arranged in a round table?
2. How many ways can 4 persons be arranged in a round table?

If 3 people will be arranged in a line, there will be 3! Or 6 permutations.

If 4 people will be arranged in a line, there will be 4! Or 24 arrangements. Will these be true in a circular manner?

INVESTIGATE:

INSTRUCTIONS:
1. Group yourselves by 4.
2. Three members of the group will simulate to be sitting in a round table (if there is no round table) while the other one does the counting and recording.
3. Rearrange yourselves and record the number of arrangements of 3 people in a round table.
4. Repeat the process where all the 4 are joined in the round table.
3. Answer the questions asked.
4. Partner with another group and compare the results.
5. Submit your paper when you're done.

Q & A
1. Was there a change in the arrangement if all the people moved clockwise?
2. How about counterclockwise?
3. What did you do to have another arrangement?
4. Where you able to monitor the counting easily?
5. Why or why not?
6. What should be the proper way to do it?
7. Do the formulae in linear permutation work? Explain.
8. How many circular arrangements are possible for 3 people? 4 people? Show your solutions.
9. Having seen the solutions of the first 2, what do you think is the number of circular permutation of 5 people? 6 people?

Circular Permutation of 3 people = 2 =	2.1	= 2!
Circular Permutation of 4 people = 6 =	3.2.1	= 3!
Circular Permutation of 5 people = 24 =	4.3.2.1	= 4!
Circular Permutation of 6 people = 120 =	5.4.3.2.1	= 5!
Circular Permutation of 7 people =	6.5.4.3.2.1	= 6!
Circular Permutation of 8 people =		= 7!
Circular Permutation of n people =		(n-1)!

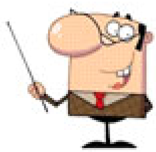
10. What formula works for circular permutation?

Check your answers by comparing with the notes that follow.

The arrangements do not change if all the people are moving clockwise or counterclockwise. For easy monitoring and counting, ***one must stay put*** while the others are moving and having another arrangement.

- Three (3) people can have 2 different circular arrangements which is equal to 2!, while
- Four (4) people can have 6 circular arrangements which is equal to 3!

Hence, **Circular Permutation = (n-1)!**



Take a look at the table that shows the difference between the linear and circular permutations:

Number of objects	Linear permutation	Circular permutation
2	2.1 = 2	1 = 1
3	3.2.1 = 6	2.1 = 2
4	4.3.2.1 = 24	3.2.1 = 6
5	5.4.3.2.1 = 120	4.3.2.1 = 24

ACTIVITY 13. Check or Understanding

Find the number of circular permutations of
1. 5 people
2. 7 people
3. 10 people

Discuss your answers in a triad. Once done, open the given website and answer the test on permutations.
http://www.mathsisfun.com/combinatorics/combinationspermutations.htm.
This contains a test on permutations.

http://www.regentsprep.org/regents/math/algtrig/ats5/pcprac.htm
This contains exercises/test on permutations and combinations.

End of FIRM UP:

In this section, the discussion was about (FCP) fundamental counting principle, the different ways of counting, permutations and how they are solved.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.

DEEPEN

Your goal in this section is to take a closer look at some aspects of the fundamental counting techniques specifically permutations and how this is used in real-life settings.

You have already covered all the lessons about counting techniques and permutations both linear and circular, so this time you will dig deeper on these topics and see how these are applied in real-life situations.

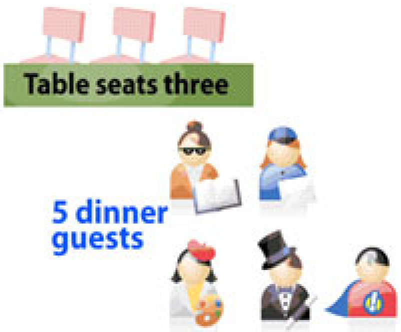
ACTIVITY 14. How Useful Is This?

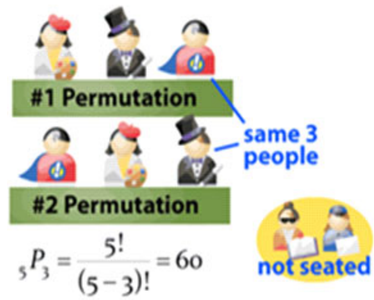
Go to the specified website, watch and listen to the video to confirm what you have just learn. Pay attention to the different situations where the concepts of permutations are used or applied.
http://www.onlinemathlearning.com/permutation-probability.html
This site contains a video on the discussion of permutations and problem solving.

Q & A:
1. Are the things you saw and heard similar to what had been discussed in class? Discuss.
2. What are the new things you learned?
3. Is counting an easy task? Justify your stand.
4. ***How can you count without counting?***

ACTIVITY 15. Counting Made Real

1. If you are the receptionist of a certain restaurant, in how many ways can 5 dinner guests be seated in 3 vacant seats in a row?





2. How many possible locks can a traveller create for his luggage if the padlock is secured even with 2 digits?

Creating a lock with only 2 digits would involve 10 choices for the first number and another 10 choices for the second number resulting to one hundred different locks.

Solution: 10x10=100

Process questions:

1. In the given situations, is it possible to act on something or make decisions without counting? Explain.
2. In no. 2, how secure is the luggage with 100 locks only?
3. How do you increase the number of security locks?
4. How many locks are there if it consist of 3 digits? 4 digits?
5. How would you strengthen the security of the luggage?
6. How important is counting in life?

3. How many possible ways can you arrange these 6 padlocks in the key cabinet?

The permutation of 6 objects $_6P_6 = 6! = 720$ ways to arrange these 6 padlocks.

4. The 8 members of the Board of Directors of the cooperative is having a round table meeting as shown below. In how many ways can the Secretary arranged them.

This case is a circular permutation;

$$P = (n-1)!$$

$$P = (8-1)!$$

$= 5040$ ways to arrange the BOD.

5. The pouch below contains 6 red marbles, 4 blue marbles and 3 green marbles. In how many ways can they be arranged in a row?

Marbles of the same color are non-distinguishable. So exchanging their places does not change the arrangement, and such would not be counted as another arrangement. Therefore, to count the number of ways to arrange them is, $P = \frac{13!}{6!4!3!} = 60{,}060$

ACTIVITY 16. Do The Counting! (Practice Exercise)

Discuss with 3 other classmates the following situations and count the number of possible outcomes.

1. In how many ways can a President, Vice –President, Secretary and Treasurer be chosen from among the 12 parents present?

2. How many 3 digit numbers can be formed from the digits 2, 3, 5, 6, 7 and 9 which are divisible by 5 and none of the digits is repeated?

3. There are three places P, Q and R such that 3 roads connects P and Q and 4 roads connects Q and R. In how many ways can one travel from P to R?

4. How many 8 digit mobile numbers can be formed if any digit can be repeated and 0 can also start the mobile number?

   After the activity, rate youself according to your readiness towards the application of the lesson or topic. (formative assessment)

COLOR YOUR READINESS!

ARE YOU READY TO MOVE ON? **Color the section that applies**

Yes   No

ACTIVITY 17. Challenge to Innovate

1. Due to so many cases of scam, as the CEO of one of the strongest bank you have decided to strengthen the security of the accounts of the clients of about 80,000.

   a. What measures are you going to implement with regards to their PIN?
   b. At least how many digits will you require for the PIN to avoid duplication?
   c. With the number of digits required, how many possible PINs can be created?
   d. What is the reason behind such requirement?
   e. Is counting important in this situation? Explain why.
   f. What is affected by the outcomes as a result of counting?
   g. How can outcomes be predicted?

2. Sheila has a dilemma with regards to what type of service will she offer in her food- stop whether she will go for value meals or a la carte. She advertizes that she has 4 dishes (pork, chicken, fish, beef) to choose from, 2 kinds of rice (steam, fried) and a drink which is either a cola or a juice. A value meal has 1 dish and 1 drink only. An a la carte meal has more than 1 dish.

   Procees Questions:
   1. How many choices of value meals are there?
   2. How many choices of meals are there other than the value meals, if there should be at least 2 dishes? If there should be at most 3 dishes?
   3. Which service type offers a lot of choices?
   4. Which is a better option, value meal or a la carte? Justify your answer.
   5. What decision will Sheila make based on her findings?
   6. Is counting important in this case? Why?
   7. *How can you count without counting?*
   8. *How can outcomes be predicted?*

ACTIVITY 18. Problem Solving

Guided Generalization:

SITUATION 1	SITUATION 2	SITUATION 3
A class of 40 students has to elect 5 responsible team leaders to take charge of the discipline of the students in the absence of the teachers. In how many ways can these 5 leaders be elected?	During the speech festival there are 10 contestants for oration. The organizers were discussing on the mechanics on how to choose the presentors. How many possible ways can the first, second and third presentors be picked up from among them?	Eight customers enter the shop & carry outlet ,after gathering their goods they immediately lined-up to pay. In how many ways can they form their line at the counter?
ANSWER:	ANSWER:	ANSWER:
SOLUTION:	SOLUTION:	SOLUTION:
CONCEPT APPLIED:	CONCEPT APPLIED:	CONCEPT APPLIED:

ESSENTIAL QUESTION: *How can you count without counting?* *How can outcomes be predicted?*	Generalization:	

ACTIVITY 19. Journal Writing

ACTIVITY 19: **Journal Writing**

In your journal, write the answers of the following questions

1. What have you learned about counting principle and permutation?
2. Why is it necessary to know how to count?
3. What are the consequences if decisions are made without knowing the outcomes or results?

End of DEEPEN:

In this section, the discussion was about the practical applications of counting and permutations.

What new realizations do you have about the topic? What connections have you made for yourself with regards to permutation?

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section…

TRANSFER

Your goal in this section is to apply your learning to real life situations. You will be given a practical task which will demonstrate your understanding.

Before accomplishing the given task, revisit your AR guide so that you can either confirm or change your initial ideas.

ACTIVITY 20. Revisiting the AR Guide

ANTICIPATION-REACTION GUIDE

Read again all the statements in the chart and put a ☆ if you agree and X mark if you disagree with the statement in the after column.

BEFORE	STATEMENTS	AFTER
	6. Fundamental counting principle is used in finding the number of possible outcomes.	
	7. Listing, the use of tables and tree diagram are methods of counting.	
	8. Permutation is used to determine the number of committees of 4 members that can be formed out of 10 members.	
	9. The notation 5! means 5x1.	
	10. P (10,4) =10.9.8.7	

ACTIVITY 21. Performance Task (FOR Lesson 1)

SCAFFOLD LEVEL 1

You are a member of the Ways and Means committee and are task to prepare a proposal for the raffle draw. You need to determine the following:

1. How much money should be raised from the raffle draw?
2. How many tickets will be sold?
3. Indicate the control numbers from beginning to end for easy accounting.
4. At what price will the tickets be sold?
   BIR rulings say "no tax will be paid for tickets sold below Php 1.00 and the organization is not willing to pay tax.
5. Based on these information, is it possible to attain the projected amount?
6. What value do you need to adjust to reach the projected amount?
7. If such number will be adjusted, is it feasible to do it?
8. What is/are your recommendation/s?

9. Present your recommendation/s in a form of a project proposal containing the ff.:
   a. purpose/objectives/beneficiaries
   b. projected amount
   c. number of tickets to prepare with the corresponding control numbers
   d. cost of the tickets
   e. projected expenses
   f. net proceeds

   You accomplish this activity with a partner.

ACTIVITY 22. Sythesis Journal

In your journal, write the answers to these questions:

1. What are your insights regarding the topics covered?
2. Did your knowledge help you accomplish the task?
3. How did you use it? What specific skills did you apply?
4. How did you find the task? Explain why?
5. How do you rate your performance?
6. How does the knowledge of permutation help you in your decision-making?

ACTIVITY 23. Self-Assessment/ Reflections

1. What are the practical things I learned about the topic?
2. How did I learn them?
3. Has my knowledge improved my decision-making skills?
4. In what manner has it improved?

Each student will submit a rating of himself and his partner.

