

# LEARNING MODULE

Mathematics | G8 | Q3

# Geometry



## NOTICE TO THE SCHOOLS

This learning module (LM) was developed by the Private Education Assistance Committee under the GASTPE Program of the Department of Education. The learning modules were written by the PEAC Junior High School (JHS) Trainers and were used as exemplars either as a sample for presentation or for workshop purposes in the JHS In-Service Training (INSET) program for teachers in private schools.

The LM is designed for online learning and can also be used for blended learning and remote learning modalities. The year indicated on the cover of this LM refers to the year when the LM was used as an exemplar in the JHS INSET and the year it was written or revised. For instance, 2017 means the LM was written in SY 2016-2017 and was used in the 2017 Summer JHS INSET. The quarter indicated on the cover refers to the quarter of the current curriculum guide at the time the LM was written. The most recently revised LMs were in 2018 and 2019.

The LM is also designed such that it encourages independent and self-regulated learning among the students and develops their 21st century skills. It is written in such a way that the teacher is communicating directly to the learner. Participants in the JHS INSET are trained how to unpack the standards and competencies from the K-12 curriculum guides to identify desired results and design standards-based assessment and instruction. Hence, the teachers are trained how to write their own standards-based learning plan.

The parts or stages of this LM include Explore, Firm Up, Deepen and Transfer. It is possible that some links or online resources in some parts of this LM may no longer be available, thus, teachers are urged to provide alternative learning resources or reading materials they deem fit for their students which are aligned with the standards and competencies. Teachers are encouraged to write their own standards-based learning plan or learning module with respect to attainment of their school's vision and mission.


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MATHEMATICS 8

# Module 3: Geometry

INTRODUCTION AND FOCUS QUESTION(S):



REASONING
CONCLUSION

Do you think it is possible to make a valid conclusion without even going through the process of investigating? What will you do if you are asked to make a decision that will affect many people? Many aspects in our life involve decisions and proof.

Geometry deals with reasoning and how to prove something. It gives us the proper way on how to prove a certain thing. In this module, you will find out the answer to the question, “What is the best argument in justifying claims or proofs?”

LESSONS AND COVERAGE:

In this module, you will examine this question when you take the following lessons:

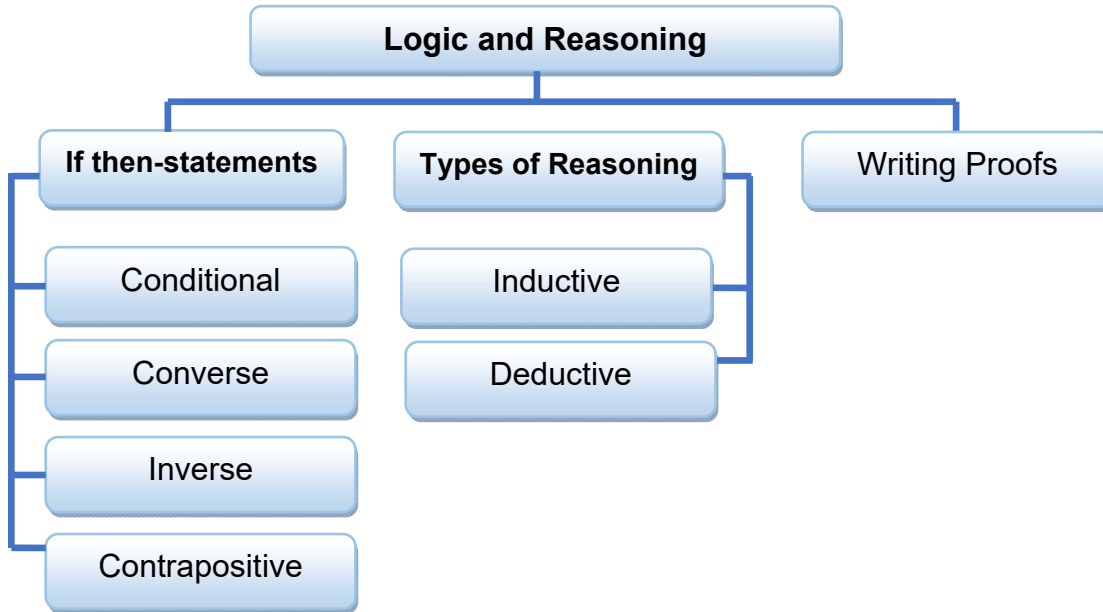
- Lesson 1* – If - then Statements
- Lesson 2* – Inductive and Deductive Reasoning  
– Writing Proof

In these lessons, you will learn the following:

<i>Lesson 1</i>	<ul style="list-style-type: none"> <li>➤ determine the relationship between the hypothesis and the conclusion of an if-then statement.</li> <li>➤ transform a statement into an equivalent if-then statement.</li> <li>➤ determine the inverse, converse, and contrapositive of an if-then statement.</li> <li>➤ illustrate the equivalences of: (a) statement and its contrapositive; and (b) the converse and inverse of a statement.</li> </ul>
<i>Lesson 2</i>	<ul style="list-style-type: none"> <li>➤ use inductive or deductive reasoning in an argument.</li> </ul>
	<ul style="list-style-type: none"> <li>➤ write a proof (both direct and indirect)</li> </ul>

**MODULE MAP:**

To guide you in this module, below is a simple map of the above lessons you will cover:



**EXPECTED SKILLS:**

To do well in this module, you need to remember and do the following:

Define terms that are unfamiliar to you.

1. Explore websites that would be of great help for your better understanding of the lessons.
2. Take down notes of the important concepts in your journal.
3. Perform and complete the exercises provided.
4. Collaborate with the teacher and peers.

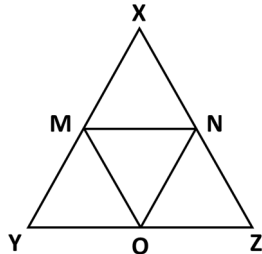
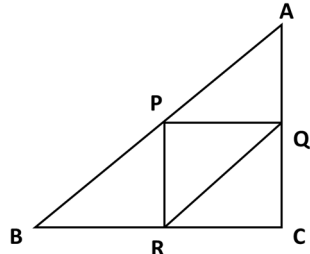
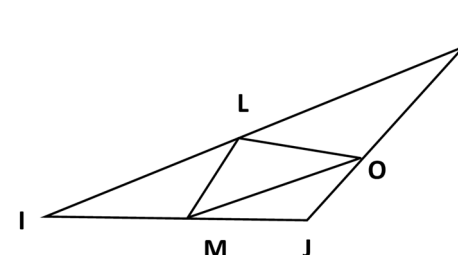
## PRE-ASSESSMENT

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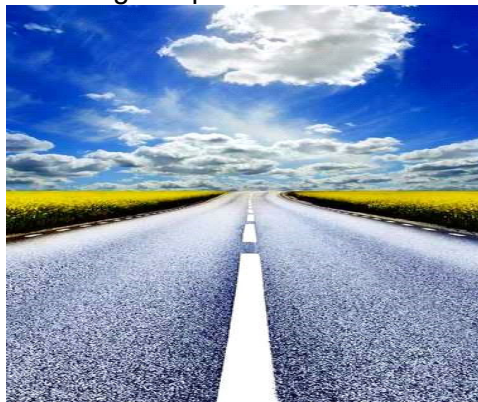
*Let's find out how much you already know about this module. Click on the letter that you think best answers the question. Please answer all items. After taking this short test, you will see your score. Take note of the items that you were not able to correctly answer and look for the right answer as you go through this module.*

- Which of the following statements is NOT true about conditional statements?
  - It is written in an if-then form.
  - The first part of the statement is the hypothesis and the second part of the statement is the conclusion.
  - A conditional has related conditionals such its converse, inverse, and contrapositive.
  - A conditional statement is defined to be always true.
- From the choices select the hypothesis of the statement.  
***If the baby is wearing blue booties, then it is a baby boy.***
  - If the baby is wearing blue booties.
  - The baby is wearing blue booties
  - Then it is a baby boy.
  - It is a baby boy
- Which of the following statements is FALSE?
  - The inverse and contrapositive are formed using negations of the hypothesis and the conclusion.
  - If a given statement is true, the converse and inverse are necessarily true.
  - A conditional statement and its contrapositive are logically equivalent.
  - The converse and inverse of a conditional statement are logically equivalent.
- Which of the following is the if-then form of the statement below?  
"Equilateral triangles are equiangular."
  - If a triangle is equilateral, then it is equiangular.
  - If a triangle is equiangular, then it is equilateral.
  - If a triangle is not equilateral, then it is not equiangular.
  - If a triangle is not equiangular, then it is not equilateral.
- Which of the following is the converse conditional below?  
"If two angles have the same measure, then the angles are congruent."
  - If two angles are congruent, then the angles have the same measure.
  - If two angles do not have the same measure, then the angles are not congruent.
  - If two angles are not congruent, then the angles do not have the same measure.
  - If two angles do not have the same measure, then the angles are congruent.
- Which of the following is the inverse of the conditional below?  
"If segments are congruent, then the segments have equal length."
  - If the segments have equal length, then the segments are congruent.
  - If the segments are not congruent, then the segments do not have equal length.

- C. If the segments do not have equal length, then the segments are not congruent.  
 D. If the segments are congruent, then the length of the segments are not equal.
7. Which of these statements is logically equivalent to the conditional below?  
 If two points are collinear, then the points lie on the same line.  
 A. If the two points lie on the same line, then the points are collinear.  
 B. If two points are not collinear, then the points do not lie on the same line.  
 C. If the two points do not lie on the same line, then the points are not collinear.  
 D. If the two points lie on the same line, then the points are collinear.
8. Determine which of the following situations is NOT based deductive reasoning.  
 A. Taking a person's life is always wrong. Capital punishment involves taking a person's life. Therefore, capital punishment is always wrong.  
 B. Six in ten children who are allowed to drink at home with their parents become alcoholics later in life. Therefore, attitudes towards drinking are formed by others near to us.  
 C. Every human being has rights. John is a human being, therefore, John has rights.  
 D. A customer must have an advantage card in order to avail of additional discounts in all SM malls nationwide. Mae is availing an additional discount for the item she purchased. Mae has an SM advantage card.
9. What generalization can be derived from the given cases?

<p style="text-align: center;"><u>Case 1</u></p>  <p>M, N, O are midpoints          The perimeter of <math>\triangle MON</math> is twice the perimeter of <math>\triangle XYZ</math>.</p>	<p style="text-align: center;"><u>Case 2</u></p>  <p>P, Q, R are the midpoints          The perimeter of <math>\triangle PQR</math> is twice the perimeter of <math>\triangle ABC</math>.</p>
<p><u>Case 3</u></p>  <p>L, M, O are the midpoints          The perimeter of <math>\triangle LMO</math> is twice the perimeter of <math>\triangle HIJ</math>.</p>	

- A. In a triangle, twice the midpoints form equal angles.  
 B. In a triangle, the side opposite a certain midpoint is twice longest side.  
 C. The perimeter of a triangle formed by the midpoints is twice the perimeter of the triangle.  
 D. The perimeter of a triangle is twice the perimeter of the triangle formed by joining the midpoints of its side.
10. Which of the following stated conclusions is valid based on the given information?  
***If two angles are vertical, then they are congruent.***
- A. Given:  $\angle A$  and  $\angle B$  are vertical angles  
 Conclusion:  $\angle A \cong \angle B$
- B. Given:  $\angle C$  and  $\angle D$  are nonadjacent angles  
 Conclusion:  $\angle C \cong \angle D$
- C. Given:  $\angle E \cong \angle F$   
 Conclusion:  $\angle E$  and  $\angle F$  are vertical angles
- D. Given:  $\angle G \cong \angle H$   
 Conclusion:  $\angle G$  and  $\angle H$  are nonadjacent angles
11. Which of the following statements is FALSE?
- A. If two angles are complementary, then the measure of each acute angle is 45.  
 B. If the sum of the measures of two angles is 90, then the angles are complementary.  
 C. If the sum of the measures of two angles is 180, then the angles are supplementary.  
 D. If two angles are nonadjacent and are formed by the intersecting lines, the angles are vertical angles.
12. Which of the following statements has a FALSE truth value?
- A. If two lines intersect, then they are coplanar.  
 B. If two lines intersect, then two pairs of vertical angles are formed.  
 C. If two lines intersect, then they are perpendicular to each other.  
 D. If two lines intersect, then they intersect at exactly one point.
13. Deduce conclusion from the given picture below.



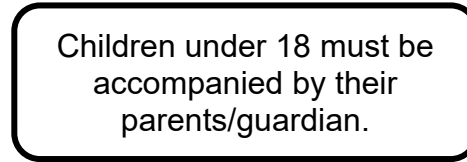
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- A. The sides of the road are intersecting.  
 B. The sides of the road are parallel.  
 C. The sides of the road are concurrent  
 D. The sides of the road has end.

14. What is the best conclusion for the following statements?
1. If you run everyday, then you will not be a couch potato.
  2. Alex run everyday.
- A. Alex will not be a couch potato.
  - B. Alex will be a couch potato.
  - C. Alex will run everyday.
  - D. No conclusion can be made.
15. *Your classmate told you that:*
1. *Students need to be self-disciplined.*
  2. *If you are self-disciplined, then you have good study habits.*
  3. *If you have good study habits, then you get good grades.*

What does your classmate want you to realize?

- A. Students who are self-disciplined have good study habits.
  - B. Students who are self-disciplined get good grades.
  - C. Students who have good study habits are self-disciplined people.
  - D. Students who get good grades will be given honor certificates.
16. Two reminders are posted for the movie that is now on showing on a cinema.



You really wanted to watch the movie but you are only 15 years old. What will you do?

- A. I will go to the cinema alone and I will tell them that I am 18 years old.
  - B. I will go to the cinema with my friends.
  - C. I will go to the cinema and ask any adult stranger to pretend that he/she is my parent.
  - D. I will ask my parents to join me in watching the movie.
17. If you were asked to make a mathematical investigation regarding a certain phenomenon, which of the following criteria would you take into consideration to assure the success of your investigation?
- A. visual aid, powerpoint presentation and grammar
  - B. grammar, content and the use of technology
  - C. coherence, clarity and powerpoint presentation
  - D. conclusion, clarity and coherence
18. As the head writer of Math Magazine, you were tasked to present an investigation about the current issue in math that will be the magazine's next theme. It will be evaluated first by the chief editor and executive committee. What do you think of assessing your investigation?
- A. coherence, clarity and sound judgment
  - B. coherence, clarity and visual aid
  - C. sound judgment, clarity and the use of technology
  - D. visual aid, the use of technology and coherence



19. You are a debater and days from now you will represent your school in an inter-school debate contest. How will your arguments be prepared?
- A. It must be coherent.
  - B. It must be clear.
  - C. It must be coherent and clear.
  - D. It must follow the rules of inductive reasoning only.
20. You were told by your family doctor that Paracetamol is a medicine given to patients with fever. Your 2 year-old sister has fever and you were told to give the lowest dosage for her age bracket as shown below.

Age	Dose (mL)
3 months to 1 year	2.5 to 5 mL
1 to 5 years	5 to 10 mL
6 to 12 years	10 to 20 mL

How many mL of Paracetamol will you give to your sister?

- A. 2.5 mL
- B. 3 mL
- C. 5 mL
- D. 10 mL

## Lesson 1: Reasons Behind Reasoning



### EXPLORE

Let's start the module by answering the problem below.

#### ACTIVITY 1. Agree or Disagree?

Bella, Kaela, and Rhea are having conversation below.

Girls, let's go out tonight and dine at Café Andre. They serve the most delicious Filipino dishes in town.

Based from your experience Rhea, I can conclude that Café Andre is not the best place in town that serves the most delicious Filipino dishes. I think it would be better if we try another place to dine. Do you agree with me?

I think not Bella, I've been in that Café last week. The Bulalo I've ordered was not really good.

#### Process Questions:

1. What are the three girls talking about?

2. Who among them is reasonable? Why

3. Why is reasoning important in our daily conversation?

4. How are clear and coherent arguments and proofs formulated? How are they proved or disproved?

**ACTIVITY 2. Anticipation Reaction Guide**

Respond to each statement twice: once before the lesson and again after the lesson.

Write **A** if you agree with the statement.

Write **B** if disagree with the statement.

<i>Before Lesson</i>	<i>Statement</i>	<i>After Lesson</i>
	1. A conditional statement and its related conditionals can be rewritten in “if-then” form.	
	2. If then statements should always be written in the “if-then” form.	
	3. The “if” part of a conditional statement is its conclusion.	
	4. The contrapositive is logically equivalent to the original conditional.	
	5. A conditional statement is defined to be true unless a true hypothesis leads to a false conclusion.	
	6. When a conditional statement and its converse are both true, you can write them as one statement called a biconditional statement.	

**End of EXPLORE:**

You gave your initial ideas on the questions and statements given above.

Let's find out how others would answer the above questions and statements and compare their ideas to our own. As you compare, you will find out if your ideas are in line with the standard. You will also learn other concepts which will help you complete a required project found at the end. This project is about making false and true arguments/statements. We will start by doing the next activity.



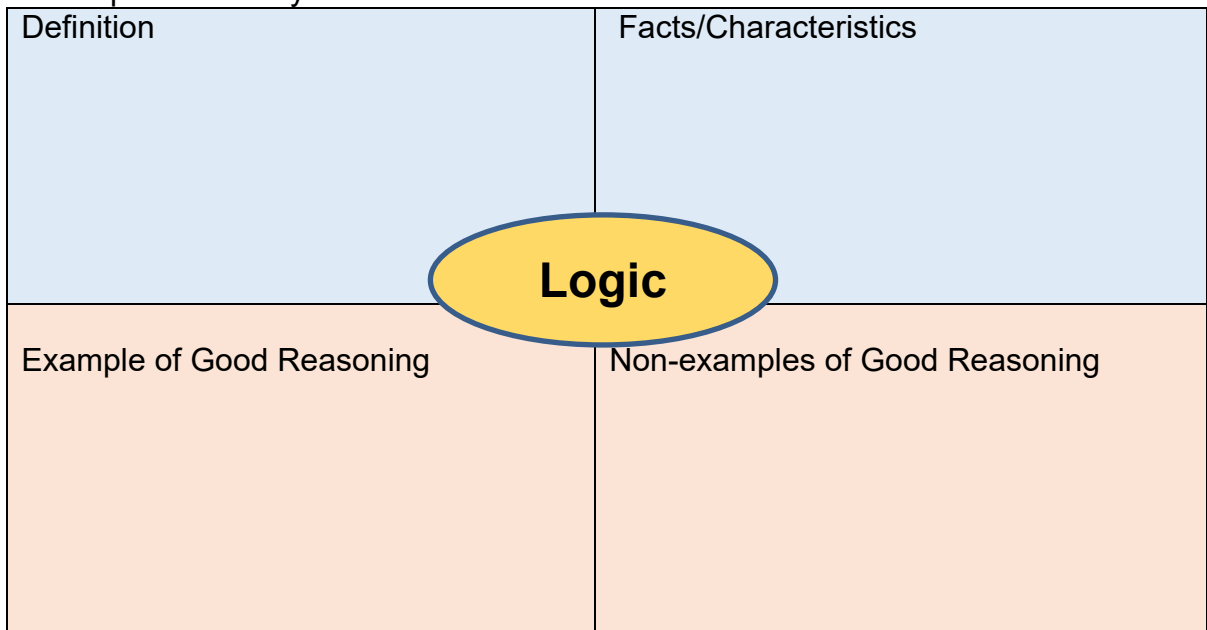
**FIRM-UP**

Your goal in this section is to learn and understand key concepts of an if-then statement and when to conclude that it is true or false.

**ACTIVITY 3. The Basic of Concepts of Logic**

- A. Read an article on <http://www.manyworldsoflogic.com/whatIsLogic.html> and read its subtopics:
  - a. Introduction
  - b. But Isn't All Reasoning Equal?
  - c. The Value of Logic
  - d. What Does a Standard of Reasoning Look Like?

Then complete the Frayer Model below.



**Process Questions:**

1. Why is reasoning important in our daily conversation?

2. *How are clear and coherent arguments and proofs formulated? How are they proved or disproved?*



Now that you have learned the meaning of logic and arguments, it is time for you to learn the how statements used in arguments formulated. In the following activities, you will learn conditional statements and its related conditionals.

**ACTIVITY 4. If-then Statements**

- A. Read an article on <http://www.mathgoodies.com/lessons/vol9/conditional.html> which contains input/idea on conditional statement and 5-item exercises with immediate feedback.
- B. Read an article and watch 3 videos in <http://www.ck12.org/book/CK-12-Basic-Geometry-Concepts/r6/section/2.3/> about conditional statements. After these, click on **Practice** on the upper part of the page to answer a **15-item interactive quiz**. Immediate feedback of your answers will be given.

The screenshot shows a learning module interface. At the top left, there are navigation arrows and the title '2.3: If-Then Statements'. Below the title, it says 'Difficulty Level: Basic | Created by: CK-12'. There are two breadcrumb links: 'CK-12 Basic Geometry Concepts' and 'Reasoning and Proof'. On the right, there is a progress indicator showing '0%' and a 'Practice If-Then Statements' button with a 'Practice' sub-button. Below the title, there are four tabs: 'Read', 'Practice', 'Resources', and 'Details'. The 'Read' tab is currently selected.

Jot down important concepts in your Ever Note.

**Process Questions:**

1. What is a conditional statement?

2. How will you identify the hypothesis and conclusion of a conditional statement?

3. When will a conditional becomes false?

4. How did you score in the 15-item interactive quiz?

5. If you scored high, what were the reasons why you were able to achieve it? If you scored low, what went wrong along the way?



Do not proceed to the next part of this activity if you did not score well in the 15-item interactive quiz. Watch the three videos again and study the examples. Then take the 15-item interactive quiz again.

**Post your answers in the Discussion Forum. Discuss with your classmates and compare it to yours.**

**Sum-up what you have learned and have gained from your classmates by completing each statement below.**

A conditional statement is an \_\_\_\_\_ statement, symbolized by \_\_\_\_\_ in which p is a \_\_\_\_\_ and q is a \_\_\_\_\_. The conditional is defined to be true unless a \_\_\_\_\_ hypothesis leads to a \_\_\_\_\_ conclusion.

**ACTIVITY 5. More Skills Practice**

A. Identify the hypothesis and the conclusion for each of the following. Then determine its truth value. If it false, give a counterexample.

1. If you are a Filipino, then you live in the Philippines.		
Hypothesis:	Conclusion:	<b>Truth Value</b>
Counterexample:		

2. If the weather is hot, then it is summer.		
Hypothesis:	Conclusion:	<b>Truth Value</b>
Counterexample:		

3. If you pass the requirements on time, then your teacher will be happy.		
Hypothesis:	Conclusion:	<b>Truth Value</b>
Counterexample:		

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4. If a polygon is triangle, then it has three sides.		
Hypothesis:	Conclusion:	<b>Truth Value</b>
Counterexample:		

5. If $x + 12 = 25$ , then $x = 13$ .		
Hypothesis:	Conclusion:	<b>Truth Value</b>
Counterexample:		

B. Rewrite each statement into an “if-then” statement. Then determine its truth value. If it false, give a counterexample.

1. A punctual student is never late in coming to school.	
If-then form:	<b>Truth Value</b>
Counterexample:	

2. Children who are well feed are healthy.	
If-then form:	<b>Truth Value</b>
Counterexample:	

3. A person who has low self-esteem is always bullied.	
If-then form:	<b>Truth Value</b>
Counterexample:	



4. A quadrilateral has four sides.	
If-then form:	Truth Value
Counterexample:	

5. Perpendicular lines are two intersecting lines that form 90-degree angles.	
If-then form:	Truth Value
Counterexample:	

C. You are in your way going to Davao City. Along the way, you can see the sign “ Speed Limit: 60 kph” and somebody told you that “***If you drive faster than 60 kilometers per hour, then you will receive a speeding ticket.***” Determine the truth value of the following statement for each set of conditions. If it is false, justify your answer.

1. You drive 70 kilometers per hour, and you receive a speeding ticket.
2. You drive 62 kilometers per hour, and you do not receive a speeding ticket.
3. You drive 58 kilometers per hour and you receive a speeding ticket.

**Process Questions:**

1. When will a conditional's truth value becomes true?

2. When does it become false?

3. How are clear and coherent arguments and proofs formulated? How are they proved or disproved?

**D.**

- a. *True.*
- b. *False. I should receive a speeding ticket since my speed is more than the given limit.*
- c. *False. I should not receive a speeding ticket since my speed is lower than the given limit.*

**ACTIVITY 6.**

**The Converse, Inverse, Contrapositive, and Biconditional Statements**

- A. Read an article and watch 3 videos in <http://www.ck12.org/book/CK-12-Basic-Geometry-Concepts/r6/section/2.4/> about the converse, inverse, and contrapositive of a conditional statement. After these, click on **Practice** to answer a **15-item interactive quiz**. Immediate feedback of your answers will be given.

Use also the table below to guide you determine the truth value of the related conditionals if the original conditional is either true or false.

$p$	$q$	Conditional $p \rightarrow q$	Converse $q \rightarrow p$	Inverse $\sim p \rightarrow \sim q$	Contrapositive $\sim q \rightarrow \sim p$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

- B. Watch a video from <https://www.youtube.com/watch?v=GovRqzKh6Kw>. This video shows how to write biconditional statements and how to separate a biconditional into its two conditional statements.

Jot down important concepts in your Ever Note.

**Process Questions:**

- How will you rewrite the converse, inverse, and contrapositive of a given conditional statement?

- How are the inverse and contrapositive statements formed?

- How do you call statements with the same truth value?

- Which of the related conditionals of a conditional statement is logically equivalent to it?

- Converse
- Inverse
- Contrapositive

5. When can you write a biconditional statement?

6. How did you score in the 15-item interactive quiz?

7. If you scored high, what were the reasons why you were able to achieve it?  
If you scored low, what went wrong along the way?



Do not proceed to the next part of this activity if you did not score well in the 15-item interactive quiz. Watch the three videos again and study the examples. Then take the 15-item interactive quiz again.

**Post your answers in the Discussion Forum. Discuss with your classmates and compare it to yours.**

**Sum-up what you have learned and have gained from your classmates by completing each statement below.**

To write the converse of a given conditional statement, \_\_\_\_\_ the hypothesis and conclusion. The inverse and contrapositive are formed using \_\_\_\_\_ of the hypothesis and the conclusion.

If a given statement is true, the \_\_\_\_\_ and \_\_\_\_\_ are not necessarily true. However, the \_\_\_\_\_ of a true conditional is always \_\_\_\_\_, and the \_\_\_\_\_ of a false conditional is always \_\_\_\_\_. If the original statement and its converse are both true then the statement is \_\_\_\_\_.

Statements with the \_\_\_\_\_ truth values are said to be logically equivalent. Hence, a conditional and its \_\_\_\_\_ are logically equivalent so as its \_\_\_\_\_ and \_\_\_\_\_.

**ACTIVITY 7. More Practice**

A. Write the converse, inverse, and contrapositive of the following conditional statements. Then tell whether the conditional, its converse, its inverse, or its contrapositive is *True* or *False*. If it is *False*, give a counterexample.

1.

Conditional:	If you are in Manila, then you are in Luzon.
True or False with Counterexample:	
Converse:	
True or False with Counterexample:	
Inverse:	
True or False with Counterexample:	
Contrapositive:	
True or False with Counterexample:	

2.

Conditional:	If two line segments have the same lengths, then they are congruent.
True or False with Counterexample:	
Converse:	
True or False with Counterexample:	
Inverse:	
True or False with Counterexample:	
Contrapositive:	
True or False with Counterexample:	

3.

Conditional:	If $x^2 = 36$ , then $x = 6$ .
True or False with Counterexample:	
Converse:	
True or False with Counterexample:	
Inverse:	
True or False with Counterexample:	
Contrapositive:	

True or False with Counterexample:	
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B. Write each biconditional as a conditional and its converse.

Bi-conditional	1. Two angles are complementary if and only if the sum of their measures is 90.
Conditional	
Converse	
Bi-conditional	2. Two lines are parallel if and only if they do not intersect.
Conditional	
Converse	
Bi-conditional	3. $x + 7 = 13$ if and only if $x = 6$ .
Conditional	
Converse	
Biconditional	4. $x^2 = 100$ if and only $x = -10$ or $x = 10$ .
Conditional	
Converse	
Bi-conditional	5. A person is morally upright if and only if he/she is living out the moral teachings.
Conditional	
Converse	

**C. Interactive Quiz**

Click <http://www.regentsprep.org/regents/math/geometry/gp2/pracrc.htm>  
 In this quiz, you will be given a statement and will be asked of its related conditional. You will be asked to choose the correct answer from the given choices. A prompt will appear if you clicked the correct or wrong answer. Just click the **EXPLANATION** if you want to be clarified of your answer.

D. You are evaluating a conditional statement in which the hypothesis is true, but the conclusion is false. Is the inverse of the statement true or false? Explain your reasoning.

My Reasoning:

**Process Questions:**

1. Among the three items in **A**, which item has all true truth values?

2. Write its equivalent biconditional statement.

3. Can you conclude that the original statement is valid/justifiable? Why?

4. How about the two other original conditionals? Can you conclude that each is valid/justifiable?

5. How are clear and coherent arguments and proofs formulated? How are they proved or disproved?

**ACTIVITY 8. Skills Readiness Check**

Reflect on the level of your performance for this lesson. Check the first column if you **need more practice** or you are now **ready to move on to the next activity**.

<u>I Need more practice</u> (if most of your answers are incorrect)	<u>I am ready to move on to the next activity</u> (if you incur only few errors in the first three skills practice)
Go back to the videos you have viewed to review the input and examples. Then answer again the three skills practices.	You may proceed to the next activity

## End of Firm- Up

In this section, the discussion was about identifying the if and then parts of conditional statements. You write its converse, inverse, and contrapositive. Also you dealt with biconditional statements and write it as a conditional and as a converse statements.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.



## DEEPEN

Your goal in this section is to have a deeper understanding on if-then statements.

### ACTIVITY 9. Misconception Check

- Nicka and Paula are evaluating the conditional "If 15 is a prime number, then 20 is divisible by 4." Both think that the **conditional is true**, but their reasoning differs. Is either of them correct? Explain.

#### **Nicka**

The conclusion is true, because 20 is divisible by 4, so the conditional is true.

#### **Paula**

The hypothesis is false, because 15 is not a prime number, so the conditional is true.

***My answer and explanation.***

**Process Question:**  
***How are arguments proved or disproved?***



**ACTIVITY 10. Making Proofs**

Show that the following statements are logically valid or not. Make use of the conditional, converse, inverse, contrapositive and biconditional statements in making your proofs. If the conditional and its related conditionals is/are false, write a counter-example.

<p><b>Statement:</b> <i>An animal that can fly is a bird.</i></p> <p><b>Conditional:</b></p> <p><b>Converse:</b></p> <p><b>Inverse:</b></p> <p><b>Contrapositive:</b></p> <p><b>Biconditional:</b></p> <p><b>Conclusion:</b> <i>The given statement is logically_____.</i></p>	<p><b>Statement:</b> <i><math>x = 28</math> when <math>x - 5 = 23</math>.</i></p> <p><b>Conditional:</b></p> <p><b>Converse:</b></p> <p><b>Inverse:</b></p> <p><b>Contrapositive:</b></p> <p><b>Biconditional:</b></p> <p><b>Conclusion:</b> <i>The given statement is logically_____.</i></p>
--	--

**Process Questions:**

1. How will you conclude that an argument is logically valid or not? What are bases?
2. How are arguments proved or disproved?

**Post Your answer to the process questions in the Discussion Forum and discuss with your classmates.**

**ACTIVITY 11. Test of Understanding**

**Instruction:** In the previous section, we looked at different statements. Let's put together in the table below our answers to the essential question that we asked for each problem.

Conclude if the given situation is logically true or false.

Essential Question:	Situation 1	Situation 2	Situation 3
<b>How are clear and coherent arguments formulated? How are they</b>	Conclude whether the given situation is logically true or false.	Conclude whether the given situation is logically true or false.	Conclude whether the given situation is logically true or false.
	Liza reads an article that says: <b>When you</b>	Dino was taught by her sister that:	Maya reads in a paper that: <b>People who tan</b>

<p><i>proved</i> <i>disproved?</i></p>	<p><b>exercise regularly, you'll be in good shape.</b> From this information, can she conclude that she'll be in good shape if she exercises regularly?</p> <p><b>Conditional:</b></p> <p><b>Converse:</b></p> <p><b>Inverse:</b></p> <p><b>Contrapositive:</b></p> <p>The statement "<b>When you exercise regularly, you'll be in good shape..</b>" was _____ (proven to be true or disproved) because the _____ statements' truth values are (true or false). Therefore Liza _____ (can or cannot) conclude that she'll be in good shape if she exercises regularly.</p> <p>The formulation of the given statement was _____ (based or not based) on sound logic and reasoning.</p>	<p><b>Supplementary angles are two angles whose sum of their measures is 180.</b> From this concept, can he conclude that two angles 85 and 95 are supplementary?</p> <p><b>Conditional:</b></p> <p><b>Converse:</b></p> <p><b>Inverse:</b></p> <p><b>Contrapositive:</b></p> <p>The statement '<b>Supplementary angles are two angles whose sum of their measures is 180.</b>' was _____ (proven to be true or disproved) because the _____ statements' truth values are (true or false). Therefore, Dino _____ (can or cannot) conclude that the two angles measuring 85 and 95 are supplementary because their sum is 180.</p> <p>The formulation of the given statement was _____ (based or not based) on sound logic and reasoning.</p>	<p><b>themselves under the sun for extended periods are at increased risk of skin cancer.</b> From this information, can she conclude that she will not increase her risk of skin cancer if she avoids tanning for extended periods of time?</p> <p><b>Conditional:</b></p> <p><b>Converse:</b></p> <p><b>Inverse:</b></p> <p><b>Contrapositive:</b></p> <p>The statement <b>People who tan themselves under the sun for extended periods are at increased risk of skin cancer</b> _____ (proven to be true or disproved) because the _____ statements' truth values are (true or false). Therefore, Maya _____ (can or cannot) conclude that she will not increase her risk of skin cancer if she avoids tanning for extended periods of time.</p> <p>The formulation of the given statement was _____ (based or not based) on sound logic and reasoning.</p>
--	---	--	--

**Process Questions:**

1. What do all answers in the above table have in common?

2. Are the truth values of the conditionals and related their related conditionals the same? If yes, explain. If not, what causes the difference?

3. Complete the following statements and support your answer with the examples from the above problems.

***Clear and coherent arguments and proofs depend on ...***

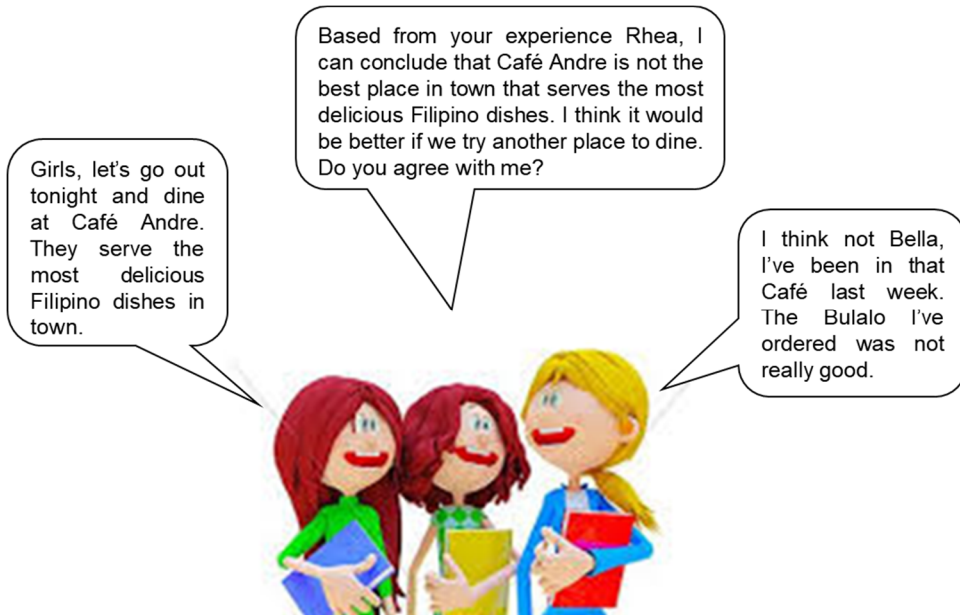
***A statement logically false if ...***

***Supporting reasons and examples:***

**ACTIVITY 12. Revisiting Activity No. 1: Agree or Disagree.**

Revisit your answers to Activity No. 1. Make necessary revisions if you think your previous answers were not convincingly correct.

Bella, Kaela, and Rhea are having conversation below.



**Process Questions:**

1. What are the three girls talking about?

2. Who among them is reasonable? Why

3. Why is reasoning important in our daily conversation?

4. How are clear and coherent arguments and proofs formulated? How are they proved or disproved?

**ACTIVITY 13. Revisiting Activity No. 2: Anticipation Reaction Guide**

Read the statement again, and write your final answer on the **After Lesson** column.

Write **A** if you agree with the statement.  
Write **B** if you disagree with the statement.

<i>Before Lesson</i>	<i>Statement</i>	<i>After Lesson</i>
	1. A conditional statement and its related conditionals can be rewritten in “if-then” form.	
	2. If then statements should always be written in the “if-then” form.	
	3. The “if” part of a conditional statement is its conclusion.	
	4. The contrapositive is logically equivalent to the original conditional.	
	5. A conditional statement is defined to be true unless a true hypothesis leads to a false conclusion.	
	6. When a conditional statement and its converse are both true, you can write them as one statement called a biconditional statement.	

*Process Questions:*

1. *Are your new answers different from your previous answers? Discuss.*
2. *How did you score?*
3. *Can you now say that you are knowledgeable on logic and reasoning? Why?*

My new score is \_\_\_\_\_ (higher or lower) than my previous scores because

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

### End of DEEPEN

In this section, you were given activities to assess the level of concepts and skills mastery of the if-then statements. You also applied it in making valid conclusions if given real-life situations.

What new realizations do you have about the topic? What new connections have you made for yourself?



### TRANSFER

Your goal in this section is apply your learning to real life situations. You will be given a practical task which will demonstrate your understanding.

#### **ACTIVITY 14. The Logic Behind Advertising Slogans**

Watch the video <https://www.youtube.com/watch?v=RqgCBFzHoec> on how to do the project. Pass your output to your teacher in your next face-to-face meeting.

#### **Process Questions:**

1. *What have you learned from this activity?*

2. *Would you buy a product based on the ad alone?*

3. *How will you prove that ads are not logically true?*

4. *How are clear and coherent arguments formulated? How are they proved or disproved?*

*I've learned that...*

**ACTIVITY 15. Truth is a Must**

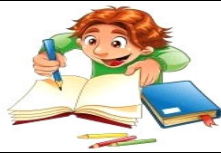
1. Write a conditional statement in which the converse, inverse, and contrapositive are all true. Explain your reasoning.
2. Write a conditional statement in which the converse, and inverse are all false. Explain your reasoning.

***Process Questions: How are clear and coherent arguments formulated? How are they proved or disproved?***

***Place your output in a short bond paper and pas it to your teacher in your next face-to-face meeting.***

***Post your answer in the Process Question in the Discussion Forum.***

**ACTIVITY 16. Cornell's Note**



Complete the table below to sum-up and assess what you have learned in this module.

Topics	Things I learned
Question I want to be answered.	

## Lesson 2: Inductive and Deductive Reasoning Writing Proofs

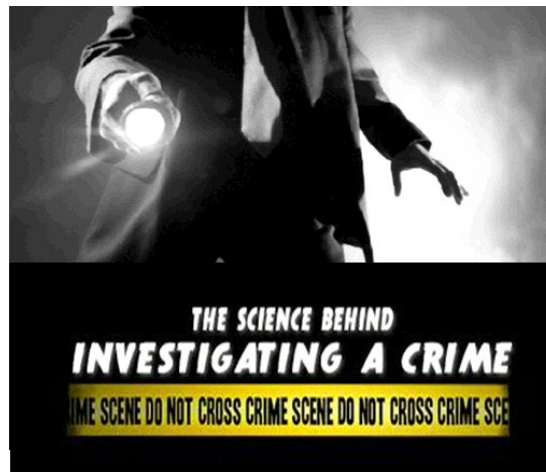


### EXPLORE

Let's start the lesson by answering the problem below.