End of TRANSFER:

In this section, your task was to summarize your learning, findings, insights and realizations.

How did you find the performance task? How did the task help you see the real world use of the topic?

GLOSSARY OF TERMS USED IN THIS LESSON:

N! or n factorial is the product of n and all the positive numbers lesser than n.

FUNDAMENTAL COUNTING PRINCIPLE
If an event can happen in p ways, the next event can happen in r ways and a third event can happen in s ways and so on, then the sequence of events can happen in p X r X s X . . .ways.

Permutation is an ordered arrangement of objects or events.

REFERENCES AND WEBSITE LINKS USED IN THIS LESSON:

A. Printed

Blitzer, Robert. Algebra and Trigonometry. Prentice-Hall, Inc. 2007.

Ogena, Ester, et al. Our Math. McGraw- Hill Co. Inc. & Vibal Publishing House, Inc. 2013.

Oronce, Orlando and Marilyn Mendoza. E-Math. Rex Book Store, Inc. Manila. 2015.

Stewart, James, et al. Algebra and Trigonometry. Brooks/Cole. 2007.

B. Non Printed

http://www.onlinemathlearning.com/permutation-probability.html
This site contains a video on the discussion of permutations and problem solving.

http://www.careerbless.com/aptitude/qa/permutations_combinations.php
This contains permutations and combination problem exercises.

http://www.regentsprep.org/regents/math/algtrig/ats5/pcprac.htm
This site contains interactive exercise on permutations.

http://www.mathsisfun.com/combinatorics/combinationspermutations.htm.
This contains a test on permutations.

Lesson 2: Combination

☑ Lesson Overview:

This lesson will help you develop the concepts of combination and to compute for the possible outcomes based on the concepts of combination. In this lesson, you will know the relationship between permutation and combination. You will also use interactive videos and references for better understanding and fun learning.

EXPLORE

In this lesson you will learn to know the difference between permutation and combination, how to derive the formula for combination, to use combination in various real life situations whenever applicable and to solve problems that uses combinations.

Let us start with this activity.

ACTIVITY 1 A Big Difference!

In this activity, you will know how to differentiate permutation from combination. Read and understand the following situations.

Situation A:

You are working in a factory that manufactures number "combination locks". The lock should have a four-digit number code for it to be opened. How many possible numeric codes can you make if repetition of a digit is allowed? (*The digits are from 0 to 9.*)

Process Questions:

1. How many possibilities are there for the first slot?

   ____

2. How about the second, third and fourth slots?

   ____

3. How many codes can you make?

   ____

4. **How did you predict the number of outcomes accurately?**

   ____
   ____
   ____

5. **How many numeric codes can you make if repetition is allowed?**

   ____
   ____
   ____

Model: ☐ × ☐ × ☐ × ☐ = ☐ codes

1ST slot   2ND slot   3RD slot   4TH slot

Situation B:

You are working in a restaurant and there are three seats available. But there are five guests waiting to be seated. In how many ways can the five guests be seated three at a time?
Make a table to show the groupings you made.

Sample Visualization:

= ONE POSSIBILITY

Complete the table below. The grouping that you will make should not have the same persons.

Group	Possible Arrangements			Number of Outcomes
1	Guest 1	Guest 2	Guest 3	1
2	Guest 1	Guest 2	Guest 4	1
3	Guest 2	Guest 1	Guest 3	Not counted; the same persons in group 1
4	Guest 1	Guest 2	Guest 5	1
5	Guest 2	Guest 5	Guest 1	Not counted; the same persons in group 4
6	Guest 1	Guest 3	Guest 4	1
7	Guest 1	Guest 3	Guest 5	1
8	Guest 1	Guest 4	Guest 5	1
9				
10				
11				
12				

Process Questions:

1. How many possible groupings are there?
   ____
   _____

2. From the table above, why do you think group 1 and group 3 are the same?
   ____
   _____

3. Do you think that the order among the members of group 1 and group 3 matters? Why or why not?
   ____
   ____
   ____

4. **How did you count for the number of possibilities** of five guests to be seated three at a time?

____
____
____

The first activity possibly gave you the idea of what combination is. But before formalizing you definitions, answer the activity below to check your prior knowledge on what combination is. Try your hunches!

ACTIVITY 2 Anticipation-Reaction Guide

Write your response for each statement in the **Before Lesson** Column only.
Write **A** if you agree with the statement.
Write **B** if you disagree with the statement.

Before Lesson	Statements	After Lesson
	1. Combination does not consider the arrangement of members in a group.	
	2. The order of members in a group is not important in combination.	
	3. ABC, ACB, BAC, BCA, CAB and CBA are counted as one in combination.	
	4. 567 and 765 are the same in the concept of combination.	
	5. The number of combinations is equal to the number of permutations of ***n*** objects taken ***r*** at a time divided by ***r!***.	
	6. Combination only applies when you choose distinct objects from the group of objects being selected from.	
	7. Combination is useful when you choose the possible number of subcommittees considering a particular number of members per subcommittee from the total number of persons being selected.	
	8. Combination is the number of different ways that a certain number of objects as	

	a group can be selected from a larger number of objects	
	9. Combination is an ordered list of possible outcomes.	
	10. Combination is the number of different ways that a certain number of objects can be arranged in order from a larger number of objects.	

END OF EXPLORE

After giving your initial answers to the activity above, compare your work with others to see if you have similar or different ideas. As you compare your work, you will know if your ideas are attuned with the standards. You will also learn related concepts that will help you do the project by the end of this module. The project is to organize a fund-raising project.

FIRM-UP

Your goal in this section is to learn and understand better the difference between permutation and combination and how the formula for combination is derived. You will know how to visualize the basic concept of combination. You will also apply these concepts in solving real life situations.

For you to know more on the difference between permutation and combination, perform the next activity and answer the questions that follow.

ACTIVITY 3 Permutation VS Combination

Watch these helpful videos to familiarize more what combination is and how it is visualized. But before clicking the links below, note the questions first so that you can be guided.

1. https://www.youtube.com/watch?v=qaFNhwNBY3k
   This website is a 20-minute lecture video which discusses lessons on Statistics 101. It points the difference between Permutation and Combination. Illustrative examples are given and sample applications. This website will help you distinguish the differences between the two counting techniques.

2. https://www.youtube.com/watch?v=PSS3mCS_Ef8
   This is a 12-minute video lesson that focuses on Combination and its formula. In this website you will learn how to use the formula for Combination through various examples. Sample calculations using the formula are shown so that you will learn how to substitute the unknown properly. It also shows relationships between permutation and combination. In addition, you will know when combination or permutation can be used.

Process Questions:

1. From the video clips, what is the meaning of combination?
   ____
   ____
   ____

2. How is it different from permutation?
   ____
   ____
   ____

3. What is the formula for combination?

____

ACTIVITY 4 TEXT ANALYSIS: Finding the Meaning of Combination and Certain Situations Where It Is Applicable

Check the following websites. After studying the contents of these websites, you are to complete the **Frayer Model** below. Put the word "COMBINATION" inside the oval as the central idea.

1. http://www.mathsisfun.com/combinatorics/combinations-permutations.html
   This website discusses and shows examples of permutation and combination with or without repetition. There are illustrative examples and computational exercises. This will help you form the correct concepts and will help you make your own examples. At the bottom of the page is an activity which you may answer or just leave. You can visit this site again after the end of FIRM UP section.

2. https://www.youtube.com/watch?v=qaFNhwNBY3k
3. https://www.youtube.com/watch?v=1stIqr0FGiE
   These are the same websites in Activity 3 which can help you answer the graphic organizer below. You can revisit the sites and pick up important points to supplement what you learned in the first web site of this activity.

The Frayer Model

Definition	Facts/Characteristics
Examples	Non-examples

Process Questions:

1. How did you determine the examples and non-examples of a combination?

____
____
____
____
____

2. How does combination differ from permutation that you have learned in the previous lesson?

   ____
   ____
   ____
   ____
   ____

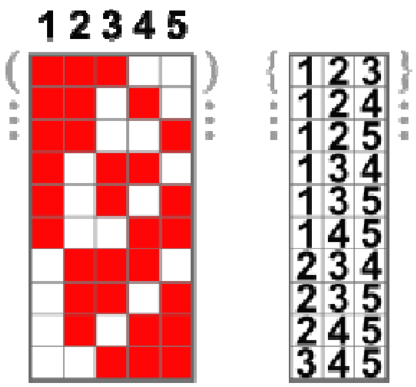
3. What are other situations you can think where combination can be applied? Discuss three or more.

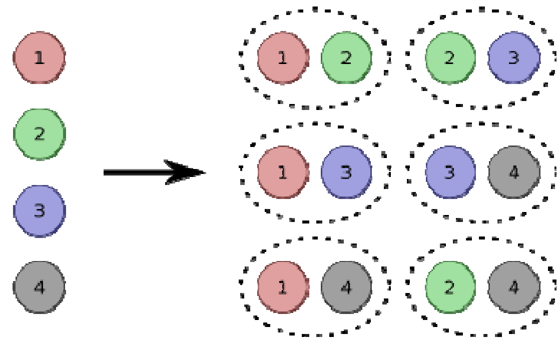
   ____
   ____
   ____
   ____
   ___________________.

ACTIVITY 5 Visualizing Combination

Aside from the visualization provided in the video in Activity 3, this activity will help you visualize what combination is. Take a look…

Model A Combination of 5 objects taken 3 at a time	Model B Combination of 4 objects taken 2 at a time





Using any real objects that will represent the numbers, you can further understand the meaning of combination.

Let us look at Model A.

You are in a restaurant and you have 5 food choices but only 3 different menus can be served in a single setting. No repetition of menu is allowed. The following are the menu:

Menu 1: Adobo
Menu 2: Fried Chicken
Menu 3: Sinigang na Sugpo
Menu 4: Crispy Pata
Menu 5: Bulalo

Process Questions

1. What are the possible sets you can order?

Set 1	Adobo, Fried Chicken and Sinigang na Sugpo

2. How many sets can you make? Illustrate your answer.

3. How does the visualization of combination help you look at the possible outcomes?

   ____
   ____
   ____

4. **How did you predict the number of outcomes accurately?**

   ____
   ____
   ____

5. **Using the formula for combination in activity 3, how many possible sets are there?**

   ____
   ____
   ____

6. What do you think is the limitation to this technique? What if you will have two orders of menu 1 and one order of menu 2, would that count as one of the possible outcomes? Explain.

____
____
____
____

Now, let us look at Model B.

You have four colored shirts. You can only bring two shirts every day. In how many days can you provide a two-colored shirt combination?

Shirt 1: purple Shirt 2: light green Shirt 3: lavender Shirt 4: indigo

Process Questions

1. What are the possible combinations can you make?

2. How does combination help you look at the possible outcomes?

____
____
____
____

3. How did you predict the number of outcomes accurately?

____
____
____
____

4. By using the formula in Activity No. 3, do you think there is a limitation to this technique in this situation? Explain.

____
____
____
____

The previous activity helps you visualize how the formula of combination can be used. However, it is important to relate permutation to combination. Study the activity below so that you can understand better how the formula for combination came to be.

ACTIVITY 6 Deriving the Combination Formula from Permutation

In this activity you will understand how the formula for combination came to be.

Example:

We will determine the number of combinations of the four numbers, ***1,2,3,4*** taken 3 at a time. Note that each combination or group consisting of three numbers determine 3! = 6 permutations of the numbers in the combination:

Combinations	Permutations
123	123, 132, 213, 231, 312, 321
124	124, 142, 214, 241, 412, 421
134	134, 143, 314, 341, 413, 431
234	234, 243, 324, 342, 423, 432

Thus the number of combinations multiplied by 3! equals the number of permutations:

Steps	Explanation
C(4, 3) or ${}_4C_3$	Number of ways to select ***groups*** or ***sets*** of size 3 from 4 different objects
3!	The number of permutations for every combination or set (considering 3 objects)
C(4, 3) times 3! = ${}_4P_3$	The number of permutations equals the number of combinations times the factorial of the number of objects taken at a time
${}_4P_3 = \dfrac{4!}{(4-3)!}$	Substituting in the permutation formula: $P(n,r) = n\Pr = \dfrac{n!}{(n-r)!}$
${}_4C_3 \bullet 3! = \dfrac{4!}{(4-3)!}$	Combining steps 3 and 4

${}_4C_3 = \dfrac{4!}{3!(4-3)!} = 4$	Dividing both sides with 3! to eliminate the number of permutations for each unique group since each group of 3 has a permutation of 3!