#### ACTIVITY 1. The Detective

---



Watch videos on how detectives solve a case. Click

1. <https://www.youtube.com/watch?v=zrTUkfEjPkA>
2. <https://www.youtube.com/watch?v=rDbQbJCLTMl>

#### Process Questions:

1. How do detectives in the video gather evidences of the crime?

2. What other evidences might a detective collect?

3. What kind of arguments do they use to in presenting their proofs or evidences?



4. What kind of reasoning do they use in identifying who is most likely responsible for a crime?

5. How are clear and coherent arguments and proofs formulated? How are they proved or disproved?

**ACTIVITY 2. Anticipation Reaction Guide**

Respond to each statement twice: once before the lesson and again after the lesson.

Write **A** if you agree with the statement.  
Write **B** if you disagree with the statement.

<i>Before Lesson</i>	<i>Statement</i>	<i>After Lesson</i>
	1. Inductive reasoning is the process of drawing conclusions based on many clues, or pieces of evidence.	
	2. Scientist and doctors make use of the inductive process in making and recommending treatments.	
	3. Deductive reasoning is the process of drawing conclusion from facts.	
	4. Deductive reasoning is the basis of the scientific method.	
	5. Indirect Proof is also known as Proof by contradiction.	

## End of Explore

You gave your initial answers to the questions and statements given above.

Let's find out how others would answer the above and compare their ideas to your own. As you compare, you will find out if your ideas are in line with the standard. You will also learn other concepts which will help you complete a required project found at the end. This project is about making arguments for a debate.



## FIRM-UP

Your goal in this section is to learn and understand the key concepts of inductive and deductive reasoning and writing proofs. You will also be asked to solve problems that would require the applications of the concepts mentioned above. Then keep on thinking about this question:  
**How are clear and coherent arguments and proofs formulated?**  
**How are they proved or disproved?**

### ACTIVITY 3. Inductive Reasoning from Patterns

Read first the Process Questions before watching the three videos from <http://www.ck12.org/book/CK-12-Basic-Geometry-Concepts/section/2.1/>. Read also the Guidance Article and try to answer Guided Practice.

Take down notes in your Ever Note or in your notebook.

Click on the red-colored Practice button on top of the page or click this link <http://www.ck12.org/book/CK-12-Basic-Geometry-Concepts/section/2.1/> to practice inductive reasoning from patterns. This is a **15-item interactive quiz** which provides immediate feedback. Just click start if you are ready to answer. You can get a hint if you have difficulty in answering a problem.

#### Process Questions:

1. What is inductive reasoning?

2. How do you call a conclusion drawn from inductive reasoning? How is it disproved?

3. What are the two types of examples given in the page that provide good examples of inductive reasoning?

4. How did you score in the 15-item interactive quiz?

5. If you scored high, what were the reasons why you were able to achieve it? If you scored low, what went wrong along the way?



Do not proceed to the next part of this activity if you did not score well in the 15-item interactive quiz. Watch the three videos again and study the examples. Then take the 15-item interactive quiz again.

**Post your answers in the Discussion Forum. Discuss with your classmates and compare it to yours.**

**Sum-up what you have learned and have gained from your classmates by completing each statement below.**

Inductive reasoning entails making conclusions based upon \_\_\_\_\_ and \_\_\_\_\_.

A \_\_\_\_\_ is a conclusion reached by using inductive reasoning. It can be disproved using a \_\_\_\_\_.

\_\_\_\_\_ and \_\_\_\_\_ provide good example of inductive reasoning.



In the previous activity, you have learned about inductive reasoning and conjectures. In this part you will learn additional problems involving visual and number patterns and a real-life situation. From these, you will make your own conjectures and counterexamples.

**Example 1:**

Use inductive reasoning to find the sum of the first 20 even numbers.

**Solution:**

1. Find the first few sums. Notice that each sum is the square of the number of terms plus the number of terms.
2. Look for a pattern.
3. Make a conjecture

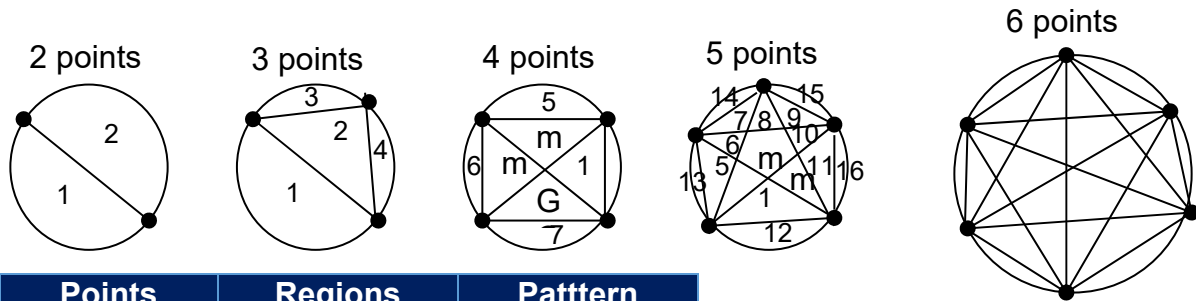
n (number of terms)		sum	pattern
1	2	2	$1^2 + 1$
2	2+4	6	$2^2 + 2$
3	2+4+6	12	$3^2 + 3$
4	2+4+6+8	20	$4^2 + 4$
			<b>CONJECTURE: <math>n^2 + n</math></b>



The sum of the first 20 even numbers is  $20^2 + 20 = 420$ . The expression  $n^2 + n$  will help you find easily the sum of the nth terms of even numbers.

**Example 2:**

If six points on a circle are joined by as many segments as possible, how many non-overlapping regions will the segments determine?



Points	Regions	Pattern
2	2	2 x 0
3	4	2 x 2
4	8	4 x 2
5	16	8 x 2
6	31	16 x 2



The table shown above shows the data for 2, 3, 4 and 5 points. The number of regions appears to double at each stage. Inductive reasoning would predict that there are 32 regions for 6 points on the circle. But as shown in the fifth diagram (**counterexample**) above, there are only 31 regions. In this case, the **conjecture** is **incorrect**.

**Example 3: Geometric Conjecture**

M is the midpoint of  $\overline{AB}$ . Make a conjecture and draw a figure to illustrate your conjecture.

**Solution:**

Given: M is the midpoint of  $\overline{AB}$ .

Conjecture:  $\overline{AM} \cong \overline{BM}$

Illustration:



**Example 4: Relating to the real world.**

Melvin has started working out regularly. When he first started exercising, he could do 10 push-ups. After the first week, he could do 14 push-ups. After the second week, he could do 19, and after the third, week he could do 25.



How many push-ups would you predict he will be able to do after the fifth week of working out? Are you sure about your prediction? Why or why not?

Week	Number of push-ups
1	14
2	19
3	25
4	32
5	40



The predicted number of push-ups that Melvin can do after the fifth week is 40.

**Counterexample:** He may or may not be able to follow the pattern since humans have limitations. If he is sick, most probably he cannot do 40 push-ups or if he is physically conditioned he can do more than 40 push-ups.

Since there is a counterexample, then the conjecture is **not valid**.

**Process Questions:**

1. How is inductive reasoning used in predicting the unknown value in each of the examples above?
2. How will you disprove a conjecture?
3. Why are the conjectures drawn from real-life situations is not valid?
4. **How are clear and coherent arguments and proofs formulated? How are they proved or disproved?**

Post your answers to these questions in the Discussion Forum.

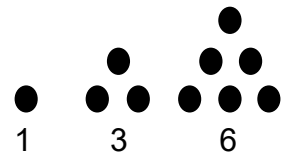
**ACTIVITY 4. Skills Practice**

Use inductive reasoning to answer the given problems.

- Find the sum of the first 20 odd numbers and make a conjecture out of it. Complete the table below.

n (number of terms)		sum	pattern
1	1		
2	1+3		
3	1+3+5		
4	1+3+5+7		
			<b>CONJECTURE:</b>

- A triangular number can be represented by a triangular arrangement of dots. The first three triangular numbers are 1, 3 and 6.



- What are the next three triangular numbers?

- What is the tenth triangular number?

- Which of the following algebraic expressions represents the nth triangular number?

A.  $n(n + 1)$       B.  $n(n - 2)$       C.  $\frac{n(n+1)}{2}$       D.  $\frac{n(n-1)}{2}$

The conjectures below are not valid. Give a counterexample for each.

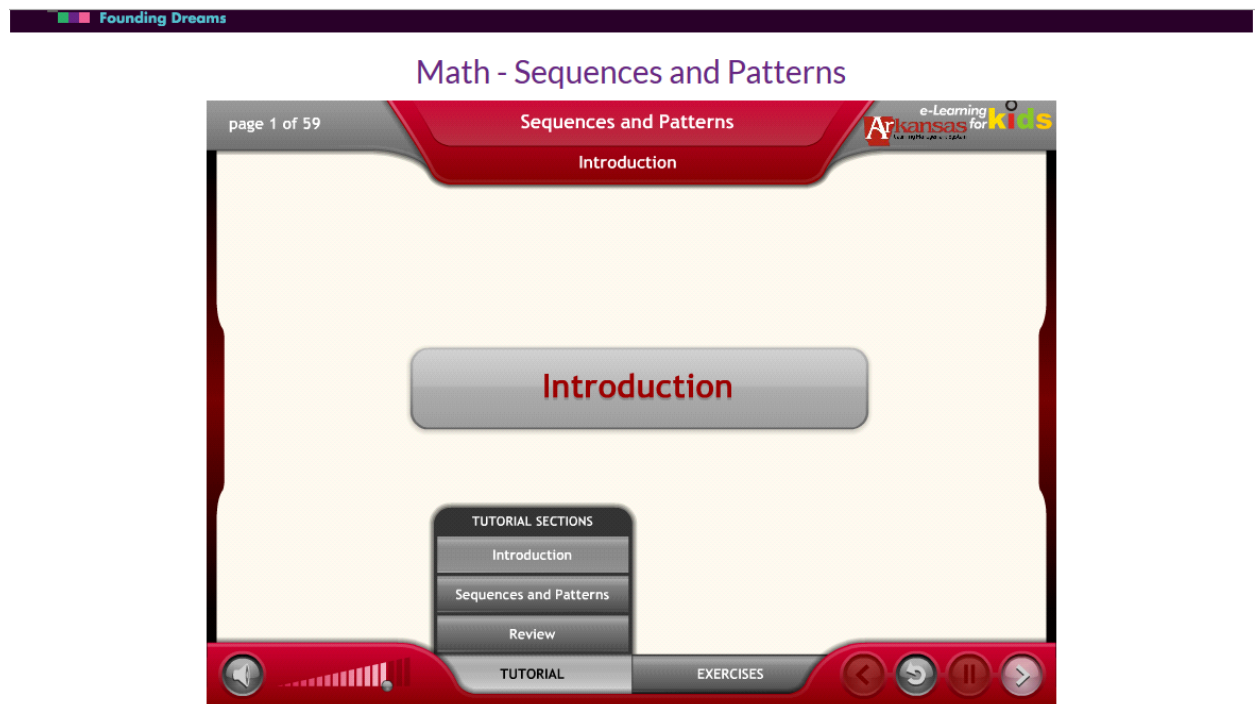
3. Given:  $\angle 1$  and  $\angle 2$  are complementary angles.  
Conjecture:  $\angle 1$  and  $\angle 2$  form a right angle.

4. Given: Points A, B, C, and D  
Conjecture: Points A, B, C, and D are collinear

5. Given:  $n$  is a real number  
Conjecture:  $n^2 + 1$  is an even number

**Interactive practice**

<http://www.e-learningforkids.org/math/lesson/sequences-and-patterns/>





Founding Dreams

Math - Sequences and Patterns

page 37 of 59 Sequences and Patterns Exercises

Exercises

TUTORIAL EXERCISES



You have just learned about inductive reasoning and conjectures. If you want to prove a conjecture, you need to use another type of reasoning which is *Deductive Reasoning*.

### ACTIVITY 5. Deductive Reasoning

Read first the Process Questions before watching the three videos from <http://www.ck12.org/geometry/Deductive-Reasoning/lesson/Deductive-Reasoning/> Read also the Guidance Article and try to answer Guided Practice.

Take down notes in your Ever Note or in your notebook.

Click on the red-colored **Practice** button on top of the page or click this link <http://www.ck12.org/book/CK-12-Basic-Geometry-Concepts/section/2.1/> to practice inductive reasoning from patterns. This is a **10-item interactive practice** which provides immediate feedback. Just click start if you are ready to answer. You can get a hint if you have difficulty in answering a problem.

**Process Questions:**

1. What is deductive reasoning?

2. What does the law of detachment say?

3. What does the law of contrapositive say?

4. What does the law of syllogism say?

5. How important are the laws of detachment, contrapositive, and syllogism in making arguments?

6. How did you score in the 10-item interactive practice?

7. If you scored high, what were the reasons why you were able to achieve it? If you scored low, what went wrong along the way?



Do not proceed to the next part of this activity if you did not score well in the 10-item interactive practice. Watch the three videos again and study the examples. Then take the **10-item interactive practice** again. Click on **Keep on Practicing** if you want have more practice problems.

**Post your answers in the Discussion Forum. Discuss with your classmates and compare it to yours.**

**Sum-up what you have learned and have gained from your classmates by completing each statement below.**

Inductive reasoning entails drawing conclusions from \_\_\_\_\_.

The law of detachment states that: \_\_\_\_\_.

The law of contrapositive states that: \_\_\_\_\_.

The law of syllogism states that: \_\_\_\_\_.

The three laws are useful in making \_\_\_\_\_ conclusions.

**ACTIVITY 6. More Practice**

Determine the logical conclusion and state which law you used (Law of Detachment, Law of Contrapositive, or Law of Syllogism). If no conclusion can be drawn, write “no conclusion.”

1. People who vote for Jane Wannabe are smart people. I voted for Jane Wannabe.

2. If Rae is the driver today then Maria is the driver tomorrow. Ann is the driver today.

3. All equiangular triangles are equilateral.  $\triangle ABC$  is equiangular.

4. If North wins, then West wins. If West wins, then East loses.

5. If  $z > 5$ , then  $x > 3$ . If  $x > 3$ , then  $y > 7$ .

6. If I am cold, then I wear a jacket. I am not wearing a jacket.

7. If it is raining outside, then I need an umbrella. It is not raining outside.

8. If a shape is a circle, then it never ends. If it never ends, then it never starts. If it never starts, then it doesn't exist. If it doesn't exist, then we don't need to study it.

9. If you text while driving, then you are unsafe. You are a safe driver.

10. If you wear sunglasses, then it is sunny outside. You are wearing sunglasses.

11. If you wear sunglasses, then it is sunny outside. It is cloudy.

12. I will clean my room if my mom asks me to. I am not cleaning my room.

13. Write the symbolic representation of #8. Include your conclusion. Does this argument make sense?

14. Write the symbolic representation of #10. Include your conclusion.

15. Write the symbolic representation of #11. Include your conclusion.

**Process Questions:**

1. How did you find the activity?
2. Can you now make logical arguments and conclusions?
3. ***How are clear and coherent arguments and proofs formulated?  
How are they proved or disproved?***

**Sum-up what you have learned and have gained from your classmates by completing each statement below.**  
*Clear and coherent arguments and proofs are formulated by/through\_\_\_\_\_.*  
*It can be proved or disproved if/by/through\_\_\_\_\_.*



Sometimes it is confusing to tell whether arguments/statements follow an inductive or deductive reasoning. Learn how to identify each by doing the next set of activities.

**ACTIVITY 7. Inductive and Deductive Reasoning: A Comparison**

Watch a video and read more examples from [http://www.ck12.org/geometry/Types-of-Reasoning/lesson/Inductive-and-Deducting-Reasoning/?referrer=featured\\_content](http://www.ck12.org/geometry/Types-of-Reasoning/lesson/Inductive-and-Deducting-Reasoning/?referrer=featured_content) .

Take down notes in Ever note or in your notebook.

**Process Questions:**

1. How do inductive and deductive reasoning differ from each other?
2. Choose one concrete situation where inductive and deductive reasoning can be used in making conclusions/generalization.
3. ***How are clear and coherent arguments and proofs formulated? How are they proved or disproved?***

*Post your answers to these questions in the Discussion Forum and discuss with your classmates. Fill- in the box below as an evidence of your understanding.*

**Sum-up what you have learned and have gained from your classmates by completing each statement below.**

*Inductive reasoning differs from deductive reasoning because*  
 \_\_\_\_\_.

*A concrete example where each can be differentiated is shown below:*  
*Inductive:* \_\_\_\_\_  
*Deductive:* \_\_\_\_\_.

*Clear and coherent arguments and proofs are formulated by/through \_\_\_\_\_.*

*It can be proved or disproved if/by/through \_\_\_\_\_.*

**ACTIVITY 8. Skills Practice: Interactive Quiz**

Take the interactive quiz for inductive and deductive reasoning by clicking <http://www.thatquiz.org/tq/previewtest?F/Z/J/V/O3UL1355243858> . Take the practice test first before clicking the Take This Now button.

**ACTIVITY 9. More Practice**

For each question, state whether the reasoning is an example of inductive or deductive logic.

Reasoning	Logic
1. All housecats are felines. All felines have claws. Therefore all housecats have claws.	
2. My dog has fleas. My neighbor’s dog has fleas. Therefore all dogs must have fleas.	
3. All cows like hay. My cow will like hay.	
4. My Mac laptop is fast. All Mac laptops are fast.	
5. My tennis shoes are comfortable. My friend’s tennis shoes are comfortable. All tennis shoes are comfortable.	
6. The scalloped potatoes I took from the oven were cheesy. The enchiladas I took from the oven were cheesy. If I take cookies from the oven, they will be cheesy.	
7. Everything cooked on the stove gets hot. If I cook macaroni on the stove, it will get hot.	
8. iPads are popular. iPhones are popular. Every phone or tablet is popular.	
9. Roses are red. Tomatoes are red. All red things come from plants.	

Reasoning	Logic
10. Rock music is loud. Sayber listens to rock music. Sayber's music is loud.	
11. Milk is good with cookies. Snicker doodles are cookies. Milk is good with snicker doodles.	
12. Hummers use a lot of gas. Suburbans use a lot of gas. Large SUV's use a lot of gas.	
13. My garden has pumpkins. My dad's garden has pumpkins. All gardens have pumpkins.	
14. Prob and Stats students are smart. You are a Prob and Stats student. You are smart.	
15. Students who study hard get good grades. You are a student who studies hard. You will get good grades.	



You have just learned about inductive and deductive reasoning. In the part of the lesson, you will apply deductive reasoning in writing proofs: direct and indirect.

### ACTIVITY 10. Direct and Indirect Proofs

Read the content of the pages for each type of proof and watch videos embedded on it.

- a. Direct proof/ two-column proof. [http://www.ck12.org/geometry/Two-Column-Proofs/lesson/Two-Column-Proofs/?referrer=featured\\_content](http://www.ck12.org/geometry/Two-Column-Proofs/lesson/Two-Column-Proofs/?referrer=featured_content)

Watch 4 other related videos from [http://www.ck12.org/geometry/Two-Column-Proofs/?referrer=featured\\_content&by=all&difficulty=all#assessment](http://www.ck12.org/geometry/Two-Column-Proofs/?referrer=featured_content&by=all&difficulty=all#assessment)

- b. Indirect proof. [http://www.ck12.org/geometry/Indirect-Proof-in-Algebra-and-Geometry/lesson/Indirect-Proof-Intermediate/?referrer=concept\\_details](http://www.ck12.org/geometry/Indirect-Proof-in-Algebra-and-Geometry/lesson/Indirect-Proof-Intermediate/?referrer=concept_details)

Jot down notes in Ever Note.



Process Questions:

1. *What is a direct proof?*
2. *What is the advantage of using a direct proof? What is its disadvantage?*
3. *What is an indirect proof?*
4. *What is the advantage of using an indirect proof? What is its disadvantage?*
5. ***How are clear and coherent arguments and proofs formulated? How are they proved or disproved?***

Post your answers in the Discussion Forum and discuss with your classmates. Fill in the box below to write your understanding about direct and indirect proofs?

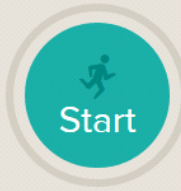



*I've learned that ...*

**ACTIVITY 11. Interactive Practice**

**Direct Proof/Two-column proof:**

- a. [http://www.ck12.org/geometry/Two-Column-Proofs/?referrer=featured\\_content&by=all&difficulty=all#assessment](http://www.ck12.org/geometry/Two-Column-Proofs/?referrer=featured_content&by=all&difficulty=all#assessment) Click on start.

Two-Column Proofs

<b>PRACTICE</b>		
 <b>GOAL:</b>	Get 10 correct answers	
 <b>PROGRESS:</b>	0%	
 <b>TIME:</b>	Unlimited	

b. <http://feromax.com/cgi-bin/Provelt.pl>

## Provelt! Two-Column Proofs Practice Tool


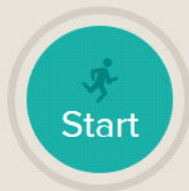


Select a proof from the list below to get started. To see and record your progress, log in [here](#).

Title	Difficulty	Solved By	Date Added
<a href="#">Complementary Angles 1</a>	easy	306 (84%)	2008-12-27
<a href="#">Complementary Angles 2</a>	easy	307 (84%)	2008-12-27
<a href="#">Supplementary Angles 1</a>	easy	301 (82%)	2008-12-27
<a href="#">Supplementary Angles 2</a>	medium	266 (73%)	2008-12-27
<a href="#">Straight Angle 1</a>	easy	256 (70%)	2008-12-24
<a href="#">Vertical Angles 1</a>	medium	235 (64%)	2008-12-27
<a href="#">Parallel Lines 1</a>	easy	145 (40%)	2008-12-27
<a href="#">Parallel Lines 2</a>	easy	141 (39%)	2008-12-27
<a href="#">Parallel Lines 3</a>	easy	137 (38%)	2008-12-27
<a href="#">Parallel Lines 4</a>	hard	101 (28%)	2008-12-28
<a href="#">Converse of Parallel Lines 1</a>	easy	111 (30%)	2008-12-28
<a href="#">Converse of Parallel Lines 2</a>	medium	88 (24%)	2008-12-28
<a href="#">Converse of Parallel Lines 3</a>	medium	69 (19%)	2008-12-28
<a href="#">Converse of Parallel Lines 4</a>	hard	44 (12%)	2008-12-28
<a href="#">Converse of Parallel Lines 5</a>	medium	65 (18%)	2008-12-28
<a href="#">Parallelogram 1</a>	easy	37 (10%)	2008-12-29
<a href="#">Parallelogram 2</a>	easy	33 (09%)	2008-12-29
<a href="#">Parallelogram 3</a>	medium	27 (07%)	2008-12-29
<a href="#">Parallelogram 4</a>	easy	31 (08%)	2008-12-29
<a href="#">Parallelogram 5</a>	hard	17 (05%)	2008-12-29
<a href="#">Parallelogram 6</a>	easy	24 (07%)	2008-12-29
<a href="#">Parallelogram 7</a>	medium	21 (06%)	2008-12-29
<a href="#">Parallelogram 8</a>	hard	13 (04%)	2008-12-29
<a href="#">Parallels and Perpendiculars 1</a>	medium	19 (05%)	2009-01-14
<a href="#">Parallels and Perpendiculars 2</a>	medium	16 (04%)	2009-01-18
<a href="#">Parallels and Perpendiculars 3</a>	medium	15 (04%)	2009-01-18
<a href="#">Triangle Sum Theorem 1</a>	easy	22 (06%)	2009-01-14
<a href="#">Triangle Sum Theorem 2</a>	medium	15 (04%)	2009-01-18
<a href="#">Triangle Sum Theorem 3</a>	hard	16 (04%)	2009-01-18
<a href="#">Triangle Exterior Angle Theorem 1</a>	easy	16 (04%)	2009-01-18
<a href="#">Quadrilateral Sum Theorem 1</a>	easy	13 (04%)	2009-01-18
<a href="#">Similar Triangles 1</a>	easy	21 (06%)	2009-02-08
<a href="#">Similar Triangles 2</a>	easy	13 (04%)	2009-02-08
<a href="#">Similar Triangles 3</a>	medium	7 (02%)	2009-02-08
<a href="#">Similar Triangles 4</a>	hard	6 (02%)	2009-02-08
<a href="#">Congruent Triangles 1</a>	easy	23 (06%)	2009-02-08
<a href="#">Congruent Triangles 2</a>	medium	17 (05%)	2009-02-08
<a href="#">Congruent Triangles 3</a>	hard	13 (04%)	2009-02-08
<a href="#">Congruent Triangles 4</a>	hard	9 (02%)	2009-02-08

Indirect Proof: <http://www.ck12.org/geometry/Indirect-Proof-in-Algebra-and-Geometry> Click on start.

Indirect Proof in Algebra and Geometry

PRACTICE

 <b>GOAL:</b>	Get 10 correct answers	
 <b>PROGRESS:</b>	0%	
 <b>TIME:</b>	Unlimited	

Process Questions:

- How did you score in each Practice Test?

- If you scored well, what contributed to your success? If not, what hinders you to get high scores?

**ACTIVITY 12. Skills Evaluation**

Listed below are the skills you need to master before moving to the next part of the lesson. Write **YES** if you have a full understanding of the lessons and write **NOT** if you are still confused.

1. I know how to arrive in conclusions/conjectures using inductive reasoning.	
2. I know how to apply the laws of detachment, contrapositive, and syllogism in making logical conclusions.	
3. I can recognize whether the given set of statements follow an inductive or deductive reasoning.	
4. I can prove statements using the direct/two-column proof.	
5. I can prove statements using the indirect proof/proof by contradiction.	

For the skills which you have written **NOT**, go back to the videos and answer again interactive practices.



**DEEPEN**

Your goal in this section is to relate inductive and deductive reasoning to real-life problems or situations. Before proceeding to the next section, consider the questions: **How are clear and coherent arguments and proofs formulated? How are they proved or disproved?**

**ACTIVITY 13. Test of Understanding**

**B. The Power of Inductive and Deductive Reasoning in Solving Problems**

**Instruction:** In the previous section, we looked at different problems about sequences and series. Let's put together in the table below our answers to the essential question that we asked for each problem.

Essential Question	Article 1	Article 2	Article 3
<p><b>How are clear and coherent arguments and proofs formulated? How are they proved or disproved?</b></p>	<p><b>The Science of Induction (Inductive Reasoning)</b> Read an article and watch two videos from <a href="http://www.ck12.org/geometry/inductive-reasoning-from-patterns/rwa/The-Science-of-Induction/">http://www.ck12.org/geometry/inductive-reasoning-from-patterns/rwa/The-Science-of-Induction/</a></p> <p><b>Process Questions:</b></p> <ol style="list-style-type: none"> <li>1. What kind of reasoning do doctors, naturalists and inventors use in the practice of their professions?</li> </ol>	<p><b>Give Me One Reason</b> <a href="http://www.ck12.org/geometry/Two-Column-Proofs/rwa/Give-Me-One-Reason?referrer=assessment_dashboard_page">http://www.ck12.org/geometry/Two-Column-Proofs/rwa/Give-Me-One-Reason?referrer=assessment_dashboard_page</a></p> <p><b>Process Questions:</b></p> <ol style="list-style-type: none"> <li>1. How would you formulate or build your statements in order to convince people to see things your way?</li> <li>2. What kind of proof will you use to make sure that the reasons you give are true?</li> <li>3. <u>How are these arguments (statements/ conjectures/conclusion</u></li> </ol>	<p><b>A contradictory Evidence</b> Read an article and watch two videos from <a href="http://www.ck12.org/geometry/Indirect-Proof-in-Algebra-and-Geometry/rwa/Contradictory-Evidence/">http://www.ck12.org/geometry/Indirect-Proof-in-Algebra-and-Geometry/rwa/Contradictory-Evidence/</a></p> <p><b>Process Questions:</b></p> <ol style="list-style-type: none"> <li>1. What kind of reasoning do lawyers use in defending his/her client?</li> <li>2. In what way are the proofs presented when they lack direct evidence?</li> <li>3. <u>How are these arguments (statements/ conjectures/conclus</u></li> </ol>

	2. How are these arguments (statements/conjectures/conclusions) proved or disproved?	<u>s) proved or disproved?</u>	<u>ions) proved or disproved?</u>
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**Process Questions:**

1. Look at your answers to the essential question in the above table. What do all the answers have in common?

2. How do the answers differ?

3. Complete the following statement and support your answer with the examples from the above problems.

Clear and coherent arguments and proofs depend on ...

They are proved or disproved if/by ...

**C. Deductive Reasoning: In Making Decisions and Conclusions**

**Instruction:** In the previous section, we looked at different problems about sequences and series. Let's put together in the table below our answers to the essential question that we asked for each problem.

Essential Question	Situation 1	Situation 2	Situation 3								
<p><b>How are clear and coherent arguments and proofs formulated? How are they proved or disproved?</b></p>	<p>Liz has an appointment to see a financial advisor on the fifteenth floor of an office building. When she gets to the building, the people at the front desk tell her that if she wants to go to the fifteenth floor, then she must take the red elevator. While looking for the elevator, a guard informs her that if she wants to find the red elevator he must find the replica of the Virgin Mary. When she finally got to the fifteenth floor, her financial advisor greeted her asking, "What can you say about the replica of the Virgin Mary?" How did Liz's financial advisor conclude that Liz must have seen the replica of the Virgin Mary?</p> <p><b>The situation can be written in the</b></p>	<p>In the hospital, doctors may prescribe medicine to help patients get better. They use a dose chart like the one shown below to determine the correct amount of medicine the patient should take.</p> <table border="1" style="margin: 10px auto;"> <thead> <tr> <th style="background-color: #d9e1f2;">Age</th> <th style="background-color: #d9e1f2;">Dose (mL)</th> </tr> </thead> <tbody> <tr> <td>Children over 3 years</td> <td>5</td> </tr> <tr> <td>Children over 6 years</td> <td>10</td> </tr> <tr> <td>Children over 12 years</td> <td>15</td> </tr> </tbody> </table> <p>If you have an age of 10 years old, what dose will a doctor give you based on the chart?</p> <p><b>The situation can be written in the</b></p>	Age	Dose (mL)	Children over 3 years	5	Children over 6 years	10	Children over 12 years	15	<p>Two signs are posted on a horror train.</p> <div style="border: 1px solid gray; padding: 5px; margin: 10px auto; width: fit-content;"> <p style="text-align: center;">NO ONE <b>UNDER 5</b> ALLOWED</p> </div> <div style="border: 1px solid gray; padding: 5px; margin: 10px auto; width: fit-content;"> <p style="text-align: center;">NO ONE <b>UNDER</b> 8 ALLOWED WITHOUT A PARENT</p> </div> <p>Inside the train, you find a child with his parent. What can you deduce about the age of the child based on the rules?</p> <p><b>The situation can be written in the following logical statements:</b></p> <div style="border: 2px solid blue; height: 100px; width: 100%; margin: 10px auto;"></div> <p><b>A conclusion can be deduced by using the Law of</b></p> <hr style="width: 20%; margin-left: auto; margin-right: 0;"/> <p><b>Can the conclusion</b></p>
Age	Dose (mL)										
Children over 3 years	5										
Children over 6 years	10										
Children over 12 years	15										

	<b><i>following logical statements:</i></b>	<b><i>following logical statements:</i></b>	<b><i>deduced above be disproved? How?</i></b>
	<div style="border: 2px solid blue; height: 100px; width: 100%;"></div> <p><b><i>Liz's financial advisor was able to conclude or reason that Liz must have seen the replica of the Virgin Mary by using the Law of _____.</i></b></p> <p><b><i>Can Liz's advisor's conclusion be disproved? How?</i></b></p>	<div style="border: 2px solid blue; height: 100px; width: 100%;"></div> <p><b><i>The doctor can determine the dose he/she has to give to the patient by using the Law of _____.</i></b></p> <p><b><i>Can a doctor alter the dosage which is not based on the given chart?</i></b></p>	

**Process Questions:**

1. Look at your answers to the essential question in the above table. What do all the answers have in common?

2. How do the answers differ?

3. Complete the following statement and support your answer with the examples from the above problems.

Clear and coherent arguments and proofs depend on \_\_\_\_\_.

It can be disproved if /by \_\_\_\_\_.

**ACTIVITY 14. Mastery Test**

The law says that if you are under 21, then you are not allowed to drink alcoholic beverages and if you are under 18, then you are not allowed to vote. For each problem, give the possible ages of the person described or state the person cannot exist.

- a. Emil cannot drink wine legally but is allowed to vote.

- b. Andre cannot vote legally but can drink beer legally.

- II. Explain how doctors make use of inductive and deductive reasoning in treating patients' illnesses?

**Scoring Rubric:**

- 3 – The answer was correct with a clear explanation.
- 2 – The answer was correct but explanation was not so clear.
- 1 – The answer was correct but explanation is not given.
- 0 – No answer and explanation was given.

- III. Write 2 examples (for each law) to illustrate the correct use of the Law of Detachment, Law of Contrapositive, and the Law of Syllogism.

Law of Detachment	Law of Contrapositive	Law of Syllogism



**IV. Proving**

1. Prove the following statements directly/using a two-column proof.

Fill in the blanks in the proofs below.

1. Given:  $\angle ABC \cong \angle DEF$  and  $\angle GHI \cong \angle JKL$

Prove:  $m\angle ABC + m\angle GHI = m\angle DEF + m\angle JKL$

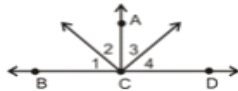
Statement	Reason
1.	1. Given
2. $m\angle ABC = m\angle DEF$ $m\angle GHI = m\angle JKL$	2.
3.	3. Addition <i>PoE</i>
4. $m\angle ABC + m\angle GHI = m\angle DEF + m\angle JKL$	4.

2. Given:  $M$  is the midpoint of  $\overline{AN}$  .  $N$  is the midpoint  $\overline{MB}$

Prove:  $AM = NB$

Statement	Reason
1.	Given
2.	Definition of a midpoint
3. $AM = NB$	

Prove:  $\angle 2 \cong \angle 3$



Statement	Reason
1. $\overline{AC} \perp \overline{BD}$ , $\angle 1 \cong \angle 4$	1.
2. $m\angle 1 = m\angle 4$	2.
3.	3. $\perp$ lines create right angles
4. $m\angle ACB = 90^\circ$ $m\angle ACD = 90^\circ$	4.
5. $m\angle 1 + m\angle 2 = m\angle ACB$ $m\angle 3 + m\angle 4 = m\angle ACD$	5.
6.	6. Substitution
7. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	7.
8.	8. Substitution
9.	9. Subtraction <i>PoE</i>
10. $\angle 2 \cong \angle 3$	10.

2. Prove the following statements indirectly.

- If  $n$  is an integer and  $n^2$  is even, then  $n$  is even.
- The base angles of an isosceles triangle are congruent.

**Pass your output to your teacher in your next face-to-face meeting.**

**ACTIVITY 15. The Reasons Why**

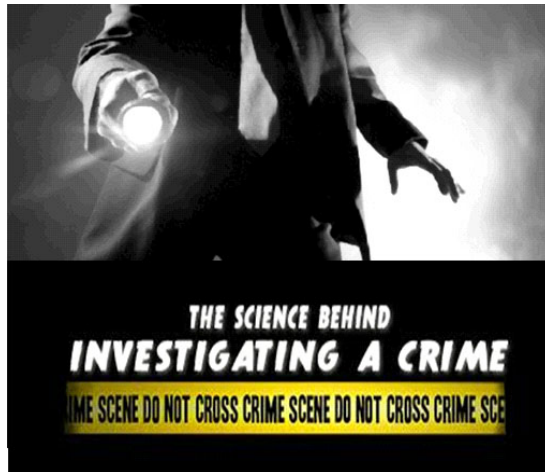
You are to conduct a survey among your classmates and find out what they think the reasons why (select one among the listed situations):

- a. students have a low grade in math
- b. friends quarrel
- c. a student is bullied

Analyze the results, come up with a valid conclusion, and formulate a plan on how to avoid the situation you have selected above to happen.

**ACTIVITY 16. Revisiting Activity No 1: The Detective**

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Watch again the videos on how detectives solve a case. Click

- 1. <https://www.youtube.com/watch?v=zrTUKfEjPkA>
- 2. <https://www.youtube.com/watch?v=rDbQbjCLTMl>

**Revisit your answers to the Process Questions. Make necessary revisions.**

- 1. *How do detectives in the video gather evidences of the crime?*

- 2. *What other evidences might a detective collect?*

- 3. *What kind of arguments do they use to in presenting their proofs or evidences?*

4. *What kind of reasoning do they use in identifying who is most likely responsible for a crime?*

5. *How are clear and coherent arguments and proofs formulated? How are they proved or disproved?*

**ACTIVITY 17. Revisiting Activity No. 2: Anticipation Reaction Guide**

Write your final response for each statement below in the After Lesson column.

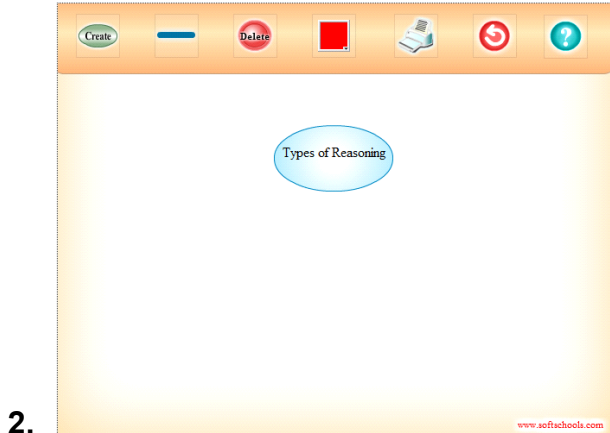
Write **A** if you agree with the statement.  
Write **B** if you disagree with the statement.

<i>Before Lesson</i>	<i>Statement</i>	<i>After Lesson</i>
	1. Inductive reasoning is the process of drawing conclusions based on many clues, or pieces of evidence.	
	2. Scientist and doctors make use of the inductive process in making and recommending treatments.	
	3. Deductive reasoning is the process of drawing conclusion from facts.	
	4. Deductive reasoning is the basis of the scientific method.	
	5. Indirect Proof is also known as Proof by contradiction.	

**ACTIVITY 18. Concept Map**

Map your understanding for this lesson.

[http://www.softschools.com/teacher\\_resources/concept\\_map\\_maker/](http://www.softschools.com/teacher_resources/concept_map_maker/)



4.



5. Just repeat the process if you want to more subtopics and lines.

### End of DEEPEN:

In this section, the discussion was about making decisions and explanation by making use of inductive and deductive reasoning.

What new realizations do you have about the topic? What new connections have you made for yourself?

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.



### TRANSFER

*Your goal in this section is apply your learning to real life situations. You will be given a practical task which will demonstrate your understanding.*

#### **ACTIVITY 19. Join a worldwide debate**

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**Option 1:** You are a member of an online debate which is participated by the different debaters worldwide. Anybody can post title/idea to be debated. **You are challenged to make a stand on one of the big issues discussed worldwide on either PRO or AGAINST positions. You will post a title/idea to be debated and wait for an opponent to respond.** While waiting, you will prepare arguments for either side. You have to convince your co debaters and the jurors of your idea. You are to present arguments that are based on coherence and clarity.

**Option 2:** You are a member of an online debate which is participated by the different debaters worldwide. **You are challenged to respond to one of the big issues discussed worldwide on either PRO or AGAINST positions.** While waiting, you will prepare arguments for either side. You have to convince your co debaters and the jurors of your idea. You are to present arguments that are based on coherence and clarity.

**SCORING RUBRIC**

Criteria	Advanced 4	Proficient 3	Progressing 2	Beginning 1
Coherence	Arguments are built on compelling facts, and follow logical concepts and rules.	Arguments are adequate and follow logical concepts and rules.	Some of the arguments are inconsistent and show lapses in logic.	Arguments are erroneous and show lack of logical reasoning.
Clarity	The arguments/ideas are clearly stated and insightful.	The arguments/ideas are clearly stated.	Some arguments/ideas are not clearly stated.	Most of the arguments/ideas are vague.

**ACTIVITY 20. Synthesis Journal**

Complete the table below by answering the questions then click “Save”.

How did I find the performance task?	What are the values I learned from the performance task?	What made my task successful?	How will I use these learning/insights in my daily life?

### End of TRANSFER:

In this section, your task was...

How did you find the performance task? How did the task help you see the real world use of the topic?

You have completed this lesson. Before you go to the next lesson, you have to answer the following post-assessment.

### GLOSSARY OF TERMS USED IN THIS LESSON:

**Biconditional Statement**– if the condition is logically equivalent to its converse

**Conditional Statement**– a statement written in if-then form

**Contrapositive**– both premise and conclusion are negated

**Converse** – formed by converting the premise to a conclusion and conclusion to premise

**Counterexample** – a single example that shows a generalization to be false

**Deductive Reasoning**– begins with a general statement then conclusion

**Inductive Reasoning** – begins with a conclusion then presents specific Cases

**Negation** – a statement denying another statement

**Reasoning** – the process of drawing conclusions from the information

### REFERENCES AND WEBSITE LINKS USED IN THIS LESSON:

Thinking kids

[http://lonavi.files.wordpress.com/2008/03/ist2\\_2334825\\_puzzled\\_kids\\_cartoon.jpg](http://lonavi.files.wordpress.com/2008/03/ist2_2334825_puzzled_kids_cartoon.jpg)

you tube- wrong judgement

[http://www.youtube.com/watch?v=muc1TMh6\\_LU](http://www.youtube.com/watch?v=muc1TMh6_LU)

scientist in blue

[http://t1.gstatic.com/images?q=tbn:ANd9GcRrGBqy07YyKNiypKyuZX-82FGibiUAKrIUK\\_rVa9ztlBsBij6S](http://t1.gstatic.com/images?q=tbn:ANd9GcRrGBqy07YyKNiypKyuZX-82FGibiUAKrIUK_rVa9ztlBsBij6S)

white kitten

<http://t2.gstatic.com/images?q=tbn:ANd9GcR6RsMuDRb89QB1YB74ihxj2fCR7-3K8s1jEafBzZm3Hbvj1ra1Gg>

multiplication kid



<http://t3.gstatic.com/images?q=tbn:ANd9GcRJeohw0dUxRYGe7gn42NMq07Q78C27La2YRPhD7qIzSk8Z2AHP7g>

study girl

[http://t0.gstatic.com/images?q=tbn:ANd9GcRyW76Rrj\\_m8Ykb8uIVm6s25Mrw8IzZwZJlxEyv2J4\\_RtYloDIBgg](http://t0.gstatic.com/images?q=tbn:ANd9GcRyW76Rrj_m8Ykb8uIVm6s25Mrw8IzZwZJlxEyv2J4_RtYloDIBgg)

recycle-students

[http://t1.gstatic.com/images?q=tbn:ANd9GcTa6svmXdlcSnrcz\\_0Jv9NQA6b1mDGlxlvXhsXrJCbK2XqDf4L4Q](http://t1.gstatic.com/images?q=tbn:ANd9GcTa6svmXdlcSnrcz_0Jv9NQA6b1mDGlxlvXhsXrJCbK2XqDf4L4Q)

clean Philippines

[http://t3.gstatic.com/images?q=tbn:ANd9GcS5IVSntHkyBxevrvUWC1KSpS8PkHOpsKsjlligEU4qPPz8kNC\\_A](http://t3.gstatic.com/images?q=tbn:ANd9GcS5IVSntHkyBxevrvUWC1KSpS8PkHOpsKsjlligEU4qPPz8kNC_A)

cellphone

<http://t2.gstatic.com/images?q=tbn:ANd9GcRdmNuykhPN9tcbiY1OnAAjxKGI9PLkJVXMO1C-iPvmw3S9Sx1V>

pinoy kid

<http://www.google.com.ph/imgres?q=pinoy+kid&num=10&hl=en&safe=active&tbo=d&biw=930&bih=392&tbnid=Jn-xtX7JxEksSM:&imgrefurl=http://www.nextping.com/education/filipino-kid-kesz-valdez-wins-international-childrens-peace-price-2012/&docid=lpHfNe7D4KBGxM&imgurl=http://www.nextping.com/wp-content/uploads/2012/09/kesz-valdez.png&w=338&h=343&ei=AZS6UPH7BO2YiAewwoCYAQ&zoom=1&iact=hc&vpx=669&vpy=68&dur=314&hovh=227&hovw=223&tx=161&ty=127&sig=114386800899313423233&page=1&tbnh=138&tbnw=123&start=0&ndsp=10&ved=1t:429,r:4,s:0,i:92>

ipod

[http://t3.gstatic.com/images?q=tbn:ANd9GcQaJZoaAJ2IUcniNb3XeX5oOdyVTAvfOcnsbYi-ZqikY\\_3fyTQAHw](http://t3.gstatic.com/images?q=tbn:ANd9GcQaJZoaAJ2IUcniNb3XeX5oOdyVTAvfOcnsbYi-ZqikY_3fyTQAHw)

tablet

<http://t1.gstatic.com/images?q=tbn:ANd9GcQOvghn511--9AeX5EzqBbv8jZ6qZHKuG5U9B6j3zkgQc0qdxzrpw>

mp3

<http://www.google.com.ph/imgres?q=mp3&hl=en&safe=active&tbo=d&biw=930&bih=392&tbnid=L0L-hzWpzaAozM:&imgrefurl=http://electronics.howstuffworks.com/mp3-player.htm&docid=mIMSt2wDI1ZwYM&imgurl=http://static.ddmcdn.com/gif/mp3-player-1b.jpg&w=400&h=200&ei=FJS6UJjoBOqWiQfl-4GgDQ&zoom=1&iact=hc&vpx=4&vpy=4&dur=292&hovh=160&hovw=319&tx=8>

[9&ty=37&sig=114386800899313423233&page=1&tbnh=132&tbnw=264&start=0&ndsp=9&ved=1t:429,r:0,s:0,i:77](http://www.google.com/search?q=detective+cartoon&hl=en&safe=active&tbo=d&biw=930&bih=431&tbnid=Tv82I15R7jzJsM:&imgrefurl=http://www.dreamstime.com/royalty-free-stock-images-detective-cartoon-illustration-image11853829&docid=hEe-Dt90lmFyXM&imgurl=http://www.dreamstime.com/detective-cartoon-illustration-thumb11853829.jpg&w=376&h=450&ei=FpS6UM2FE9GeiQfQzoCgCQ&zoom=1&iact=hc&vpx=4&vpy=60&dur=235&hovh=247&hovw=206&tx=91&ty=160&sig=114386800899313423233&page=1&tbnh=148&tbnw=144&start=0&ndsp=12&ved=1t:429,r:0,s:0,i:77)

detective with hat

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detective with foot prints

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detective kid

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kid with cam

[https://encrypted-tbn0.gstatic.com/images?q=tbn:ANd9GcQOjZEQQN6Ou7jPeQUeP\\_C8UQoNs1cmg4E3HC1mjMPnejmavKKpKA](https://encrypted-tbn0.gstatic.com/images?q=tbn:ANd9GcQOjZEQQN6Ou7jPeQUeP_C8UQoNs1cmg4E3HC1mjMPnejmavKKpKA)

medal

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treasure chest

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trophy

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ribbon

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road

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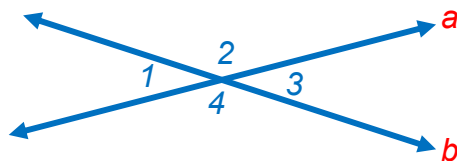
## POST-ASSESSMENT

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*It's now time to evaluate your learning. Click on the letter of the answer that you think best answers the question. Your score will only appear after you answer all items. If you do well, you may move on to the next module. If your score is not at the expected level, you have to go back and take the module again.*

1. If a conditional and its converse are true, what kind of statement is formed when you combine them?
  - A. Biconditional
  - B. Converse
  - C. Conclusion
  - D. Truth value
2. What is the term used to refer to a statement which is formed by switching a conditional's hypothesis and conclusion?
  - A. Biconditional
  - B. Converse
  - C. Conclusion
  - D. Truth value
3. Which of the following statements is true?
  - A. If a conditional is true then the inverse is false.
  - B. If a conditional is false then the converse is false.
  - C. If a conditional is true then the contrapositive is true.
  - D. Every true statement will have at least one counterexample.
4. Which of the following is the if-then form of the statement below?  
"It is time for dinner if it is 7 o'clock."
  - A. If you want to eat dinner, then you must eat at 7 o'clock.
  - B. If it is time for dinner, then it is 7 o'clock.
  - C. If it is 7 o'clock, then it is time for dinner.
  - D. If it is not time for dinner, then it is not 7 o'clock.
5. What is the converse of the following statement?  
**If a whole number is divisible by 2, then it is even.**
  - A. If a whole number is not divisible by 2, then it is not even.
  - B. If a whole number is even, then it is divisible by 2.
  - C. If a whole number is not even 2, then it is not divisible by 2.
  - D. If a whole number is not even, then it is divisible by 2.
6. What is the inverse of the following statement?  
If Liza makes a 50 on the test, then Liza failed the test.
  - A. If Liza makes a 50 on the test, then Liza failed the test.
  - B. If Liza failed the test, then Liza made a 50 on the test.
  - C. If Liza did not make a 50 on the test, then Liza did not fail the test.
  - D. If Liza did not fail the test, then Liza did not make a 50 on the test.

7. Which of these statements is logically equivalent to the conditional below?  
 "If the measure of an angle is less than 90, then it is acute."  
 A. If an angle is acute, then its measure is less than 90.  
 B. If the measure of an angle is not less than 90, then the angle is not acute.  
 C. If an angle is not acute, then its measure is not less than 90.  
 D. If an angle is not acute, then its measure is less than 90.
  
8. Determine which of the following situations is NOT based inductive reasoning.  
 A. I heard lots of barking last night. The neighbor's dog must've been pretty upset about something, since he rarely barks.  
 B. No whale lives in fresh water, and the lake is fresh water, so there are no whales living there.  
 C. Some cookies are burnt. Some burnt things are good to eat. So some cookies are good to eat.  
 D. Crackers almost always have salt on them, so the crackers at this restaurant will most likely be salted in some way.
  
9. Complete the proof showing that if two angles are vertical angles, then they have the same measure by identifying the correct reason for statement **c**.



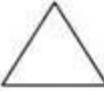



Given : lines a and b intersect;  $\angle 1$  and  $\angle 3$  are vertical angles  
 Prove:  $m\angle 1 = m\angle 3$

Statements	Reasons
lines a and b intersect;	given
$\angle 1$ and $\angle 2$ are a linear pair of angles and $\angle 3$ and $\angle 4$ are a linear pair.	Definition of a linear pair
$m\angle 1 + m\angle 2 = 180$ $m\angle 3 + m\angle 2 = 180$	?
$m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$	Substitution
$m\angle 1 = m\angle 3$	Subtraction Property of Equality

- A. Definition of Adjacent Angles
  - B. Definition of Congruent Angles
  - C. Definition of Complementary Angles
  - D. Definition of Supplementary Angles
- 
10. The following are examples of deductive reasoning through syllogisms.  
 Which of the following possesses a valid and harmonious conclusion?  
 A. Monkey eats banana. Ben eats banana

- Ben is monkey.
- B. Square is quadrilateral  
Trapezoid is quadrilateral  
Trapezoid is square.
- C. Rectangle is a parallelogram
- Square is parallelogram  
Square is a rectangle.
- D. John is hospitable.  
John is Filipino  
Filipino is hospitable.

11. What is the best conclusion that can be drawn from the given below?  
[www.kidsnewsroom.org](http://www.kidsnewsroom.org)

Number of Sides	Diagram	Number of Diagonals
3		0
4		2
5		5
6		9

n is the number of sides

- A.  $(n(n - 3)) / 2$
- B.  $n(n - 3)$
- C.  $(n - 3) / 2$
- D.  $(n(n + 3)) / 2$
12. Advertisements make use of if-then statements to promote products or services. An ad for car repair shop reads, *If you're looking for fast and reliable car repair, visit CarCare Center.* Does the advertisement say that CarCare Center is fast and reliable?
- A. Yes. Advertisements are always true.
- B. Yes. The if-then statement is true.
- C. No. The conclusion is implied.
- D. A conclusion cannot be derived.
13. Which of the following statements is TRUE?
- A. If an animal is a panther, then it lives in the forest.
- B. If tomorrow is Friday, then today is Thursday.
- C. If an animal is spotted, then it is a Dalmatian.
- D. If you live in Tagaytay, then you live in Luzon.

14. Determine if the third statement of the following statements is valid.
1. If the taxi driver has a long drive, then the fare will be large.
  2. The taxi driver has a long drive.
  3. The fare will not be large.
- A. Invalid. The fare will be large.  
B. Invalid. The taxi driver did not charge a fare.  
C. Valid  
D. Invalid. The taxi driver has a short drive.
15. You are a debater. How will you prepare your arguments in order to win?
- A. It must be based on facts, coherent and clear.  
B. It must be based on what of the majority of the people will say about the topic.  
C. It must be based on the information provided by the social media.  
D. It must be based on my own judgment.
16. Suppose that you an investigator who will present the result of your investigation. How do you think your investigation will be assessed?
- A. coherence, clarity and sound judgment  
B. coherence, clarity and visual aid  
C. sound judgment, clarity and the use of technology  
D. visual aid, the use of technology and coherence
17. You are a lawyer. You have been assigned to defend someone accused of violating a law uptown at 10:00 A.M. on March 24. You argue that if guilty, your client must have been there at that time. You have a video of your client being interviewed by a TV reporter at a political rally on the other side of town at the same time. Can you conclude that your client is not guilty?
- A. My client is guilty.  
B. My client is not guilty.  
C. I am doubtful.  
D. I cannot decide.
18. Last week Cathy saw a cat on her way and met an accident on the road. It is belief that once a cat passes your way, something bad will happen. You saw a cat passed by as you are on your way to your uncle's house. Will you still go to your uncle's house?
- A. No. I strongly believe on the belief.  
B. No. Something bad might happen to me.  
C. Yes. I will just go carefully to my uncle's house.  
D. Yes. There was no established evidences and facts regarding the belief.

19. Your mother told you that once you drop a spoon on the floor, a female friend will visit you and if it is a fork, a male friend. You were expecting your friend Lolita to visit you and it so happened that during your meal time you dropped your fork? Will you conclude that Lolita cannot come anymore?
- A. No. What my mother told me was only a belief and was not based on facts.
  - B. No. I do not believe on beliefs.
  - C. Yes. Logically, it can be concluded that Lolita cannot come.
  - D. Yes. Based on the law of detachment, Lolita cannot really come.
20. You are a field reporter. How will you present your news so that it will be reliable and convincing?
- A. coherence, clarity and sound judgment
  - B. coherence, clarity and video documentation
  - C. sound judgment, clarity and the use of technology
  - D. visual aid, the use of technology and coherence



## Lesson 2: Stability: An Obligation or an Option?

### ☑ INTRODUCTION AND FOCUS QUESTION(S):



Have you ever wondered how these structures are designed? What factors are being considered in the designs? Designing these structures entails the knowledge of congruent triangles, specifically their properties and principles. In this module, the definition of congruent triangles, the triangle congruence postulates and theorems, and the proving of the congruency of triangles will be tackled. These concepts and skills will equip you to communicate, formulate, investigate, analyze, and solve real-life problems related to structure stability.

This module seeks to find the answers to the question: how are problems on structure stability solved?

**LESSON COVERAGE:**

In this lesson, you will examine this question when you take the following topics:

*Lesson 2.1 - Axiomatic Structure in Geometry*

*Lesson 2.2 – Triangle Congruence*

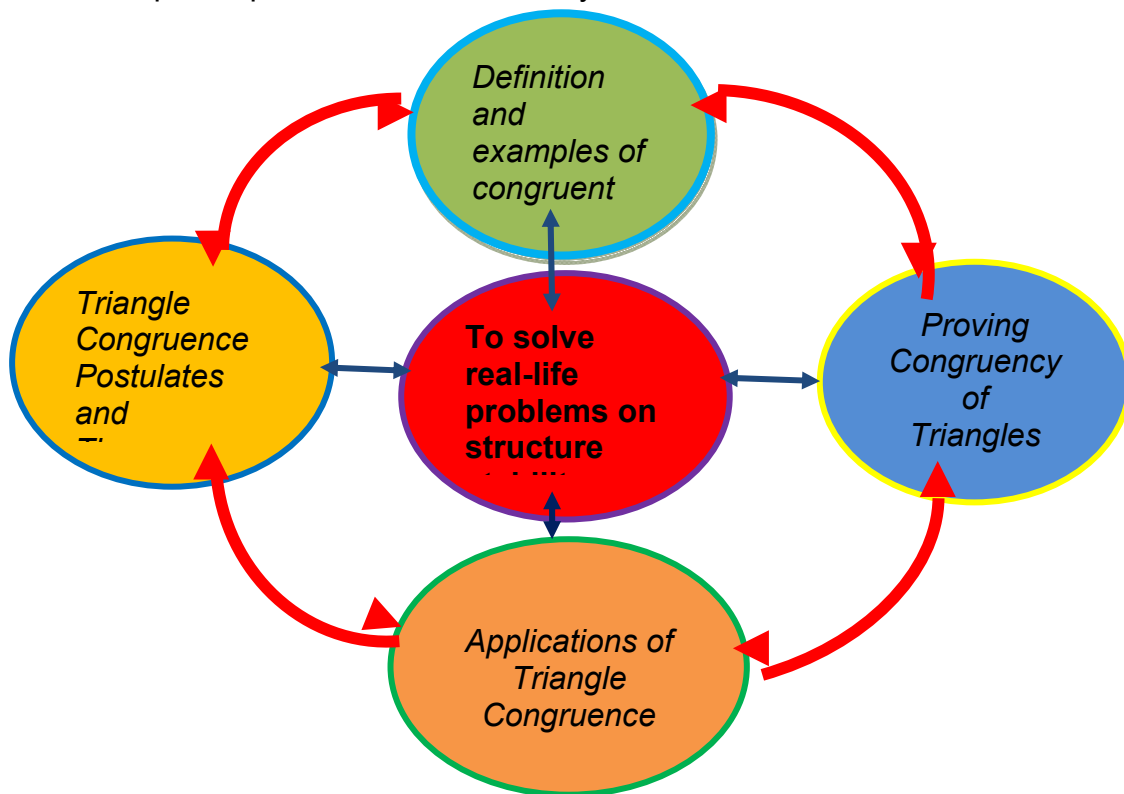
- a. *Definition and Examples of Congruent Triangles*
- b. *Conditions for Triangle Congruence*
  - *SAS Congruence Postulate*
  - *SSS Congruence Postulate*
  - *ASA Congruence Postulate*
- c. *Proving Congruence of Triangles*
- d. *Applications of Triangle Congruence*

In these topics, you will learn the following:

<i>Lesson 2.2</i>	<ul style="list-style-type: none"> <li>➤ Illustrates triangle congruence.</li> <li>➤ States and illustrates the SAS, ASA and SSS congruence postulates.</li> <li>➤ Applies the postulates and theorems on triangle congruence to prove statements on congruence, including right triangles.</li> <li>➤ Applies triangle congruence to geometric constructions to perpendicular and angle bisector.</li> </ul>
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**MODULE MAP:**

Here is a simple map of the above lessons you will cover:



**EXPECTED SKILLS:**

To do well in this lesson, you need to remember and do the following:

5. Define congruent triangles and other related terms about triangles and their parts; state the different congruence postulates and theorems.
6. Explore certain websites indicated in the module that would be of great help for your better understanding of the lessons on congruent triangles and work on the interactive activities.
7. Take down notes of the important concepts and follow the flow chart in proving congruency of triangles.
8. Perform the specific activities or tasks and complete the exercises and assessments provided.
9. Collaborate with the teacher and peers.

**PRE-ASSESSMENT:**

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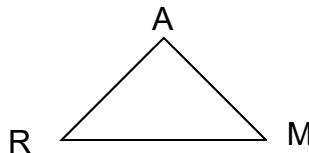
Let's find out how much you already know about this lesson. Click on the letter that you think best answers the question. Please answer all items. After taking this short test, you will see your score. Take note of the items that you were not able to correctly answer and look for the right answer as you go through this lesson.

1. In the congruent triangles  $\triangle MRN$  and  $\triangle SRT$ , the corresponding vertex of R is

- A. R
- B. M
- C. N
- D. S

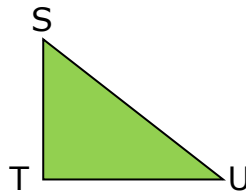
2. In  $\triangle MAR$ , the angle included between AM and AR is

- A.  $\angle RAM$
- B.  $\angle ARM$
- C.  $\angle AMR$
- D.  $\angle MRA$



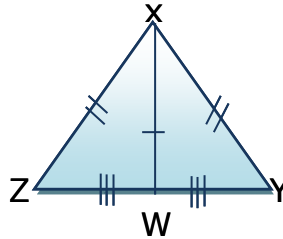
3. In  $\triangle STU$ , the side included between  $\angle T$  and  $\angle U$  is

- A.  $\overline{ST}$
- B.  $\overline{SU}$
- C.  $\overline{TU}$
- D.  $\overline{TS}$

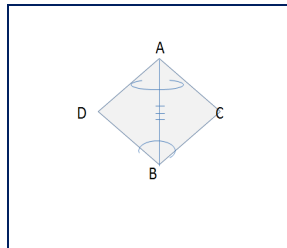


4. In the given figure, the triangle congruent to  $\triangle XYW$  is

- A.  $\triangle XWZ$
- B.  $\triangle XZW$
- C.  $\triangle WZX$
- D.  $\triangle ZXW$

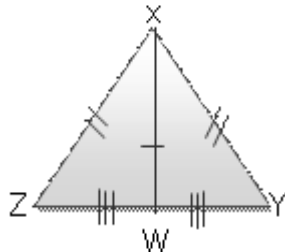


5. The postulate which supports the congruency of  $\triangle ABC$  and  $\triangle ABD$  is



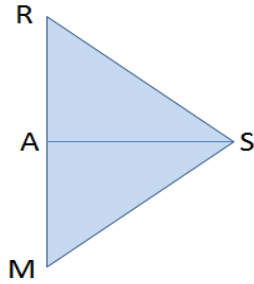
- A. SSS Congruence
- B. SAS Congruence
- C. AAA Congruence
- D. ASA Congruence

6. The two triangles in the drawing are congruent due to the

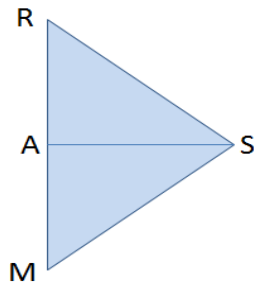


- A. SSS Congruence Postulate
- B. SAS Congruence Postulate
- C. LL Congruence Theorem
- D. HL Congruence Theorem

7. If A is the midpoint of RM, then the result is

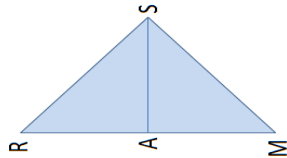


- A.  $RA \cong MA$   
 B.  $AR \cong MA$   
 C.  $RS \cong MS$   
 D.  $AS \cong AS$
8. Refer to the same figure and answer in no.7 and  $AS \cong AS$ , using the SAS Congruence Postulate to prove the congruency of  $\Delta RAS$  &  $\Delta MAS$ , the third pair of congruent corresponding parts needed is



- A.  $\angle SRA \cong \angle SMA$   
 B.  $\angle RSA \cong \angle MSA$   
 C.  $\angle RAS \cong \angle MAS$   
 D.  $RS \cong MS$
9. If two triangles are congruent and the perimeter of one triangle is 15 feet, then the perimeter of the second triangle is
- A. 10 feet  
 B. 15 feet  
 C. 20 feet  
 D. 30 feet

10. If the area of the first triangular garden to be landscaped is 30 sq. meters, and its base is 10 meters, what is the area of another triangular garden having exactly the same shape as the first and whose base is also 10 meters?
- 15 sq. m
  - 20 sq. m
  - 30 sq. m
  - 60 sq. m
11. What is the implication to the cost of landscaping, granting that the materials and styles used of the first garden are exactly the same?
- one-third of the cost of the first
  - halved
  - same cost
  - doubled
12. Roof trusses allow a roof to withstand the stress of heavy loads. A roof truss for a particular building makes a  $42^\circ$  angle on the right and the triangles are to be congruent in order for it to be rigid. What should be the measure of the angle on the left? or  $\angle R$ ?



- $38^\circ$
  - $42^\circ$
  - $48^\circ$
  - $96^\circ$
13. If the measure of AR is 12 feet, what is the measure of AM?
- 6 ft.
  - 9 ft.
  - 12 ft.
  - 15 ft.
14. What is the measure of  $\angle SAM$ ?
- $42^\circ$
  - $48^\circ$
  - $90^\circ$
  - $96^\circ$

15. Therefore, RM and SA are line segments which are
- concurrent
  - parallel
  - perpendicular
  - skew
16. What happen if the triangles in the roof trusses are not congruent?
- The truss will be lopsided
  - The truss will not be stable and may cause the structure to collapse
  - The truss will not resist the stress of the load
- I only
  - II only
  - III only
  - A,B and C
17. A four-legged stool is more stable than a stool with lesser number of legs, but it is easily shaken when subjected to too much movement. It will be strengthened by putting a brace at the bottom which will
- be parallel to the other legs
  - be perpendicular to the other legs
  - form a diagonal of the rectangular base
  - intersect the two legs of the stool
18. What figures are evident from a top view of an airplane?  
A pair of triangles with
- varied shape and size
  - the same size
  - the same shape
  - the same shape and size
19. The strength of the migrating birds lies in their formation during flying, what do their formation look like? They
- fall in one horizontal line
  - fall in one vertical line
  - fly with their partners and form two lines
  - fly forming an inverted V
20. If you are ask to present a design of a structure, your work will be judged based on
- stability, originality, accuracy and mathematical reasoning
  - creativity, clarity, accuracy, voice and diction
  - attractiveness, accuracy, organization and completeness
  - accuracy, originality, attractiveness, communication

## Lesson 2.1 Axiomatic Structure of Geometry



### EXPLORE

Let's start the module by answering the problem below.

#### **ACTIVITY 1. Geometric Figures in Construction**

---



1. What are some of the geometric figures that you can see in the structure of the building?
2. How do these figures define the structure and the stability of the building?
3. How is Math used in the design of different structures?
4. ***How are problems on structure stability solved?***



**ACTIVITY 2. Anticipation/ Reaction Guide**

Instruction: Respond to each statement and write your answer in the Before the Lesson Column.

Write **A** you if you agree with the statement

Write **B** you if you disagree with the statement

Before the Lesson	TOPIC: The undefined terms in Geometry	After Lesson
	1. Points, lines, and planes are the bases in defining other terms in Geometry.	
	2. An axiomatic system is a set of axioms/postulates, from which some or all axioms/postulates can be used together to logically derive and prove theorems.	
	3. A theorem is a statement that is accepted only after it has been proven.	
	4. Points that lie on the same line are coplanar.	
	5. A postulate is a statement that is accepted only after it has been proven.	
	6. Two points always determine a line.	
	7. At least three noncollinear points determine a plane.	
	8. Points, lines and planes can be modeled by real-life objects and situations.	

*Click on SAVE if you have responded to the six statements in the first column.*

### End of EXPLORE:

You gave your initial answers to the questions and statements given above.

Let's find out how others would answer the above, and compare their ideas to our own. As you compare ideas, you will find out if your ideas are in line with the standard. You will also learn other concepts which will help you complete a required project found at the end. This project is about making a sketch of any structural designs such as buildings, houses, etc. and defend its stability.



### FIRM-UP

Your goal in this section is to learn and understand the axiomatic structure of geometry which includes the key concepts of the undefined terms, defined terms, postulates, and theorems. Put into mind this question: " **How are problems on structure stability solved?**" as you answer each activity.

### ACTIVITY 3. Axiomatic System in Geometry

Watch the two videos below. Read first the process questions before watching each. Do not forget to take down notes.

<https://www.youtube.com/watch?v=w2v0GNIS-5E> This video explains the components of axiomatic system in geometry.

<https://www.youtube.com/watch?v=DzcTI3urU8I> This video explains the importance of axiomatic system in geometry in designing games and structures.

#### Process Questions:

1. What are the four parts of an axiomatic system?
2. Where do we use these four parts?
3. What are the undefined terms, defined terms and postulates and theorems?

**Post your answers to these question in the Discussion Forum.**



*You have just learned the axiomatic system in geometry and its importance. Let us now discuss each part of the system.*

#### **ACTIVITY 4. Undefined and Defined Terms in Geometry**

1. Read an article from <http://www.gradeamathhelp.com/undefined-terms-in-geometry.html>. This article explains the undefined terms in geometry. Specific examples of each in the real world are given so that you can easily understand each.
2. Watch a video from <https://www.youtube.com/watch?v=LJ4YxqumMkc> In this video, Mr. T describes the difference between “defined” and “undefined” terms in Geometry. He describes points, lines, planes, line segments, and rays, as well as what it means for sets of points to be collinear and coplanar.

#### **Process Questions:**

1. What are the undefined terms in geometry?

2. Why are they considered the building blocks of Geometry?

3. How do you name a:

- a. point;

- b. line;

- c. plane?

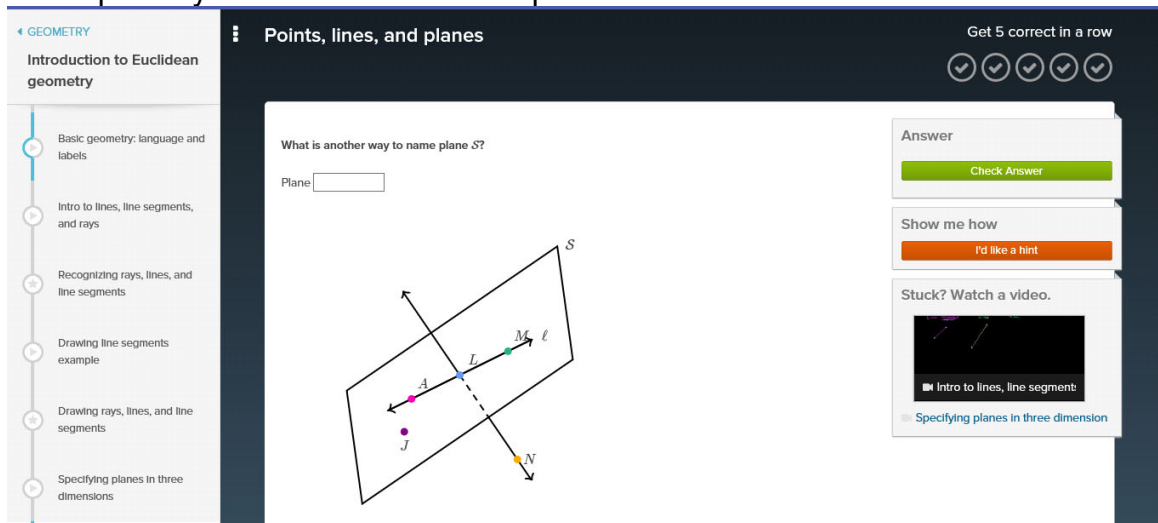
4. What is the difference between the undefined and the defined terms in Geometry?

5. What are some of the defined terms that you have learned from the article and from the video?

### ACTIVITY 5. Interactive Practice

Click on

- [https://www.khanacademy.org/math/geometry/intro\\_euclid/e/points\\_lines\\_and\\_planes](https://www.khanacademy.org/math/geometry/intro_euclid/e/points_lines_and_planes) and answer the first five questions. If you are able to correctly answer all five, you can proceed to next three levels of practice. If you need some help, click the **show me how button** for some hint. If you get stuck, you may watch the suggested video to the right. Do not stop until you have mastered this part.



The screenshot shows the Khan Academy interface for the topic 'Points, lines, and planes'. On the left is a navigation menu with a progress indicator. The main area contains a question: 'What is another way to name plane S?'. Below the question is a text input field labeled 'Plane'. A diagram shows a 3D perspective of a plane labeled 'S'. A line with arrows at both ends passes through the plane. Points are labeled: 'A' (pink dot on the line), 'J' (purple dot below the line), 'L' (blue dot on the line), 'M' (green dot on the line), and 'N' (yellow dot below the line). A line segment is labeled 'ℓ'. On the right side of the interface, there are five checkmarks indicating progress, an 'Answer' section with a 'Check Answer' button, a 'Show me how' section with an 'I'd like a hint' button, and a 'Stuck? Watch a video.' section with a video player and a video title 'Intro to lines, line segment'.

- [https://www.khanacademy.org/math/geometry/intro\\_euclid/e/recognizing\\_rays\\_lines\\_and\\_line\\_segments](https://www.khanacademy.org/math/geometry/intro_euclid/e/recognizing_rays_lines_and_line_segments) This is an interactive quiz for recognizing rays, lines, and line segments. Answer the first five questions. If you are able to correctly answer all five, you can proceed to next three levels of practice. If you need some help, click the **show me how button** for some hints. If you get stuck, you may watch the

suggested video to the right. Do not stop until you have mastered this part.

3. [https://www.khanacademy.org/math/geometry/intro\\_euclid/e/geometric-definitions](https://www.khanacademy.org/math/geometry/intro_euclid/e/geometric-definitions) This is an interactive quiz for recognizing defined terms in geometry. Answer the first five questions. If you are able to correctly answer all five, you can proceed to next three levels of practice. If you need some help, click the **show me how button** for some hint. If you get stuck, you may watch the suggested video to the right. Do not stop until you have mastered this part.

**ACTIVITY 6. Skills and Readiness Check**

Check the column which reflects your readiness to go further in this lesson.

I am still confused	I got it but I still need more practice	I am ready to go
Go back to the previous activities, watch and/or read the article again.	Go to ...look for another interactive website for practice.	You may now take the Mastery Quiz

**ACTIVITY 7. Mastery Quiz**

This quiz will test your knowledge and skills mastery on the undefined and defined terms in Geometry. Analyze each figure first before answering.

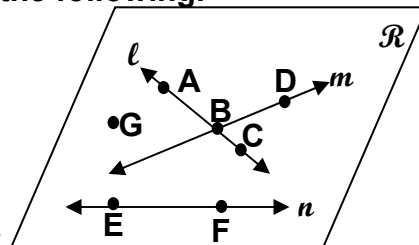
I. Refer to the figure at the right to answer the following.

1. Name a line that contains point A.

2. Name a plane that contains lines  $m$  and  $n$ .

3. Name the intersection of lines  $l$  and  $m$ .

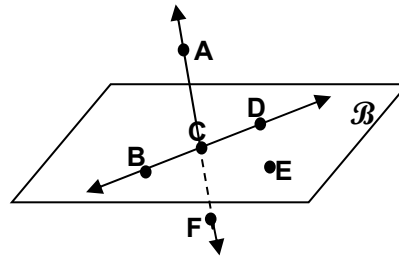
4. What is the other name for line  $n$ .



**II. Use the figures to your right to answer the following.**

Are the points collinear? (Yes or No)

1. B, C, D \_\_\_\_\_
2. B, C, E \_\_\_\_\_
3. A, D, E \_\_\_\_\_
4. A, C, F \_\_\_\_\_



Are the points coplanar? (Yes or No)

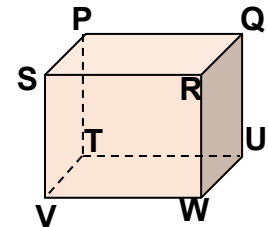
5. B, C, D, E \_\_\_\_\_
6. B, C, D, A \_\_\_\_\_
7. A, C, F, E \_\_\_\_\_

Give 2 names for plane B.

8. \_\_\_\_\_
9. \_\_\_\_\_

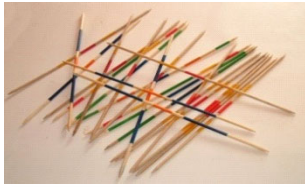
Tell whether the following are coplanar or noncoplanar?

10. points P, Q, and R  
\_\_\_\_\_
11. points T, U, W, and V  
\_\_\_\_\_
12. line PS and line RW  
\_\_\_\_\_
13. point V and line QR  
\_\_\_\_\_



**II. Determine what undefined term is being modeled by each picture.**

1.




2.




3.




4.




5.




6.

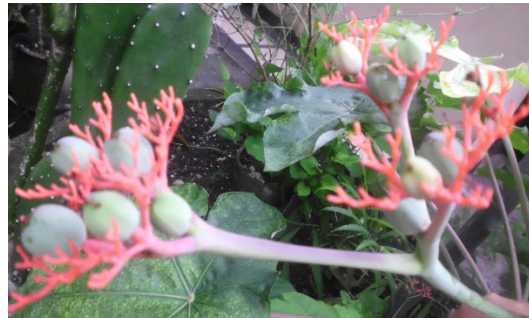



**III. Which of the following pictures/situations below illustrate**

1. Collinear points



A.



B.

1.

2. Coplanar points





- A. The three cups
- B. Three cups and the saucer
- C. The three cups, the saucer, and the teapot

2.


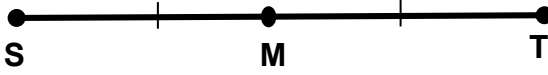



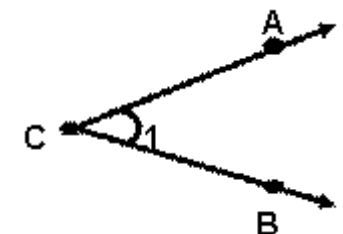
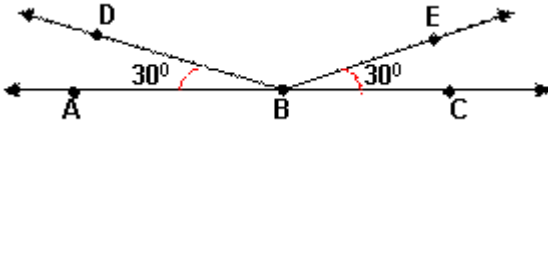

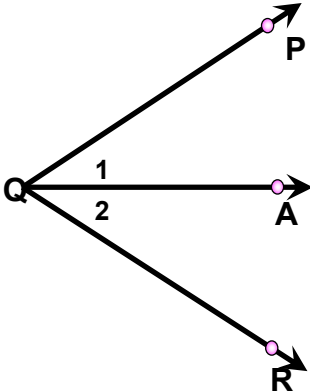
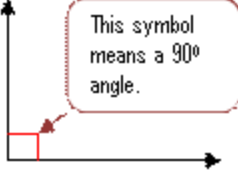
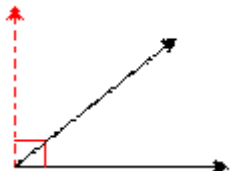
The definitions you have learned in Math 7 will help you learn the next lesson in triangle congruence. Review these definitions.