So, ${}_4P_3 = 4 \bullet 3 \bullet 2 = 24$ and $3! = 6$; therefore ${}_4C_3 = \frac{{}_4P_3}{3!} = \frac{4 \bullet 3 \bullet 2}{6} = 4$ as seen from the example above.

<u>In a more general discussion:</u>

Since each combination of ***n*** objects taken ***r*** at a time determines **r!** permutations of the objects, we conclude that P(n, r) = r!C(n, r)

Thus we obtain, $nCr = \dfrac{P(n,r)}{r!}$

The number of combinations is equal to the number of permutations divided by r! to eliminate those counted more than once because the order is not important.

Replacing,

$$P(n,r) = \frac{n!}{(n-r)!}$$

So,

$$nCr = \frac{P(n,r)}{r!} = \frac{n!}{r!(n-r)!}$$

$nCr = \dbinom{n}{r} = \dfrac{n!}{r!(n-r)!}$ (Note: No two objects are alike. Combination without repetition)

Check these websites out to understand better the proof.

http://statistics.about.com/od/Formulas/a/How-To-Derive-The-Formula-For-Combinations.htm

http://www.algebra.com/algebra/homework/Permutations/Proof-of-the-formula-on-the-number-of-Combinations.lesson
(detailed proof)

These websites require you to read the materials. These focus on the idea of multiplication principle and how the formula for combination is derived from permutation. There are web links that you can visit in order to refresh some needed vocabulary and prerequisite concepts. There are words related to counting techniques that are highlighted which you can click. Upon clicking, this will direct to you to some definitions and examples. These show proofs of deriving the formula for combination by definition and simple algebra.

Try this!

Make a combination and permutation of the following objects (▲, ♣,♥,♦), taken 3 at a time. Fill out the table below. The first one is done for you.

Combinations (*Group*)	**Permutations** (*Different orders of members in a group*)
▲,♣,♥	▲,♣,♥, ▲,♥, ♣, ♣,♥,▲, ♣,▲,♥, ♥,▲,♣, ♥,♣,▲

Process Questions:

1. What is the total permutation from all the sets/groups?
   ____
2. How many permutations are there for every set/group?

   ____
3. How much is the quotient of the total permutation and the number of permutations per set?

   ____

4. **How can you solve the situation without listing?**
   ____
   ______

5. Do you think the formula for combination is helpful? Why or why not?
   ____
   ______

In this activity you are going to apply the concept of combination to various real life situations. In counting possibilities, check if combination can help. Try this out!

ACTIVITY 7 Revisiting Activity 5 Using the Formula

$$nCr = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Situation A:

You are in a restaurant and you have 5 food choices but only 3 menus can be served in a single setting. The following are the menus:

Menu 1: Adobo
Menu 2: Fried Chicken
Menu 3: Sinigang na Sugpo
Menu 4: Crispy Pata
Menu 5: Bulalo

Answer the following questions.

1. Using the formula $nCr = \binom{n}{r} = \frac{n!}{r!(n-r)!}$
   What is ***n*** equal to? ________________
   What is ***r*** equal to? ________________
   How many possible choices are there? ___________

2. If four menus can be served at time, how many possible choices are there?

3. What if all menus can be served at a time, how many choices are there?

Situation B.

You have four colored shirts. You can only bring two shirts every day. In how many days can you provide a two-colored shirt combination?

Shirt 1: purple
Shirt 2: light green
Shirt 3: lavender
Shirt 4: indigo

Answer the following question:

Using the formula, $nCr = \binom{n}{r} = \frac{n!}{r!(n-r)!}$, what is your answer to the given situation?

Try This!

You are organizing a fund raising project for a certain orphanage. You are to assign students in various committees. In how many ways can you select a committee of 10 students out of 20 students?

Answer the following question:

Using the formula, $nCr = \binom{n}{r} = \frac{n!}{r!(n-r)!}$, what is your answer to the given situation?

So, did combination help in the previous activity? At this point you will compare and contrast permutation and combination. It is good to know how the two differs from each other. Do this activity.

ACTIVITY 8 Identifying Similarities and Differences

Activity 8: Identifying Similarities and Differences

Compare Permutation and Combination by filling in the blanks with the correct concepts and formula.

PERMUTATION and **COMBINATION** are similar because they both ______________________________ and ______________________________.

PERMUTATION and **COMBINATION** are different because
Permutation __________________, but Combination ______________________.

Permutation __________________, but Combination ______________________.

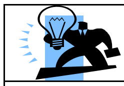
Permutation __________________, but Combination _______________________.

The formula for Permutation is _____________________________ while the formula for Combination is ____.

At this point, pause first and take time to reflect about your learning so far.

Answer this activity so that you will know what to learn more and what to review. Submit to your teacher after answering.
.

ACTIVITY 9 **3-2-1**

	Instruction	Answer
3	Write three key ideas you think are important.	
2	Write two things you need to study or learn more.	
1	Write a question you have in mind.	

END OF FIRM-UP:

In this section, the discussion was about the differences between permutation and combination. It also points out how the formula for combination is derived. More than the formula was the visualization activity that gives you a clearer picture of combination. Certain examples from real life were also used to see that combination is applicable to our life. You were also asked to solve certain real life problems which use the concepts of combination. Look at your initial answers in the previous section in Activity 2. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.

DEEPEN

Your goal in this section is to relate combinations to real life problems or situations. You will intensify your understanding of the topic by answering varied exercises and applications to real world setting. You will also construct your own real world situation that will use combination to solve for certain outcomes. But before continuing, think of an answer to this question: **How can you predict outcomes accurately?**

ACTIVITY 10 Let's Compare

Put a check mark if the description or example falls for permutation or combination.

PERMUTATION	DESCRIPTION/EXAMPLE	COMBINATION
	The individual order of each member in the group counts.	
	The individual order of each member in the group does not count.	
	123 is different from 231.	
	123 is the same as 231.	
	123 is taken three at a time without considering the order.	
	123 is taken three at a time considering the order.	

From the above descriptions or examples, how is combination different from permutation?

____
____
____

ACTIVITY 11 Practice Makes Perfect

Solve the following problems related to combination. Show your solutions.

Questions	Answers
4. How many 4–element subsets can be formed from the set {1, 2, 3, 4, 5, 6, 7}?	
5. How many different committees of 4 can be chosen from 12 people?	
6. How many different committees of 4 can be chosen from 12 people?	
7. How many different 3–card hands can be chosen from a 52–card deck?	
8. Nine students are eligible to play doubles in lawn tennis. How many different 2–person teams can be chosen?	
9. How many lines are determined by 8 points, no 3 of which are collinear?	
10. A box contains 8 flashlights, all of different colors. How many different sets of 3 flashlights can be chosen?	
11. In lottery, 4 winners will get equal prizes. If 20 people join the lottery, how many different groups of 4 winners can be chosen?	

Fill in the blanks below.

The questions above were solved using ________________. The outcomes were solved and computed by

____

____

____

__________________.

List down the steps you did to solve and compute for the answers to the questions:

____
____
____
____
____
____________________________.

How certain are you with your answers? (Compare your answers with the answer key.)

____
____
____
_________________.

Did you get all answers correct?

____

If no, why?

____
____
__________.

The next activity will help you reinforce what you learned in solving and computing for the outcomes of particular events. Do this activity to have more practice. Each exercise has an answer key. So you can assess your own work.

ACTIVITY 12 Permutation or Combination (Situations Where Permutation or Combination is Applied)

Visit this website http://www.intmath.com/counting-probability/4-combinations.php
and answer the exercises on combination. In this website you can see the solution on how the exercises are solved. Learn from this and note your mistakes if there are. There are five examples that you need to work on. Answer first before clicking the answer button. Write your answers on the blanks provided below.

Write your answers here:

1) ______________________________

2) ______________________________

3) ______________________________

4) ______________________________

5) ______________________________

ACTIVITY 13 Permutation or Combination

Put a check mark if the situation calls for permutation or combination.

PERMUTATION	DESCRIPTION/EXAMPLE	COMBINATION
	Making alphanumeric codes	
	A 5-man committee selected from a certain number of persons	
	Zip codes	
	Basketball team members selection	
	Telephone numbers	
	Three-color paints from five choices	
	Pizza toppings of your choice	
	Fruit mix or fruit salad	
	Plate numbers	
	Three-member group chosen from 15 persons	
	Selecting nominees for a Council	
	Arranging elected Council members in the Council meeting	

Process Questions

1. Why do you think that in making plate numbers count for permutation and not combination?
____
____
______

2. Why do you think in the selection of the members of a committee count for combination and not permutation?
____
____
______

ACTIVITY 14 Here's What, So What, Now What

Here's What	Describe one very important concept/skill that you learned during this lesson.
Answer	
So What	How can you practice or use this concept/skill so you will know that you understand/remember it?
Answer	
Now What	How can you use this concept/skill to help you become a better problem solver?
Answer	

Answer the activity below to check your understanding of the lesson. At this point, you should have a close to perfect score.

ACTIVITY 15 Reinforcing Understanding

1. In how many ways can you select a committee of 3 students out of 10 students?

2. A committee including 3 boys and 4 girls is to be formed from a group of 10 boys and 12 girls. How many different committees can be formed from the group?

3. A committee of 5 persons is to be formed from 6 men and 4 women. In how many ways can this be done when:
   a) At least 2 women are included?
   b) At most 2 women are included?

In this activity you are tasked to note down your ways of solving and computing for the answers in every situation. However, to solicit a generalization, you are only to choose three situations that will be highlighted in *Activity 17*.

ACTIVITY 16 Combination in Situations!

Situation 1

You are to find the number of subsets of the set {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} having 4 elements per group. What will you do?

Answer:

Situation 2

In architecture and designing, you are to determine how many cyclic quadrilaterals can be drawn by using 8 points on the circle. You are to draw the figure.

Answer:

Situation 3

In a box, there are 9 black pens, 8 white pens and 10 red pens. In how many ways can 5 black pens, 4 white pens and 6 red pens can be chosen?

Clues:

Number of ways of choosing 5 black pens from 9 black pens = ______________

Number of ways of choosing 4 white pens from 8 white pens = ______________

Number of ways of choosing 6 red pens from 10 red pens = ________________

By the Counting Principle, 5 black pens, 4 white pens, and 6 red pens can be chosen in

$\square \times \square \times \square =$ _______________

ways

Situation 4
There are 5 goalies, 10 forwards, 10 defenders and 12 midfielders in a particular football team. In every game that they will join only 1 goalie, 3 forwards, 4 defenders and 4 midfielders will participate. How many possible groupings can be made out of the 37 players?

Answer:

Situation 5
You went into a customized pizza store where you can select different toppings that delight your taste! You can choose 10 different toppings from a variety of 24 toppings. How many choices can you make with 10 different toppings?

Answer:

So let's, make some conclusions. A generalization is an inference that you can form after looking into some common observations.

ACTIVITY 17 Guided Generalization Table

Pick up three situations in Activity 16. Rewrite them in the boxes below by paraphrasing them. Then answer the questions that follow.

Situation 1	Situation 2	Situation 3

Process Questions:

1. Are those situations familiar? Do you encounter them from day to day?

____
____

2. How did the idea of combination help you answer the situations?

____
____
____.

GENERALIZATION:

Form your generalization by answering the question, how can outcomes of various real life situations that involve counting be predicted accurately?

____
____
____.

ACTIVITY 18 Combination with Repetition.

(Refer back to Activity 5 on visualizing combinations)

For you to know how to do this please refer to the following website for a discussion and sample for combination with repetitions. http://www.mathsisfun.com/combinatorics/combinations-permutations.html. This website discusses the difference between combination without repetition and with repetition. There are questions that you can click for exercises.

The formula used in dealing with combinations when repetition is allowed is:

$$\binom{n+r-1}{r} = \frac{(n+r-1)!}{r!(n-1)!}$$

where ***n*** is the number of things to choose from, and we choose ***r*** of them (Repetition allowed, order doesn't matter)

Example:

You went into an ice cream shop. They are offering the following flavors: rocky road, vanilla, chocolate, buko pandan, fruit salad, melon, mango, banana, strawberry and durian. They are offering a promo of Php 25.00 per three scoops. How many possible variations would there be when you can have three scoops of the same flavor?

Let n = the total number of flavors

r = number of flavors chosen in a single order

There are **n=10** things to choose from, and you choose **r =3** of them. (The order does not matter and you can repeat.)

By using the formula, $\binom{n+r-1}{r} = \frac{(n+r-1)!}{r!(n-1)!}$

$$\frac{(n+r-1)!}{r!(n-1)!} = \frac{(10+3-1)!}{3!\,(10-1)!} = \frac{12!}{3!9!} = \frac{12 \bullet 11 \bullet 10 \bullet 9!}{3!9!} = \frac{12 \bullet 11 \bullet 10}{3!} = \frac{12 \bullet 11 \bullet 10}{6} = 220$$

Your Turn!

You are in a restaurant and you have 5 food choices but only 3 menus can be served in a single setting. The following are the menus:

Menu 1: Adobo
Menu 2: Fried Chicken
Menu 3: Sinigang na Sugpo
Menu 4: Crispy Pata
Menu 5: Bulalo

Combination Without Repetition	Combination With Repetition

Process Questions

1. Which among the two conditions give you more choices?
   ____
   ____
   ____
2. Is combination helpful? Why or why not?
   ____
   ____
   ____
3. **How can outcomes of various real life situations that involve counting be predicted accurately?**

Combination Without Repetition	Combination With Repetition

END OF DEEPEN

In this section, the discussion was about the real world use of combination.

What new realizations do you have about the topic? What new connections have you made for yourself? What helped you make these connections?

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.

TRANSFER

Your goal in this section is to apply your learning to real life situations. You will be given practical tasks which will demonstrate your understanding.

ACTIVITY 19 The Exhbit

You are assigned to compute for the possibilities of the following event:

The class is organizing a mathematics exhibit. There are 40 students in your class in which 18 are boys and 22 are girls.

Task 1: You are to make an 8-member group in which 3 are boys and 5 are girls. Make 10 listings of the sample groupings.

Task 2: You will design a layout of the possible arrangements of each group in which four groups of similar concepts will be placed side by side each other. **You will make a visual model of the layout using Gliffy.**

To have an organized flow of answers, refer to the questions below and answer them completely with detailed solutions.

1. How did you compute for the number of groupings?

   ____

2. What are your considerations? Why did you decide for such groupings?

   ____

3. How will you plan for the layout?

____
______

4. What are your considerations? Why did you decide for such layouts?
____
______

5. How did you compute for the numbers of possibilities in arranging the groups?
____
______

6. **How can you predict outcomes of task 1 and task 2 accurately?**
____
______

After the series of activities that you did in the DEEPEN section, it is your time to show what you really know by making your own problem. Remember to be CREATIVE!

ACTIVITY 20 Problem Construction

Construct a situation that reflects a real world application of combination. Make the situation original. You may use a story, skit or a dialogue to deliver the situation. Then identify what formula to use and show correct answer.

Be guided with the scoring rubric below:

Criteria	*Excellent (4)*	*Satisfactory (3)*	*Developing(2)*	*Beginning (1)*
Quality of Problem (60%)	The situation made is original and thought-provoking.	The situation made is original.	The situation made is adapted from other sources.	The situation made is completely copied from other sources.
Accuracy of Answer (40%)	The answer is correct and presented in a logical order.	The answer is correct.	The answer is correct but not presented in a logical order.	The answer has some errors and not presented in a logical order.

Situation:
What formula to use?
Solution and Answer:

ACTIVITY 21 Reflection Journal

I learned that combination is

____
____
____
____
_____________________.

ACTIVITY 22 What VALUES did I learn?

List and explain three values that you learn from this lesson.

What would happen if your computations are wrong, what is/are the consequence/s of not learning this lesson right?

____
____
____
____
____
______________________________.

ACTIVITY 23 Anticipation-Reaction Guide

Write your response for each statement in the After Lesson Column only.

Write **A** if you agree with the statement.
Write **B** if you disagree with the statement.

Before Lesson	Statements	After Lesson
	1. Combination does not consider the arrangement of members in a group.	
	2. The order of members in a group is not important in combination.	
	3. ABC, ACB, BAC, BCA, CAB and CBA are counted as one in combination.	
	4. 567 and 765 are the same in the concept of combination.	
	5. The number of combinations is equal to the number of permutations of ***n*** objects taken ***r*** at a time divided by ***r!***.	
	6. Combination only applies when you choose distinct objects from the group of objects being selected from.	
	7. Combination is useful when you choose the possible number of subcommittees considering a particular number of members per subcommittee from the total number of persons being selected.	
	8. Combination is the number of different ways that a certain number of objects as	

	a group can be selected from a larger number of objects	
	9. Combination is an ordered list of possible outcomes.	
	10. Combination is the number of different ways that a certain number of objects can be arranged in order from a larger number of objects.	

END OF TRANSFER

After giving your final answers to the activity above, compare your work with
your initial answers in the explore section. Are there changes in your thinking? What are the changes? What realizations have you made?

GLOSSARY OF TERMS USED IN LESSON 2:

1. **Combination –** a counting technique used to choose items from a collection or set wherein the order of selection (arrangement of members) does not matter
2. **Permutation –** a counting technique used to choose items from a collection or set wherein the order of selection (arrangement of members) matters
3. **Outcome –** with reference to the lesson on combination, it is defined as the quantitative result after a certain computation is applied

4. **Prediction –** the process of determining a reasonable future quantitative outcome based on observed quantitative behaviors achieved by applying useful formula/s or from observed pattern/s

REFERENCES AND WEBSITE LINKS USED IN LESSON 2:

Videos

1. https://www.youtube.com/watch?v=qaFNhwNBY3k
This website is a 20-minute lecture video which discusses lessons on Statistics 101. It points the difference between Permutation and Combination. Illustrative examples are given and sample applications.

2. https://www.youtube.com/watch?v=PSS3mCS_Ef8
This is a 12-minute video lesson that focuses on Combination, its formula and how it is used. In addition, you will know when combination or permutation can be used.

3. https://www.youtube.com/watch?v=1stIqr0FGiE
A 14-minute video about permutation, factorial and non-examples of combination.

References

1. http://www.mathsisfun.com/combinatorics/combinations-permutations.html
This website discusses and shows examples of permutation and combination with or without repetition. There are illustrative examples and computational exercises.

2. http://statistics.about.com/od/Formulas/a/How-To-Derive-The-Formula-For-Combinations.htm
   This article discusses how the formula of combination is derived from the idea of permutation.

3. http://www.algebra.com/algebra/homework/Permutations/Proof-of-the-formulaon- he- number-of-Combinations.lesson
   This website requires you to read the materials. It focuses on the idea of multiplication principle and how the formula for combination is derived from permutation.

4. http://www.intmath.com/counting-probability/4-combinations.php
   This website requires you to solve the exercises and submit your answers online.
   You can check if your answer is correct by clicking the answer button.

Lesson 3: Probability of Union and Intersection of Two Events

☑ INTRODUCTION AND FOCUS QUESTION(S):

Have you thought of those times when you made decisions based on chances? Have you experienced answering a multiple choice test by guessing? Do you consider the number of characters in creating your passwords? How about choosing to bet on a 6/42 or 6/45 lotto, which has more chances of winning? Would you have more choices using OR or AND?

In this lesson, you will find out the various applications of probability in real life and its usefulness in making decisions. As you go through this lesson, remember to search for the answer to the following questions: "**How do you predict outcomes accurately?**" and **"In what ways can knowledge in predicting outcomes help you in making conclusions and decisions in your daily life?"**

☑ LESSONS AND COVERAGE:

In this module, you will examine this question when you take this lesson:

Lesson 3 – <u>Events, Union and Intersection and Probability of Union and Intersection of Two Events</u>

In this lesson, you will learn the following:

Lesson 3	<u>Illustrate events, union and intersection of events.</u> <u>Illustrate the probability of a union of two events.</u> <u>Find the probability of union and intersection of two events.</u> <u>Illustrate independent events.</u> <u>Solve problems involving probability.</u>

☑ MODULE MAP:

Here is a simple map of the above lessons you will cover: <u>(Provide diagram of module map)</u>

☑ EXPECTED SKILLS:

To do well in this lesson, you need to remember and do the following:

1. take note of the definition of the different terms
2. master the skills in counting learned in lessons 1 and 2
3. check out all recommended websites for different solutions of problems
4. solve all the problems with accuracy

EXPLORE

In grade 8, you learned that probability is the likelihood of occurrence of an event. You also learned how to find the probability of events and how this concept is used to solve real-life problems. In this section, you will learn more about probability in a new context which is in combination of two events. Your initial ideas on this topic will be assessed. Be reminded also that you will be introduced to the different types of events so it is important to take note of their definitions. As you go through this lesson, keep on thinking about this question: ***How do you predict outcomes accurately? In what ways can knowledge in predicting outcomes help you in making conclusions and decisions in your daily life?***

Let us begin our lesson by answering the problem below.

ACTIVITY 1 Survey Says

DESCRIPTION: Analyze the results of the survey and answer the questions that follow.

Mr. Real conducted a survey on his homeroom. He asked his students what math course and what science course they will be taking in advance this summer. The results of the survey will help the academic coordinator plan the courses to be offered for summer and prepare the schedule. The following are the results:

18 students will take Math
9 students will take Science
4 will take both Math and Science
6 will take neither of the two courses

Process Questions:

1. How many students were surveyed?
2. If a student is selected at random, what is the probability that the student will be taking:
   a. Math only?
   b. Science only?
   c. Math or Science?
   d. Math and Science?
3. What is the probability that the student will not be taking
   a. Math?
   b. Math or Science?
4. Find the probability of a student taking Science given that the student is not taking Math.

Every now and then, we will be referring back to this problem when we discuss different types of events and how to compute its probabilities. As you go through with this lesson, keep in mind the question "How do you predict outcomes accurately?"

Let us continue by answering the KWLH Chart.

ACTIVITY 12 KWHL Chart

DESCRIPTION: This activity will help you organize your initial ideas about predicting outcomes accurately of two or more events. Your task is to complete the chart by writing in the first column with **What I Know** and the second column with **What I Want to Know** about combination of two events and its probability. You will be asked to fill out the other columns in the different sections of this lesson.

PROCESS QUESTIONS: What are the key concepts that you learned from the previous lessons and activities? What do you want to learn more from this lesson?