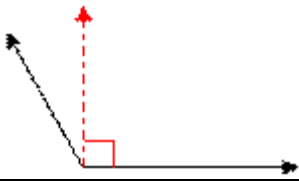
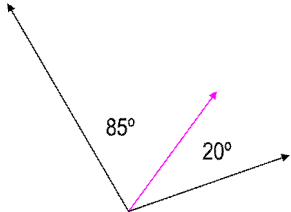
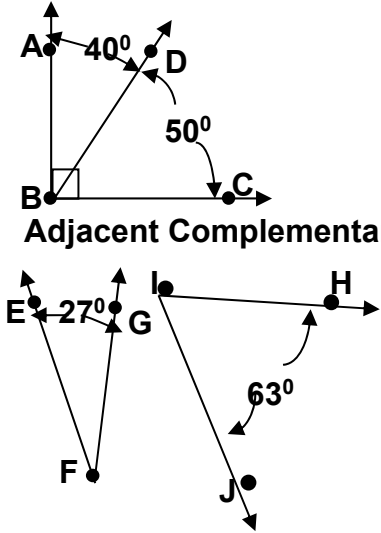
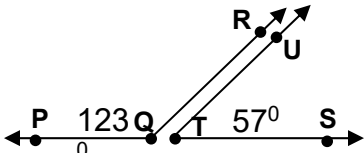
**ACTIVITY 8. Skills Review on the definitions learned in Math 7**

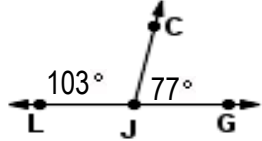
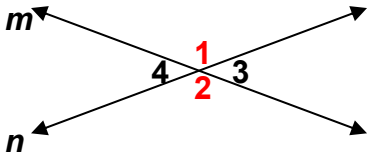
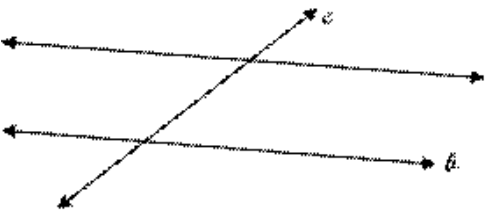
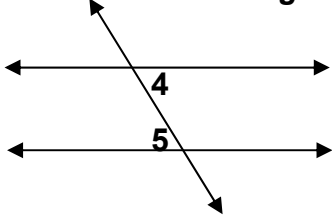
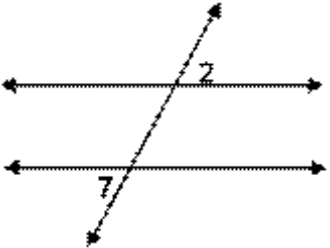
Read the content below.

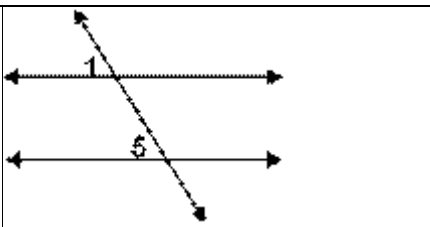
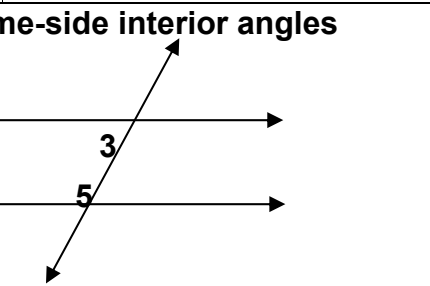
**Definitions Learned in Mathematics 7**

Term	Definition
<p><b>LINE SEGMENT</b></p> 	<p>Segment <math>AB</math>, denoted by <math>\overline{AB}</math> or <math>\overline{BA}</math> is the union of points <math>A</math>, <math>B</math>, and all the points between them. <math>A</math> and <math>B</math> are called the <i>endpoints</i> of the segment.</p>
<p><b>MIDPOINT OF A SEGMENT</b></p> 	<p>Point <math>M</math> is the midpoint of a segment if and only if <math>M</math> is between <math>S</math> and <math>T</math> and <math>\overline{SM} \cong \overline{TM}</math> which means that <math>SM = TM</math>.</p>
<p><b>SEGMENT BISECTOR</b></p> 	<p>It is a segment, ray, or line that contains the midpoint of the segment.</p>
<p><b>PERPENDICULAR BISECTOR OF A SEGMENT</b></p>	<p>It is a segment, ray, or line that is perpendicular to a segment at its midpoint.</p>

	<p>Recall: Two lines are said to be perpendicular if they form <math>90^\circ</math> (right) angles.</p>
<p style="text-align: center;"><b>ANGLE</b></p> 	<p>An angle is the union of two noncollinear rays that have a common endpoint called the vertex.</p> <p>The two rays make up the sides of the angle.</p>
<p style="text-align: center;"><b>CONGRUENT ANGLES</b></p> 	<p>Two angles are congruent if and only if their measures are equal.</p> <p>In symbols, <math>\angle ABD \cong \angle CBD</math>, if and only if <math>m\angle ABD = m\angle CBD</math>.</p> <p><b>Arcs in the figure also indicate which angles are congruent.</b></p> 
<p style="text-align: center;"><b>ANGLE BISECTOR</b></p> 	<p>The bisector of an angle is a ray with its endpoint at the vertex of the angle, extending into the interior of the angle.</p> <p>The bisector divides the angle into two congruent angles.</p> <p><math>\overline{QA}</math> is the angle bisector of <math>\angle PQR</math></p> <p><math>\angle 1 \cong \angle 2</math> or <math>\angle PQA \cong \angle RQA</math></p>
<p><b>Definition of Different Angles</b></p>	
<p style="text-align: center;"><b>RIGHT ANGLE</b></p> 	<p>A right angle is an angle with a measure of 90.</p>
<p style="text-align: center;"><b>ACUTE ANGLE</b></p> 	<p>An acute angle is an angle with a measure greater than 0 but less than 90.</p>

<p style="text-align: center;"><b>OBTUSE ANGLE</b></p> 	<p>An obtuse angle is an angle with a measure greater than 90 but less than 180.</p>
<p style="text-align: center;"><b>ADJACENT ANGLES</b></p> 	<p>These are two angles that share a common side (ray) and a common vertex.</p>
<p style="text-align: center;"><b>Complementary Angles</b></p>  <p style="text-align: center;"><b>Adjacent Complementary Angles</b></p> <p style="text-align: center;"><b>Non-Adjacent Complementary Angles</b></p>	<p>Two angles whose sum of their measures is <math>90^\circ</math>.</p> <p>It can be adjacent or non-adjacent.</p>
<p style="text-align: center;"><b>Supplementary Angles</b></p> 	<p>Two angles whose sum is <math>180^\circ</math>.</p>
<p style="text-align: center;"><b>Linear pair of Angles (Adjacent Supplementary Angles)</b></p>	<p>Two angles form a linear pair if they are adjacent, and the noncommon sides form a line.</p>

	
<p><b>VERTICAL ANGLES</b></p> 	<p>A pair of non-adjacent angles formed by the <u>intersection</u> of two straight lines.</p> <p>2 pairs of vertical angles are</p> <ol style="list-style-type: none"> <li><math>\angle 1</math> and <math>\angle 2</math></li> <li><math>\angle 3</math> and <math>\angle 4</math></li> </ol>
<p><b>Transversal</b></p> 	<p>A line (or segment) that intersects two or more coplanar lines (or segments) at distinct points.</p> <p><i>Line c</i> is the transversal of <i>lines a</i> and <i>b</i>.</p>
<p><b>Alternate interior angles</b></p> 	<p>These are two angles in the interior of the parallel lines, and on opposite (alternate) sides of the transversal.</p> <p>Alternate interior angles are non-adjacent and congruent.</p> <p><math>\angle 4 \cong \angle 5</math></p>
<p><b>Alternate exterior angles</b></p> 	<p>These are two angles in the exterior of the parallel lines, and on opposite (alternate) sides of the transversal.</p> <p>Alternate exterior angles are non-adjacent and congruent.</p> <p><math>\angle 2 \cong \angle 7</math></p>
<p><b>Corresponding angles</b></p>	<p>These are two angles, one in the interior and one in the exterior that are on the same side of the transversal.</p>

		<p>Corresponding angles are non-adjacent and congruent.</p> $\angle 1 \cong \angle 5$
<p><b>Same-side interior angles</b></p>		<p>These are two interior angles lying on the same side of the transversal</p> <p>Same-side interior angles are always supplementary.</p> $m\angle 3 + m\angle 5 = 180$



You have just learned how to name and identify some of the defined terms. Let us now move on to the Basic Postulates and Theorems. In this topic, you will see the difference between a postulate and a theorem and how they are applied in the real world.

## THE BASIC POSTULATES AND THEOREMS

**POSTULATE-** is a statement that is accepted without proof.

**THEOREM-** is a statement whose truth is accepted only after it has been proven.

### Postulates

*Postulate 1:* A line contains at least two points.

*Postulate 2:* A plane contains at least three noncollinear points.

*Postulate 3:* Through any two points, there is exactly one line.

*Postulate 4:* Through any three noncollinear points, there is exactly one plane.

*Postulate 5:* If two points lie in a plane, then the line joining them lies in that plane.

*Postulate 6:* If two planes intersect, then their intersection is a line.

Postulate 7: A space contains four noncoplanar points.

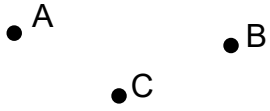

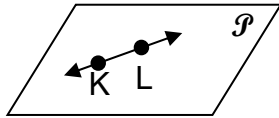
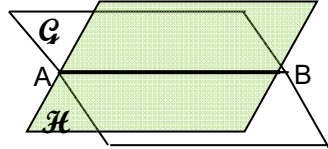
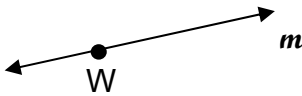
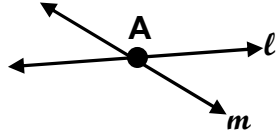
**Theorems**

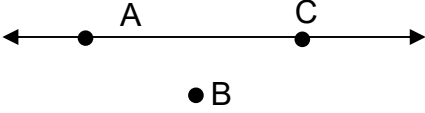
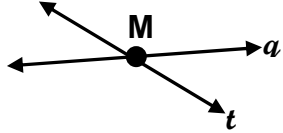
*Theorem 1:* If two lines intersect, then they intersect in exactly one point.

*Theorem 2:* If a point lies outside a line, then exactly one plane contains both the line and the point.

*Theorem 3:* If two lines intersect, then exactly one plane contains both lines.

**Example 3:** State the postulate or theorem you would use to justify the statement made about each figure.

<p>a. One plane contains points A, B, and C.</p>  <p>Answer: Through any three noncollinear points, there is exactly one plane (<i>Postulate 4</i>).</p>	<p>b. Only one line contains points Q and T.</p>  <p>Answer: Through any two points, there is exactly one line (<i>Postulate 3</i>).</p>
<p>c. <math>\overleftrightarrow{KL}</math> lies in plane P.</p>  <p>Answer: If two points lie in a plane, then the line joining them lies in that plane (<i>Postulate 5</i>).</p>	<p>d. Plane G and plane H intersect along <math>\overline{AB}</math>.</p>  <p>Answer: If two planes intersect, then their intersection is a line (<i>Postulate 6</i>).</p>
<p>e. There is another point besides point W on line <math>m</math>.</p>  <p>Answer: A line contains at least two points (<i>Postulate 1</i>).</p>	<p>f. One plane contains lines <math>l</math> and <math>m</math>.</p>  <p>Answer: If two lines intersect, then exactly one plane</p>

<p>g. One plane contains <math>\overline{AC}</math> and B.</p>  <p>Answer: If a point lies outside a line, then exactly one plane contains both the line and the point (<i>Theorem 2</i>).</p>	<p>contains both lines (<i>Theorem 3</i>).</p> <p>h. Lines <math>q</math> and <math>t</math> intersect at <math>M</math> and no other point.</p>  <p>Answer: If two lines intersect, then they intersect in exactly one point (<i>Theorem 1</i>).</p>
---	--

The notes in Example 3 was taken from the site.

[http://www.cliffsnotes.com/study\\_guide/Postulates-and-Theorems.topicArticleId-18851,articleId-18770.html](http://www.cliffsnotes.com/study_guide/Postulates-and-Theorems.topicArticleId-18851,articleId-18770.html)

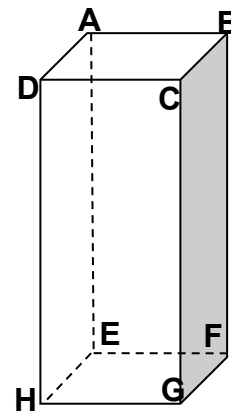
**ACTIVITY 9. Skills Practice**

Use the postulates and theorems to answer the following.

1. At least how many points are contained in a line? \_\_\_\_\_
2. At least how many points determine exactly one plane? \_\_\_\_\_
3. What is the intersection of two distinct planes? \_\_\_\_\_
4. What is the intersection of two coplanar lines? \_\_\_\_\_
5. What can contain a line and a point not on the line? \_\_\_\_\_

Use the figure on the right to answer the following.

6. What is the intersection of plane EFGH and plane CBFG? \_\_\_\_\_
7. What is the intersection of plane ABFE and  $\overline{BC}$ ? \_\_\_\_\_
8. Name another point that is in the same plane as points E, H and D. \_\_\_\_\_



**Complete the statement with the word always, sometimes or never to make the given statement true.**

9. Intersecting lines are \_\_\_\_\_ coplanar.
10. Two planes \_\_\_\_\_ intersect in exactly one point.
11. Three points are \_\_\_\_\_ collinear.
12. A line and a point not on the line are \_\_\_\_\_ coplanar.

**ACTIVITY 10.**

**Revisiting Activity No. 1: Geometric Figures in Construction**

This is to revisit activity 1, so most parts are the same with #1, although question 1-3 are additional questions.

Analyze the picture below and answer the following questions.



1. Which parts of the structure of the building models a point? How will you describe these points?
2. Which parts of the structure of the building models a line?
3. Which parts of the structure of the building models a plane?
4. How do these figures define the structure and the stability of the building?
5. How is Math used in the design of different structures?
6. ***How are problems on structure stability solved?***

**ACTIVITY 11. Take a Pic**

Take at least 2 pictures of the parts of your house that you consider to be strong and stable. Identify the undefined and the defined terms/figures that define each.

**Process Questions:**

1. What geometric figures are used to define each part of your house?
2. How do each contribute to its stability? What postulates and theorems support this?



3. *How do problems on structure stability solved?*

**End of FIRM UP**

In this section, the discussion was about how the points, lines, and planes are called the undefined terms in Geometry. We also learned how to name and represent each undefined and defined term using models. Real-life models and situations were also identified to connect what you have learned to the real-world.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.






**DEEPEN**

Your goal in this section is to relate geometric figures to real-life problems or situations. Also, you will organize and assess your learning in this topic. Before proceeding to the next section, consider the following questions: ***How are***

**ACTIVITY 12. Test of Understanding**

**Instruction:** In the previous section, we looked at different problems on structure stability. Let's put together in the table below our answers to the essential question that we asked for each problem.

Essential Question	Problem 1:	Problem 2:	Problem 3:
<b>How do problems on structure stability solved?</b>	Furniture 	Equipment 	Construction 
	How many legs does the table have? What do you think is the reason why the maker of this table make use of this number of legs?	How many legs does the tripod have? What do you think is the reason why the maker of this tripod make use of this number of legs?	What kind of lines are used in constructing the foundation of the house? What do you think is the reason why the engineers made use of this kind of lines?

**Process Questions:**

- Look at your answers to the essential question in the above table. What do all the answers have in common?
- What made each furniture/equipment/structure stable in each problem? Do they have the same basis for their stability? If yes, explain. If not, cite basis/bases of stability which is/are not found on the other furniture/equipment/structure?
- Complete the following statement and support your answer with the examples from the above problems.

**Problems on structure stability is solved using ...**

**Supporting reasons and examples:**

**ACTIVITY 13. Revisiting Activity 1**

Go back to your answers in Activity No.1 and revise them.



1. What are some of the geometric figures that you can see in the structure of the building?
2. How do these figures define the structure and the stability of the building?
3. How is Math used in the design of different structures?
4. ***How are problems on structure stability solved?***

**ACTIVITY 14. Revisiting Activity 2**

Go back to your answers in Activity No.2 and revise them.

**ACTIVITY 2: Anticipation/ Reaction Guide**

**Instruction: Respond to each statement and write your answer in the After the Lesson Column.**

Write **A** you if you agree with the statement

Write **B** you if you disagree with the statement

Before the Lesson	TOPIC: The undefined terms in Geometry	After Lesson
	1. Points, lines, and planes are the bases in defining other terms in Geometry.	
	2. An axiomatic system is a set of axioms/postulates, from which some or all axioms/postulates can be used together to logically derive and prove theorems.	

	3. A theorem is a statement that is accepted only after it has been proven.	
	4. Points that lie on the same line are coplanar.	
	5. A postulate is a statement that is accepted only after it has been proven.	
	6. Two points always determine a line.	
	7. At least three noncollinear points determine a plane.	
	8. Points, lines and planes can be modeled by real-life objects and situations.	

### End of Deepen

In this section, you have seen and witness how geometric figures relate with real-life situations.

What have you realized about the topic? What new learnings have you made for yourself?

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section...



### TRANSFER

Your goal in this section is to apply your learning to real life situations. You will demonstrate your understanding by accomplishing the given practical task below.

#### **ACTIVITY 15. My own structural design**

Make a sketch of any structures (house, school, commercial building, etc.) you want to design. Explain how the design becomes stable.

**ACTIVITY 16. Writing a Synthesis Journal**

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**ACTIVITY 16: Writing a Synthesis Journal**

Complete the table below.

<b>Synthesis Journal</b>			
<b>Lesson</b>	<b>What I did.</b>	<b>What I learned.</b>	<b>How I can use it</b>

## Lesson 2.2 Triangle Congruence



### EXPLORE

You began by engaging in an activity which will explore the concepts of congruent triangles and how they are applied in the real world.

#### ACTIVITY 1. Brainstorming Activity

**DESCRIPTION:** Watch a video clip on an acrobat show featuring tightrope walking. After viewing, look for a partner and answer the following questions;

1. What ability/quality does the tightrope walker have which enables him to walk on the rope?
2. How do you feel while watching it?
3. What cause him not to fall at the beginning?
4. Where does the balance lie?
5. How does this work?
6. What figure constitutes the stability of the tightrope walker?
7. In your opinion, how are problems about stability solved?

Share your answers to the class and note the similarities and the differing points of view or intriguing ideas.



*You gave your initial answers to the questions and statements given above. Let's find out how others would answer the above and compare their ideas to our own. As you compare, you will find out if your ideas are in line with the standard. You will also learn other concepts which will help you complete a required project found at the end. This project is about making a sketch of any structural designs such as buildings, houses, etc. and defend its stability.*



<http://www.dailymail.co.uk/news/article-2170881/Chinese-tightrope-walker-plummets-ground-trying-high-wire-stunt-backwards-AND-blindfolded.html>

This site shows a video clip on tightrope walking.

**ACTIVITY 2. How much do I know?**

**GENERALIZATION TABLE**

After hearing your thoughts on the situation given in the previous activity, it would be better to have a record on the flow of your ideas. Fill up the first column of the generalization table, the initial thoughts about congruent triangles which you will revisit as you progress in the lesson.

My Initial Thoughts	My Findings And Corrections	Supporting Evidence	Qualifying Conditions	My Generalizations

You have just given your initial ideas about congruent triangles; now proceed by sharing them to 2 of your classmates.

**End of EXPLORE:**

You have heard how others answered the questions in the previous activity and expressed what you know about congruent triangles, so you will start finding out the answers by doing the next activity.



**FIRM-UP**

Your goal in this section is to learn and understand key concepts of triangle congruence, so let's start by knowing what congruent triangles are.

**ACTIVITY 3. What Are You?**

**CONCEPT BUILDING ABOUT CONGRUENT TRIANGLES**

**MODELING WITH MANIPULATIVES**

Materials:	Barbecue sticks	Protractor
	Ruler	cutter and pair of scissors
	Markers	cartolina

Instructions: Work with a partner:

1. Cut 2 pieces of barbecue sticks with measure 5, 7 & 9 inches each with a total of 6 pieces.
2. Arrange the 3 pieces with measure 5,7 and 9 inches to form a triangle.
3. Name the 2 triangles as  $\triangle ABC$  and  $\triangle XYZ$ .
4. Measure the 3 angles of the 2 triangles.
5. Trace the triangles in a cartolina and cut the figures with the triangular shape.
6. Place 1 cut-out figure on top of the other such that all the corresponding parts will coincide.

Answer the following questions:

1. What can you say about the size and shape of the 2 triangles; cut-out figures?
2. What have you noticed about the measure of the corresponding angles?
3. What have you observed of the corresponding sides and angles?
4. What are the correspondence of the vertices, angles and sides?
5. What is your conclusion about the 2 triangles?
6. What are congruent triangles?
7. What happens if you change the pairing of the vertices?
8. What is the effect if only the 3 corresponding angles of the triangles are congruent but their corresponding sides are not?
9. If there is a slight movement of the sides, what is its effect to the angles? What about the other way around?
10. How stable is a triangle? Explain why?
11. Based on this activity, how is stability problem be solved?

Exercise:

1. Draw 3 pairs of congruent triangles of any kind and size, name and cut them.
2. Put one triangle on top of the other.
3. Record the correspondence of the vertices, the congruent corresponding sides and angles.
4. Write the definition of congruent triangles.

You may check your definition by clicking this site;


<http://elearning.skola.edu.mt/wpcontent/uploads/2010/03/congruenttriangles.pdf>



This provides a focused instruction about congruent triangles.

OR CLICK this URL for a video explanation of congruent triangles.

<http://www.onlinemathlearning.com/geometry-congruent-triangles.html>



**CONGRUENT TRIANGLES**

are triangles whose vertices can be made to correspond so that the 3 corresponding angles and the 3 corresponding sides are congruent.

**ACTIVITY 4. Is it True?**

**INVESTIGATIVE ACTIVITY ON THE CONGRUENCE POSTULATES**

You have already defined congruent triangles and you have seen how tedious it is to prove that 2 triangles are congruent. But is there a way of knowing whether the two triangles are congruent with lesser number of congruent corresponding parts? You will find out the answer after doing this investigation.

**INVESTIGATE:**

You look for two other buddies and perform this investigation.

Directions:

- A.
  1. In the previous activity, what are the parts with the given measures? How many parts?
  2. As a result, what happened to the rest of the parts?
  3. What conjecture can you make about the 2 triangles?
  
- B.
  1. Trace a  $36^\circ$  angle whose 2 sides measure 6 in. and 9 in. Connect the 2 distinct endpoints of the segments and cut the resulting figure.
  2. Repeat the process of tracing another figure with exactly the same measure of the angle and the 2 sides, connect and cut.
  3. Compare the 2 triangles and record your observation.
  4. State your generalization.
  
- C.
  1. Draw an 8 inches segment whose endpoints are the vertices of the 2 angles whose measures are  $40^\circ$  and  $60^\circ$ .
  2. Extend the sides of the angles such that the sides will intersect.
  3. Cut the resulting figure.
  4. Duplicate the drawing and cut.
  5. Place 1 figure on top of the other and write your conclusion.

Report your findings, compare with the answers of the other groups and state your generalizations. You can also give the meaning of the side included between 2 angles and the angle included between 2 sides.

The generalization of the first investigation is referred to as the



### **SSS CONGRUENCE POSTULATE**

If the three sides of one triangle are congruent to the 3 corresponding sides of another triangle, then the two triangles are congruent.

The second investigation is summarized as the



### **SAS CONGRUENCE POSTULATE**

If the 2 sides and the included angle of one triangle are congruent to the corresponding 2 sides and the included angle of another triangle, then the two triangles are congruent.

The result of the third investigation is referred to as the



### **ASA CONGRUENCE POSTULATE**

If the 2 angles and the included side of one triangle are congruent to the corresponding 2 angles and the included side of another triangle, then the 2 triangles are congruent.



### **DEFINITION**

An angle included between 2 sides – is an angle whose vertex is the common endpoint of the 2 sides.

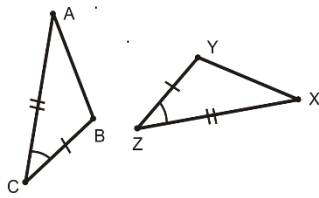
A side included between 2 angles – is a line segment whose endpoints are the vertices of the 2 angles.

**ACTIVITY 5. Hwat's My Perfect Partner?**

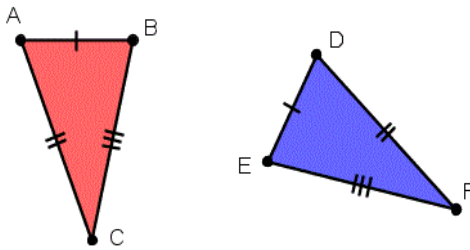
**IDENTIFYING THE CONGRUENCE POSTULATES USED IN THE CONGRUENT TRIANGLES**

From the given figures, spot the congruent corresponding parts of the triangles. Identify the Congruence Postulate which will justify the congruency of the triangles and explain why.

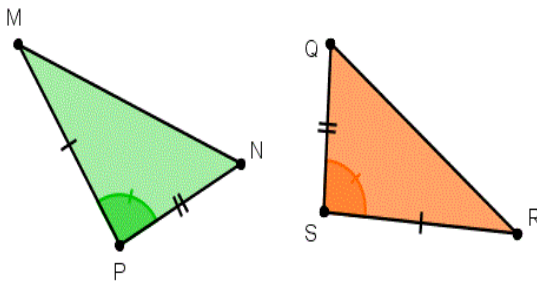
1.)



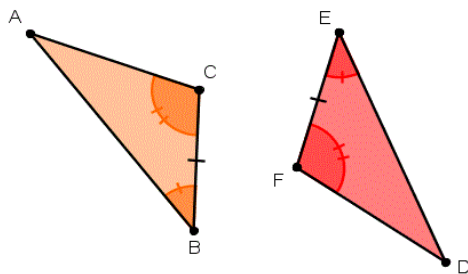
2.)



3.)



4.)



**ACTIVITY 6. Draing Out the Congruence Parts of the Triangles**

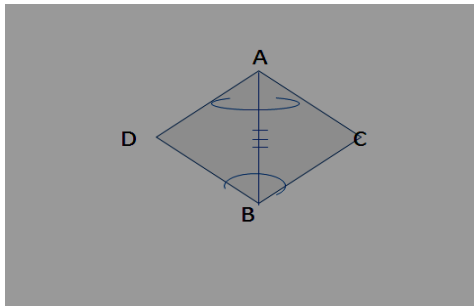
Give the congruent parts which you can derive from the given hypothesis and state the supporting reason:

1. If  $\triangle ACE$  is an isosceles triangle with  $\overline{CE}$  as the base, then \_\_\_\_\_.  
Reason \_\_\_\_\_
2. If  $\overline{AM}$  bisects  $\angle CAE$ , then \_\_\_\_\_.  
Reason \_\_\_\_\_
3. If  $\overline{MN}$  &  $\overline{PR}$  bisect each other at O, then \_\_\_\_\_.  
Reason \_\_\_\_\_
4. If X is the midpoint of  $\overline{WY}$ , then \_\_\_\_\_.  
Reason \_\_\_\_\_
5. If  $\overline{ST} \perp \overline{PR}$  at O, then \_\_\_\_\_.  
Reason \_\_\_\_\_
6. If  $\overline{AB}$  is the perpendicular bisector of  $\overline{CE}$ , then \_\_\_\_\_.  
Reason \_\_\_\_\_

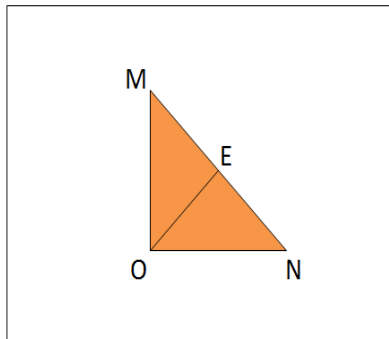
A discussion of the answers will follow after performing the entire activity 6 which include the items that follow.

Using the specified congruence postulate, name the congruent parts which will complete the triangle congruence:

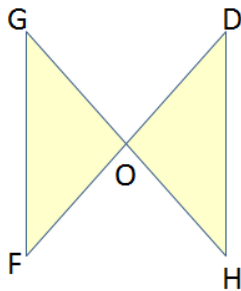
1. ASA



2.



3. SAS



MATRIX

HYPOTHESIS	CONCLUSION	REASON
1. AB bisects $\angle CAD$	$\angle CAB \cong \angle DAB$	Angle bisector divides an angle into 2 congruent parts
2. M is the midpoint of ST	$MS \cong MT$	Midpoint divides a line segment into equal parts
3. $XY \perp PR$ at Y	$\angle XYP$ and $\angle XYR$ are right angles	Perpendicular line segments intersect and form right angles
4. $\angle XYP$ and $\angle XYR$ are right angles	$\angle XYP \cong \angle XYR$	All right angles are congruent
5. $\triangle ABC$ is an isosceles triangle where BC is the base	$AB \cong AC$	An isosceles triangle has two congruent legs or sides

**ACTIVITY 7. Focused Instruction On Proving Triangle Congruence**

<http://www.youtube.com/watch?v=TPL12Tk7L6U>

This site contains a video explaining the congruency of triangles using the congruence postulates.



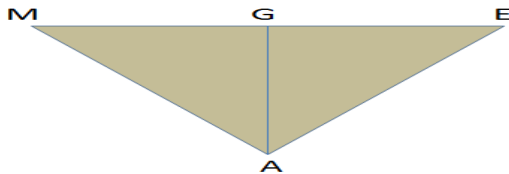
Study very well the flow of the proof of congruent triangles in the following examples:

Example 1:

Given:  $\triangle AME$  is an isosceles  $\triangle$  where  $ME$  is the base

$G$  is the midpoint of  $ME$

Prove:  $\triangle AGE \cong \triangle AGM$



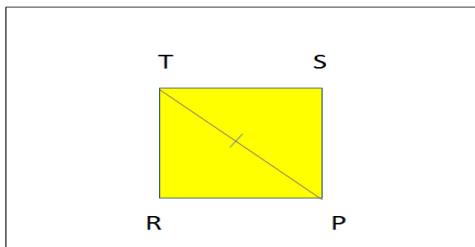
STATEMENTS	REASONS
1. $\triangle AME$ is an isosceles $\triangle$ where $ME$ is the base	1. Given
2. $AM \cong AE$ <span style="color: red; margin-left: 150px;">S*</span>	2. Isosceles triangle has two congruent sides.
3. $G$ is the midpoint of $ME$	3. Given
4. $GM \cong GE$ <span style="color: red; margin-left: 150px;">S*</span>	4. Midpoint divides a line segment into two congruent segments.
5. $AG \cong AG$ <span style="color: red; margin-left: 150px;">S*</span>	5. Reflexive Property of Equality
6. $\triangle AGE \cong \triangle AGM$	6. SSS Congruence Postulate

S\* = side

Example 2:

Given:  $TP$  is a diagonal of the parallelogram  $TSPR$

Prove:  $\triangle TSP \cong \triangle PRT$



STATEMENTS	REASONS
1. TP is a diagonal of the parallelogram TSPR	1. Given
2. $\angle STP \cong \angle RPT$ <span style="color: red;">A*</span>	2. If lines cut by the transversal are parallel, then alternate interior angles are congruent.
3. $\angle SPT \cong \angle RTP$ <span style="color: red;">A*</span>	3. If lines cut by the transversal are parallel, then alternate interior angles are congruent.
4. $PT \cong TP$ <span style="color: red;">S*</span>	4. A line segment is congruent to itself (Reflexivity)
5. $\triangle TSP \cong \triangle PRT$	5. ASA Congruence Postulate

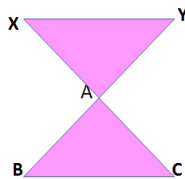
A\*=angle

Example 3:

Given: XC bisect YB at A;

A is the midpoint of YB

Prove:  $\triangle XYA \cong \triangle CBA$



STATEMENTS	REASONS
1. XC bisect YB at A	1. Given
2. $AY \cong AB$ <span style="color: red;">S*</span>	2. Bisector divides the line segment into two congruent segments.
3. A is the midpoint of YB	3. Given
4. $YA \cong BA$ <span style="color: red;">S*</span>	4. Midpoint divides a line segment into two equal parts.
5. $\angle YAX \cong \angle BAC$ <span style="color: red;">A*</span>	5. Vertical angles are congruent.
6. $\triangle XYA \cong \triangle CBA$	6. SAS Congruence Postulate

**ACTIVITY 8. Proof Flowcharting**

Study the examples below. Follow the arrow. It indicates the consequence of the given statement.

Example 1:

STATEMENTS	REASONS
1. $\triangle AME$ is an isosceles $\triangle$ where $ME$ is the base	1. Given
2. $AM \cong AE$ <span style="color: red;">S*</span>	2. Isosceles triangle has two congruent sides.
3. $G$ is the midpoint of $ME$	3. Given
4. $GM \cong GE$ <span style="color: red;">S*</span>	4. Midpoint divides a line segment into two congruent segments.
5. $AG \cong AG$ <span style="color: red;">S*</span>	5. Reflexive Property of Equality
6. $\triangle AGE \cong \triangle AGM$	6. SSS Congruence Postulate

Example 2:

STATEMENTS	REASONS
1. $TP$ is a diagonal of the parallelogram $TSPR$	1. Given
2. $\angle STP \cong \angle RPT$ <span style="color: red;">A*</span>	2. If lines cut by the transversal are parallel, then alternate interior angles are congruent.
3. $\angle SPT \cong \angle RTP$ <span style="color: red;">A*</span>	3. If lines cut by the transversal are parallel, then alternate interior angles are congruent.
4. $PT \cong TP$ <span style="color: red;">S*</span>	4. A line segment is congruent to itself (Reflexivity)
5. $\triangle TSP \cong \triangle PRT$	5. ASA Congruence Postulate

Example 3:

STATEMENTS	REASONS
1. $XC$ bisect $YB$ at $A$	1. Given
2. $AY \cong AB$ <span style="color: red;">S*</span>	2. Bisector divides the line segment into two congruent segments.
3. $A$ is the midpoint of $YB$	3. Given
4. $YA \cong BA$ <span style="color: red;">S*</span>	4. Midpoint divides a line segment into two equal parts.
5. $\angle YAX \cong \angle BAC$ <span style="color: red;">A*</span>	5. Vertical angles are congruent.
6. $\triangle XYA \cong \triangle CBA$	6. SAS Congruence Postulate



**ACTIVITY 9. Practice On Proving Triangle Congruence**

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Answer this exercise with a partner.

Write a 2-column proof of the ff. and sketch the figure:

- Given:  $MN \perp MR$  &  $NP$  where  $R$  &  $P$  are on the opposite sides of  $MN$   
 $O$  is the midpoint of  $MN$

Prove:  $\triangle RMO \cong \triangle PNO$

- Given:  $\triangle XWY$  is isosceles where  $XY$  is the base  
 $Z$  is the midpoint of  $XY$

Prove:  $\triangle WZX \cong \triangle WZY$

**ACTIVITY 10. Interactive Activity on Proving Congruency of Triangles**

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[http://www.mathwarehouse.com/classroom/worksheets/congruent\\_triangles/Angle-Angle-Side-euclidean-proof-worksheet-activities.pdf](http://www.mathwarehouse.com/classroom/worksheets/congruent_triangles/Angle-Angle-Side-euclidean-proof-worksheet-activities.pdf)

This site provides interactive activities on proving congruency of triangles.

[http://www.mathwarehouse.com/geometry/congruent\\_triangles/side-angle-side-postulate.php](http://www.mathwarehouse.com/geometry/congruent_triangles/side-angle-side-postulate.php)

This website provides interactive activities on proving congruency of triangles using the SAS congruence postulate.

Explore these sites and answer as many exercises as you wish until you can master the skill in proving congruency of triangles.

Reflections: Answer these questions in your reflection log.

- How do you find the exercises?
- Is learning how to prove congruency of triangles taking place? Explain.
- What have you realized after doing the exercises?
- Do they improve your logical thinking?
- Do you think this can help you in your day to day activity? How?
- How can the concepts of congruency of triangles affect stability of a structure?

Click on **SAVE** if you have responded to the questions asked

**ACTIVITY 11. Exercises on Proving**

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On your own, sketch the figure of the given triangles and make a 2-column proof of the ff.

1. Given: AE and CD bisect each other at M  
Prove:  $\triangle AMC \cong \triangle EMD$

STATEMENTS	REASONS

2. Given: PR is the  $\perp$  bisector of ST at R  
Prove:  $\triangle PRS \cong \triangle PRT$

STATEMENTS	REASONS

When you're done, find a learning partner and take turns in commenting your answers.

Pause to reflect on the following questions:

How is your experience in proving congruency of triangles? Can you manage to do it on your own? If you do, then you are ready to use it in the next activity which will prepare you to do the succeeding tasks.

### **ACTIVITY 12. Paper Planes**

Watch this video before you work on the activity so you will have an idea on how to proceed with it.

[http://www.youtube.com/watch?v=y3\\_VMPkzI8A](http://www.youtube.com/watch?v=y3_VMPkzI8A)

This video shows how to make paper plane for scaffold level 1



During the Math Fair, one of the activities is a symposium in which the delegates will report on an inquiry about an important concept in Math. You will report on how congruent triangles are applied in real-life. Your query revolves around this situation;

1. Design at most 5 different paper planes using congruent triangles.
2. Let it fly and record the flying time and compare which one is the most stable.
3. Point out the factors that affect the stability of the plane.
4. Explain why such principle works. Provide a flow chart for your reasoning.
5. Draw out some insights and make recommendations.

### End of FIRM UP:

In this section, the discussion was about congruency of triangles and the possible ways to prove them using the different congruence postulates. Flow charting helps you see the coherence of your thoughts.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.



### DEEPEN

Your goal in this section is to take a closer look at some aspects of the topic particularly the connections of congruent triangles to the real world situations and their significance to life.

### ACTIVITY 13. Got It?

In the construction of different structures, varied geometric figures are used. You are to infer which of the polygons will contribute to their balance and stability. Take a look at the 2 pictures below.



**Think, Pair and Share on the answer of the question which follows;**

**Which of the two doorways do you think is more stable? The one with the brace or no brace ? Explain why.**

**To check whether you got the right ideas, read the text inside the box.**



**In the doorway with the diagonal brace, the wood forms triangles whose sides have fixed lengths. The SSS Congruence Postulate guarantees that these triangles are rigid, because a triangle with given side lengths has only one possible size and shape. The doorway without the brace is unstable because there are many possible shapes for a four-sided figure with the given side lengths.**

**ACTIVITY 14.**

**Picture Analysis On Congruent Triangles In Real –Life Setting**

Group yourselves by 3 and take a look at the different pictures. Identify the geometric figure/s shown, answer the questions at the end and share it in your group.



**Q & A**

1. What parts of the picture illustrates congruent triangles?
2. How do you know that these triangles are congruent?
3. Why do these triangles need to be congruent?
4. Does congruency of triangles affect stability of structure? Explain why.
5. Will this knowledge help solve stability problems? In what manner?
6. What do you think will happen if a certain pair of triangles are not congruent?

Volunteers report the answers of the questions which they share in their respective groups.

**ACTIVITY 15.**

**Proving Congruency of Triangles From The Pictures In The Real-World**

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Look for 3 group mates, and together identify the different congruent triangles in the objects found in the picture. Discuss how and why they are congruent. Justify the answers by explaining the appropriate postulates which are applicable. Post your flow chart showing the specific congruent parts and the supporting reasons and the class will go on a Gallery Walk. A member of a group will stay behind as a curator to explain why these triangles are congruent.

OBJECT	CONGRUENT PARTS	JUSTIFICATIONS
1. Butterfly	wings	The wings of the butterfly are symmetrical which means one side is the mirror image of the other side, thus the measure of all the three sides of one wing have exactly the same measure as the corresponding 3 sides of the other.
2. Roof trusses		
3. Seaplane		
4. Window		

(The instructions on what to do are already given above.)

To further prepare you to do the task on coming up with the proposal of a design of stable stools with fewer legs, perform the next activity.

**ACTIVITY 16. Parachute: Work out this activity within a group of 4 members.**



In the upcoming City Festival, there will be a parachute flying. You are to submit a certain design of a parachute and an instruction guide of how it operates. The designer who can come up with a parachute which can fly the longest wins a prize. Present the mechanics and show the flow on how you come up with such a design.

Process Questions:

1. How was the balance of the parachute achieved?
2. From what figure did you pattern the design?
3. What were the conditions of the parts?
4. Why were they like that?
5. How did you know such conditions exist? Justify your answer.
6. Are the concepts of triangle congruency important in the making of the design?
7. How did these contribute to the stability of the structure?

Revisit your generalization table and fill in the new ideas you gain, findings and evidence of congruent triangles.