Probability of Combination of Events			
What I Know	What I Want to Know	What I Learned	How Can I Learn More

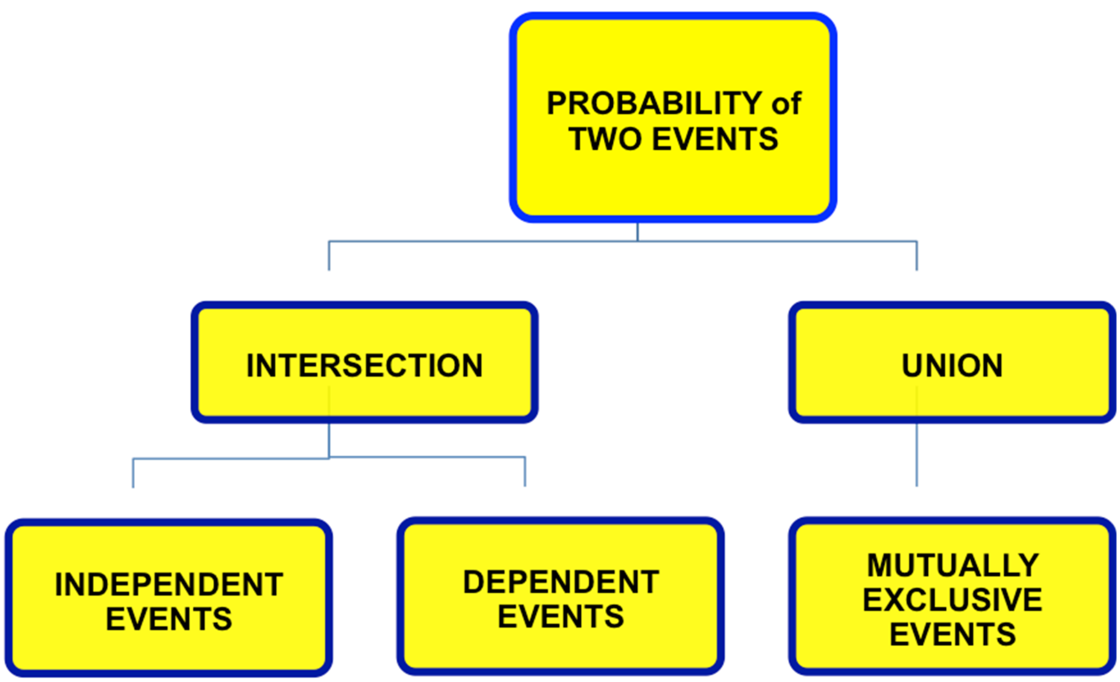
End of EXPLORE:

You gave your initial ideas on how to measure the likelihood of occurrence of a combination of two or more events by answering the KWLH Chart. Let's find out how others would answer the above and compare their ideas to our own. As you compare, you will find out if your ideas are in line with the standard. You will also learn other concepts that will help you complete a required project found at the end. This project is about formulating conclusions and sound decisions based on possible outcomes.

We will start by doing the next activity.

FIRM-UP

Your goal in this section is to learn how to combine two events using union and intersection and solve its probability. In the course of determining the probability, you will be introduced to the different types of events and how the type affects the probability. Below is a simple map of the concepts that will be covered in this lesson.



ACTIVITY 3 Memory Check

DESCRIPTION: You will recall first the definition of key terms used in the study of probability which you learned in grade 8. Click the following links.

http://www.mathgoodies.com/lessons/vol6/intro_probability.html

- This link gives the definitions and examples of experiment, outcomes, events and probability. There is also a set of exercises which you may try answering.

http://www.mathgoodies.com/lessons/vol6/sample_spaces.html

- This link gives the definition and examples of sample space. You may answer the exercises found after the discussion.

PROCESS QUESTIONS:
Using "Activity 1: Survey Says", answer the following questions.

1. Identify the following:
   a. experiment
   b. sample space
   c. event
   d. outcome

2. How did you identify the sample space?
3. What is the probability of a student taking Math?
4. How do you predict outcomes accurately?

In the survey problem, the sample space is the students in Mr. Real's class, and the events are choosing a math course and choosing a science course. The terms sample space and event will be used throughout this lesson. Thus, it is important to take note of the meaning of these terms.
The survey problem also involves two events which are combined by "or" and "and". You will learn how to combine two events through the following examples.

Sample space and events are sets. Therefore, two events can be combined using union and intersection.

The following examples illustrate finding $A \cup B$ and $A \cap B$.

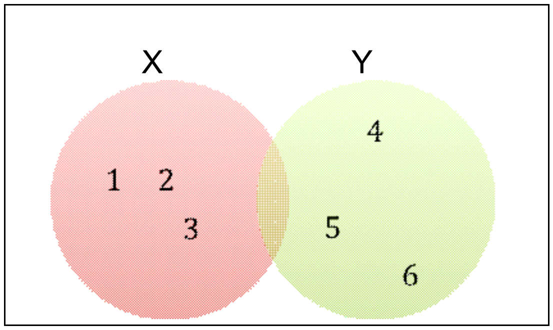
Example 1:

Consider the experiment of tossing a die. Let X be the event that an odd number comes out and let Y be the event that an even number comes out.

Determine the possible outcomes of the events X and Y and draw the Venn diagram to illustrate $X \cup Y$ and $X \cap Y$.

Solution:

Sample space S = {1, 2, 3, 4, 5, 6}
X = {1, 3, 5}
Y = {2, 4, 6}



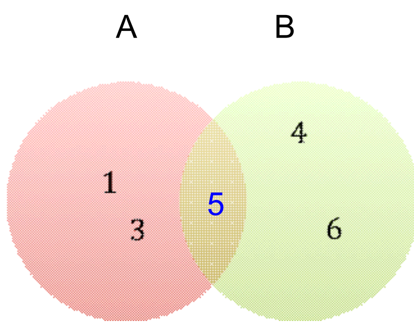
$X \cup Y = \{1, 2, 3, 4, 5, 6\}$
$X \cap Y = \varnothing$

Example 2:

A die is tossed. Let A be the event that an odd number turns up and let B be the event that a number greater than 3 turns up.
Find $A \cup B$ and $A \cap B$.
Draw a Venn diagram to illustrate $A \cup B$ and $A \cap B$.

Solution:

sample space S = {1, 2, 3, 4, 5, 6}
A = {1, 3, 5}
B = {4, 5, 6}



$A \cup B = \{1, 3, 4, 5, 6\}$
$A \cap B = \{5\}$

Example 3:

Two dice are tossed, one red and one blue. Let M be the event that the sum of the dots is equal to 6 and let N be the event that an equal number of dots appears on both dice.

Solution:

The sample space S is defined below.

	1	2	3	4	5	6
1	1, 1	2, 1	3, 1	4, 1	5, 1	6, 1
2	1, 2	2, 2	3, 2	4, 2	5, 2	6, 2
3	1, 3	2, 3	3, 3	4, 3	5, 3	6, 3
4	1, 4	2, 4	3, 4	4, 4	5, 4	6, 4
5	1, 5	2, 5	3, 5	4, 5	5, 5	6, 5
6	1, 6	2, 6	3, 6	4, 6	5, 6	6, 6

M = {(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)}
N = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}

$M \cup N$ = {(1, 5), (2, 4), (3, 3), (4, 2), (5, 1), (1, 1), (2, 2), (4, 4), (5, 5), (6, 6)}

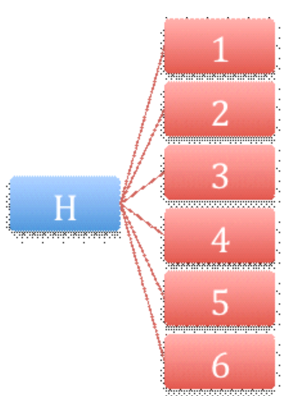
$M \cap N$ = {(3, 3)}

Example 4:

A die and a coin are tossed. Let A be the event that a tail comes out and B the event that a 5 comes out. Find $A \cup B$ and $A \cap B$.

Solution:

Using the tree diagram,



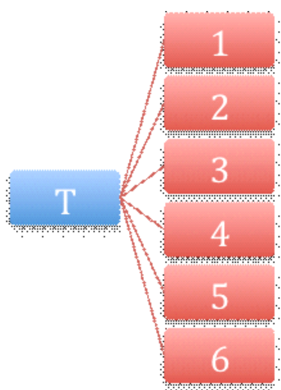
$S = \{H1, H2, H3, H4, H5, H6,$
$T1, T2, T3, T4, T5, T6\}$

$A = \{1T, 2T, 3T, 4T, 5T, 6T\}$

$B = \{5T, 5H\}.$

$A \cup B = \{1T, 2T, 3T, 4T, 5T, 5H, 6T$

$A \cap B = \{5T\}$



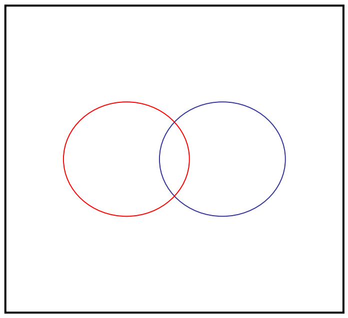
PROCESS QUESTIONS:

1. How would you define the union of two events? the intersection?
2. In the Venn diagram, how would you describe the union and intersection?
3. What mathematical concept was used in determining the outcomes of the events?
4. How do you predict outcomes accurately?

ACTIVITY 4 Venn Diagrams in Survey

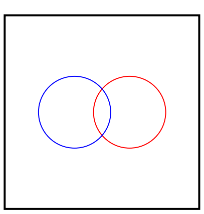
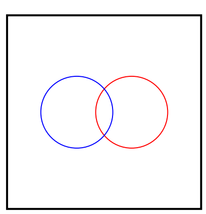
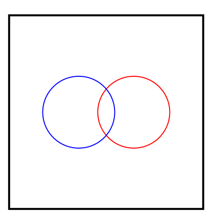
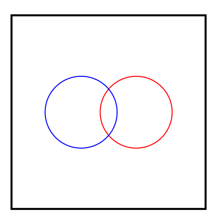
DESCRIPTION: You just learned from the above examples how to use Venn diagrams in determining the union and intersection of two events. Apply now what you have learned by making the Venn diagram of the survey results

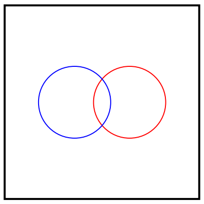
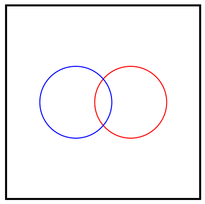
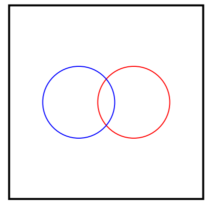
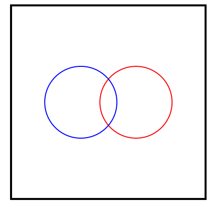
18 students will take Math 10
9 students will take Science 10
4 will take both Math and Science
6 will take neither of the two courses



Process Questions:

1. How many students were surveyed?
2. How many students will take
   a. Math only?
   b. Science only
   c. Math or Science?
   d. Math and Science?
3. How many students will not take
   a. Math?
   b. Science?
   c. Math or Science?
   d. Math and Science?
4. Using Venn diagrams, shade the region represented by each case in numbers 2 and 3. Print the Venn diagrams below and shade the region. Then pin it on Pinterest. Go to www.pinterest.com and create your own account. Do not forget to describe the shaded region before posting it.

ACTIVITY 5 Online Quiz on Venn Diagrams

DESCRIPTION: It is now time to test your skill in using Venn diagrams to determine the intersection and union of two events. Answer an online quiz by clicking the following link:
http://glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-02-833240-7&chapter=13&lesson=2

The sample problems above presented two types of experiments, simple and composite experiments. Tossing a die (in examples 1 and 2) is a simple experiment. Rolling two dice or tossing a die and a coin (in examples 3 and 4) is a composite experiment. Notice that the method used in identifying the outcomes of a simple experiment is different from that of a composite experiment. Now apply what you have learned from the examples and activities by using the appropriate method for determining outcomes to find the union and intersection in the next set of activities.

ACTIVITY 6 Finding Union and Intersection

DESCRIPTION: In the problems you solved, the situations involved events that are either both or one of the two. In this next set of problems, you will apply determining the union and intersection of events involving simple experiments.

Events	$A \cup B$	$A \cap B$
1. A card is drawn from a standard deck of 52 cards. 2. A is the event that a face card is drawn and B is the event that a 3 is drawn. 1.		

3. A die is tossed once. A is the event that an even number comes out and B is the event that a prime number comes out.		
4. A 1-peso coin is tossed. A is the event that the coin falls head and B is the event that the coin falls tail.		

ACTIVITY 7 IS IT "AND" or "OR"?

DESCRIPTION: The previous set of problems involved simple experiments. In this activity, the experiments are composite. Find the union and intersection of events A and B defined by the following.

1. A 3-section spinner marked red, green, blue is spun once and a coin is tossed once. A is the event of spinning a red and B is the event of getting a head.

2. A die is tossed twice. A is the event of getting equal dots and B is the event of getting a sum of 11.

3. A coin is tossed three times. A is the event that at least 2 heads come up and B is the event that only one head comes up.

PROCESS QUESTIONS:
1. What method did you use the most in determining the outcomes of the events?
2. Can you easily identify the events having no intersection? How?

ACTIVITY 8 Muddiest Point

DESCRIPTION: Before proceeding to the next activity, complete the journal below.

The part of the lesson that I still find confusing is ________________   because _________________.

Do not hesitate to ask from your classmate or your teacher for clarification in your next meeting.

ACTIVITY 9 MUTUALLY EXCLUSIVE and PROBABILTY

DESCRIPTION: Now that you have learned how to find the union and intersection of two events, you are now ready to learn probability of the union of two events. In determining the probability of union, you need to know what are mutually exclusive events. Click the following link to watch videos showing what are mutually exclusive events and how to solve for probability of union of two events. Then complete the Frayer's model below.

http://www.onlinemathlearning.com/mutually-exclusive-events.html

- The link discusses what are mutually exclusive events and how to compute their probabilities. The link also includes videos.

Definition	Facts/Characteristics
Examples	Non-Examples

MUTUALLY EXCLUSIVE EVENTS

PROCESS QUESTIONS:
1. How did you come up with the list of facts and characteristics of mutually exclusive events?
2. Do all the examples that you have given have all the essential characteristics?
3. How did you come up with the non-examples? Why do you consider these as non-examples? Support your answer.
4. How did the listing down of non-examples help you understand fully the concept?
5. In the survey activity, are the events of choosing a math course and choosing a science course mutually exclusive? Explain.

ACTIVITY 10 Which Are Mutually Exclusive?

DESCRIPTION: Having just learned mutually exclusive events, let us check your understanding through the next activity. Identify whether the following pairs of events are mutually exclusive or not by checking the appropriate column. Explain why the events are not mutually exclusive.

	Mutually Exclusive	Not Mutually Exclusive
1. Two dice are rolled. The event that the sum of the numbers is odd or greater than 7.		
2. Two cards are drawn from a deck of 52 cards. The event that the card drawn is a red card or a spade.		
3. A drawer contains 2 sets of black socks and 3 sets of white socks. You pull at random 2 sets of socks. The event that the 2 sets of socks picked are of the same color.		
4. A student in a certain high school is selected as contestant in the math quiz show. The event that the student is a female or in grade 10.		
5. A couple has three children. The event that at least 2 of their children are boys and the event that all three children are girls.		

6. A box contains slips of paper numbered 1 to 15. The event of drawing an odd number or a prime number.		

The formula for $P(A \cup B)$ is referred to as the Addition Rule. You are going to practice using this formula through the next activity.

ACTIVITY 11 To Add Or Not To Add?

DESCRIPTION: Knowing how to distinguish mutually exclusive events from not mutually exclusive, apply now the Addition Rule in finding the probability of the union of the following events.

Determine the likelihood of occurrence of the following events.

1. A die is rolled once. What is the probability of getting a 6 or an odd number?
2. A coin is tossed twice. What is the probability of getting both heads or both tails?
3. A bag contains 3 red balls, 5 green balls and 7 blue balls. One ball is drawn. What is the probability that the ball is either red or green?
4. A card is drawn from a deck of 52 cards. What is the probability that the card drawn is a face card or a spade?
5. The possibility that Josie passes math is 0.5 and the probability that she passes science is 0.7. If the probability that she passes both subjects is 0.3, what is the probability that she will pass at least one of these subjects?
6. The table below shows the enrollment of ABC High School.

	Grade 7	Grade 8	Grade 9	Grade 10
Male	85	80	80	70
Female	95	90	100	80

One student is selected at random. What is the probability of selecting

a. a male or a female student?
b. a grade 7 or a grade 9 student?
c. a male or a grade 10 student?
d. a female or a grade 8 student?

PROCESS QUESTIONS

1. How did you find the activity?
2. Did you encounter any difficulty? If yes, which part of the lesson is it? What did you do to solve the difficulty?
3. How do you predict outcomes accurately?
4. In what ways can knowledge in predicting outcomes help you in making conclusions and decisions in your daily life?

ACTIVITY 12 Comfort Zones

DESCRIPTION: Before proceeding to the next activity, let us check your progress in learning the skills so far. Check the "Comfort Zone" column if you have mastered the skill and "Stretch" column if you still have difficulties or confusion.

SKILLS	COMFORT ZONE	STRETCH
Illustrate events, and union and intersection of events.		
Illustrate probabilty of union of two events.		
Identify mutually exclusive events and not mutually exclusive.		
Find the probability of $A \cup B$.		

If you have skills identified as "STRETCH", do not hesitate to ask help from any person who is knowledgeable of the lesson. You may also ask for clarification from your teacher in your next face-to-face meeting, or post your question in the discussion room.

If the events are combined using intersection, how do you find its probability?

ACTIVITY 13 Independednt Events

DESCRIPTION: After learning what are mutually exclusive events and its relation to probability of union, it is now time to learn another type which is independent events and its use in finding the probability of intersection of two events. Click the following links to compare independent and dependent events and how their probabilities are computed. Then complete the table below.

https://www.youtube.com/watch?v=lFyEjgfSwsU
- This link defines independent and dependent events.

https://www.youtube.com/watch?v=WeiQgFEOCqo
- This link gives the formula for finding the probability of dependent events and examples.

http://www.mathgoodies.com/lessons/vol6/independent_events.html
- This link gives the formula for finding probability of independent events and examples.

http://www.shmoop.com/video/independent-and-dependent-events
- This link compares independent and dependent events with real-life examples.

	Independent Events A & B	Dependent Events A & B
Definition		
Formula for probability		
Example		

PROCESS QUESTIONS:
1. How did you determine the examples and non-examples of independent and dependent events?
2. How will you know that two events are independent or dependent?
3. In the activity "Survey Says", are there any events which are independent or dependent? Explain.

ACTIVITY 14 Skill Builder

DESCRIPTION: Click on the following links for an interactive quiz to practice your skill in finding the probability of independent and dependent events.

http://www.onlinemathlearning.com/independent-probability.html

- This link includes a worksheet on finding probability of independent events.

http://www.onlinemathlearning.com/dependent-probability.html

- This link contains an interactive quiz on finding probability of dependent events.

Process Questions:

1. How many problems did you get correctly?
2. What are the difficulties that you encountered?
3. What did you do to solve the difficulties?

ACTIVITY 15 Conditional Probability

DESCRIPTION: In the previous activity, you used the formula P $(A \cap B)$ = P(A) x P(B/A) for probability of dependent events. However, there are cases when situations call for finding the probability of an event given that another event has happened, that is P(B/A). This is called Conditional Probability. Now, click the following links to learn more about conditional probability through examples.

http://www.virtualnerd.com/algebra-2/probability-statistics/compound-events/conditional-probability-contingency-tables/conditional-probability-definition

- This link defines conditional probability and presents examples.

http://www.mathgoodies.com/lessons/vol6/conditional.html

- This link shows sample problems on conditional probability.

http://www.dnatube.com/video/11865/What-is-Conditional-Probability

- This video shows how to compute for conditional probability.

PROCESS QUESTIONS:

1. Derive the formula for conditional probability.
2. When do we use conditional probability?
3. How would you describe in the Venn diagram conditional probability?
4. In the survey activity, which event/s show conditional probability, if there are any?
5. How do you predict outcomes accurately?
6. In what ways can knowledge in predicting outcomes help you in making conclusions and decisions in your daily life?

ACTIVITY 16 Skill Builder

DESCRIPTION: Click on the following links for an interactive quiz to practice your skill in finding the conditional probability.

http://www.regentsprep.org/regents/math/algebra/apr3/praccond.htm

http://algebralab.org/lessons/lesson.aspx?file=Algebra_ConditionalProbability.xml

ACTIVITY 17 Skills Check

Description: At this point, let us check your progress in this lesson. Based on the results of your quiz, determine your level of mastery in the skills listed below.

SKILLS	I learned much on this	I still need to learn more on this	I have not learned any on this
Illustrate independent events.			
Differentiate independent and dependent events.			
Find the probability of A $\cap$ B.			
Find the probability of B given A.			

ACTIVITY 18 Conditional Probability In The Real World

DESCRIPTION: Now that you know what is conditional probability, you are going to watch the following video showing its applications to real life.

http://study.com/academy/lesson/applying-conditional-probability-independence-to-real-life-situations-lesson-quiz.html#lesson

http://www.virtualnerd.com/algebra-2/probability-statistics/compound-events/conditional-probability-contingency-tables/conditional-probability-example

- This link is a tutorial showing one example of conditional probability in a real world problem.

Process Questions:

1. How is conditional probability used in real life?
2. How do you predict outcomes accurately?
3. In what ways can knowledge in predicting outcomes help you in making conclusions and decisions in your daily life?

ACTIVITY 19 My Own Conditional Probability Scenarios

DESCRIPTION: Now it is your turn to give examples of conditional probability in real life. Go to www.pinterest.com. Create your own account. Select scenarios that show conditional probability and Pin it. Do not forget to add a caption describing the scenario before posting it.

ACTIVITY 20 More Pratice

Description: It is now time to test your skill to the next level by answering the following questions.

1. In a deck of cards, cards are drawn one at a time and replaced after each drawing. What is the probability of drawing exactly 2 aces when 3 cards are drawn?
2. A jar contains four red balls and four blue balls. What is the probability of getting a red ball first and a blue ball second if no replacement is done?
3. The probability that the school's basketball team will win the first game is ½. The probability that it will win the second game is 2/3. What is the probability that it will win both games?
4. In a school, 36% of the students own a cellular phone and 24% of the students own a cellular phone and a laptop computer. What is the probability that a student owns a laptop computer given that the student owns a cellular phone?

ACTIVITY 21 Muddiest Point

DESCRIPTION: After doing practice problems on probability of union and intersection of two events, you will now complete the journal below.

The part of the lesson that I still find confusing is _______________

because _________________.

ACTIVITY 22 Mutually Exclusive Vs Independent Events

DESCRIPTION: You are now going to put all together the concepts you learned in the study of probability through a comparison table. Complete the following statements to compare mutually exclusive events and independent events.

MUTUALLY EXCLUSIVE EVENTS AND INDEPENDENT EVENTS are similar because they both

____

____

____

MUTUALLY EXCLUSIVE EVENTS AND INDEPENDENT EVENTS are different because

MUTUALLY EXCLUSIVE EVENTS are ______________________________, but
INDEPENDENT EVENTS are__________________________________.

MUTUALLY EXCLUSIVE EVENTS are ______________________________, but
INDEPENDENT EVENTS are__________________________________.

MUTUALLY EXCLUSIVE EVENTS are ______________________________, but
INDEPENDENT EVENTS are__________________________________.

PROCESS QUESTIONS:

1. What are the bases of your comparison?
2. Which of the following do you find easiest to answer? hardest to answer? Why?
3. Was the comparison helpful in understanding the concepts about mutually exclusive events and independent events?
4. For example, a die and a coin are tossed. If A is the event of obtaining heads and B is the event of rolling a 6, are events A and B mutually exclusive, independent or both? Can mutually exclusive events be independent events? Explain.

ACTIVITY 23 Flowchart

DESCRIPTION: As another summary, make a flowchart on how to determine the probability of union and intersection of two events relating all the concepts you have learned so far. You will use gliffy.com. E-mail the flowchart to your teacher.

PROCESS QUESTIONS:
1. What did you consider when making the flowchart?
2. How did the flowchart help you put together the concepts that you learned so far?
3. What did you learn from making the flowchart?

ACTIVITY 24 My Flowchart vs Other Flowchart

DESCRIPTION: How thorough is your flowchart? Well, let us check by looking at a flowchart found in the following link.

http://faculty.mdc.edu/mshakil/Probability_&_Counting_Flow_Chart.pdf

The link shows a flowchart on how to find the probability of two events. Compare the flowchart that you just made with the link's flowchart. Post your comparisons in the discussion forum.