MY INITIAL THOUGHTS	MY FINDINGS AND CORRECTIONS	SUPPORTING EVIDENCE	QUALIFYING CONDITIONS	MY GENERALIZATIONS

**ACTIVITY 17. Video Clip on the Practical Applications of Congruent Triangles**

<http://www.nexuslearning.net/books/ml-geometry/Chapter4/ML%20Geometry%204-3%20SSS%20and%20SAS.pdf>

**This website contains real-life applications of congruent triangles.**

Take time to watch this video clip and answer the ff. questions:

1. How relevant are congruent triangles in the real-world? Explain.
2. Do people always measure things to ensure congruency? Explain.
3. What do they do if such measure is inaccessible?
4. Is it possible that this congruency of triangles will not be applied?
5. What will happen to the structures?



6. Is the knowledge of triangle congruency important? How does it solve stability problems?

### **ACTIVITY 18. Wooden Stools**

To further enhance your skills on the congruency of triangles and prepare you of the final task on designing a hanging bridge, you should perform this task together with your group mates.



A certain barangay wanted to build wooden stools to be distributed to its constituents who were victims of flash flood. Because a bigger portion of the budget goes to the provisions of the victims, the barangay has limited funds for this project, so, bidding for the project must be done. As a draftsman, you are to submit a proposal for the design which maximizes the number of stools without sacrificing its stability. The basis for judging your proposal includes accuracy, mathematical communication, stability and practicality.

After the submission of your work output, answer the following process questions;

1. What is the best stable design that maximizes the number of stools made?
2. What supports the stability of the structure?
3. What are your findings about the lesson?
4. What new connections do you have regarding triangle congruence?
5. Is this lesson significant? Why or why not?
6. How does it solve problems on stability?

#### **End of DEEPEN:**

In this section, the discussion was about the applications of congruent triangles in the real-world.

What new realizations do you have about the topic? What new connections have you made for yourself?

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section...

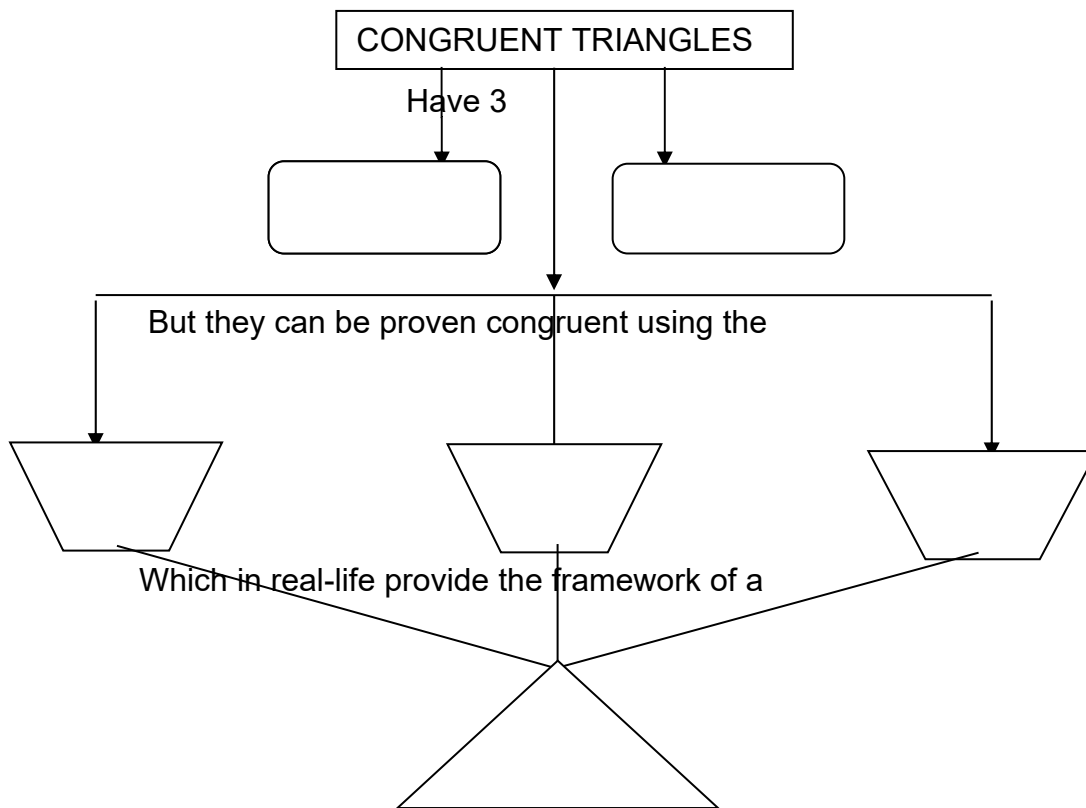


## TRANSFER

Your goal in this section is to apply your learning to real life situations. You will be given a practical task which will demonstrate your understanding. Before doing the specific PT, accomplish the next activity.

### ACTIVITY 19. Concept Mapping

Provide a visual representation of the summary of the concepts of congruent triangles. Complete the map by filling in the blank shapes.



After completing the concept map, compare your answers with the text below.

Congruent triangles have 3 pairs of congruent corresponding angles and 3 pairs of congruent corresponding sides. But they can be proven congruent using the SSS, SAS and ASA congruence postulates which in real-life provide the framework of a stable structure.

**SUMMATIVE TEST**

To check on how much you know about the concepts and skills of congruent triangles, answer the following set of questions; Please refer to the post-assessment below.

A clearer and better understanding of the lesson on congruent triangles is made evident in the performance of this given task.

**ACTIVITY 20. Performance Task**

**GROUP ACTIVITY:**

Do this task as instructed and submit it on the third meeting.



One of the moves of the City Council for economic development is to connect a nearby island to the mainland with a hanging bridge for easy accessibility of the people. Those from the island can deliver their produce, and those from the mainland can enjoy the beautiful scenery and beaches of the island.

As one of the staff of the engineers of the DPWH who is commissioned by the Special Project Committee, you are tasked to present a design/blueprint of a hanging bridge to the City Council together with the City Engineers. Your presentation will be evaluated according to its accuracy, creativity & originality, stability and mathematical communications.

After submitting your work, revisit the generalization table and complete the entry by filling in the last 2 columns; qualifying conditions and generalizations.

MY INITIAL THOUGHTS	MY FINDINGS AND CORRECTIONS	SUPPORTING EVIDENCE	QUALIFYING CONDITIONS	MY GENERALIZATIONS

You have already realized the importance of the what, how and why of the topic, now you will proceed to the knowing of oneself and how you may affect the lives of other people.

**VALUE INTEGRATION:**

1. Is your project worth the effort? Justify your answer.
2. How do you find such kind of work?

3. Do you think that the knowledge you gain about congruent triangles is applicable in this job?
4. Is the application of such knowledge of congruent triangles an option or an obligation? Why?
5. Whose responsibility is the structure stability?
5. What is your accountability of that bridge?
6. What is your reaction if the bridge you design will collapse?
7. How do you feel if people will like your design and the structure will be proven to be very stable?
8. What do you do if there is a certain structure which is found to be unstable?

### **ACTIVITY 21. Synthesis Journal**

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In your journal, write the answers to these questions:

1. What are your insights regarding the task?
2. Do you find the task useful? How?
3. How prepared are you with the given task?
4. How do you rate the experience?
5. How does the knowledge of congruent triangles help in solving stability problems?

### **ACTIVITY 22. Self-Assessment/ Reflections**

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1. What did I learn about the topic on congruent triangles?
2. How did I learn them?
3. How is my logical thinking? Has it improved?
4. Is logical thinking beneficial to me? Cite specific way/s?

Each member of your group will submit the ratings of all the members of the group according to the participation/cooperation.

### **End of TRANSFER:**

In this section, your task was to summarize your learning, findings, insights and realizations.

How did you find the performance task? How did the task help you see the real world use of the topic?

You have completed this lesson. Before you go to the next lesson, you have to answer the following post-assessment.

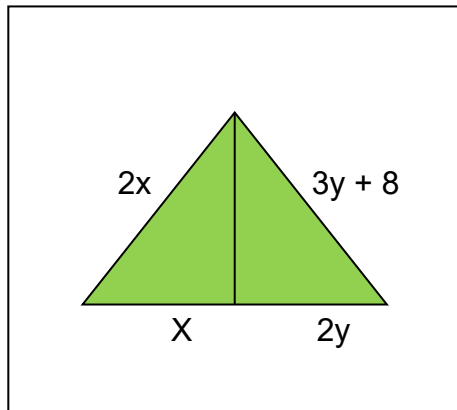
**POST-ASSESSMENT:**

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*It's now time to evaluate your learning. Click on the letter of the answer that you think best answers the question. Your score will only appear after you answer all items. If you do well, you may move on to the next module. If your score is not at the expected level, you have to go back and take the module again.*

1.  $\triangle WAS \cong \triangle RAY$ , the corresponding vertex of S is  
 A. W                                      B. A                                      C. R                                      D. Y
  
2. The following description/s are true about congruent triangles. They have  
 A. congruent corresponding angles                      C. the same size and shape  
 B. congruent corresponding sides                      D. all of a, b, & c
  
3. If you are tasked to make a table that will be placed in an uneven area, at least how many legs would be needed in order for it to stand?  
 A. 2    C. 4  
 B. 3    D. 5
  
4. Which of the following points are **NOT** coplanar?  
 A. B, C, D    C. C, D, E  
 B. B, C, E    D. B, D, F
  
5. Which of the following is a statement that is accepted as true without proof?  
 A. definition of a term  
 B. postulate  
 C. Theorem  
 D. Undefined term
  
6. Triangle ABC is congruent to triangle DEF. Therefore, segment AB is congruent to segment \_\_\_\_\_.  
 A. EF    C. DE  
 B. DF    D. ED
  
7. Triangle GHI is congruent to triangle JKL. Therefore, angle L is congruent to angle \_\_\_\_\_.  
 A. G    C. H  
 B. I    D. J
  
8. Triangle MNO is congruent to triangle PQR. Therefore,  $QP = \underline{\quad}$ .  
 A. MO                                      B. NM                                      C. MN                                      D. ON

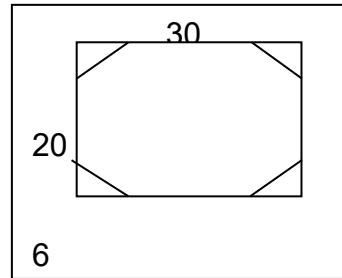
9. Triangle STU is congruent to triangle VWX. Therefore, triangle XVW is congruent to triangle \_\_\_\_.
- A. UST                      B. UTS                      C. STU                      D. SUT
10. Jason wants to prove triangle ABC congruent to triangle XYZ. He knows that  $AB = XY$  and  $AC = XZ$ . What other information must he know to prove the triangles congruent?
1. Angle C is congruent to angle Z
  2. Angle A is congruent to angle X
  3. Angle B is congruent to angle Y
  4.  $BC = YZ$
- A. Choice 1 or 4 will either work                      C. Choice 3 or 4 will either work  
 B. Choice 2 or 4 will either work                      D. None of the above
11. The 2 friends Lordan and Gigi are evaluating the congruent figures, Lordan says  $\triangle ABC \cong \triangle PQR$  and Gigi says  $\triangle CAB \cong \triangle RPQ$ . Who is correct?
- A. Gigi                      B. Lordan                      C. both of them                      D. no one
12. Which of the following statements is/are true about triangles?
- I. If two triangles are congruent, then their perimeters are equal.
  - II. If two triangles have equal perimeters, then they are congruent.
- A. I only                      B. II only                      C. I & II                      D. None at all
13. Find  $x$  and  $y$  in the figure showing 2 congruent triangles.



- A.  $x = 2$   
 $y = 1$
- B.  $x = 6$   
 $y = 3$
- C.  $x = 12$   
 $y = 6$
- D.  $x = 16$   
 $y = 8$

14. Shiela cut 4 congruent triangles off the corners of a rectangle to make an octagon as shown in the figure. What is the area of the octagon?

- A.  $456 \text{ cm}^2$                       C.  $552 \text{ cm}^2$   
 B.  $528 \text{ cm}^2$                       D.  $564 \text{ cm}^2$



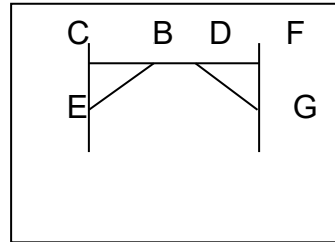
15. If 2 cut-out triangles are laid one on top of the other so the parts of the same measures are match up. What is true about the triangles?

- I. They have the same size.  
 II. They have the same shape.  
 III. They are congruent.

- A. I                                      B. II                                      C. III                                      D. I, II & III

16. Allan is using a brace to support a table top, namely BE and DG so that  $\triangle BCE \cong \triangle DFG$ . If the  $m\angle CEB = 50^\circ$ , then the  $m\angle FGD$  is

- A.  $40^\circ$                                       C.  $60^\circ$   
 B.  $50^\circ$                                       D.  $90^\circ$



17. Why should the brace of the table be triangular?



- I. to resist the load put on the table  
 II. to make it more stable  
 III. to avoid the table to collapse

- A. I only                                      B. II only                                      C. III only                                      D. a, b & c

18. Vikki is preparing plain square cloths of different colors so that her quilt will show geometric patterns. If she needs to have 2 triangular cloths of the

same size and shape from the available square cloths, how will she cut it without measuring?

- A. Cut it along the diagonal.
  - B. Cut it lengthwise
  - C. Cut it crosswise
  - D. Cut it lengthwise and then crosswise
19. Fitness equipment such as bike uses triangles in the frame in order to
- A. add aesthetic appearance
  - B. maintain its form
  - C. add strength to its structure
  - D. maintain its alignment
20. To ensure that sailboat races are fair, the boats and their sails are required to have
- I. the same size
  - II. the same shape
- A. I only                      B. II only                      C. both I & II                      D. neither I  
nor II

### **GLOSSARY OF TERMS USED IN THIS LESSON:**

(List in alphabetical order the key terms and give their definitions)

1. **Angle bisector** – is a ray/segment which divides the angle into 2 equal/congruent parts.
2. **Angle included between two sides** – it is an angle whose vertex is the common endpoint of the two sides/segments.
3. **Congruent triangles** – are triangles whose vertices are paired such the three corresponding sides and the three corresponding angles are congruent.
4. **Isosceles triangle** – is a triangle with two congruent sides.
5. **Midpoint** – is a point which divides a line segment into two congruent parts.
6. **Perpendicular bisector** – is a line/ray/segment which divides a segment into two congruent segments and form right angles.
7. **Side included between two angles** – it is the side/segment whose endpoints are the vertices of the two angles.



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<http://www.dailymail.co.uk/news/article-2170881/Chinese-tightrope-walker-plummets-ground-trying-high-wire-stunt-backwards-AND-blindfolded.html>

1. video on a tightrope walker

<http://elearning.skola.edu.mt/wpcontent/uploads/2010/03/congruenttriangles.pdf>

2. focused instructions about congruent triangles

[http://www.analyze-math.com/Geometry/congruent\\_triangles.html](http://www.analyze-math.com/Geometry/congruent_triangles.html)

3. discussion of congruent triangles

4. <http://www.onlinemathlearning.com/geometry-congruent-triangles.html>  
video explanation on congruent triangles
5. [http://www.mathwarehouse.com/geometry/congruent\\_triangles/](http://www.mathwarehouse.com/geometry/congruent_triangles/)  
how do we prove congruency of triangles
6. <http://www.youtube.com/watch?v=TPL12Tk7L6U>  
video explaining congruency of triangles using all the postulates
7. [http://www.mathwarehouse.com/classroom/worksheets/congruent\\_triangles/Angle-Angle-Side-euclidean-proof-worksheet-activities.pdf](http://www.mathwarehouse.com/classroom/worksheets/congruent_triangles/Angle-Angle-Side-euclidean-proof-worksheet-activities.pdf)  
interactive activity on proving using the postulates
8. [http://www.mathwarehouse.com/geometry/congruent\\_triangles/side-angle-side-postulate.php](http://www.mathwarehouse.com/geometry/congruent_triangles/side-angle-side-postulate.php)  
interactive activity on proving congruency of triangles using the different congruence postulates
9. <http://www.nexuslearning.net/books/ml-geometry/Chapter4/ML%20Geometry%204-3%20SSS%20and%20SAS.pdf>  
real-life applications of congruent triangles
10. [http://www.youtube.com/watch?v=y3\\_VMPkzI8A](http://www.youtube.com/watch?v=y3_VMPkzI8A)  
video on how to make paper plane for scaffold level 1

## Lesson 3: Triangle Inequalities

### ☑ INTRODUCTION AND FOCUS QUESTION(S):



Have you ever wondered how the different mechanical machines found in amusement parks, and tools and equipment used in home and in work with triangular features work and designed? What mathematical explanations will justify their functions? And have you ever asked yourself how to determine the shortest path in going to different places? Find out the answers to these questions by understanding and applying the theorems on triangle inequality in solving real-life problems.

In this module the theorems of triangle inequalities will be tackled. The concepts, skills, and attitudes this lesson teaches will shape your logical thinking, strengthen your reasoning skills, and polish your mathematical communication.

Remember to find out the answers to this question: ***How can you justify inequalities in triangles? How can the knowledge of inequalities in triangles help us solve problems related to design and estimation of distances?***

**LESSON COVERAGE:**

In this lesson, you will take the following topics:

**Lesson 3 – Triangle Inequality**

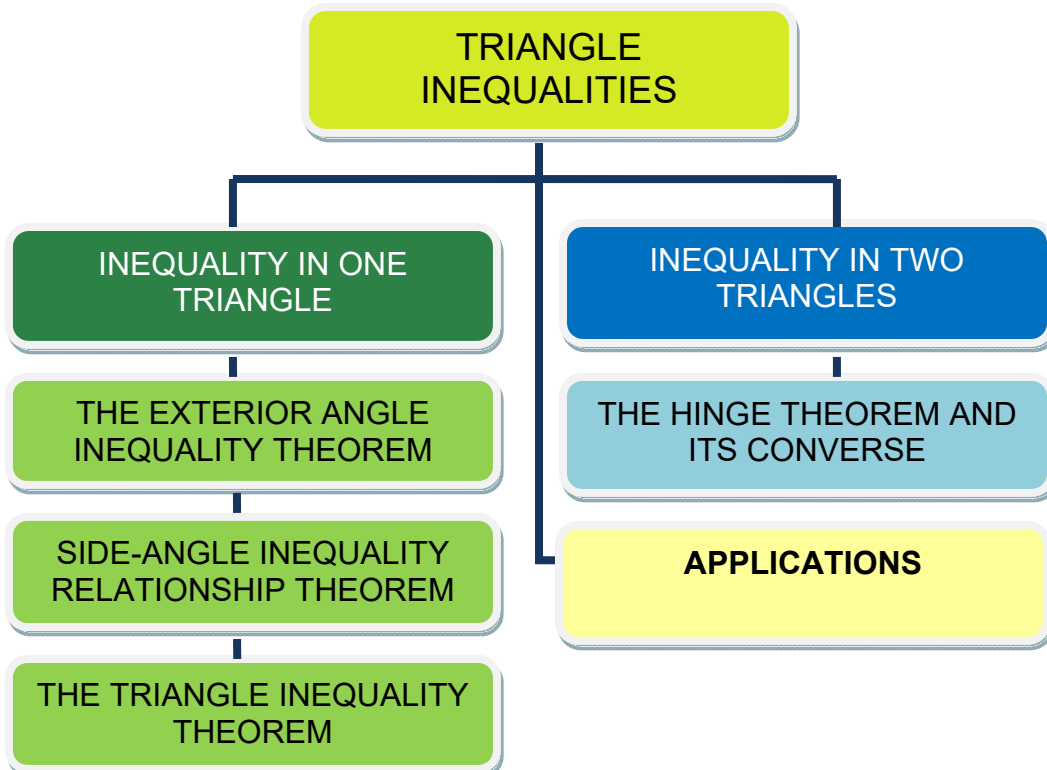
- 3.1 *Inequalities for Sides and Angles of a Triangle*
- 3.2 *Theorems on Triangle Inequality*
- 3.3 *Applications of the Theorems on Triangle Inequality*

In these lesson, you will learn the following:

<b>Lesson 3</b>	<ul style="list-style-type: none"> <li>➤ States and illustrates the theorems on triangle inequalities such as exterior angle inequality theorem, triangle inequality theorem, hinge theorem.</li> <li>➤ Applies theorems on triangle inequalities to:             <ul style="list-style-type: none"> <li>a. determine possible measures for the angles and sides of triangles.</li> <li>b. justify claims about the unequal relationships between side and angle measures.</li> </ul> </li> <li>➤ Applies the theorems on triangle inequalities to prove results involving triangle inequalities.</li> </ul>
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**MODULE MAP:**

Here is a simple map of the above lessons you will cover:



**EXPECTED SKILLS:**

To do well in this lesson, you need to remember and do the following:

10. *Define terms that are unfamiliar to you.*
11. *Explore websites that would be of great help for your better understanding of the lessons.*
12. *Take down notes of the important concepts in your journal.*
13. *Perform and complete the exercises provided.*
14. *Collaborate with the teacher and peers.*

**LESSON NO. 3.1: INEQUALITIES IN ONE TRIANGLE**  
**LESSON NO. 3.2: INEQUALITIES IN TWO TRIANGLES**

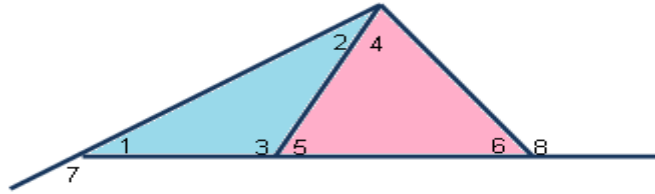
**PRE-ASSESSMENT:**

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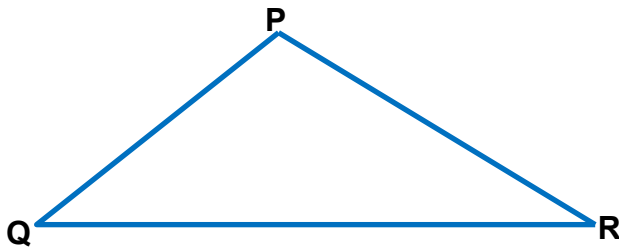
*Let's find out how much you already know about this module. Click on the letter that you think best answers the question. Please answer all items. After taking this short test, you will see your score. Take note of the items that you were not able to correctly answer and look for the right answer as you go through this module.*

1. The measure of the exterior angle of a triangle is always
  - A. greater than its adjacent interior angle.
  - B. lesser than its adjacent interior angle.
  - C. greater than its two remote interior angles
  - D. lesser than its two remote interior angles.
  
2. Which holds true about the sum of any of the lengths of the two sides of the triangle in relation to the length of its third side?
  - A. It is greater than the length of the third side.
  - B. It is lesser than the length of the third side.
  - C. It is equal to the length of the third side.
  - D. It is greater than or equal to the length of the third side.
  
3. Which of the following theorems states that "If two sides of the corresponding parts of a triangle are congruent, and the included angle of the first is bigger than the included angle of the second, then the length of the third side of the first triangle is longer than the length of the second triangle."?
  - A. The Exterior Angle Inequality Theorem
  - B. The Hinge Theorem
  - C. The Triangle Inequality Theorem
  - D. The Triangle-Angle-Sum Theorem

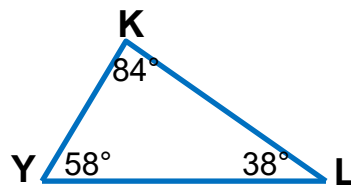
- 1.
2. 4. Using the Exterior Angle Inequality Theorem, what are the angles lesser than  $\angle 3$ ?



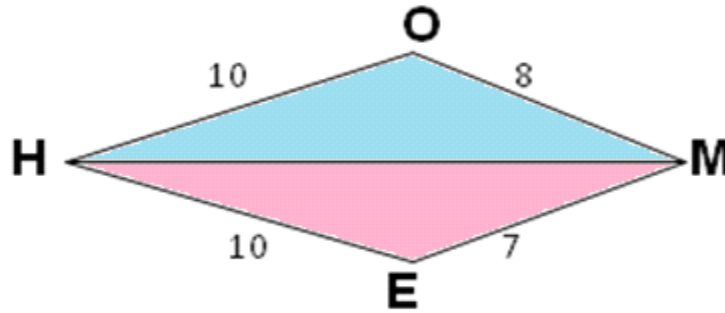
- A.  $\angle 1$  and  $\angle 2$
  - B.  $\angle 4$  and  $\angle 5$
  - C.  $\angle 4$  and  $\angle 6$
  - D.  $\angle 7$  and  $\angle 8$
4. In  $\triangle PQR$ ,  $PQ = 13$ ,  $QR = 17$ . The length of  $PR$  cannot be



- A. 5
  - B. 14
  - C. 20
  - D. 30
5. List the sides of  $\triangle LYK$  in order from least to greatest measure.



- A.  $\overline{LY}, \overline{KY}, \overline{KL}$
  - B.  $\overline{KY}, \overline{LY}, \overline{LK}$
  - C.  $\overline{KY}, \overline{KL}, \overline{LY}$
  - D.  $\overline{LY}, \overline{KL}, \overline{LY}$
6. From the inequalities in the triangles shown, a conclusion can be reached using the converse of hinge theorem. Which of the following is the last statement?

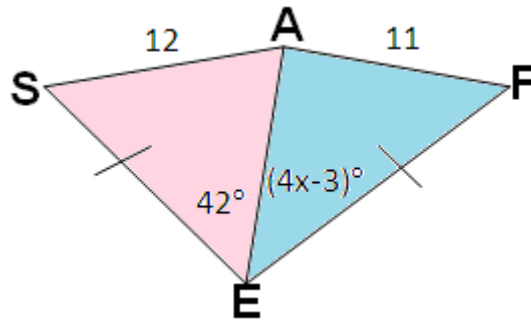


3.

- A.  $\overline{HM} \cong \overline{HM}$
- B.  $\angle OHM > \angle EHM$
- C.  $\overline{HO} \cong \overline{HE}$
- D.  $\angle EHM > \angle OHM$

7. What is the range of values for x in the figure?

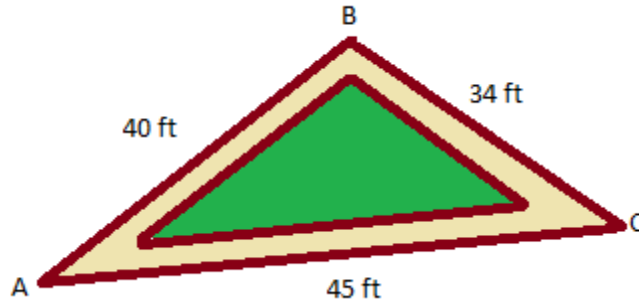
4.



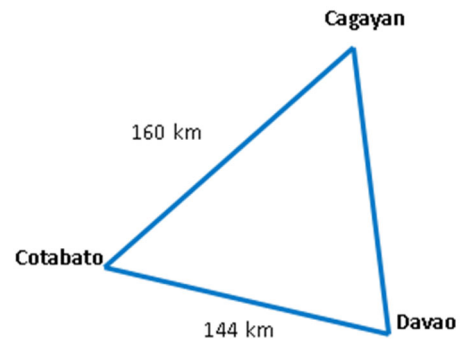
5.

- A.  $x < 11.25$
- B.  $x > 11.25$
- C.  $x \leq 11.25$
- D.  $x \geq 11.25$

8. Vince is tasked to place a big potted tree on the biggest corner of the patio. He is only given a diagram of the patio. In which corner will he place the potted tree?

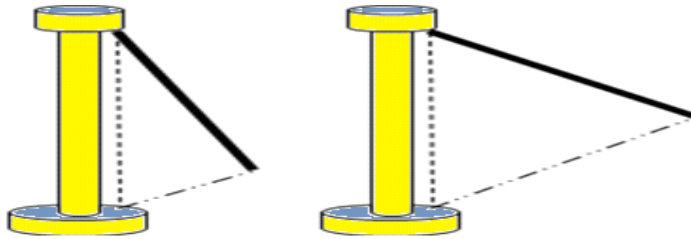


- A. Corner A  
B. Corner B  
C. Corner C  
D. at the center
- 6.
9. Clara, Dana, Jane, and Liza were instructed to create a triangle. They were each given a piece of stick of 18 in long. Each cut the stick in their own chosen lengths as follows: Clara (6 in, 6in, 6in ), Dana ( 4in ,5in, 9in), Jane, (7in, 5in, 6in), and Liza (3 in, 7 in, 10 in). Who among them was not able to make a triangle?
- A. Clara  
B. Dana  
C. Jane  
D. Liza
- 7.
11. From a station, hikers Oliver and Ruel, who both have uniform hiking, speed walk in opposite directions—Oliver speed walks eastward whereas Ruel speed walks westward. After walking 3 kilometers each, both of them take left turns at different angles—Oliver at an angle of  $30^\circ$  and Ruel at  $40^\circ$ . Both continue hiking and cover another 4 kilometers each before taking a rest. Which of the hikers is farther from their station?
- A. Ruel  
B. Oliver  
C. It cannot be determined.  
D. Ruel is as far as Oliver from the rendezvous.
12. Elena Travels from Davao to Cotabato for 144 km, Cotabato to Cagayan for 160 km, How far may she travel from Cagayan to Davao?
- A. between 16 km to 144 km  
B. between 16 km to 160 km  
C. between 16 km to 200 km  
D. between 16 km to 302 km\*

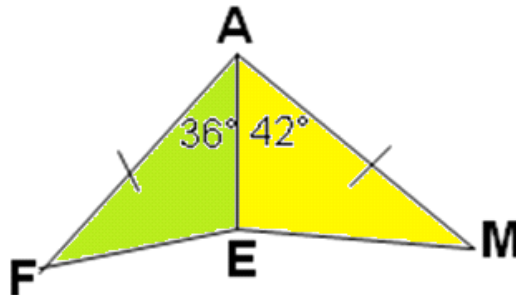




13. The chairs of a swing ride are farthest from the base of the swing tower when the swing ride is at full speed. What conclusion can you make about the angles of the swings at different speeds?

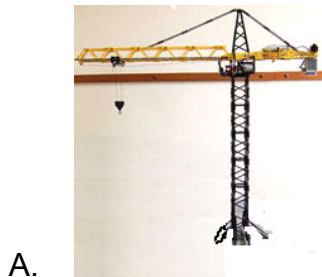


8. A. 9. The angles of the swings remain constant whether the speed is low or full.
10. B. 11. The angles of the swings are smaller at full speed than at low speed.
12. C. 13. The angles of the swings are larger at full speed than at low speed.
14. D. The angles of the swings are larger at low speed than at full speed.
14. Will you be able to conclude that  $EM > EF$  if one of the following statements is **not** established:  $\overline{AE} \cong \overline{AE}$ ,  $\overline{AF} \cong \overline{AM}$ ,  $\angle MAE > \angle FAE$ ?

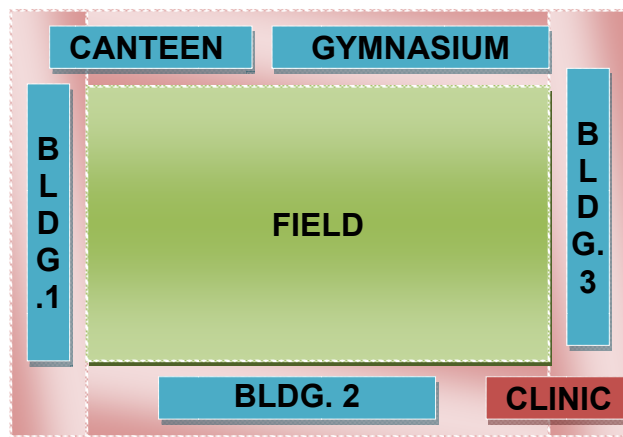


15. A. 16. Yes, I will.
17. B. 18. No, I won't.
19. C. 20. It is impossible to decide.
21. D. 22. It depends on which statement is left out.
- 23.
15. Your best friend has saved enough money to build a vacation house. In the original plan, the design was based on a larger area. Now, your friend would like it to be built on a smaller lot and the design is patterned after a simple A-Frame Style house. In connection with this, he is looking for someone who could explain to him in detail the possible adjustments and changes that have to be made to the design. Having experienced contracting construction jobs, your best friend asks you to do the project, if he is convinced that your modification is the best. What major modification will you make?

- A. adjust the included angle formed by the two braces of the frame so that it will be smaller than the original\*
  - B. adjust the included angle formed by the two braces of the frame so that it will be larger than the original
  - C. adjust the lengths of the sides by cutting it into equal measurements
  - D. adjust the length of one sides to make t shorter
16. Which of the following standards will you consider in the design modification?
- A. Creativity
  - B. Mathematical justification
  - C. Originality
  - D. Practicality
17. You are a furniture owner and a designer. One customer ordered a portable foldable table. What part of the table needs to be considered so that the table can be folded?
- A. the size of the table top
  - B. the materials to be used
  - C. the length and the number of legs
  - D. the hinge that adjusts the angles between the legs
18. You are a contractor. Your new project is a 32-storey building. This is the tallest building that you will have attempted to construct since the time you started in the construction business. Hence, there is a need for you to update your equipment in lifting loads. Which of the following cranes should be purchased?



19. In choosing the equipment in item 18, what is the main criterion to be considered to ensure quality of the product?
- Efficiency
  - Flexibility
  - Elegance of the design
  - Low cost
- 24.
20. One of your classmates fainted in the canteen. You are one of those students who assisted her in going to the clinic. The clinic is far from the gym as shown in the map below. In what path will your group pass so that you can arrive at the clinic right away?



- Pass through buildings 1 and 2
- Pass through the gym and building 3
- Pass through the field following a diagonal path\*
- Pass through building 1 then pass across the field going to building 3.

## LESSON 3.1 INEQUALITY IN ONE TRIANGLE



### EXPLORE

Let's start the module by getting your initial ideas about triangle inequality in the statements given. Then keep in mind these questions:

**How can the knowledge of inequality in one triangle help us solve problems related to design and distances?**

**How can you justify inequalities in one triangle?"**

#### ACTIVITY 1. Anticipation / Reaction Guide

**Instruction:** Respond to each statement twice: once before the lesson, and again after the lesson.

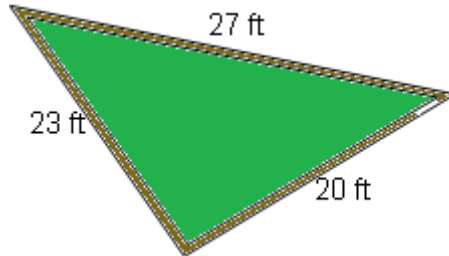
Write **A** if you agree with the statement.

Write **B** if you disagree with the statement.

Before Lesson	Statement	After Lesson
	1. You can form a triangle using your chosen lengths even without doing a trial and error process.	
	2. The measure of the exterior angle of a triangle is always greater than the measure of its two remote interior angles.	
	3. Segments with lengths 2 in, 2 in, and 4 in can form a triangle.	
	4. The sum of the lengths of any two sides of a triangle is always greater than its third side.	
	5. If you want to find for the biggest angle of a triangle, look for the longest side.	

**ACTIVITY 2. Which Is The Largest Corner?**

A landscape engineer is designing a triangular area. He wants to place potted trees in the two largest corners. In which corners will he place the two potted trees?



**Process Questions:**

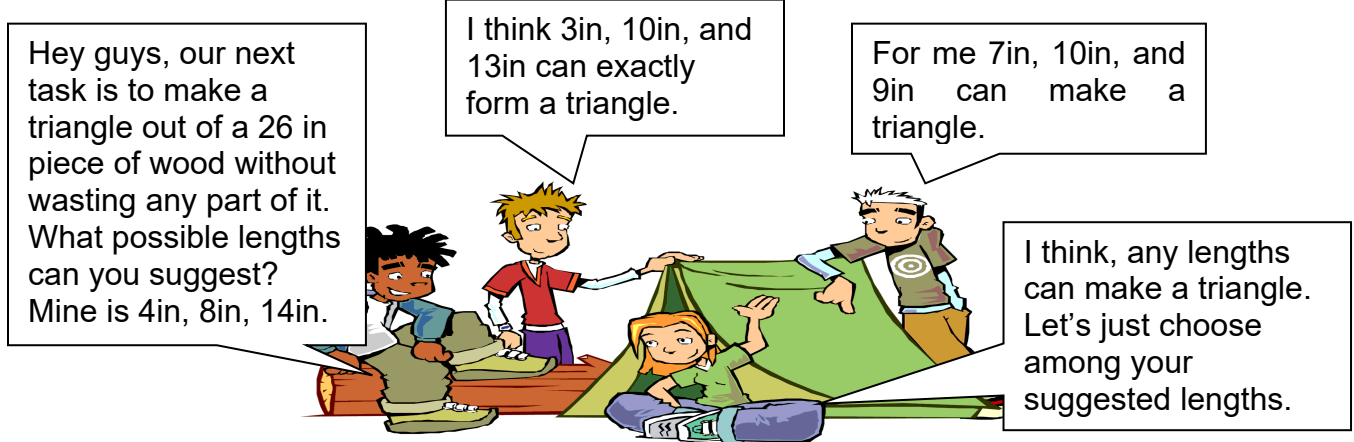
1. *What idea led you to arrive at your answer?*

2. *Can you see the relationship between sides and angles of the given triangular area? What relationship have you seen?*

3. *How can the knowledge on inequality in one triangle help us solve problems in the real-life?*

**ACTIVITY 3. Let's Make a Triangle!**

Read the conversation of the four campers: Adrian, Kenneth, Loyd, and Miko (arranged from left to right) as they solve a given task.



**Process Questions:**

1. Which of these boys is correct? Why?

2. If you are in their situation, will it be easy for you to decide which set of lengths to choose?

3. Is inequality of triangle present in their suggested lengths?

4. *How can you justify inequalities in one triangle?*

**5. How can triangle inequalities helped solve problems in the real-life?**

**END OF EXPLORE:**

You gave your initial ideas to the statements and questions given above.

Let's find out how others would answer the question and compare their ideas to our own. As you compare, you will also learn other concepts which will help you complete the required project. This project is about designing and making a miniature of mechanical equipment. We will start by doing the next activity.



**FIRM-UP**

Your goal in this section is to learn and understand key concepts on inequalities in one triangle. These are: the relationship between the measure of the exterior angle to the measure of its two remote interior angles, the relationship between the sides and angles of a triangle, and the possible lengths of the sides of a triangle.

Before you start this part, make sure that you have **installed** a pdf reader. (If not installed, download **adobe reader** which can be downloaded for free).

**Create a folder** in the desktop and name it **TRIANGLE INEQUALITIES MATERIALS**. It is where all you downloaded pdf materials will be saved.

**Download and save the following pdf files by clicking on the links below :**

1. For your reference,  
<http://images.pcmac.org/SiSFiles/Schools/AL/MobileCounty/BryantHigh/Uploads/Forms/chap05.pdf> Rename it as **Glencoe Geometry Chapter 5**.
2. For more practice problems <http://glencoe.mcgraw-hill.com/sites/dl/free/0078738181/518676/geonot2.pdf>  
Rename it as **Glencoe Interactive Study Notebook**.
3. For a study guide and practice problems, Copy the URL,  
[www.glencoe.com/sec/math/geometry/ca/ca\\_05/pdfs/gcasgw\\_SE\\_6232.pdf](http://www.glencoe.com/sec/math/geometry/ca/ca_05/pdfs/gcasgw_SE_6232.pdf) Rename it as **Glencoe Study Guide**.

**DO NOT START WITH THE FIRST ACTIVITY IF YOU HAVE NOT ACCOMPLISHED THE AFOREMENTIONED REQUIREMENTS.**



Now, let's have a review on the key concept of inequality and the properties of inequalities for real numbers. Read **page 247** of Glencoe Geometry Chapter 5 for the properties of inequalities in real numbers.

For more detailed explanation and situational examples of the properties of inequalities, click on this site <http://www.mathsisfun.com/algebra/inequality-properties.html>

The properties of inequalities that you have learned in algebra can be applied to the measures of angles and segments. You should familiarize yourself with these properties because it will be used in our proving activities.

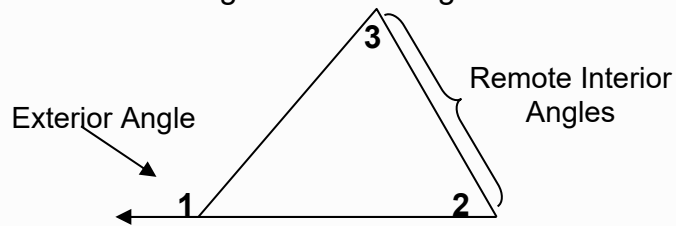
Know the exterior and remote interior angles of a triangle by reading the definition below.

After this, we will explore the relationship between the measures of the exterior angle of a triangle to the measure of its two remote interior angles.

## The Exterior Angle Inequality

### The Exterior and Angles of a Triangle

An **EXTERIOR ANGLE** of a polygon is an angle which is adjacent and supplementary to one of the angles of the triangle.



For each exterior angle of a triangle, the two non-adjacent interior angles are called its **REMOTE INTERIOR ANGLES**.



**ACTIVITY 4. Compare Angle Measures**

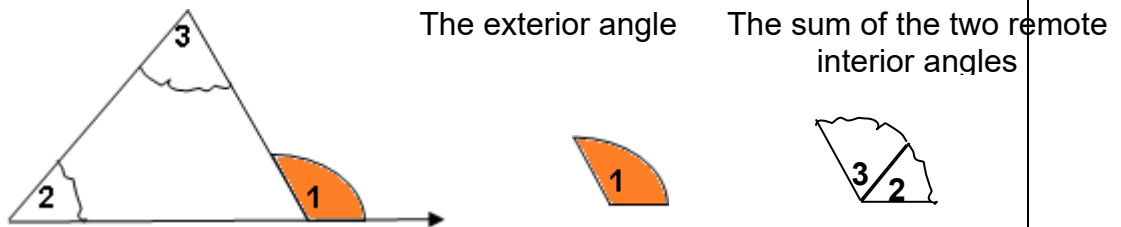
This activity will help you find out the relationship between

1. the measure of the exterior angle of a triangle to the sum of the measures of its two remote interior angles
2. the measure of the exterior angle to the measure of each remote interior angle.

**INVESTIGATION**

1. Draw and label a large triangle like the one shown below. Cut the two remote interior angles and put them together using an adhesive tape. Place the two remote interior angles on top of the exterior angle 1. What do you observe? (Note: only look at the two sides of the angles if they fit together. Disregard the arcs.)

Another option. The remote interior angles are combined together. Drag it over to angle 1. What did you observe? Did they fit together? (Note: Look only at the two sides of the angles if they fit together. Disregard the arcs.)




2. Make a conjecture about the measure of an exterior angle of a triangle in relation to the sum of the measures of its two remote interior angles.


3. Compare the measure of the exterior angle to the measure of each remote interior angle by placing each remote interior angle over the exterior angle 1. What inequalities have observed between the measures of the exterior angle to the measures of its two remote interior angles?

Another Option: Drag each remote interior angle over the exterior angle. What did you observe with the measure of the exterior angle to the measure of each of its two remote interior angles?



4. Make a conjecture about the measure of an exterior angle of a triangle in relation to the measure of each two remote interior angles.

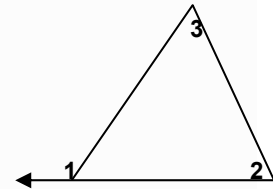
**Discuss results with other students and with your teacher in the discussion forum.**



The conjectures you've made in the investigation above would lead you to the Exterior Angle Inequality Theorem. Click on the link <http://images.pcmac.org/SiSFiles/Schools/AL/LeeCountySchools/Smit hsStationHigh/Uploads/DocumentsCategories/Documents/5.3 Inequalities in One Triangle.ppt> to view a PowerPoint file. Answer first the 5-minute check that will serve as a review of the past lessons. Then, view the slides up to the coverage of the Exterior Angle Inequality only.


**Theorem 8.1. The Exterior Angle Theorem**

The measure of the exterior angle of a triangle is equal to the sum of the measures of its two remote interior angles. That is,  
 $m\angle 1 = m\angle 2 + m\angle 3$



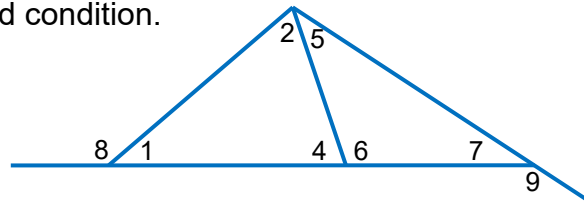
**Theorem.8.2. The Exterior Angle Inequality Theorem**

The measure of an exterior angle of a triangle is greater than the measure of either of its two remote interior angles.  
 $m\angle 1 > m\angle 2$  and  $m\angle 1 > m\angle 3$

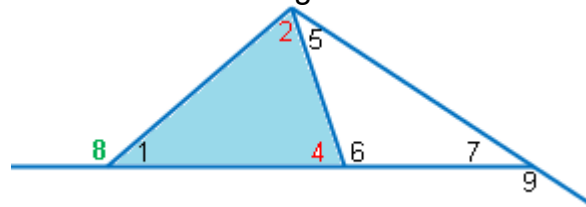


Study the additional examples below

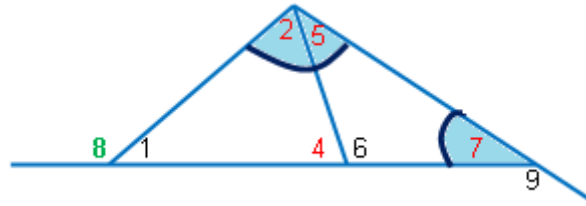
**Example:** Use the Exterior Angle Inequality Theorem to list all of the angles that satisfy the stated condition.



1. All angles whose measures are greater than  $\angle 8$ .

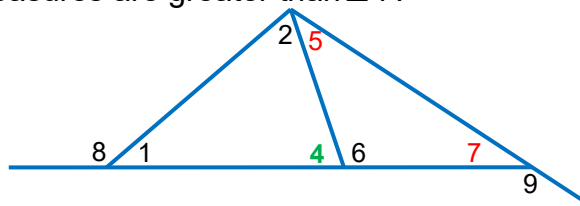


- a.  $\angle 2$  and  $\angle 4$  are the two remote interior angles of the exterior  $\angle 8$  in the smaller triangle.  
By the Exterior Angle Inequality Theorem,  $m\angle 8 > m\angle 2$  and  $m\angle 8 > m\angle 4$ .



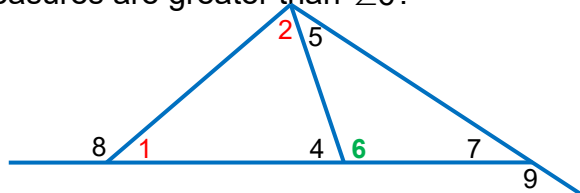
- b.  $(\angle 2 + \angle 5)$  and  $\angle 7$  are the two remote interior angles of  $\angle 8$  in the biggest triangle.  
By the Exterior Angle Inequality Theorem,  $m\angle 8 > m\angle 2$ ,  $m\angle 8 > m\angle 5$  and  $m\angle 8 > m\angle 7$ .  
Therefore, the angles that are lesser than  $\angle 8$  are  $\angle 2$ ,  $\angle 4$ ,  $\angle 5$  and  $\angle 7$ .

2. All angles whose measures are greater than  $\angle 4$ .



- $\angle 5$  and  $\angle 7$  are the two remote interior angles of the exterior  $\angle 4$ .  
By the Exterior Angle Inequality Theorem,  $m\angle 4 > m\angle 5$  and  $m\angle 4 > m\angle 7$ .  
Therefore  $\angle 5$  and  $\angle 7$  are the angles that are lesser than  $\angle 4$ .

3. All angles whose measures are greater than  $\angle 6$ .

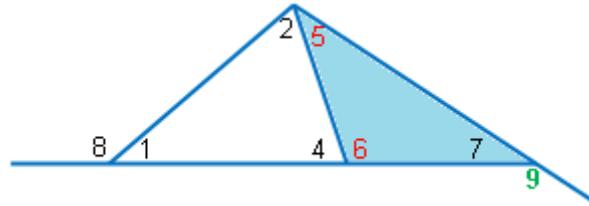


$\angle 1$  and  $\angle 2$  are the two remote interior angles of the exterior  $\angle 6$ .

By the Exterior Angle Inequality Theorem,  $m\angle 6 > m\angle 1$  and  $m\angle 6 > m\angle 2$ .

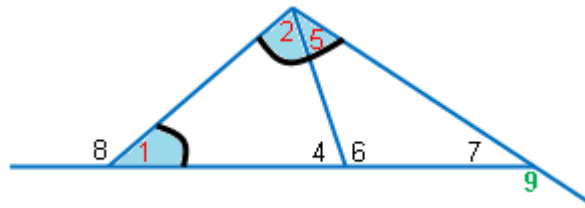
Therefore  $\angle 1$  and  $\angle 2$  are the angles that are lesser than  $\angle 6$ .

4. All angles whose measures are greater than  $\angle 9$ .



- a.  $\angle 5$  and  $\angle 6$  are the two remote interior angles of the exterior  $\angle 9$  in the small

By the Exterior Angle Inequality Theorem,  $m\angle 9 > m\angle 5$  and  $m\angle 9 > m\angle 6$ .



- b.  $(\angle 5 + \angle 2)$  and  $\angle 1$  are the two remote interior angles of the exterior  $\angle 9$  in the

By the Exterior Angle Inequality Theorem,  $m\angle 9 > m\angle 5$ ,  $m\angle 9 > m\angle 2$ , and  $m\angle 9 > m\angle 1$ .  
Therefore, the angles that are lesser than  $\angle 9$  are  $\angle 5$ ,  $\angle 6$ ,  $\angle 2$  and  $\angle 1$ .



I hope the examples really helped you in determining the relationships between the angle measures of the triangle.

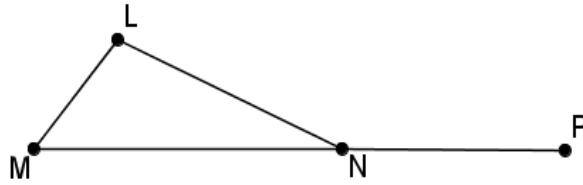
Let us now move on to the indirect proof of the Exterior Angle Inequality Theorem by clicking <http://mrpilarski.wordpress.com/2010/01/10/how-to-write-indirect-proofs-external-angle-inequality-theorem/>

Another indirect proof of this theorem is found on page 257 of Glencoe Geometry Chapter 5 pdf (main reference file saved in your computer).

#### Two-Column Proof of the Triangle Inequality Theorem

**Given:**  $\triangle LMN$  with exterior angle  $\angle LNP$

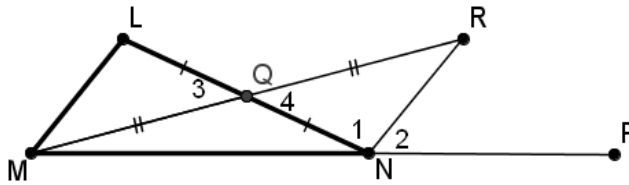
**Prove:**  $m\angle LNP > m\angle MLN$



**Proof:**

Let us prove that  $\angle LNP > \angle MLN$  by constructing the following:

1. midpoint Q on  $\overline{LN}$  such that  $\overline{LQ} \cong \overline{NQ}$
2.  $\overline{MR}$  through Q such that  $\overline{MQ} \cong \overline{QR}$



25. Statements	26. Reasons
1. $\overline{LQ} \cong \overline{NQ}$ 27. $\overline{MQ} \cong \overline{QR}$	28. Given
2. $\angle 3 \cong \angle 4$	29. Vertical angles are congruent
3. $\triangle LMQ \cong \triangle RNQ$	30. SAS Postulate
4. $\angle MLN \cong \angle 1$	31. ASA Postulate
5. $m\angle LNP \cong m\angle 1 + m\angle 2$	32. Angle Addition Postulate
6. $m\angle LNP > m\angle 1$	33. Exterior Angle Inequality Theorem
7. $m\angle LNP > m\angle MLN$	34. Substitution Property

**Process Questions:**

1. In the given examples, how can you justify inequality in triangles?

2. How were the postulates, theorems, and properties of inequalities used in the proof?

Post your answers in the **Discussion Forum** and discuss with your online classmates and teacher.

**ACTIVITY 5.**

**Practice Problems on the Inequality between the Measures of the Exterior Angle and its Two Remote Interior Angles of a Triangle**

In this activity, you will be given more practice problems for you to master the skills in identifying and applying the relationship between the measures of the exterior angle and its two remote interior angles in solving problems.

Open the Glencoe Geometry Interactive Study Notebook found in the TRIANGLE INEQUALITIES MATERIALS folder in the desktop.

Find the page navigator at the upper part of the page, change the page to 130 - 131 to view pages 122 and 123 of the book.

Fill all the boxes to complete the solution to every problem. You can print the file so that it will be very easy for you to answer. Just press CTRL + P or click the printer icon. Set the Print range from 130 to 131, then click OK.



*If you think your skill in identifying and applying the relationship between the measures of the exterior angle and its two remote interior angles in solving problems is not enough, make use of the **Glencoe Study Guide pdf material found in your TRIANGLE INEQUALITIES folder.***

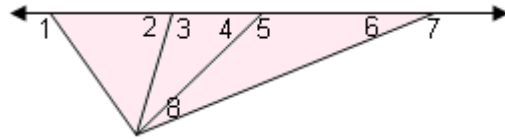
*Go to the page navigator and change it to 43 to view page 39. Read again the theorem and study the examples given. Then, answer the problems. You can print the page.*

**ACTIVITY 6. Short Quiz on the Exterior Angle Inequality**

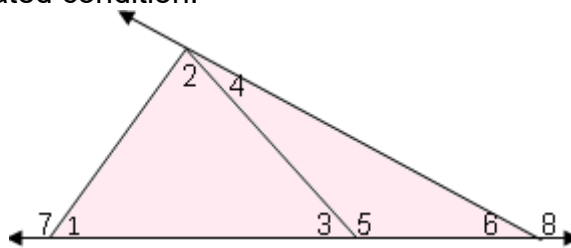
This activity will enable you to apply the Exterior Angle Inequality Theorem on your own. Write all answers in the provided box.

A. Determine which angle has the greatest measure.

1. $\angle 1, \angle 2, \angle 4$	<input type="text"/>
2. $\angle 5, \angle 7, \angle 8$	<input type="text"/>
3. $\angle 2, \angle 4, \angle 6$	<input type="text"/>
4. $\angle 3, \angle 5, \angle 7$	<input type="text"/>
5. $\angle 2, \angle 6, \angle 8$	<input type="text"/>

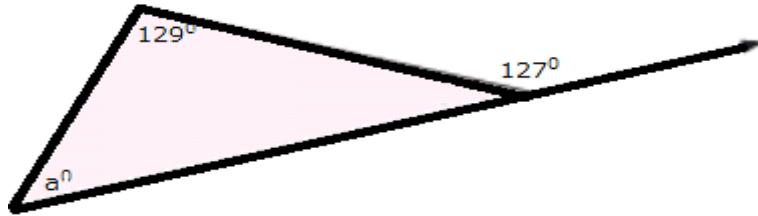


B. Use the Exterior Angle Inequality theorem to list all of the angles that satisfy the stated condition.



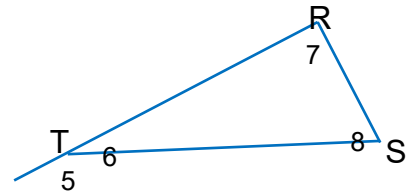
a. The angles which are less than $m\angle 8$ .	<input type="text"/>
b. The angles which are less than $m\angle 7$ .	<input type="text"/>
c. The angles which are greater than $m\angle 6$ .	<input type="text"/>

C. **Developing Proof.** What's wrong with the triangle below? Explain



D. **Writing Proofs.**

- Given:  $\angle 5$  is an exterior angle of  $\triangle RST$   
 Prove:  $m\angle 5 > m\angle 7$  and  $m\angle 5 > m\angle 8$



Indirect Proof:

**Step 1.** Assume that  $m\angle 5 < m\angle 7$  and  $m\angle 5 < m\angle 8$

In other words  $m\angle 5 \leq m\angle 7$  and  $m\angle 5 \leq m\angle 8$

**Step 2:** Show that the assumption leads to a contradiction of known facts or of the given information.

$m\angle 5 \leq m\angle 7$  and  $m\angle 5 \leq m\angle 8$  means that either  $m\angle 5 = m\angle 7$  and  $m\angle 5 < m\angle 7$

**Case 1:**  $m\angle 5 = m\angle 7$

**Case 2:**  $m\angle 5 < m\angle 7$

**Step 3:** Conclude that the original conclusion is true.

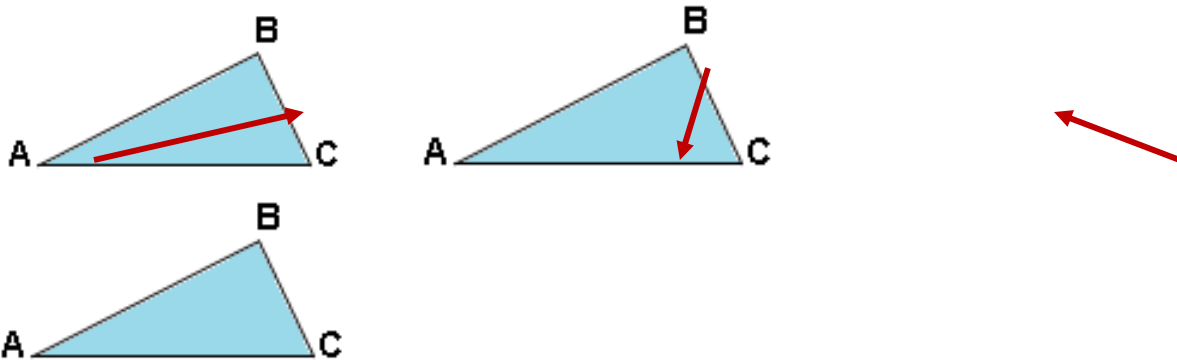




Recall that if two sides of a triangle are congruent, then the angles opposite those sides are also congruent. In the next activity, you will investigate the relationship between sides and angles of a triangle when they are not congruent.

Each angle and its corresponding opposite side of a triangle are shown below for you to easily see their relationship.

### SIDE-ANGLE RELATIONSHIPS



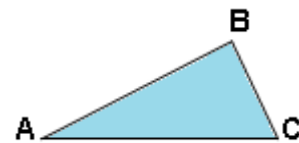
$\overline{BC}$  is the side opposite of  $\angle A$   $\overline{AC}$  is the side opposite of  $\angle B$   $\overline{AB}$  is the side opposite of  $\angle C$

### ACTIVITY 7. The Sides and Angles of a Triangle

#### INVESTIGATION

**Materials: protractor and ruler**

1. Draw an acute scalene triangle, and label the vertices A, B, and C.
2. Get the length each side of the triangle using a ruler. Record each length in a table.



Side	Length
BC	
AC	
AB	

3. Measure each angle using a protractor. Record each angle measure in a table.

Angle	Measure
A	
B	
C	

**Analyze**

- Describe the measure of the angle opposite the longest side and the angle opposite the shortest side.
- Describe length of the side opposite the biggest angle and the side opposite the smallest angle.

**Make a Conjecture**

- What can you conclude about the relationship between the lengths of the sides and measures of the angles of a triangle?

**ACTIVITY 8. Let's Do It the Interactive Way**

Verify your results in **Activity 7** by visiting the interactive website <http://www.mathopenref.com/trianglesideangle.html>. This will give you a clearer understanding of the relationship between sides and angles of a triangle.

Just click any point, then drag it around. The sides and angles of the interactive triangle will adjust accordingly. You may choose which side to show, shortest or the longest side. Just click on the box to the right of the triangle. Then, record your observations.

**Process Questions:**

1. What have you found out whenever you drag a point? What happened to the lengths of the two sides and the measures of the angles opposite them?

2. What have you noticed with the measure of the angle opposite the longest side?

3. What have you noticed with the measure of the angle opposite the shortest side?

4. What can you conclude with the relationship between the sides and angles of a triangle?

5. Was the conclusion you've made in Activity 6 is the same with the conclusion you've just made?

**Discuss results with other students and with your teacher in the discussion forum.**



The conjecture you've made in the investigation above would lead you to the following theorems below. For an input and examples, click on this link to view a ppt file

[http://images.pcmac.org/SiSFiles/Schools/AL/LeeCountySchools/SmithsStationHigh/Uploads/DocumentsCategories/Documents/5.3\\_Inequalities\\_in\\_One\\_Triangle.ppt](http://images.pcmac.org/SiSFiles/Schools/AL/LeeCountySchools/SmithsStationHigh/Uploads/DocumentsCategories/Documents/5.3_Inequalities_in_One_Triangle.ppt)

*This has been viewed in the first lesson, just continue on the slides till you reach the slides for the Angle-Side Relationships. For its two-column proof and additional examples, go to pages 249 and 250 of Glencoe Geometry, Chapter 5 pdf.*

### **Theorem 8.3.**

**If two sides of a triangle are not congruent, then the measure of the angle opposite the longer side is greater than the measure of the angle opposite the shorter side.**

### **Theorem 8.4.**

**If two sides of a triangle are not congruent, then the measure of the angle opposite the longer side is greater than the measure of the angle opposite the shorter side.**

### **ACTIVITY 9. Practice Problems on the Side-Angle-Relationships**

In this activity, you will be given more practice problems for you to master the skills in identifying and applying the side-angle relationships in one triangle in solving problems.

Open the Glencoe Geometry Interactive Study Notebook found in the TRIANGLE INEQUALITIES MATERIALS folder in the desktop.

Find the page navigator at the upper part of the page, change the page to 132 to view pages 124 of the book.

Fill all the boxes to complete the solution to every problem. You can print the file so that it will be very easy for you to answer. Just press CTRL + P or click the printer icon. Set the Print range from 132, then click OK.

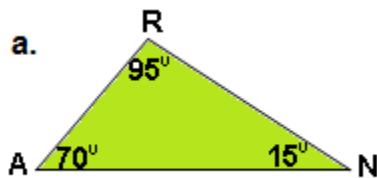


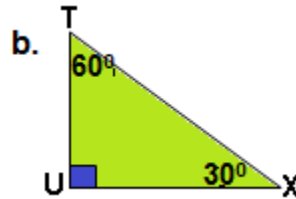
If you think your skill in identifying and applying the side-angle relationships in one triangle in solving problems, make use of the **Glencoe Study Guide pdf material found in your TRIANGLE INEQUALITIES folder.**

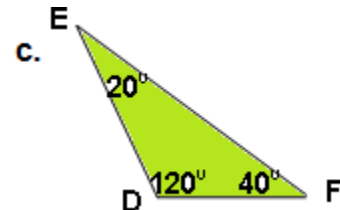
Go to the page navigator and change it to 44 to view page 40.  
Read again the theorem and study the examples given. Then, answer the problems.  
You can print the page.

### ACTIVITY 10. Short Quiz on the Side-Angle-Relationships

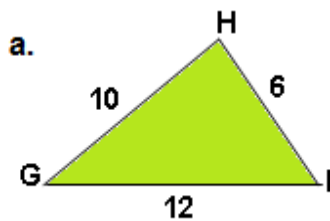
1. List the sides of each triangle from shortest to longest measure.

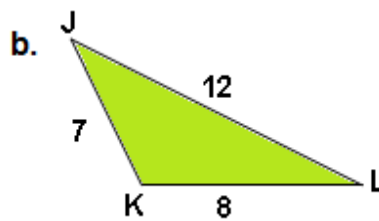


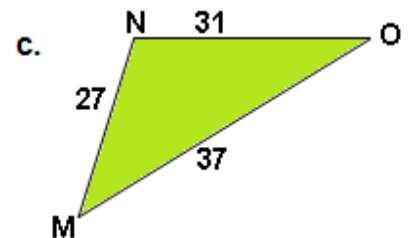




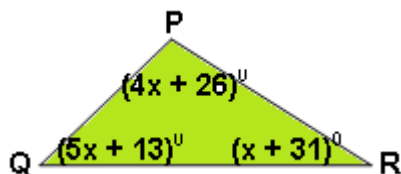

2. List the angles of each triangle from smallest to largest measure.



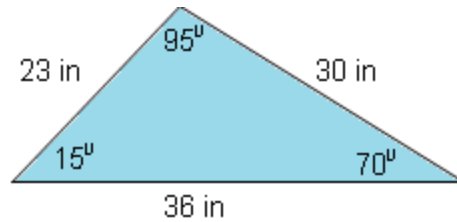





3. Determine the longest and shortest side in the given figure below.



4. Developing Proof. What's wrong with the measurements in the triangle below? Explain.



40.

41.

5. Writing Proofs.

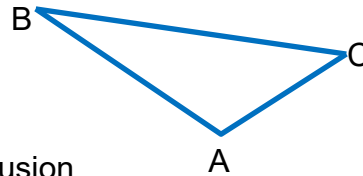
42.

- a. Given:  $m\angle A > \angle B$

Prove:  $BC > AC$

Write an Indirect Proof:

Step 1. Assume the negation of the conclusion.



Step 2. Show that the assumption leads to a contradiction of known facts or of the given information.

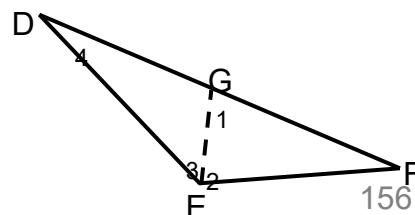
Step 3. Conclude that the original conclusion is true.

- b. Complete the two-column proof by supplying the reasons in the second column for each statement in the first column.

43.

Given:  $DF > EF$  and  $\overline{FG} \cong \overline{FE}$

Prove:  $m\angle DEF > m\angle 4$



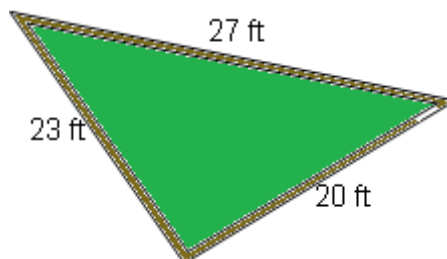
Proof:

To prove, draw first  $\overline{EG}$  such that  $\overline{FG} \cong \overline{FE}$ .

44. Statements	45. Reasons
1. $\overline{FG} \cong \overline{FE}$	
2. $m\angle 1 \cong m\angle 2$	
3. $m\angle DEF = m\angle 2 + m\angle 3$	
4. $m\angle DEF > m\angle 2$	
5. $m\angle DEF > m\angle 1$	
6. $m\angle 1 = m\angle 3 + m\angle 4$	
7. $m\angle 1 > m\angle 4$	
8. Based on the statements in items 4 and 5, $m\angle DEF > m\angle 4$	

**ACTIVITY 11. REVISITING ACTIVITY 2: Which Is The Largest Corner?**

A landscape engineer is designing a triangular area. He wants to place potted trees in the two largest corners. In which corners will He place the two potted trees?



**Process Questions:**

1. What idea led you to arrive at your answer?

2. Can you see the relationship between sides and angles of the given triangular area? What relationship have you seen?

**3. How can the knowledge on the side-angle relationships of a triangle help us solve problems in the real-life?**



Let us have a recap of the two lessons that you have just learned. In the Exterior Angle Inequality of a Triangle, you have found out that the measure of the exterior angle is always greater than the measure of each of its two remote interior angles. In the Inequality for the Sides and Angles of a Triangle, you have found out that the biggest measure of the angle is always opposite the longest side and the smallest measure of the angle is always opposite the shortest side.

Also, the longest side is opposite the biggest angle and shortest side is opposite the smallest angle.

In the next lesson, you'll find out when you can and cannot form a triangle, and determine the possible lengths of the third side of a triangle.



### The Triangle Inequality

#### ACTIVITY 12.

#### Investigating the Possible Lengths of the Sides of the Triangle

#### INVESTIGATION

**Materials:** straws, scissors, and a ruler

**Procedure:**

1. Cut the straws in to the following lengths: 1 in., 2 in., 3 in., 4 in., 5 in., and 6 in.
2. For each activity below, choose the three straws indicated in the table. With the straws touching at the ends, try to make a triangle.



**Triangle**



**Not a Triangle**



**46.** Record the results in the second column. Complete the last three columns of the table.

Sets of Straw Pieces			Do the straws form a triangle or not?	Use <, >, or = to compare the sum of any two sides to the third side.		
a	b	c		$a + b$ ___ $c$	$a + c$ ___ $b$	$b + c$ ___ $a$
1 in.	2 in.	3 in.				
2 in.	4 in.	6 in.				
3 in.	6 in.	1 in.				
2 in.	3 in.	4 in.				
3 in.	4 in.	5 in.				

3 in.	2 in.	5 in.				
-------	-------	----------	--	--	--	--

3. What did you notice with symbols used to compare the sum of any two sides to its third side when the response is **Yes**? How about in the **No** answers. Are they all greater than, less than or equal to?

47.

4. Make a conjecture about the inequality between the sum of any two sides of a triangle to the length of the third side by completing the statement below. Use greater than or lesser than.

48. **“ The sum of the lengths of any two sides of a triangle is the length of the third side.”**

49.

### ACTIVITY 13. Let's Do It the Interactive Way

Verify your results by visiting the site

<http://www.mathopenref.com/triangleinequality.html> . You can adjust the triangle by dragging the points A, B, and C. Compare the length of third side of the triangle (whether it is the shortest or the longest) to the sum of the lengths of the other two sides.

#### Follow-Up Questions:

1. We're you able to verify the conjecture you've made in **Activity 9**?

2. What is now your final conjecture regarding the relationship between the length of the third side of the triangle to the sum of its two other sides?

**Post your answers in the Discussion Forum and discuss results with your online teacher and classmates.**



The conjecture you've made in the investigation above would lead you to the theorem below. For an input and examples, click on this link [http://images.pcmac.org/SiSFiles/Schools/AL/LeeCountySchools/SmithsStationHigh/Uploads/DocumentsCategories/Documents/5.3\\_Inequalities\\_in\\_One\\_Triangle.ppt](http://images.pcmac.org/SiSFiles/Schools/AL/LeeCountySchools/SmithsStationHigh/Uploads/DocumentsCategories/Documents/5.3_Inequalities_in_One_Triangle.ppt) to view a ppt file. This has been viewed in the previous lessons. Just go to the slides on **Triangle Inequality**.

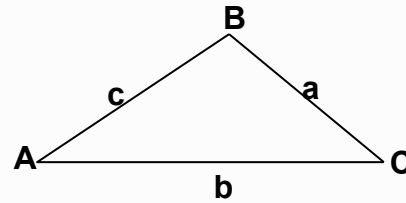
### Theorem 8.5. THE TRIANGLE INEQUALITY THEOREM

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

$$a + b > c$$

$$a + c > b$$

$$b + a > a$$



#### ACTIVITY 14. Practice Problems on Triangle Inequality

In this activity, you will be given more practice problems for you to master the skills in applying the Triangle Inequality Theorem in solving problems.

Open the Glencoe Geometry Interactive Study Notebook found in the TRIANGLE INEQUALITIES MATERIALS folder in the desktop.

Find the page navigator at the upper part of the page, change the page to 136 - 137 to view pages 128 and 129 of the book.

Fill all the boxes to complete the solution to every problem. You can print the file so that it will be very easy for you to answer. Just press CTRL + P or click the printer icon. Set the Print range from 136 to 137, then click OK.



*If you think your skill in identifying and applying the relationship between the measures of the exterior angle and its two remote interior angles in solving problems, make use of the **Glencoe Study Guide pdf material found in your TRIANGLE INEQUALITIES folder.***

*Go to the page navigator and change it to 45 to view page 41. Read again the theorem and study the examples given. Then, answer the problems. You can print the page.*

**ACTIVITY 15. Interactive Quiz**

Click <http://library.thinkquest.org/20991/textonly/quizzes/geo/g13/test.html> to answer a 6-item quiz. When the page appears, click on take the quiz. After you have selected your answer, an acknowledgement whether the answer is correct or will appear. An explanation of the answer is also given.

**ACTIVITY 16. Short Quiz on Triangle Inequality**

I. Determine if whether or not a triangle can be possibly drawn given the sides.

1. 6, 8, 13      2. 8, 10, 20      3. 12, 11, 23      4. 10, 21, 22

II. Find the possible values of  $x$  (range) given the lengths of the first two sides of a triangle.

1. 35, 48,  $x$       2. 40, 52,  $x$       3. 26, 53,  $x$       4. 58, 77,  $x$

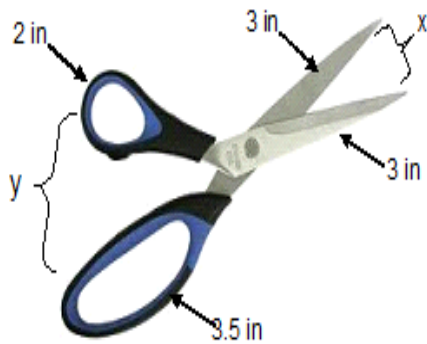
III. Find three numbers that can be the lengths of the sides of a triangle and three numbers that cannot be the lengths of the sides of a triangle. Justify your reasoning with a drawing.

**ACTIVITY 17. Inequalities Applied**

**A. Tools.**

Given are the tools that are commonly used at home, in school, and in work. Determine the changes in length X or the distance from tip to tip of the tool as it opens wide. Explain why triangle inequality is applied in their function.

1.



2.

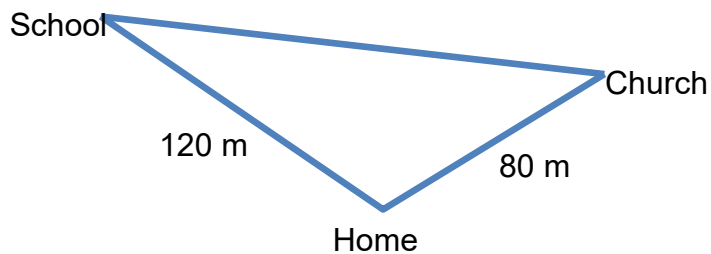


Answer:

Answer:

Explanation:

**B. Estimating Distances**



The distance Rolly walks from home to school is 120 meters and 80 meters when he goes to church from home. Kat estimates that the distance Rolly walks when he goes directly to Church, coming from school, is 180 meters. Gina's estimation is 210 meters.

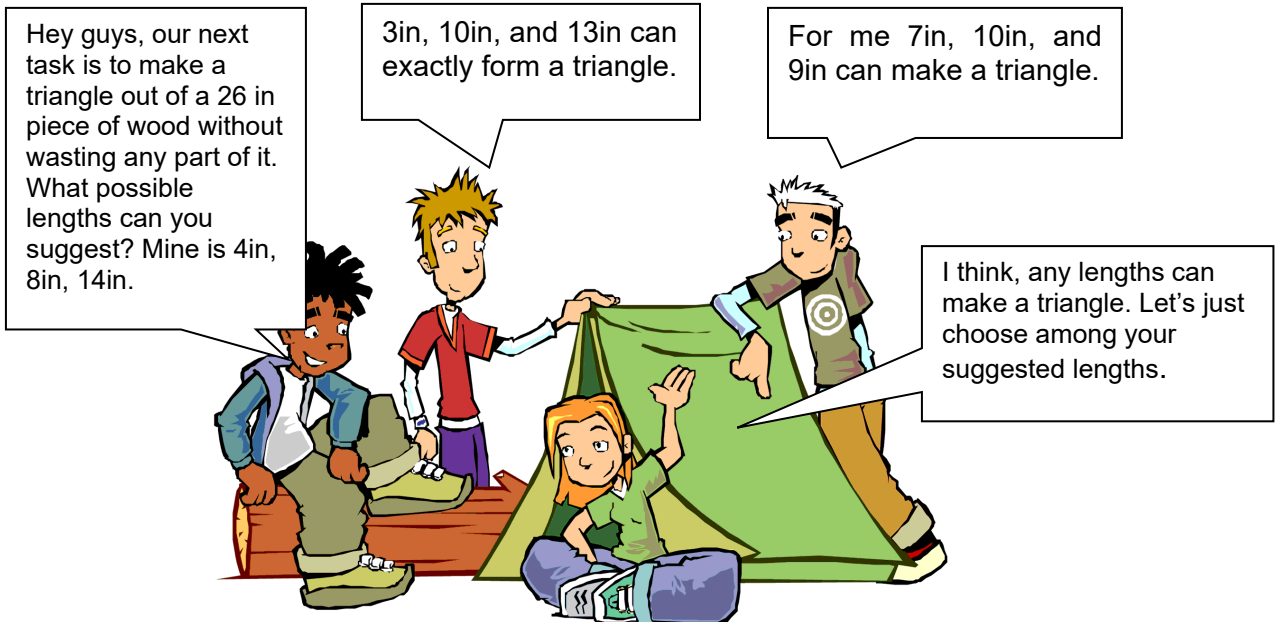
1. Which estimation is feasible? Justify your answer.

2. Which of the following paths to church is the shortest if you are from school? Justify your answer.

- Path No. 1: School to Home then to Church
- Path No. 2: School to Church

**ACTIVITY 18. Revisiting Activity 3: Let's Make a Triangle!**

Read the conversation of the four campers: Adrian, Kenneth, Loyd, and Miko (arranged from left to right) as they solve a given task.



**Process Questions:**

1. Which of these boys is correct? Why?

2. If you are in their situation, will it be easy for you to decide which set of lengths to choose?

3. Is inequality of triangle present in their suggested lengths?

4. How can you justify inequalities in one triangle?

5. How can triangle inequalities helped solve problems in the real-life?

**ACTIVITY 19. REVISITING ACTIVITY 1: Anticipation/ Reaction Guide**

Go back to the A/R guide. Answer the column on *After Lesson*.  
 Click on **SUBMIT** if you have answered all the items.

**ACTIVITY 20. 3-2-1 Chart**

Complete the chart below to sum up what you have learned in this section.

3 – things I’ve found out	
2 – interesting things I’ve learned	
1 – question I want to be answered	



### End of FIRM-UP:

In this section, the discussion was about the inequality that exist between the measure of the exterior angle to the measure of its two remote interior angles, the relationship between the sides and angles, and the possible lengths of the sides of the third side of a triangle.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.



### DEEPEN

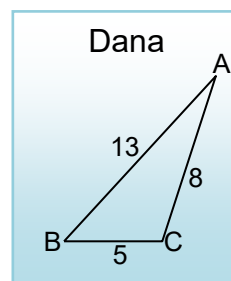
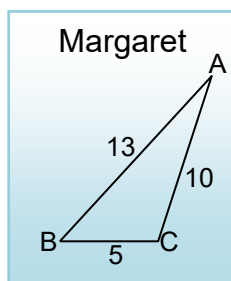
Your goal in this section is to take a closer look at some aspects of inequalities in one triangle. You will be asked to answer problems which will check if you still have misconceptions about the lesson and solve real-life problems

#### ACTIVITY 21. Checking for Understanding

Answer the following questions:

Note: In justifying or explaining your answer, cite a theorem to support it.

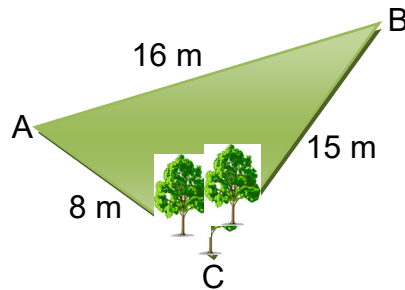
- Margaret and Dana drew  $\triangle ABC$  with  $AB = 13$  and  $BC = 5$ . Each of them chose a possible measure for  $AC$ .



Who is correct? Explain.

## APPLICATION PROBLEMS

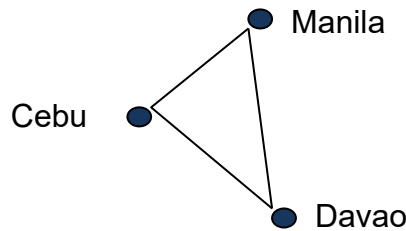
2. Nick was asked to plant the trees in the biggest corner of a triangular yard shown below. He was given a map so that he'll know where to place the trees.