ACTIVITY 25 Skills Check

DESCRIPTION: Before proceeding to the next section, let us determine your level of progress in terms of learning the skills listed below. Check the apropriate column.

SKILLS	I learned much on this	I still need to learn more on this	I have not learned any on this
Illustrate events, and union and intersection of events.			
Illustrate probabilty of union of two events.			
Find the probability of $A \cup B$.			
Illustrate independent events.			
Find the probabilty of $A \cap B$.			

End of FIRM UP:

In this section, the discussion was about finding the union and intersection of two events and determining their probabilities.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision? What new learning goal should you now try to achieve?

ACTIVITY 26 **KWHL Revisit**

DESCRIPTION: At this point, fill-out the third column What I Learned.

Probability of Combination of Events			
What I Know	What I Want to Know	What I Learned	How Can I Learn More

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.

DEEPEN

Your goal in this section is to take a closer look at how the different counting techniques are used in finding the probability of events. You will be using the Fundamental Counting Principle, permutations and combinations which you learned from the previous lessons of this module in predicting outcomes of real-life situations. As you go through this section, keep in mind the following question**: How do you predict outcomes accurately? In what ways can knowledge in predicting outcomes help you in making conclusions and decisions in your daily life?**

ACTIVITY 27 COUNTING for PROBABILITY

DESCRIPTION: You have already learned that solving probability problems involve identifying the sample space by using different counting techniques. Now, click the following links to watch videos on how the different counting techniques are used in determining probability.

https://www.youtube.com/watch?v=RNH_O2QvkWA

https://www.youtube.com/watch?v=Ed1waMzBSzg

PROCESS QUESTIONS:
1. What did you learn from the video?
2. Is there another way to solve the two problems? How?
3. How do you predict outcomes accurately?
4. In what ways can knowledge in predicting outcomes help you in making conclusions and decisions in your daily life?

ACTIVITY 28 Let Me Count The Ways

DESCRIPTION: Apply what you have learned from the video by answering the following problems.

1. PHONE NUMBERS. Ivy knows that the last four digits of her friend's 5-digit phone number are 0, 3, 5, and 6, but she cannot remember the exact order. She knows that the first digit is 5. What is the probability that Ivy randomly selects the digits in the correct order?

2. LOTTO. What is your chance of winning the Philippine lotto wherein 6 numbers are chosen – in no particular order – from the numbers 1 to 42?

3. COMMITTEES. A committee consists of 3 girls and 5 boys. Two delegates to an international conference are to be chosen from the committee.
   a. What is the probability that the group chosen will be both boys?
   b. What is the probability that the group chosen will be both girls?
   c. What is the probability that the group chosen will be a boy and a girl?

4. PASSWORDS. A password consists of 6 characters without repetition, the first three are vowels from the English alphabet and the next three are digits from 0 – 4.
   a. What is the probability that the password ends with 0?
   b. What is the probability that the password starts with a?

PROCESS QUESTIONS:

1. What are the different techniques that you used to determine the outcomes of the events?
2. What generalization can you form in terms of when you can use the different techniques?
3. How is knowledge on counting techniques useful in determining probability?
4. How do you predict outcomes accurately?
5. In what ways can knowledge in predicting outcomes help you in making conclusions and decisions in your daily life?

ACTIVITY 29 PROBABILITY in REAL LIFE

DESCRIPTION: Now, test your generalization by solving the following applications in the real world.

1. At a grocery store customers were surveyed: 25% use coupons, 43% bring their own bags, and 12% do both. What is the probability that a random shopper uses coupons or brings a bag?

2. A pizza shop has regular, hand-tossed, and thin crusts; two different cheeses; and four toppings. What is the probability that the pizza you order is hand-tossed or thin crust?

3. The survey conducted among college students who are in FAVOR and NOT in FAVOR of changing the school's name is as follows:

	Favor	Not Favor
Male	100	350
Female	60	220

   What is the probability that a student selected in random is a male who is in favor of changing the school's name?
4. Eight clerks and 5 drivers apply for positions in a company. If 4 applicants are hired in random, what is the probability that 2 are clerks and 2 are drivers?

5. Phone numbers in a small city consists of 5 digits where the first digit is not 0. What is the probability that a phone number has the same first two digits and has the same last two digits?

6. Suppose a box contains 20 fuses, 5 of which are defective. What is the probability of drawing at random two non-defective fuses in succession if the first fuse that has been drawn is not returned before the second draw?

PROCESS QUESTIONS:

1. How is knowledge of probability used in the real world?
2. How do you predict outcomes accurately?
3. In what ways can knowledge in predicting outcomes help you in making conclusions and decisions in your daily life?

ACTIVITY 30 Skills Readiness Check

Description: Reflect on the level of your performance in this lesson. Check the first column if you need more practice or you are now ready to move on to the next activity.

I Need more practice (if most of your answers are incorrect)	I am ready to move on to the next activity (if you have answered all items correctly or have incurred only one or two wrong answers)
↓	↓
Answer the interactive quizzes on this site: http://braingenie.ck12.org/subjects/118 This link includes problems on probability of union and intersection of events. You just choose which topic you still want to practice more. Click the ***practice*** icon to solve a problem and enter your answer. Click the ***try*** icon to display the solution. If you want more explanation on the solution, choose ***watch video*** icon.	You may proceed to the next activity.

At this point, you have learned how probability is used to determine how likely an event is to occur. In the next section, you will be looking at specific real life situations where probabilty helps you formulate conclusions and make decisions. Add to that, the following activities will provide you helpful ideas for your performance task.

ACTIVITY 31 Test for Understanding

DESCRIPTION: In the previous section, we looked at different problems computing for the probability of an event to occur. Let's put together in the table below our answers to the essential question that we asked for each problem:

Essential Question	Problem 1: Clothes	Problem 2: School	Problem 3: Seat Plan
How do you predict outcomes accurately?	Carla has 2 pairs of black pants, 3 pairs of blue pants, and 1 pair of tan pants. She also has 4 white and 2 red shirts. If Carla chooses a pair of pants and a shirt at random, what is the probability that she will choose a pair of black pants and a white shirt? The problem was solved using …..	The names of 24 students, of which 14 are boys and 10 are girls, in Mr. Ali's science class are written on cards and placed in a jar. Mr. Ali randomly selects two cards simultaneously to determine which students will present their lab reports today. Find the probability that two boys are selected or two girls are selected? The problem was solved using ….	Five students A, B, C, D and E are seated in a front row with 5 chairs randomly. The teacher sees the need for A and B to sit beside each other and also C and D. What is the probability of this arrangement? The problem was solved using …

PROCESS QUESTIONS:

1. Look at your answers to the essential question in the above table. What do all answers have in common?
2. When will you know if a problem calls for the application of FCP, permutation or combination?
3. Complete the following statement and support your answer with the examples from the above problems.

Outcomes can be predicted accurately by ________________________. The choice of counting technique depends on ________________________.

Supporting reasons and examples:

ACTIVITY 32 Let's Test It!

DESCRIPTION: After forming your generalization in the previous activity, you are now going to check if this generalization can be applied in the next situation. The problem is a game between the hare and the tortoise. Games are popular applications of probability. So, analyze the game and identify who is likely to win.

THE HARE AND THE TORTOISE GAME

The following are the rules of this game.

1. The tortoise and the hare start each turn at X.
2. To start play: Each player rolls a die. The highest number chooses a character, hare or tortoise, and takes the turn.
3. Each turn consists of three moves. The player will roll a die a total of three times per turn. After each roll, the player's marker is moved one place to the left if the number on the die is odd and one place to the right if the number on the die is even.
4. Scoring: The tortoise gets a point if at the end of the three moves the marker is on position G or P. The hare gets a point if the marker ends at position E, F, X, Q, R.
5. Player alternate turns until each has 10 turns. Construct a table for your results.
6. The player with the greater number of wins after 10 turns will be declared the winner.

E F G X P Q R

← **ODD START EVEN** →

PROCESS QUESTIONS:
1. Who is likely to win the game? Why?
2. What method did you use to determine the probability of the hare or tortoise to win?
3. **How do you predict outcomes accurately?**
4. **In what ways can knowledge in predicting outcomes help you in making conclusions and decisions in your daily life?**

End of DEEPEN:

In this section, the discussion was about applications of probability in the real world using the different counting techniques.

What new realizations do you have about the topic? What new connections have you made for yourself? What helped you make these connections?

Go back to the KWLH Chart and fill out the column **How Can I Learn More**.

Probability of Combination of Events			
What I Know	What I Want to Know	What I Learned	How Can I Learn More

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.

TRANSFER

Your goal in this section is apply your learning to real life situations. You will be given a practical task which will demonstrate your understanding.

ACTIVITY 33 Scaffold for Transfer

DESCRIPTION: The next activity will help you prepare for the final task.

The school is planning to have an outreach program. You will be going to a particular slum area for immersion. You will have to help the outreach coordinator formulate the groupings. The composition should be fair enough so that the boys and girls are distributed properly. Aside from the groupings that you will make, you are to compute for the budget that is required for the program. You will also have to calculate the possibility of making the program a success. You will also have to assign each group different rotational assignments considering one group with a fix assignment. You will have to determine all possibilities so that a fair distribution of responsibilities is observed. You will be working closely with your teacher and consult if the groupings and assignments are fair enough.
You will present your pre-plan/draft to your teacher for corrections and modifications. The final plan will be required by your teacher for submission.

ACTIVITY 34 Performance Task

The SK in your barangay will be holding a Family Day wherein students of your age are encouraged to design fund-raising activities that are profitable. The proceeds of the activity are for the children who are cancer patients under Kapwa Ko Mahal Ko Foundation. You are a sports tournament organizer or a game designer/developer. As a sports organizer, you are to formulate a random and fair grouping. You will have to include in your proposal the projected winnings of each team based on its composition. You are encouraged to be creative and innovative in terms of your ideas so that the activities are very interesting and are giving new impressions on the level of festivity of the event. The most feasible proposals will be approved by the SK Council. As part of your proposal, a reasonable fee is to be collected and prizes are to be given. You are to formulate a projected income of your activity. The proposal will be judged according to the following criteria: justification of the proposal which includes the planning and execution of the activity, feasibility, accuracy of computations and use of mathematical concepts. Below is the rubric to be used.

Criteria	*Excellent*	*Satisfactory*	*Developing*	*Beginning*
Justification	Explanations have varied and advanced supporting details.	Explanations have adequate supporting details.	Explanations have limited and in some cases, partially correct supporting details.	Explanations have totally erroneous supporting details or no details are given at all.
Feasibility	The proposal plan is profitable and appealing to the people due to its uniqueness and innovativeness.	The proposal plan is profitable and appealing to the people.	The proposal plan is profitable but not appealing to the people.	The proposal plan is not profitable.
Accuracy of Solution	All calculations are made with all correct and detailed solutions.	All calculations are made with all correct solutions.	There is at least 1 error in the calculations.	There are two to four errors in the calculations.
Use of mathematical concepts	The proposal is supported extensively by appropriate mathematical concepts and in details. The use of the concepts goes beyond classroom discussion.	The proposal is supported by sufficient mathematical concepts. The use of the concepts follows the way they were taken in classroom discussions.	The use of the concepts is in certain parts confusing and erroneous.	Little or no attempt has been made to utilize concepts discussed in class. .

ACTIVITY 35 Reflection on the Performance Task

DESCRIPTION: After doing the final task, look back at your experiences as you complete the log below.

1. How was your experience when doing the performance task?
2. What have you learned?
3. How can you use the skills you learned in the real world?

ACTIVITY 36 Synthesis Journal

The unit's lesson was on ____. One key idea was ___________________________________ ________________. This is important because ____________________________________ ______________________________________. Another key idea was ____. This is also important because ____________________________________. In summary, the unit's lesson ____ ____.

CONGRATULATIONS! You have completed this module. Before you go to the next module, you have to answer the following post-assessment.

POST-ASSESSMENT:

It's now time to evaluate your learning. Click on the letter of the answer that you think best answers the question. Your score will only appear after you answer all items. If you do well, you may move on to the next module. If your score is not at the expected level, you have to go back and take the module again.

1. Find out the number of ways in which 6 rings of different types can be worn in 3 fingers?
   A. 120 B. 720 C. 125 D. 729

2. 25 buses are running between two places P and Q. In how many ways can a person go from P to Q and return by a different bus?
   A. None of these B. 600 C. 576 D. 625

3. How many 6 digit telephone numbers can be formed if each number starts with 35 and no digit appears more than once?
   A. 720 B. 360 C. 1420 D. 1680

4. How many ways can 12 couples be arranged in a 12-cage ferris–wheel?
   A. 10! B. 11! C. 12! D. 13!

5. The arrangements of 3 shirts (red, yellow and green) hang in the closet are
   A. (r, y , g)
   B. (ryg, rgy, yrg, ygr, gry, gyr)
   C (gry)
   D. (ry, rg, yg, yr, gr, gy)

6. How many ways can 5 flower bases be arranged by 2's in a shelf?
   A. 5.4 ways B. 5.4.3 C. 5.4.3.2 D. 5.2

7. Six closed- friends pose for picture taking, how many ways can they stand beside each other?
   A. 6 B. 12 C. 30 D. 720

8. How is combination different from permutation?
   A. They are just the same since we are selecting from *n* objects taken *r* at a time.
   B. The result of using combination is always greater than permutation.
   C. Combination is different from permutation since order of elements in combination does not matter.
   D. Combination is different from permutation because arrangement of elements in combination matters a lot.

9. How many lines can you draw using 4 noncollinear points W, X, Y, and Z on a plane?
   A. 3
   B. 6
   C. 9
   D. 12

10. There are 15 juniors and 20 seniors in the Photography Club. The club is to send 7 representatives to the local student photography competitions. If the members of the club decide to send 3 juniors and 4 seniors, how many different groupings are possible? Which of the following computations gives the correct answer?
    A. ${}_{15}C_3 + {}_{20}C_4$
    B. ${}_{15}C_3 \bullet {}_{20}C_4$
    C. $\dfrac{{}_{15}C_3}{{}_{20}C_4}$
    D. ${}_{15}C_3 - {}_{20}C_4$

11. Which of the following is the correct process of deriving the combination formula from permutation formula?

a)

$$_nP_r = r!(_nC_r)$$

$$\frac{n!}{(n-r)!} = r!(_nC_r)$$

$$\frac{\frac{n!}{(n-r)!}}{r!} = {}_nC_r$$

$$_nC_r = \frac{\frac{n!}{(n-r)!}}{r!}$$

$$_nC_r = \frac{n!}{(n-r)!} \bullet \frac{1}{r!}$$

$$_nC_r = \frac{n!}{r!(n-r)!}$$

b)

$$_rP_n = r!(_nC_r)$$

$$\frac{r!}{(r-n)!} = r!(_nC_r)$$

$$\frac{\frac{r!}{(r-n)!}}{r!} = {}_nC_r$$

$$_nC_r = \frac{\frac{r!}{(r-n)!}}{r!}$$

$$_nC_r = \frac{r!}{(r-n)!} \bullet \frac{1}{r!}$$

$$_nC_r = \frac{1}{(r-n)!}$$

c) $_nP_r = n!(_nC_r)$

$$\frac{n!}{(n-r)!} = n!(_nC_r)$$

$$\frac{\frac{n!}{(n-r)!}}{n!} = {}_nC_r$$

$$_nC_r = \frac{\frac{n!}{(n-r)!}}{n!}$$

$$_nC_r = \frac{n!}{(n-r)!} \bullet \frac{1}{n!}$$

$$_nC_r = \frac{1}{(n-r)!}$$

d) $_nP_r = r!(_nC_r)$

$$\frac{n!}{(n+r)!} = r!(_nC_r)$$

$$\frac{\frac{n!}{(n+r)!}}{r!} = {}_nC_r$$

$$_nC_r = \frac{\frac{n!}{(n+r)!}}{r!}$$

$$_nC_r = \frac{n!}{(n+r)!} \bullet \frac{1}{r!}$$

$$_nC_r = \frac{n!}{r!(n+r)!}$$

12. You went into a sandwich shop. They are offering the following patties: chicken, tuna, beef, pork, and bacon. They are offering a discount of 10% from their original price of Php 150 for every three patties of your choice. You may choose three patties of the same flavor. How many possible choices can you make?
    A. 15
    B. 20
    C. 30
    D. 35

$$\frac{(n+r-1)!}{r!(n-1)!} = \frac{(5+3-1)!}{3!\,(5-1)!} = \frac{7!}{3!4!} = \frac{7 \bullet 6 \bullet 5 \bullet 4!}{3!4!} = \frac{7 \bullet 6 \bullet 5}{3!} = \frac{7 \bullet 6 \bullet 5}{6} = 35$$

13. A local NGO is spearheading a concert for a cause for the benefit of old people who are living in a local home for the aged. The chairman of the said NGO plans to create a 5-man and 5-woman members for the Ways and Means committee. There is no ranking involved nor positions to consider since all ten of them are members. There are 20 men and 15 women who volunteered to be part of the selection process. How can the chairman determine the possible number of committees from which he can select 10 committees?
    A. He can just randomly select from the qualified persons to lead the committee and count all the possibilities.
    B. He can use permutation to find the number of outcomes since every position in the committee is important.
    C. He can use combination to find the number of outcomes because every member is equal in rank and ordering their position does not matter.

D. He can just multiply the number of men by the number of women to determine the number of possible outcomes since the position of every member is not important.

14. Two coins are tossed.
Event A: The first coin tossed turned up heads.
Event B: The second coin tossed turned up tails

What is A U B?
A. {HH, HT, TH, TT}
B. {HH, HT, TT}
C. {HT, TH}
D. {HT}

15. f one card is drawn from a deck of 52 cards, what is the probability that it will be a club or a face card?
A. 25/52
B. 3/52 or 1/14
C. 17/52
D. 11/26

16. A club of 9 people wants to choose a board of 3 officers: President, Vice-President and Secretary. Assuming that the officers are chosen at random, what is the probability that the officers are Maria for President, Myrna for Vice-President and Amy for Secretary?
A. 1/729 B. 1/84 C. 1/504 D. 1/3

17. A survey is conducted on the students' preferences on ice cream. 70% like chocolate, and 35% like chocolate and strawberry.
What is the probability that a student chosen in random likes strawberry given that he likes chocolate?
A. 0.35 B. 0.5 C. 0.245 D. 0.4

18. You are hired in a restaurant as a supervisor. Part of your job is to plan the menu to be served everyday. You are tasked by the owner to come up with a recommendation of a list of different choices of food to be served. Your recommendation must include an explanation why some dishes are less or more often frequently served. What mathematical concepts are used in this situation?
A. combination and permutation
B. FCP and permutation
C. combination and FCP
D. combination, permutation and FCP

19. You are the organizer of a basketball tournament in your barangay. You are to submit to the tournament committee head the budget for the referees fees. To prepare the budget, you need to determine the number of games to be played. There are 6 teams who registered. If the single round robin system is followed, how many games will be played?

A. 6! = 720 B. ${}_{6}P_{2} = 30$ c. ${}_{6}C_{2} = 15$ d. $\frac{6!}{2} = 360$

20. As the number of electronic devices increases, so does the use of rechargeable batteries. A particular manufacturer produces batteries in lots of 100. In each lot, two of the batteries will be defective. The batteries are randomly packaged in groups of four batteries. You work in the Quality Control department of that company. You want to know the probability that all of the batteries in a package will not be defective. The goal is to make this probability as low as possible to minimize the warranty costs. What is the probability that all of the batteries in a package will not be defective?

A. $\frac{{}_{98}C_{4}}{{}_{100}C_{4}}$ B. $\frac{{}_{98}P_{4}}{{}_{100}P_{4}}$ C. $\frac{{}_{4}C_{2}}{{}_{98}C_{4}}$ D. $\frac{{}_{4}P_{2}}{{}_{98}P_{4}}$

GLOSSARY OF TERMS USED IN THIS LESSON:

CONDITIONAL PROBABILITY – is the probability that an event will occur under the condition that another event occurs first.

DEPENDENT EVENTS – are events where the outcome of one event affects the outcome of another event.

EVENT – is one or more outcomes of an experiment.

EXPERIMENT – is a situation involving chance or probability that leads to results called outcomes.

INDEPENDENT EVENTS – are events where the outcome of one event does not affect the outcome of the other events.

INTERSECTION of TWO EVENTS – is the set containing the outcomes common to both events.

MUTUALLY EXCLUSIVE EVENTS– are events that cannot happen at the same time.

OUTCOME – is a possible result of an experiment.

PROBABILITY – is the measure of how likely an event is.

SAMPLE SPACE – is the set containing all the possible outcomes of an event

UNION of TWO EVENTS - is the set containing all the outcomes of the two events.

REFERENCES AND WEBSITE LINKS USED IN THIS LESSON:

ARTICLES

http://www.mathgoodies.com/lessons/vol6/intro_probability.html
- This link gives the definition and examples of experiment, outcomes, events and probability.

http://www.mathgoodies.com/lessons/vol6/sample_spaces.html
- This link gives the definition and examples of sample space.

http://www.onlinemathlearning.com/mutually-exclusive-events.html

- The link discusses what are mutually exclusive events and how to compute their probabilities.

http://www.mathgoodies.com/lessons/vol6/independent_events.html
- This link gives the formula for finding probability of independent events and examples.
-

http://www.mathgoodies.com/lessons/vol6/conditional.html
- This link shows sample problems on conditional probability.

http://faculty.mdc.edu/mshakil/Probability_&_Counting_Flow_Chart.pdf
- This is a flowchart showing how to compute for the probability of union and intersection of events.

VIDEOS

https://www.youtube.com/watch?v=lFyEjgfSwsU
- This video defines independent and dependent events.

https://www.youtube.com/watch?v=WeiQgFEOCqo
- This video gives the formula for finding the probability of dependent events and examples.

http://www.shmoop.com/video/independent-and-dependent-events
- This video compares independent and dependent events with real-life examples.

http://www.virtualnerd.com/algebra-2/probability-statistics/compound-events/conditional-probability-contingency-tables/conditional-probability-definition
- This video defines conditional probability and presents examples.

http://www.dnatube.com/video/11865/What-is-Conditional-Probability
- This video shows how to compute for conditional probability.

http://study.com/academy/lesson/applying-conditional-probability-independence-to-real-life-situations-lesson-quiz.html#lesson
- This video shows an example of a real-life situation using conditional probability.

http://www.virtualnerd.com/algebra-2/probability-statistics/compound-events/conditional-probability-contingency-tables/conditional-probability-example
- This video is a tutorial showing one example of conditional probability in a real world problem.

https://www.youtube.com/watch?v=RNH_O2QvkWA

- The video shows how to compute for probability using permutation and combination.

https://www.youtube.com/watch?v=Ed1waMzBSzg
- This video shows examples of finding probability using permutation and combination.

INTERACTIVE WEBSITES:

http://glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-02-833240-7&chapter=13&lesson=2
- This link is an online quiz on Venn diagrams.

http://www.onlinemathlearning.com/independent-probability.html
- This link includes a worksheet on finding probability of independent events.

http://www.onlinemathlearning.com/dependent-probability.html
- This link is an interactive quiz on finding probability of dependent events.

http://www.regentsprep.org/regents/math/algebra/apr3/praccond.htm
- This link is an interactive quiz on finding conditional probability.

http://algebralab.org/lessons/lesson.aspx?file=Algebra_ConditionalProbability.xml
- This link is an interactive quiz on finding conditional probability.

http://braingenie.ck12.org/subjects/118
- This link includes interactive quizzes on probability of union and intersection of events with solutions and videos.