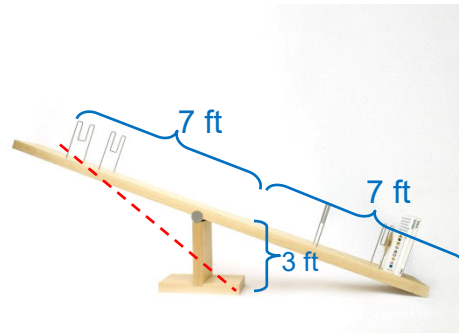
Nick planted the trees at corner C. Is he correct? Explain.

### 3. What is the Shortest Path?

Melinda travels Manila, Cebu, and Davao as part of her job as an accountant. Melinda lives in Manila and needs to get to Davao as quickly as possible. Should she take a flight that goes from Manila to Davao, or a flight that goes from Manila to Cebu, then to Davao? Justify your answer.

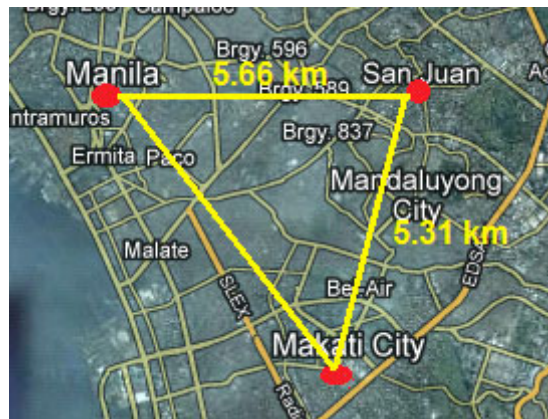


4. Given the seesaw below, what is the greatest integral distance of the tip of the board to the bottom part of the fulcrum? Explain.



**5. Finding the Possible Distance**

Given the map below, what are the possible distances from Makati City to Manila? Justify your answer.



**Process Questions:**

1. How can you justify inequalities in triangles?

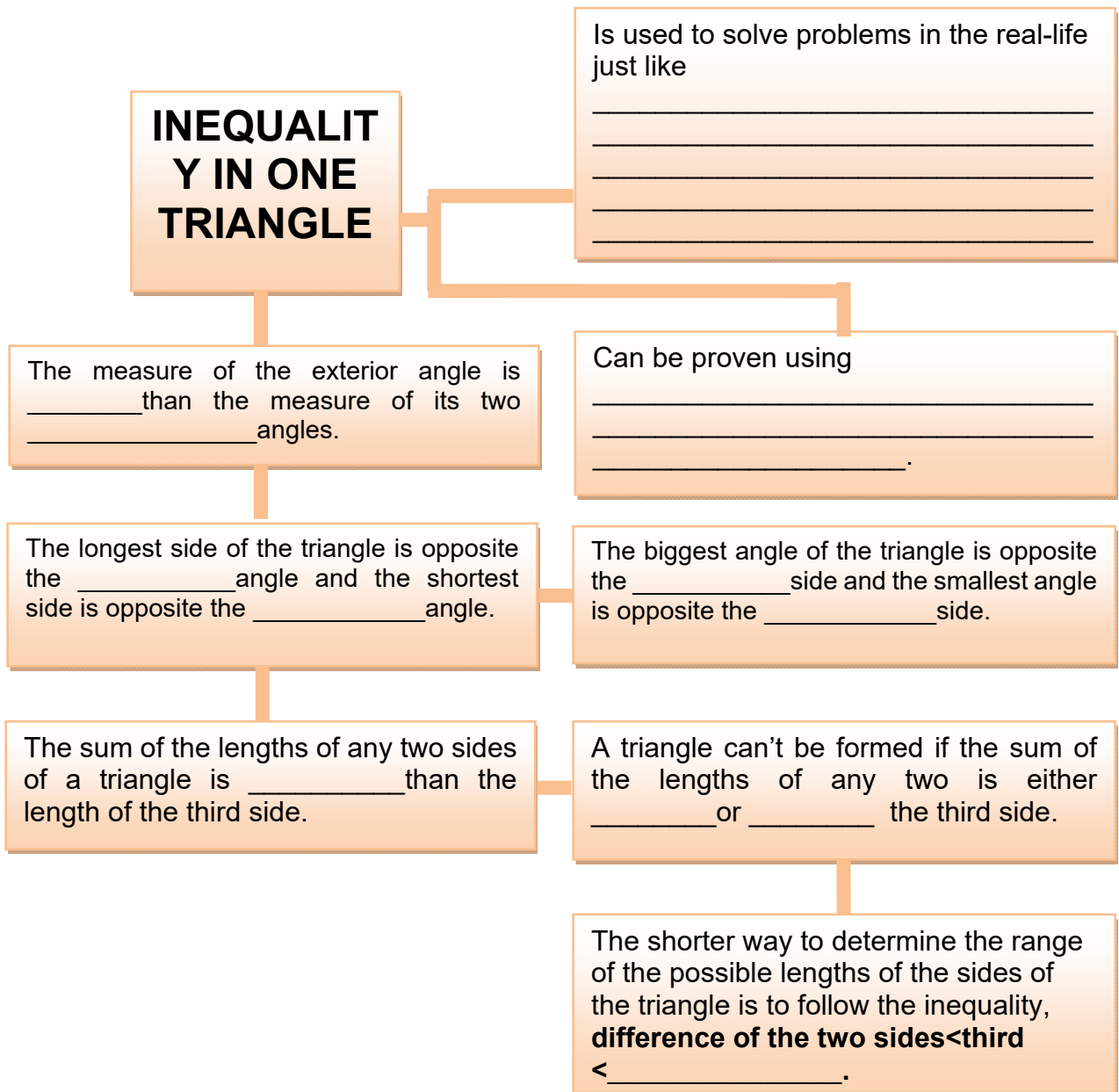
2. How can the knowledge of inequalities in triangles help you solve problems related to design and distances?

**Process Questions:**

1. Inequalities in triangles can be justified deductively.
2. Inequalities in triangles can help us estimate distances and can help us landscape or design a triangular land.

**ACTIVITY 22.**

**Organize your thoughts. Complete the statement in every box to complete the idea.**



**ACTIVITY 23. Comic Strip Making**

Think of a situation wherein you can apply the theorems on the inequalities in one triangle. Just imagine that your friend, mother, brother, or any person that you can think of needs your help in a particular situation and the solution to their problem is your knowledge on inequality in one triangle. Make a comic script out of it. You may use other application/website aside from the suggested site given below.

**Procedure:**

1. Go to <http://www.toondoo.com>.
2. Create an account (choose the free account).
3. Log in into your account.
4. After you have logged in, press the CREATE button on the upper right corner.
5. Choose a page layout for your comic.
6. Once chosen a page, choose any type of props, background, people, etc. on the tool bar on the left side of the screen b dragging it to the box.
7. If you want to change the expression or posture of the character, press the emotion or position button found on the bottom toolbar.
8. If you want to lay an object over another object, press BACK or FRONT button found on the bottom toolbar.
9. If you want to rotate your object, people, or background, press to ROTATE button on the bottom toolbar
10. If you're finished with your comics, find the Toondoo icon on the upper left corner of the page. Choose SAVE then type the title and description. Click the Publish button to save your work.
11. Email your comics or share it through Facebook.

**Process Questions:**

1. How did you find the activity?

2. Were you able to see the real-world application of the inequalities in one triangle?

3. How useful it is in solving problems in the real-life?

Rubric

CRITERIA	EXCELLENT (4)	SATISFACTORY (3)	PROGRESSING (2)	DEVELOPING (1)	RATING
Creativity (x5)	The story contains many creative details and/or description that contribute to the reader's comprehension and enjoyment of the story.	The story contains a few creative details and/or description that contribute to the reader's comprehension and enjoyment of the story.	The story contains a few creative details and/or descriptions, but they distract from the story making it somewhat difficult to understand.	There is little evidence of creativity in the story. Story is difficult to understand.	<u>20</u>
Dialogue/Text (x5)	There is an appropriate amount of dialogue and text to bring the characters to life and it is always clear which character is speaking. Grammar and usage are correct. Choice of words is correct and appropriate.	There is an appropriate amount of dialogue and text to bring the characters to life and it is always clear which character is speaking. Grammar and usage are correct. Choice of words is simple and correct	There is no appropriate dialogue or text but it is clear which character is speaking.	There is no enough dialogue or text and it is not clear which character is speaking.	<u>20</u>
Illustration (x5)	Illustrations are detailed, attractive, creative, and relate to the text on the page.	Illustrations are detailed, presentable, and relate to the text on the page	Illustrations relate to the text on the page.	Illustrations are limited and do not relate to the text.	<u>20</u>
Title (x2)	Title is very catchy. It engages the reader to read the rest of the comic strip. It also tells what the topic will be.	Title is interesting and tells what the topic will be.	Title may be catchy but does not relate to the topic.	Title is not interesting and do not relate to the topic.	<u>10</u>
Originality (3)	Comic shows originality and inventiveness.	Comic shows originality and inventiveness.	Comic shows an attempt at originality and	Comic is a rehash of other people's	

	The content and ideas are presented in a unique and interesting way.	The content and ideas are presented in an interesting way.	inventiveness in some part of the presentation.	ideas and/or images and shows very little attempt at original thought.	<u>12</u>
Content and Accuracy (5)	All content throughout the presentation is accurate. There are no factual errors and were able to connect learned ideas from previous lessons.	All content throughout the presentation is accurate. There are no factual errors.	The content is generally accurate, but one piece of information is inaccurate.	Content is confusing and contains more than one factual information.	<u>20</u>
Total					<u>102</u>

Upload output in the discussion forum so that it can also be viewed by other students and make comments and suggestions in your work.

### End of DEEPEN:

In this section, the discussion was about the use and importance of inequality in one triangle and justifies inequalities that existed on it.

What new realizations do you have about the topic? What new connections have you made for yourself?



### TRANSFER

Your goal in this section is apply your learning to real life situations. You will be given a practical task which will demonstrate your understanding.

**ACTIVITY 24. The Magic Ladder**

Watch a video about a magic ladder design in <https://www.youtube.com/watch?v=GIANsddCczg>.

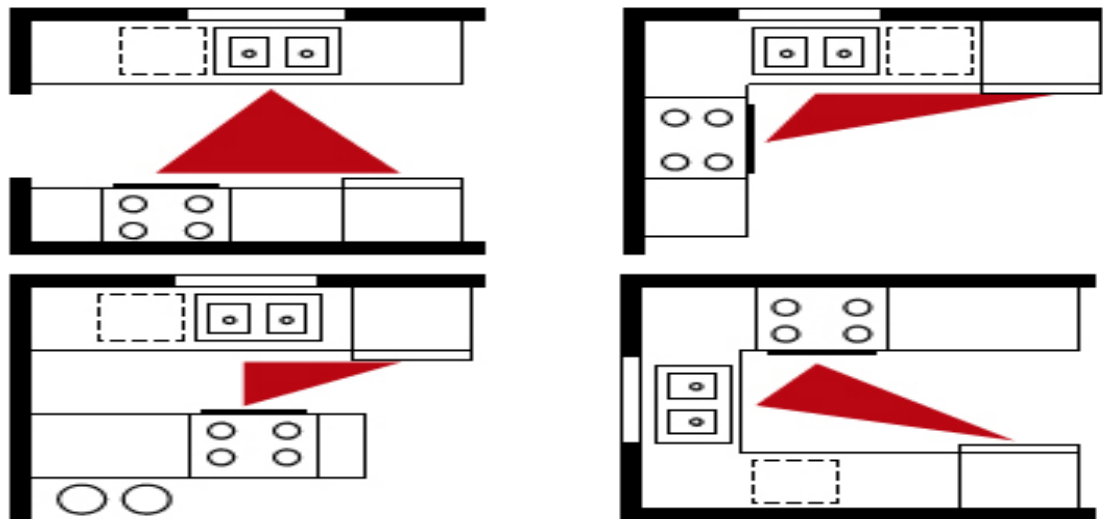
Process Questions:

1. How can you justify that inequalities in triangles exist in the design?

2. How do the designers design and make tools and equipment with triangular features?

*Post your answers in the Discussion forum and discuss with your teacher and peers.*

**ACTIVITY 25 The Kitchen Triangle**



The Kitchen Triangle was developed in the 1950's as a tool to aid designers in creating an effective kitchen lay-out. The triangle has a corner at the sink, the refrigerator, and the stove, the three essential locations in the kitchen when cooking. Most kitchen plans still include this today. The idea is to have them close enough that they can easily be moved between, but not too far to reduce



the amount of movement while cooking. The general rule is that the triangles perimeter must be at least 12 ft. but should not be greater than 26 ft.

The area inside the triangle should be completely open, making the movement between each of these easy. Most modern kitchen plans still include this.

See the whole article at <http://bathroomphotogallery.com/kitchen-plans.php~>

**For your activity, you are given the situation below.**

**Situation:** You are an architectural student and in your apprentice stage. Your boss asked to design a kitchen for a condominium. He gave you instructions on how to do it easily. These are some of his instructions:

1. Choose one of kitchen triangle designs.
2. Assign lengths for the sides of the triangle following the condition that the perimeter of the triangle is at least 12 ft. but should not be greater than 26 ft.
3. Justify that the lengths you have assigned for your triangle is possible.
4. Output can be done in MSWord or PowerPoint Presentation form.
5. Attach file in the email icon.

**Process Questions:**

1. What have you realized while doing the activity?
2. **How can you justify inequalities in triangles?**
3. ***How can triangle inequalities help you solve problems in the real-life?***

**RUBRIC**

Criteria	Excellent (4)	Satisfactory (3)	Progressing (2)	Needs Improvement (1)	RATING
<b>Accuracy (x 10)</b>	The chosen set of lengths for the sides of the triangle is possible and is suited to the design. Computations in determining	The chosen set of lengths for the sides of the triangle is possible. Computations in determining the sides are accurate.	The chosen set for the lengths of the sides of the triangle is possible but there is an error in one of the computations.	The chosen set for the lengths of the sides of the triangle is not possible. Some of the computations are erroneous.	40

	the sides are accurate.				
<b>Mathematical Justification (x10)</b>	Justification for the chosen lengths for the sides of the triangle makes use of the theorems of the inequalities in one triangle. It is also clear and logical.	Justification for the chosen lengths for the sides of the triangle makes use of the theorems of the inequalities in one triangle.	Justification for the chosen lengths of the sides of the triangle did not make use of the theorems of the inequalities in one triangle.	Justification for the chosen lengths of the sides of the triangle is vague.	— 40
<b>TOTAL</b>					— 80

Upload output in the discussion forum so that it can also be viewed by other students and make comments and suggestions in your work.

**End of TRANSFER:**

In this section, your task was to watch a video where inequality in one triangle is applied in the design. This activity will give you an idea on how you will perform the final task.

How did you find the performance task? How did the task help you see the real world use of the topic?

## LESSON 3.2 INEQUALITY IN TWO TRIANGLES



### EXPLORE

Let's begin by getting your initial ideas about inequality two triangles in the statements given. Then keep in mind these questions:

**How can you justify inequalities in two triangles?"**

**How can the knowledge of inequality in two triangles help us solve problems related to design and distances?**

### ACTIVITY 1. Anticipation / Reaction Guide

**Instruction:** Respond to each statement twice: once before the lesson and again after reading it.

Write **A** if you agree with the statement

Write **B** if you disagree with the statement

	1. There is inequality among the third sides and included angles of two triangles in which two of its corresponding sides are congruent.	
	2. If the two corresponding sides of the two triangles are congruent but the included angle of the first is larger than the first, then the length of the third side of the first triangle is shorter than the length of the third side of the second triangle.	
	3. Inequalities in two triangles can be used in designing mechanical tools and equipment.	
	4. Inequalities in two triangles can be used to compare distances.	

**ACTIVITY 2. Hurricane Ride**

Shown below is a hurricane ride. The design of this ride involves triangles. Observe how the changes in the sides and angles of a triangle as the arms are moved. Watch the video in the site <http://www.youtube.com/watch?v=3ApaQVv7Eik> .



Process questions:

1. What have you noticed with the distance of the chair from the base of the hurricane tower as the arm is moved up and moved down?

2. What have you noticed with the measure of the angle formed by the tower and the arm of the hurricane ride as the arm is moved up and moved down?

3. Can you see inequalities in the sides and angles of the triangles? What inequality have you observed?

4. How can you justify that inequality in triangles exists in this amusement ride?

5. How are triangle inequalities useful in the design of some amusement rides?

**End of EXPLORE:**

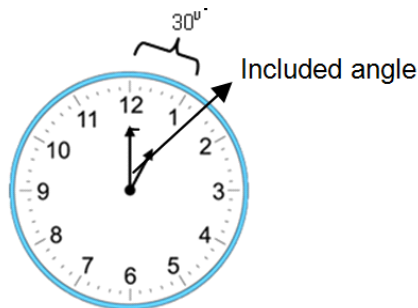
You gave your initial ideas on the questions given above.  
Let's find out how others would answer the question and compare their ideas to our own. We will start by doing the next activity.



**FIRM-UP**

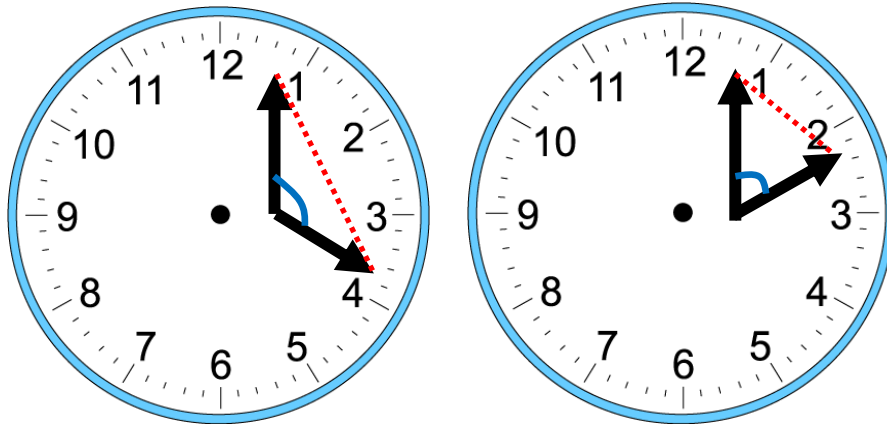
*Your goal in this section is to learn and understand key concepts on inequalities in two triangles*

*Before you do the first activity, let us recall that every space in the clock is equivalent to  $30^\circ$ .*



*An included angle is an angle that lies in between the two sides of the angle.*

**ACTIVITY 3. Investigating Inequalities in Two Triangles**



**INVESTIGATION**

**Materials: Clock and ruler**

**Procedure:**

**FIRST TRIANGLE**

1. Set the clock at 4:00.
2. Determine the measure of the angle formed.
3. Measure the distance between the tips of the arms of the clock using a ruler.

**SECOND TRIANGLE**

1. Set the clock at 2:00.
2. Determine the measure of the angle formed.
3. Measure the distance between the tips of the arms of the clock using a ruler.

**Process Questions:**

1. What are the congruent parts (equality) in the two triangles shown?

2. How will you compare the measure of the included angle of the first triangle to the measure of the included angle of the second triangle?

3. How will you compare the length of the third side of the first triangle to the third side of the second triangle?

4. What inequality have you observed between the parts of the two triangles?

5. Make a conjecture about the inequality between the included angles and opposite sides of the two triangles. Use longer than or shorter than.  
**“If two sides of one triangle are congruent to two sides of another triangle, but the included angle of the first triangle is larger than the included angle of the second, then the third side of the first triangle is \_\_\_\_\_ the third side of the second triangle.”**



The conjecture you have just completed is the Hinge Theorem. For an input and examples, click on the link

<http://www.hasdpa.net/21512042794634380/lib/21512042794634380/Geometry - Section 56.ppt> to view a ppt file on Inequalities in Two Triangles.

**ACTIVITY 4.**

**Practice Problems Applying the Hinge Theorem and its Converse**

In this activity, you will be given more practice problems for you to master the skills in applying the Hinge Theorem and its Converse in solving problems.

Open the Glencoe Geometry Interactive Study Notebook found in the TRIANGLE INEQUALITIES MATERIALS folder in the desktop.

Find the page navigator at the upper part of the page, change the page to 138 - 139 to view pages 130-132 of the book.

Fill all the boxes to complete the solution to every problem. You can print the file so that it will be very easy for you to answer. Just press CTRL + P or click the printer icon. Set the Print range from 138 to 139, then click OK.

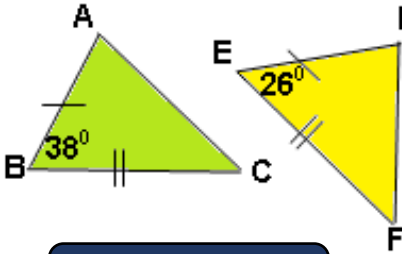
**ACTIVITY 5. Self-Check Quiz**

Click <http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-888484-5&chapter=5&lesson=6&&headerFile=7> to take a 5-item self-check quiz. Hint is given for each item to guide you in choosing the right answer. After you have finished answering all the items, click on **Check It** to view the items that you answered correctly. If you want to take again the quiz, click **RETRY**.

**ACTIVITY 6. Short Quiz**

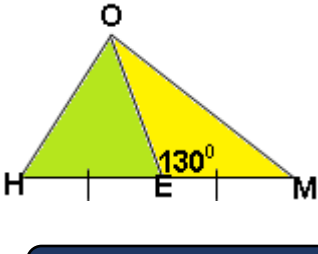
1. Using the Hinge Theorem and its converse, write the symbols  $>$ ,  $<$ , or  $=$  to make the following statement true.

a.



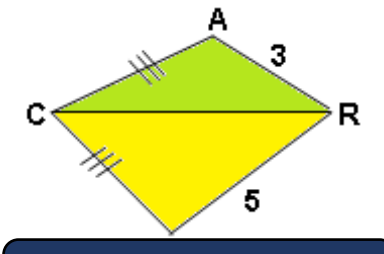
AC    DF

b.



MO    HO

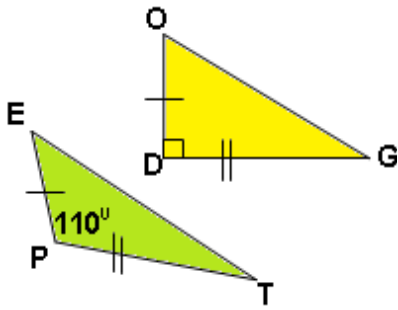
c.



$\angle ACR$      $\angle RCE$

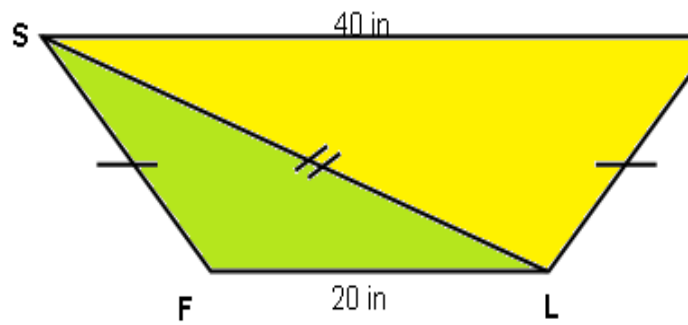


d.



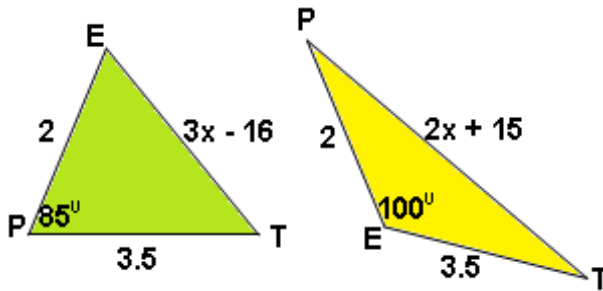
EF OG

e.



$\angle SFL$   $\angle SLE$

2. Solve for x using the Hinge Theorem and its converse.



3. Writing proofs

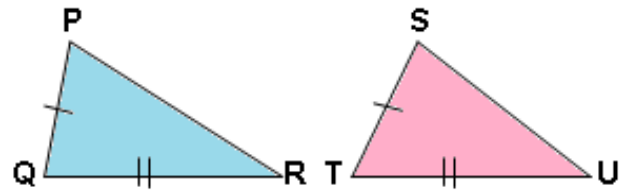
Use an indirect proof to prove the Hinge Theorem (SSS Inequality Theorem)

a. Given:

$$\overline{PQ} \cong \overline{ST}$$

$$\overline{QR} \cong \overline{TU}$$

$$PR > SU$$



Prove:  $m\angle Q > m\angle T$

**Indirect Proof:**

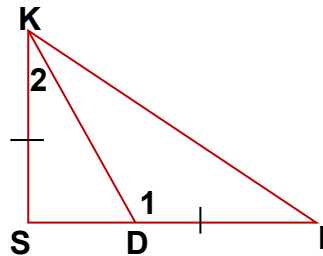
**Step 1. Assume the negation of the conclusion.**

**Step 2. Show that the assumption leads to a contradiction.**

**Step 3. Point out that the original conclusion is true**

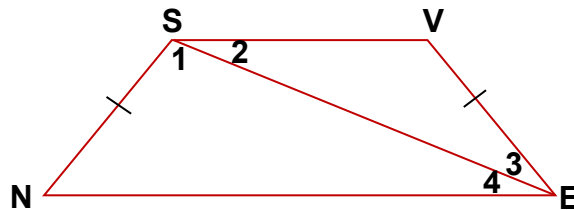
For items b and c, supply the reasons for each statement in every two-column proof below.

- b. Given:  $\triangle KIS$   
 $\overline{KS} \cong \overline{DI}$   
 Prove:  $KI > SD$



Statements	Reasons
1. $\overline{KS} \cong \overline{DI}$	
2. $\overline{KD} \cong \overline{KD}$	
3. $m\angle 1 = m\angle 2 + m\angle KSD$	
4. $m\angle 1 > m\angle 2$	
5. $KI > SD$	

- c. Given:  $\overline{SN} \cong \overline{VE}$   
 $SV < NE$   
 Prove:  $m\angle 3 < m\angle 1$



Statements	Reasons
1. $\overline{SN} \cong \overline{VE}$	
2. $\overline{SE} \cong \overline{SE}$	
3. $SV < NE$	
4. $m\angle 3 < m\angle 1$	

**ACTIVITY 7. Application in Making Tools**

Hinges are used to fasten two things together and allow one of them to pivot. Some tools and equipment just like the ones shown below have hinges. Choose one of these then use the Hinge Theorem to explain how to use it.

**ladder**



**compass**



**clock**



**back hoe**



**See saw**



**Process Questions:**

1. How do the designers design and make tools and equipment with triangular features?

2. How can you justify that inequality in triangles exist in the designs?

### Process questions

1. Designers of tools and equipment with triangular features make use of the knowledge in inequalities in two triangles so that the latter will be adjustable.
2. Inequality in triangles existed in a design if and only if there are changes in the measure of the included angle and its third side whenever the arms of the equipment or tools are adjusted. Justifying inequality in triangles is done deductively.

### ACTIVITY 8. Photo Collage

Make a collage of real-life objects that model inequality in triangles. Suggested site is [www.photovisi.com](http://www.photovisi.com) but you can also visit other sites for your collage.

#### Procedure:

1. Search for photos of objects or situations that would model the use of inequalities in triangles and save it in your computer.
2. Visit the site [www.photovisi.com](http://www.photovisi.com)
3. Register for an account.
4. Click on **Start Creating** Button.
5. Select a template for your collage.
6. Click **+ Add Item** to download pictures from your computer. Just drag the pictures in to the template.
7. Click on **Add text** button to add title.
8. If you're finished with the collage, click on **Finish** button to save it.
9. Share your collage by clicking the **Email Collage** button then type the email address of your teacher and peers. Then, click **Share this Collage**.

#### Rubric

CRITERIA	EXCELLENT (4)	SATISFAC- TORY (3)	PROGRESSING (2)	DEVELOPING (1)	RATING
Number of Pictures (x3)	There are more than 8 pictures used to model inequalities in triangles.	There are at least 5 and at most 8 pictures that are used to model inequalities in triangles.	There are 3-4 pictures used to model inequalities in triangles.	There are 1-2 pictures used to model inequalities in triangles.	$\frac{\quad}{15}$

Appropriateness of the pictures (x 5)	All of the chosen pictures are correct and unique models for inequalities in triangles.	All of the chosen pictures are correct models for inequalities in triangles.	One of the chosen pictures is not a correct model for inequalities in triangles.	Most of the chosen pictures are not models for inequalities in triangles	— 20
Visual Impact (x3)	Overall impact is very effective.	Overall visual impact is effective.	Overall impact is somewhat effective .	Overall impact is limited.	— 15
Total					— 44

**ACTIVITY 9. Revisiting Activity 27: Hurricane Ride**

Shown below is a hurricane ride. The design of this ride involves triangles. Observe how the changes in the sides and angles of a triangle as the arms are moved. Watch the video again in the site

<http://www.youtube.com/watch?v=3ApaQVv7Eik> .



**Process questions:**

1. What have you noticed with the distance of the chair from the base of the hurricane tower as the arm is moved up and moved down?

2. What have you noticed with the measure of the angle formed by the tower and the arm of the hurricane ride as the arm is moved up and moved down?

3. Can you see inequalities in the sides and angles of the triangles?

4. How can you justify that inequality in triangles exists in this amusement ride?

5. How are triangle inequalities useful in the design of some amusement rides?

**ACTIVITY 10. Revisiting Activity No. 26. A/R Guide**

Go back to the A/R Guide. Read again the statements in the second column. Then write your final response on the third column.

**Process Questions:**

1. Were there changes in your previous responses?

2. What were the concepts and insights that you have learned in the lesson?

### End of FIRM-UP:

In this section, the discussion was about the inequalities that existed in two triangles. It was also discussed how to justify and prove inequalities in triangles and how they were used to design machines, equipment, and tools.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?



### DEEPEN

Your goal in this section is to take a closer look at some aspects of the topic.

*Inequalities are found in the angles and sides of one triangle or in two triangles. Their concepts would really help us understand how and why certain designs or situations happen.*

*For you to have a deeper understanding of the lesson, you are given more challenging problems and activities.*

### **ACTIVITY 11. Swing Ride Design**

You are a design engineer of a contracting company. One day, your boss asked you to design a swing ride for a big amusement park. He told you that if the owner is convinced of the design, the implementation of the project will be given to the company. He also told you that the following details must be included during the presentation of the design:

1. Specify the height of the tower and the length of the swing arm in feet.
2. Specify the measure of the angle of the swings when it is at its full speed, medium and low speeds. Provide mathematical justification.

Your boss also told you that you have to prepare a PowerPoint presentation of the design and its details to the owner and its board members.

**Process Questions:**

1. How did you find the activity? Did you find any difficulty in designing your own swing ride?
2. How did your knowledge of inequalities in triangles helped you in designing your own swing ride?
3. How can the knowledge on the inequality of two triangles help design engineers in designing rides, tools, and equipment?

**RUBRIC**

Criteria	Excellent (4)	Satisfactory (3)	Progressing (2)	Needs Improvement (1)	RATING
<b>Accuracy (x 10)</b>	The chosen set of lengths for the arms and the tower is possible and is suited to the design. Computations in determining the sides are accurate.	The chosen set of lengths for the arms and the tower is possible. Computations in determining the sides are accurate.	The chosen set for the arms and the tower is possible but there is an error in one of the computations.	The chosen set for arms and the tower is not possible. Some of the computations are erroneous.	40
<b>Mathematical Justification (x10)</b>	Justification for the chosen lengths makes use of the theorems of the inequalities in triangles. It is also clear and logical.	Justification for the chosen lengths makes use of the theorems of the inequalities in triangles.	Justification for the chosen lengths did not make use of the theorems of the inequalities in triangles.	Justification for the chosen lengths is vague.	40
<b>Creativity (x5)</b>	The design is attractive and convincing.	The design is presentable and convincing.	The design lacks color and art.	The design is not presentable and convincing.	20
<b>TOTAL</b>					100

Upload output in the discussion forum so that it can also be viewed by other students and make comments and suggestions in your work.



### **ACTIVITY 12. The Magic Ladder**

<https://www.youtube.com/watch?v=GIANsddCczg> a video on a folding ladder design. Watch this video again to explain its function using the hinge theorem and its converse.

### **ACTIVITY 13. Comic Strip Making**

Think of situation where you can apply the theorems on the inequalities in one triangle. Just imagine that your friend, mother, brother, or any person that can you think of needs your help in a particular situation/ problem and the solution to their problem is your knowledge on inequalities in two triangles. Suggested situations and objects can be found at home, hospital, and playground. Make a comic script out of it. Make a comic script out of it. Go to [www.toondoo.com](http://www.toondoo.com). or any site where you can make your comic strips.

#### **Procedure:**

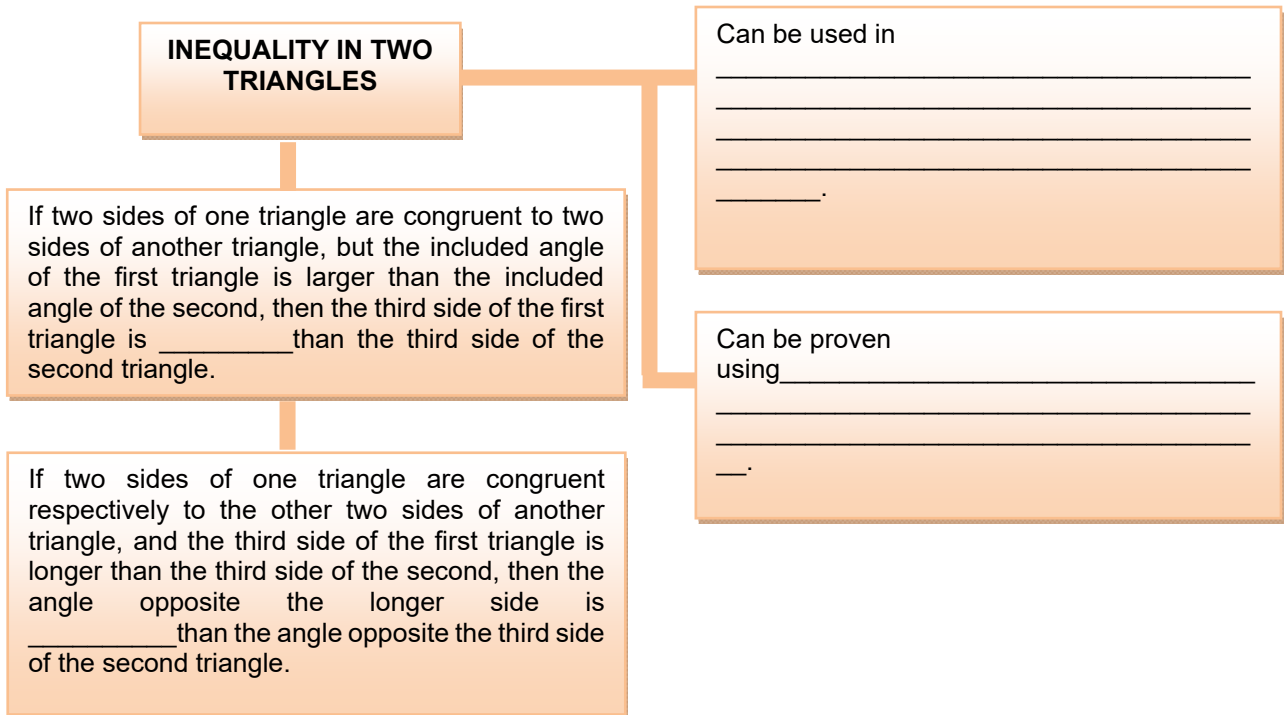
1. Go to <http://www.toondoo.com>.
2. Log in into your account.
3. After you have logged in, press the CREATE button on the upper right corner.
4. Choose a page layout for your comic.
5. Once chosen a page, choose any type of props, background, people, etc. on the tool bar on the left side of the screen b dragging it to the box.
6. If you want to change the expression or posture of the character, press the emotion or position button found on the bottom toolbar.
7. If you want to lay an object over another object, press BACK or FRONT button found on the bottom toolbar.
8. If you want to rotate your object, people, or background, press to ROTATE button on the bottom toolbar
9. If you have completed your comics, save it by clicking the Toondo icon on the left corner of the page and choose SAVE then type the title and description. Go down to SHARE OPTIONS and click share with friends then type the email address of your online teacher then click SAVE.

CRITERIA	Rubric				RATING
	EXCELLENT (4)	SATISFACTORY (3)	PROGRESSING (2)	DEVELOPING (1)	
Creativity (x2)	The story contains many creative and original details and/or description that contribute to the reader's comprehension and enjoyment of the story.	The story contains a few creative details and/or description that contribute to the reader's comprehension and enjoyment of the story.	The story contains copied details and/or descriptions, and they distract from the story making it difficult to understand.	There is no evidence of creativity in the story. Story is incoherent.	— 8
Dialogue/Text (x2)	There is an appropriate amount of dialogue and text to bring the characters to life and it is always clear which character is speaking. Grammar and usage are correct. Choice of words is correct and appropriate.	There is an appropriate amount of dialogue and text to bring the characters to life and it is always clear which character is speaking. Grammar and usage are correct. Choice of words is simple and correct	There is no appropriate dialogue or text but it is clear which character is speaking.	There is no enough dialogue or text and it is not clear which character is speaking.	— 8
Illustration (x4)	Illustrations are detailed, presentable and relate to the text on the page.	Illustrations are presentable, and relate to the text on the page	Illustrations are limited and only some relate to the text on the page.	Illustrations do not relate to the text.	— 16
Title (x1)	Title is catchy. It engages the reader to read the rest of the comic strip. It also tells what the topic will be.	Title is interesting and tells what the topic will be.	Title is interesting but does not relate to the topic.	Title is not interesting and does not relate to the topic.	— 4
Originality (3)	Comic shows originality and inventiveness in all parts. The content and ideas are presented in a unique and interesting way.	Comic shows originality and inventiveness in all parts. The content and ideas are presented in an interesting way.	Comic shows an attempt at originality and inventiveness in some part of the presentation.	Comic is a rehash of other people's ideas and/or images and shows no attempt at original thought.	— 15
Content and Accuracy	All content throughout the	All content throughout the	The content is generally	Content is confusing and	

(5)	presentation is accurate. There are no factual errors. Content shows connection of learned ideas from previous lessons.	presentation is accurate. There are no factual errors.	accurate but one piece of information is inaccurate.	contains more than one factual information.	<u>20</u>
Total					<u>68</u>

**ACTIVITY 14. Organize Your Thoughts**

Complete the statement in each box to sum up the things you have learned in this lesson.







## TRANSFER

Your goal in this section is apply your learning to real life situations. You will be given a practical task which will demonstrate your understanding.

### ACTIVITY 16. Ladder for Safety

#### Situation:

The lessons learned from the widespread flooding in many parts of the country during typhoons and monsoon season include securing tools and gadgets needed for safety. More and more people are shopping for ladders that could reach as high as 10 feet, long enough for people to gain access to their ceiling or their roof. There is a high demand for folding ladders for they can be stored conveniently. Being the design engineer of your company, the company head asks you to submit a miniature model of that ladder and justify the design. The design must include accurate computations and mathematical justifications. The product must be efficient and creative.

CRITERIA	RUBRIC				RATING
	Outstanding 4	Satisfactory 3	Developing 2	Beginning 1	
<b>Accuracy (x5)</b>	The computations are accurate and show an insightful use of the geometric concepts specifically on triangle inequalities.	The computations are accurate and show the use of geometric concepts specifically on triangle inequalities.	The computations show some errors in the use of the concepts on triangle inequalities.	The computations do not show the use of the concepts on triangle inequalities.	— 20
<b>Mathe-matical Justifi-cation (x5)</b>	Justification is logically clear, convincing, and professionally delivered. The concepts learned on triangle inequalities are applied and previously learned concepts are connected to the new ones.	Justification is clear and convincingly delivered. Appropriate concepts learned on triangle inequalities are applied.	Justification is not so clear. Some ideas are not connected to each other. Not all concepts on triangle inequalities are applied.	Justification is ambiguous. Only few concepts on triangles inequalities are applied.	— 20

<b>Efficiency (x5)</b>	The miniature is very effective and flawlessly done. It is also attractive.	The miniature is effective and flawless.	The miniature has some defects.	The miniature has many defects.	<u>20</u>
<b>Resourcefulness (x5)</b>	The miniature shows originality and inventiveness. Materials used are of low cost but of good quality.	The miniature shows originality and inventiveness. Materials used are reasonable.	The miniature is patterned from other designs. Materials used are expensive.	The miniature shows no evidence of creativity. Materials used are not appropriate and reasonable.	<u>20</u>
<b>Total</b>					<u>80</u>

**Process Questions:**

1. What have you experienced in the activity?
2. What are the insights that you have learned?
3. What does one need in order to come up with a good ladder design?
4. What will happen to the design if there are no hinges on it?
5. How can you justify inequality in triangles?

**Valuing:**

1. How could ladder illustrate the relationship between you and God?
2. Do we really need a ladder to reach Him?
3. What inequality exists between God-Man relationships? How about man-man relationships?
4. What will you do in order to promote equality among people?

**End of TRANSFER:**

In this section, your task was to make a foldable ladder that can be stored conveniently.

How did you find the performance task? How did the task help you see the real world use of the topic?

You have completed this lesson. Before you go to the next lesson, you have to answer the following post-assessment.

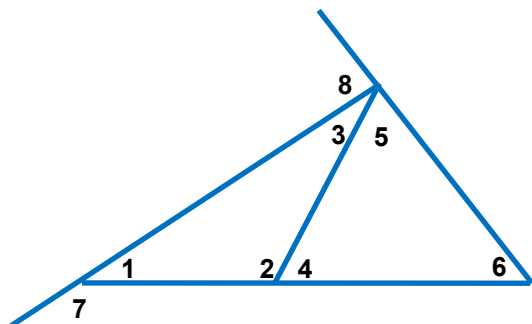
**POST-ASSESSMENT:**

*It's now time to evaluate your learning. Click on the letter of the answer that you think best answers the question. Your score will only appear after you answer all items. If you do well, you may move on to the next module. If your score is not at the expected level, you have to go back and take the module again.*

1. What holds true about the measure of the exterior angle to the measure of its two remote interior angles?
  - A. The measure of the exterior angle is greater than the measures of its two remote interior angles.
  - B. The measure of the exterior angle is less than the measures of its two remote interior angles.
  - C. The measure of the exterior angle is greater than or equal to the measures of its two remote interior angles.
  - D. The measure of the exterior angle is less than or equal to the measures of its two remote interior angles
  
2. What must be considered in determining the possible lengths of the sides of a triangle?
  - A. The sum of the lengths of any two sides must be greater than the length of the third side.
  - B. The sum of the lengths of any two sides must be lesser than the length of the third side.
  - C. The sum of the lengths of any two sides must be greater than or equal to the length of the third side.
  - D. The sum of the lengths of any two sides must be lesser than the length of the third side.
  
3. If two sides of the corresponding parts of a triangle are congruent, and the included angle of the first is bigger that the included angle of the second, then what must be true about the length of the third side of the first triangle?
  - A. It is longer than the third side of the second triangle.
  - B. It is shorter than the third side of the second triangle.
  - C. It is equal to the length of the third side of the second triangle.
  - D. Its length cannot be determined.

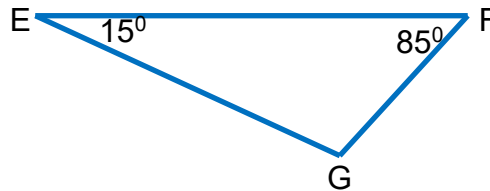
4. Using the Exterior Angle Inequality Theorem, what are the angles lesser than  $\angle 7$ ?

- A.  $\angle 2, \angle 3$
- B.  $\angle 2, \angle 3, \angle 6$
- C.  $\angle 2, \angle 3, \angle 5, \angle 6$
- D.  $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5, \angle 6$



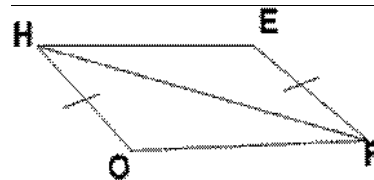


5. The two sides of a triangle have lengths 20 and 35. Which cannot be a possible length for its third side?
- A. 15  
B. 16  
C. 30  
D. 54
6. List the sides of  $\triangle EFG$  in order from least to greatest measure.



- A.  $\overline{EF}, \overline{FG}, \overline{GE}$   
B.  $\overline{FG}, \overline{GE}, \overline{EF}$   
C.  $\overline{FG}, \overline{EF}, \overline{GE}$   
D.  $\overline{GE}, \overline{FG}, \overline{EF}$
7. What theorem will complete the proof below?

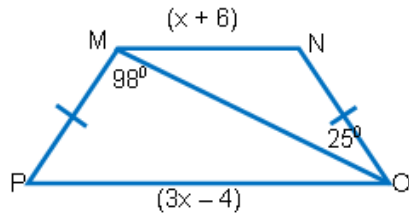
**Given:**  $\overline{HO} \cong \overline{EF}$ ,  $\angle OHP > \angle EPH$   
**Prove:**  $\overline{OP} > \overline{EH}$



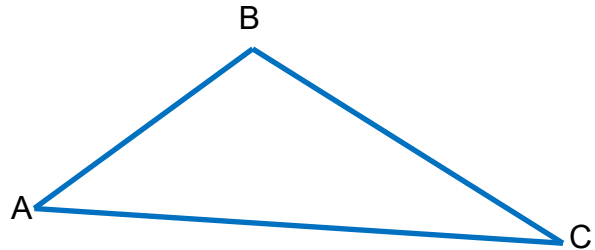
Statements		Reasons
1	$\overline{HO} \cong \overline{EF}$	Given
2	$\overline{HF} \cong \overline{HF}$	Reflexive Property of Equality
3	$\angle OHP > \angle EPH$	Given
4	$\overline{OP} > \overline{EH}$	?

- A. Converse of the Hinge Theorem  
B. Exterior Angle Inequality Theorem  
C. Hinge Theorem  
D. Triangle Inequality Theorem

8. What is the range of values for  $x$  in the figure?



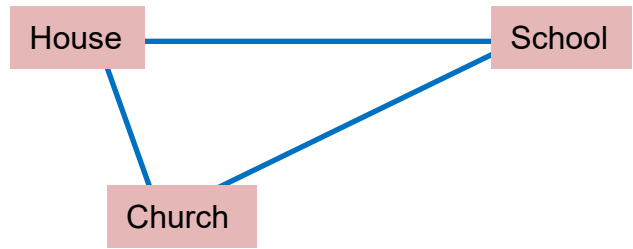
- A.  $x < 5$   
 B.  $x > 5$   
 C.  $x \leq 10$   
 D.  $x \geq 10$
9. The Cruz family owns a triangular lot shown below wherein their house is built in the middle of it. To give shade to their house, a family friend suggested that they need to plant a tree in the biggest corner of the lot. Where will the Cruz family plant the tree?



- A. Corner A  
 B. Corner B  
 C. Corner C  
 D. at the center
10. Ana, Bea, Clara, and Dina were instructed to create a triangle. They were each given a piece of stick of 21 in long. Each cut the stick in their own chosen lengths as follows: Ana (7 in, 7in, 7in), Dana ( 7in ,8in,6in), Jane, (5in, 10in, 6in), and Liza (3 in, 7 in, 11 in). Who among them was not able to make a triangle?
- A. Ana  
 B. Bea  
 C. Clara  
 D. Dina
11. Kenneth, Leo, and Michael play with their roller skates at the town oval. From the center of the oval, all of them skated 10 m east then turned south by 7 m but at different angles. Kenneth at  $68^\circ$ , Leo at  $54^\circ$  and Michael at  $70^\circ$ . Who is farther from the center of the oval?
- A. Kenneth  
 B. Leo  
 C. Michael  
 D. all of them are equally apart from the center

12. Bryan’s house is 20 m away from the church and 50 m away from his school. If Bryan went to the church first before going to school, for how many meters will he have to walk from church to school?

- A. between 20 m to 50 m
- B. between 30 m to 50 m
- C. between 30 m to 70 m\*
- D. between 50 m to 70 m

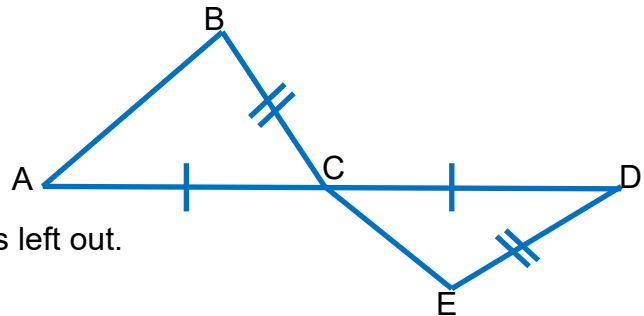


13. Jasper, Jerome, and Joel went to the field for kite flying. The length of their strings is 15 meters. The 15 meters strings were already consumed as the kite fly on the air. But Jasper’s kite made an angle of  $75^{\circ}$  with the ground while Jerome’s  $68^{\circ}$ , an Joel  $65^{\circ}$ . Which kite is higher?

- A. Jasper
- B. Jerome
- C. Joel
- D. All of them

14. Will you be able to conclude that  $AB > CE$  if one of the following statements is not established:  $\overline{AC} \cong \overline{CD}$ ,  $\overline{BC} \cong \overline{EC}$ ,  $\angle BCA > \angle EDC$ ?

- A. Yes, I will.
- B. No, I won't.
- C. It is impossible to decide.
- D. It depends on which statement is left out.



15. As the head of the physical plant department, you are asked to design foldable chairs that will be used in the new auditorium. Which of the following standards will you primarily consider in the design that affects the cause and the efficiency of the product?

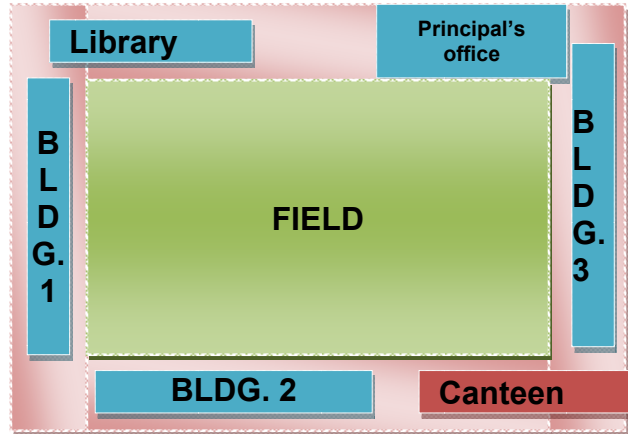
- A. Creativity
- B. Mathematical justification
- C. Originality
- D. Practicality

16. You are a pilates instructor. Some of your clients cannot perform some of the pilates moves. Because of this, you suggested that each one should buy his/her own ab rocket to aid in the stretching moves. You need to convince that they need to buy the product. Which of the following criteria will you consider in persuading them?



- A. Creativity
  - B. Efficiency
  - C. Practicality
  - D. Originality
17. You are a doctor assigned in the Emergency Room. A patient was rushed into the ER because his arms are fractured. You found out that his left arm can be bent at an angle of  $30^{\circ}$  from the bed and his right at  $75^{\circ}$ . Which of his two arms need to be given more attention in the treatment?
- A. left arm
  - B. right arm
  - C. both arms
  - D. treatment not needed
18. Your project in Livelihood and Economics is to make a fan. What material will you need to fasten the ridges together and make the fan foldable?
- A. bolt
  - B. pin
  - C. rope
  - D. string
19. In item 18, what other criteria will you consider in making your fan aside from efficiency?
- A. creativity and cost of materials\*
  - B. creativity and flexibility
  - C. creativity and originality
  - D. creativity and visual impact

20. Your teacher instructed the class to proceed to the library after recess. While in the canteen, you forget about the time and you only have few minutes left before the time. In what path will you pass so that you can immediately arrive at the library?



- A. Pass through buildings 2 and 1
- B. Pass through building 3 and the principal's office
- C. Pass through the field following a diagonal path
- D. Pass through building 3 then pass across the field going to building 1.

**GLOSSARY OF TERMS USED IN THIS LESSON:**

**Exterior Angle-** is an angle which is adjacent and supplementary to one of the angles of the triangle.

**Included Angle-** an angle that lies between two sides of a triangle

**Opposite angle –** is the angle opposite a side

**Opposite side-** is the side opposite an angle

**Remote Interior angles-** the two non-adjacent interior angles of the exterior angle of a triangle

**The Converse of the Hinge Theorem-** If two sides of one triangle are congruent respectively to the other two sides of another triangle, and the third side of the first

triangle is longer than the third side of the second, then the angle opposite the longer is larger than the angle opposite the third side of the second triangle.

**The Exterior Angle Theorem-** The measure of the exterior angle of a triangle is equal to the sum of the measures of its two remote interior angles.

**The Exterior Angle Inequality Theorem-** The measure of an exterior angle of a triangle is greater than the measure of either of its two remote interior angles.

**The Hinge theorem-**If two sides of one triangle are congruent to two sides of another triangle, but the included angle of the first triangle is larger than the included angle of the second, then the third side of the first triangle is longer the third side of the second triangle.

### **The Side-Angle- Relationship Theorems**

If two sides of a triangle are not congruent, then the measure of the angle opposite the longer side is greater than the measure of the angle opposite the shorter side.

If two sides of a triangle are not congruent, then the measure of the angle opposite the longer side is greater than the measure of the angle opposite the shorter side.

**The Triangle Inequality theorem-** The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

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## Lesson 4: Parallelism and Perpendicularity

### ☑ INTRODUCTION AND FOCUS QUESTION(S):



Have you ever wondered how carpenters, architects and engineers design their work? What factors are being considered in making their designs? The use of parallelism and perpendicularity of lines in real life necessitates the establishment of these concepts deductively.

This module seeks to find the answer to the question: *How can we establish parallelism or perpendicularity of lines?*

**MODULE LESSONS AND COVERAGE:**

In this module, you will examine this question when you study the following lessons:

*Lesson 4.1: Parallel Lines, Perpendicular Lines and Its Applications*

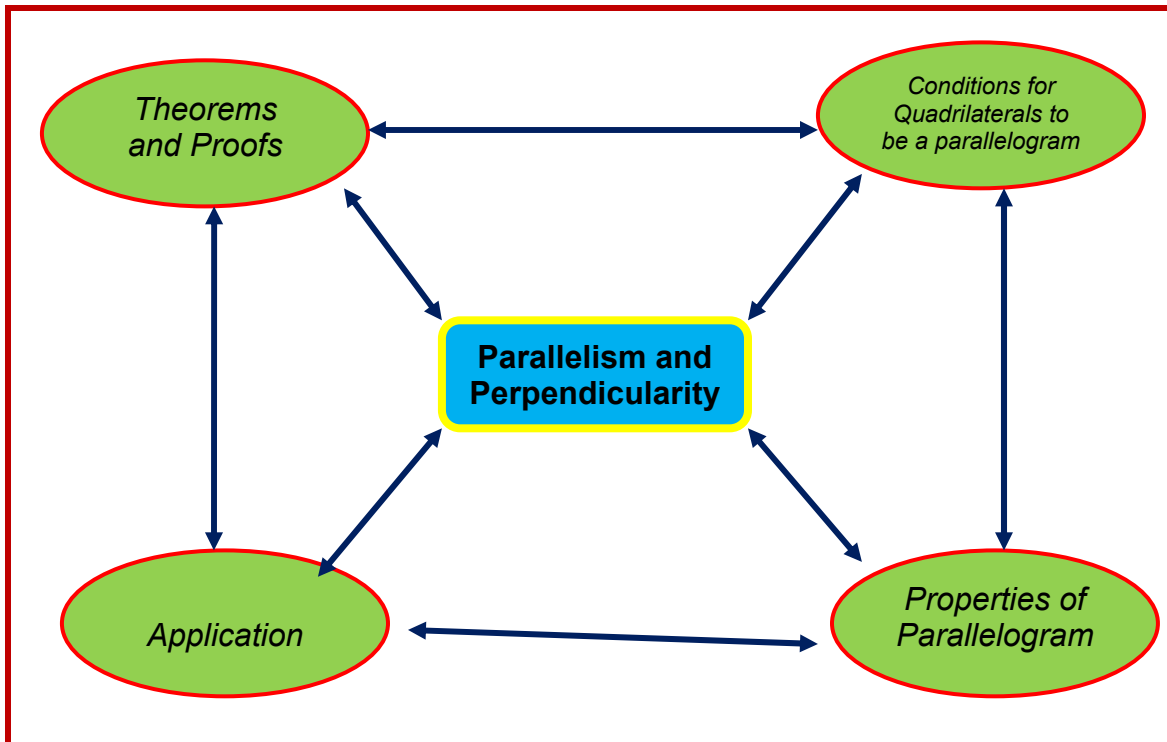
*Lesson 4.2: Parallelogram and Its Applications*

In these lessons, you will learn the following:

<p><i>Lesson 4.1</i></p>	<p>Parallel Lines, Perpendicular Lines and Its Applications</p> <ul style="list-style-type: none"> <li>• Determine which lines and segments are parallel or perpendicular</li> <li>• Illustrate parallel and perpendicular lines</li> <li>• Prove properties of parallel lines cut by a transversal</li> <li>• Use properties of parallel lines cut by a transversal to find measures of angles formed</li> <li>• Apply properties of parallel lines cut by a transversal in solving real-life problems</li> </ul>
<p><i>Lesson 4.2</i></p>	<p>Parallelogram, Its Properties and Applications</p> <ul style="list-style-type: none"> <li>• Name the different kinds of quadrilaterals and identify those that are parallelograms.</li> <li>• Determine conditions that make a quadrilateral a parallelogram</li> <li>• Prove properties that make a quadrilateral a parallelogram</li> <li>• Use properties of parallelogram to find the measures of angles, sides and other quantities</li> <li>• Apply properties of parallelogram in solving real-life problems</li> </ul>

**MODULE MAP:**

Here is a simple map of the above lessons you will cover:



**EXPECTED SKILLS:**

To do well in this module, you need to remember and do the following:

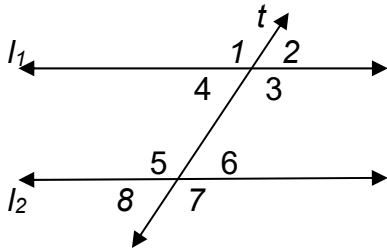
15. Define terms that are unfamiliar to you.
16. Explore websites that would be of great help for your better understanding of the lessons.
17. Take down notes of the important concepts in your journal.
18. Perform and complete the exercises provided.
19. Collaborate with your teacher and peers.

MODULE PRE-ASSESSMENT

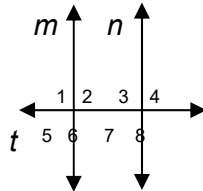


Let's find out how much you already know about this module. Click on the letter that you think best answers the question. Please answer all items. After taking this short test, you will see your score. Take note of the items that you were not able to correctly answer and look for the right answer as you go through this module.

- (A) 1. Using the figure below, if  $l_1 \parallel l_2$  and  $t$  is a transversal, then which of the following are corresponding angles?

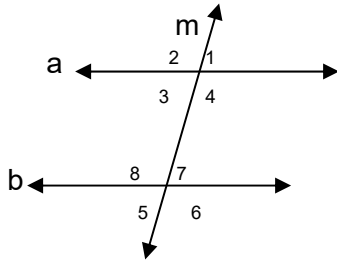


- A.  $\angle 4$  and  $\angle 6$ ,  $\angle 3$  and  $\angle 5$   
 B.  $\angle 1$  and  $\angle 7$ ,  $\angle 2$  and  $\angle 8$   
 C.  $\angle 1$  and  $\angle 5$ ,  $\angle 2$  and  $\angle 6$   
 D.  $\angle 4$  and  $\angle 5$ ,  $\angle 3$  and  $\angle 6$
- (A) 2. All of the following are properties of a parallelogram *except*:
- A. diagonals bisect each other  
 B. opposite angles are congruent  
 C. opposite sides are congruent  
 D. opposite sides are not parallel
- (A) 3. Lines  $m$  and  $n$  are parallel cut by transversal  $t$  which is also perpendicular to  $m$  and  $n$ . Which statement is *not* correct?



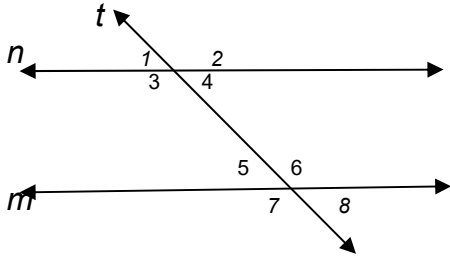
- A.  $\angle 1$  and  $\angle 6$  are congruent.  
 B.  $\angle 2$  and  $\angle 3$  are supplementary.  
 C.  $\angle 3$  and  $\angle 5$  are congruent vertical angles.  
 D.  $\angle 1$  and  $\angle 4$  form linear pair

- (A) 4. Lines  $a$  and  $b$  are parallel cut by transversal  $m$ . If  $m \angle 1 = 85^\circ$ , then what is  $m \angle 5$ ?



- A.  $80^\circ$
- B.  $85^\circ$
- C.  $95^\circ$
- D.  $100^\circ$

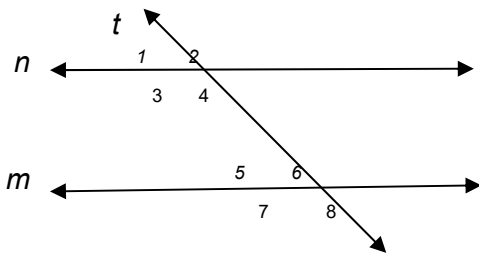
- (A) 5. Using the figure below, which of the following guarantee that  $m \parallel n$ ?



- A.  $\angle 1 \cong \angle 7$
- B.  $\angle 3 \cong \angle 5$
- C.  $\angle 4 \cong \angle 5$
- D.  $\angle 4 \cong \angle 7$

- (A) 6. In  $\square JOSH$ , the measure of  $\angle J = 57^\circ$ , find the measure of  $\angle H$ .
- A.  $43^\circ$
  - B.  $57^\circ$
  - C.  $63^\circ$
  - D.  $123^\circ$

- (A) 7. Using the figure below, if  $m \parallel n$  and  $t$  is a transversal which angles are congruent to  $\angle 5$ ?

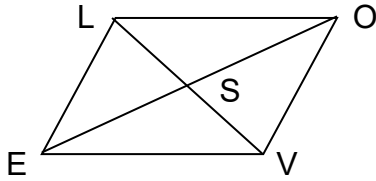


- A.  $\angle 1, \angle 2$  and  $\angle 3$



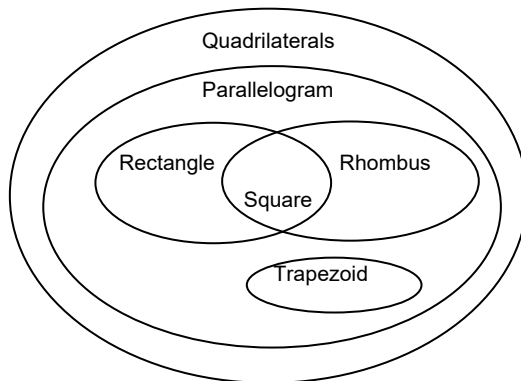
- B.  $\angle 1$ ,  $\angle 4$  and  $\angle 8$
- C.  $\angle 1$ ,  $\angle 4$  and  $\angle 7$
- D.  $\angle 1$ ,  $\angle 2$  and  $\angle 8$

(A) 8.  $\square$  LOVE is a parallelogram. If the length of SE = 6, then what is the length of SO?



- A. 3
- B. 6
- C. 12
- D. 15

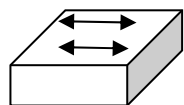
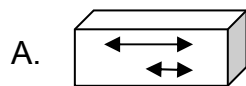
(M) 9. The Venn diagram below shows the relationships of special quadrilaterals. Which statements are true?



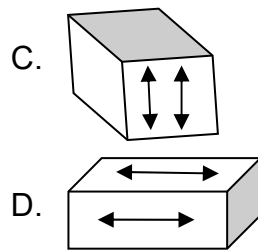
- I - Squares are rectangles.
- II - A trapezoid is a parallelogram.
- III - A rhombus is a square.
- IV - Some parallelograms are squares.

- A. I and II
- B. III and IV
- C. I and IV
- D. II and III

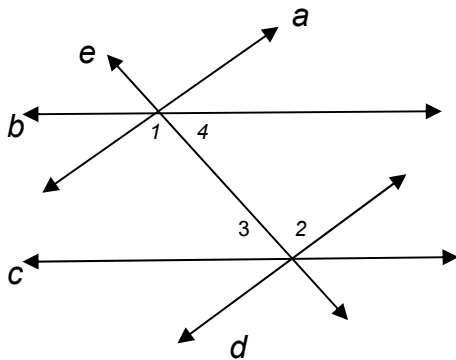
(M) 10. All of the figures below illustrate parallel lines *except*:



B.



(M) 11. In the figure below,  $a \parallel d$  with  $e$  as the transversal. What is true about  $\angle 3$  and  $\angle 4$  in order to make  $b \parallel c$ ?



- A.  $\angle 3$  is a complement of  $\angle 4$
- B.  $\angle 3$  congruent to  $\angle 4$
- C.  $\angle 3$  is a supplement of  $\angle 4$
- D.  $\angle 3$  is bigger than  $\angle 4$

(M) 12. Which of the following statements ensures that a certain quadrilateral is a parallelogram?

- A. Diagonals bisect each other.
- B. The two diagonals are congruent.
- C. The consecutive sides are congruent.
- D. Two consecutive angles are congruent.

(M) 13. Which of the following statements is always true?

- A. Lines that do not intersect are parallel lines.
- B. Two coplanar lines that do not intersect are parallel lines.
- C. Lines forming right angle are parallel lines.
- D. Two lines that are parallel will not meet no matter how long they are.

(M) 14. In  $\square STAR$  with diagonals  $\overline{AS}$  and  $\overline{RT}$ , if  $m\angle STR = (3x - 5)^\circ$  and  $m\angle ART = (x + 10)^\circ$ , then what is the  $m\angle RAT$ ?

- A.  $8^\circ$
- B.  $19^\circ$
- C.  $71^\circ$

D.  $161^\circ$

(T) 15. You are tasked to divide a blank card into three equal rows and you do not have a ruler. Instead, you will use a piece of equally lined paper and a straight edge. What is the sequence of the steps you are going to undertake in order to apply the theorem on parallel lines?

I – Mark the points where the second and third lines intersect the card.

II – Place a corner of the top edge of the card on the first line of the paper.

III – Repeat for the other side of the card and connect the marks.

IV – Place the corner of the bottom edge on the fourth line.

- A. I, II, III, IV
- B. II, III, IV, I
- C. I, III, IV, II
- D. II, IV, I, III

(T) 16. You are a student council president. You want to make a request for financial assistance for the installation of a book shelf for the improvement of your school's library. Your student council moderator requested you to submit a proposal for their approval. Which of the following will you prepare so as to ensure that your request be granted?

I. propose design of book shelf

II. research on the important of book shelf

III. estimated cost of the project

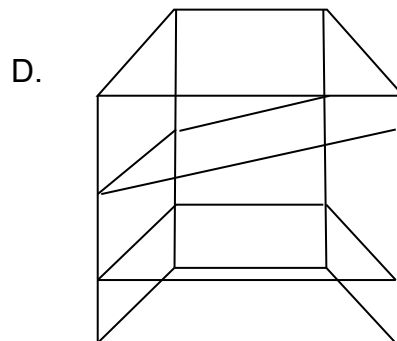
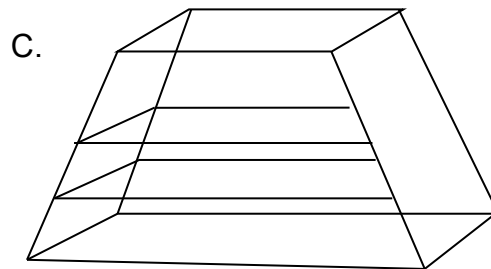
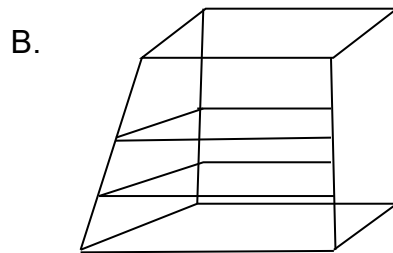
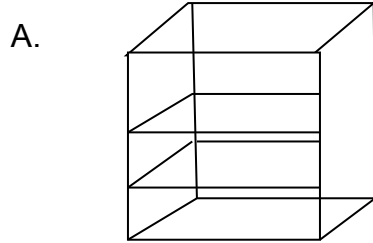
IV. pictures of the different libraries

- A. I only
- B. I and II only
- C. I and III only
- D. II and IV only

(T) 17. Based on your answer in item no. 16, which of the following standards should be the basis of your moderator in approving or granting your request?

- A. accuracy, creativity and mathematical reasoning
- B. practicality, creativity and cost
- C. accuracy, originality and mathematical reasoning
- D. organization, mathematical reasoning and cost

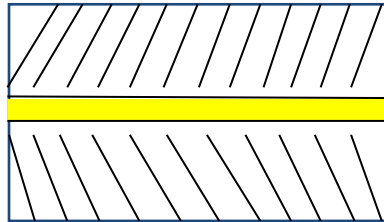
(T) 18. Based on item no. 16 design is common to all the four given options. If you are to make the design, which of the drawing below will you make to ensure stability?



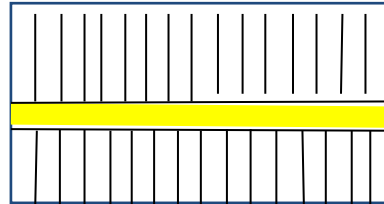
(T) 19. You are an architect of the design department of SM Mall of Asia. With the increasing number of mall-goers, the management decided to restructure their parking lot so as to maximize the space. As the head architect, you are tasked to make a design of the parking area, and this design is to be presented to higher officers of the SM Mall of Asia

management for approval. Which of the following will be the design that you are going to make so as to guarantee maximizing the available lot?

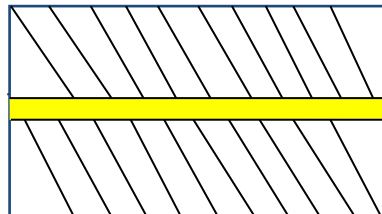
A.



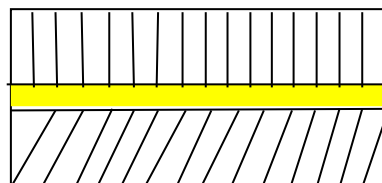
B.



C.



D.



(T) 20. Based on your answer in item no. 19, how will your immediate supervisor know that you have a good design?

- A. The design should be creative and accurate.
- B. The design should be accurate and practical.
- C. The design shows a depth application of mathematical reasoning and it is practical.
- D. The design should be realistic and accurate.

**SUBMIT**

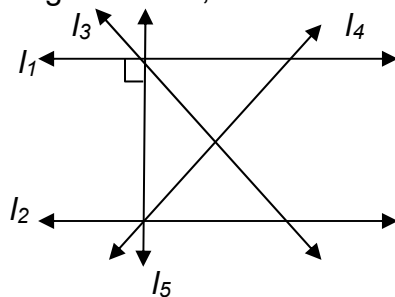
## Lesson 4.1 Parallel Lines, Perpendicular Lines and Its Application

### LESSON: PRE-ASSESSMENT

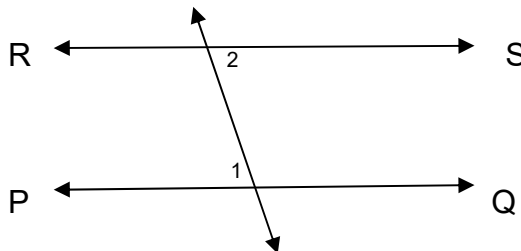


Let's find out how much you have already know about this lesson. Click on the letter that you think best answers the question. Please answer all items. After taking this short test, you will see your score. Take note of the items that you were not able to correctly answer and look for the right answer as you go through this lesson.

1. Using the figure below, which of the lines is perpendicular to  $l_1$ ?

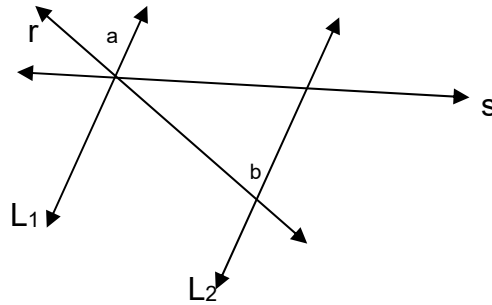


- A.  $l_2$   
B.  $l_3$   
C.  $l_4$   
D.  $l_5$
2. In the figure,  $RS \parallel PQ$ . If  $m \angle 1 = 85^\circ$ , then what is the  $m \angle 2$ ?

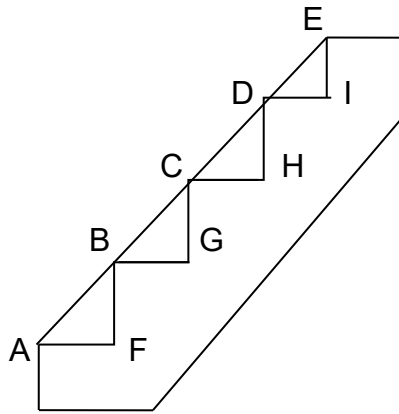


- A.  $65^\circ$   
B.  $75^\circ$   
C.  $85^\circ$   
D.  $95^\circ$
3. All of the following are properties of a parallel lines cut by a transversal *except*:
- A. corresponding angles are congruent  
B. vertical angles are congruent  
C. alternate-interior angles are congruent  
D. alternate-exterior angles are congruent

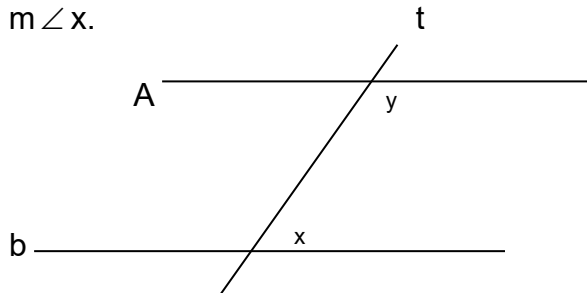
4.  $L_1 \parallel L_2$  with  $r$  and  $s$  as the transversals. What type of angle pairs is  $a$  and  $b$ ?



- A. corresponding angles  
 B. alternate-interior angles  
 C. alternate-exterior angles  
 D. interior angles on the same side of the transversal
5. A carpenter builds a stairway by cutting triangles like  $\triangle AFB$  and  $\triangle BGC$  from a piece of lumber.  $\angle GBC$  and  $\angle HCD$  are corresponding angles relative to what pair of parallel lines?



- A.  $\overline{AF}$  and  $\overline{BG}$   
 B.  $\overline{BG}$  and  $\overline{CH}$   
 C.  $\overline{CH}$  and  $\overline{DI}$   
 D.  $\overline{AF}$  and  $\overline{CH}$
6. In the figure  $a \parallel b$  with transversal  $t$  and  $\angle x$  is  $72^\circ$  less than  $\angle y$ . Find the  $m\angle x$ .

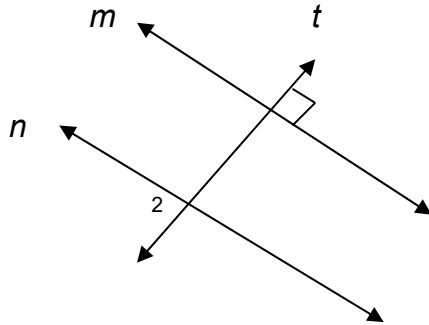


- A.  $54^\circ$   
 B.  $72^\circ$   
 C.  $128^\circ$   
 D.  $180^\circ$

7. If two lines  $r$  and  $s$  are cut by a transversal forming corresponding angles with measures  $3x - 25$  and  $2x + 10$ , what value of  $x$  will guarantee that  $r$  and  $s$  are parallel?

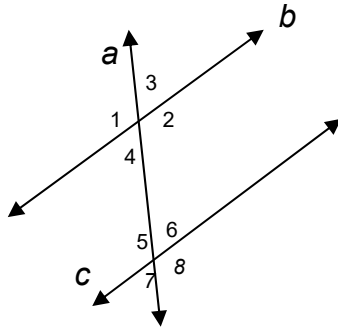
- A. 15
- B. 25
- C. 35
- D. 45

8. In the figure,  $m \parallel n$  and transversal  $t$  is perpendicular to  $m$ . If  $\angle 2 = 3x - 6$ , find  $x$ .



- A. 8
- B. 16
- C. 24
- D. 32

For items 9 and 10, refer to the figure below:



9. What do you call  $\angle 3$  and  $\angle 7$ ?
- A. corresponding angles
  - B. same-side interior angles
  - C. alternate exterior angles
  - D. alternate interior angles
10. Suppose  $m\angle 4 = 48^\circ$ . Find the  $m\angle 5$ .
- A.  $42^\circ$
  - B.  $48^\circ$
  - C.  $142^\circ$
  - D.  $148^\circ$





## EXPLORE

Let's start the module by taking a look at the figures below and then answer the questions given.

### ACTIVITY 1. Optical Illusion

Below are three different figures. Figures 1 and 2 are in connection with the concept on parallel lines, whereas figure 3 is related to perpendicular lines. Right after the given figures, answer the questions provided then "SAVE" your answer.

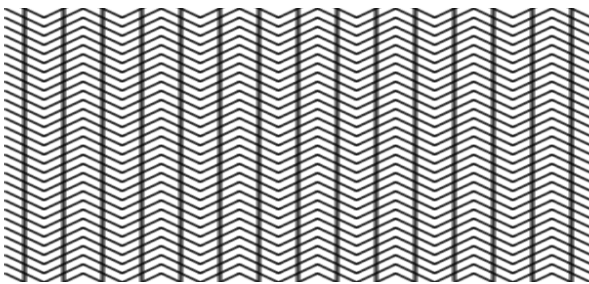


Figure 1

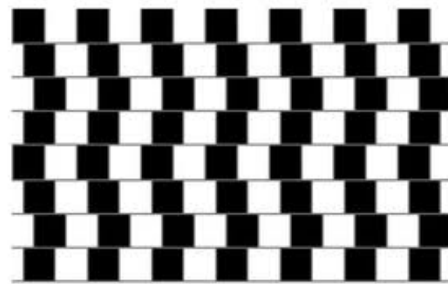


Figure 2

- *Can you see straight lines in the pictures above?*

- *Are these lines parallel? Why?*

- *Can you describe what parallel lines are?*

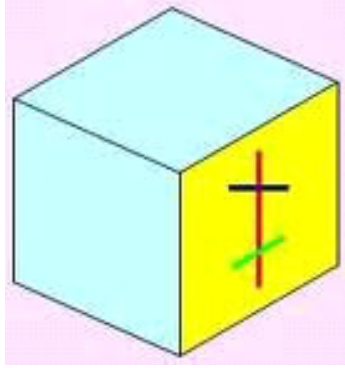


Figure 3

- *What can you say about the edges of the box?*

- *Are these lines perpendicular? Why?*

- *Can you describe what perpendicular lines are?*



*You have just tried describing parallel and perpendicular lines. In our next activity, your prior knowledge on parallelism and perpendicularity will be elicited.*

**ACTIVITY 2. Generalization Table**

Fill in the first column of the generalization table below by stating your initial thoughts on the question, then save your answer.

**“How can parallelism or perpendicularity of lines be established?”**

My Initial Thoughts	My Findings and Corrections	Supporting Evidence	Qualifying Conditions	My Generalizations

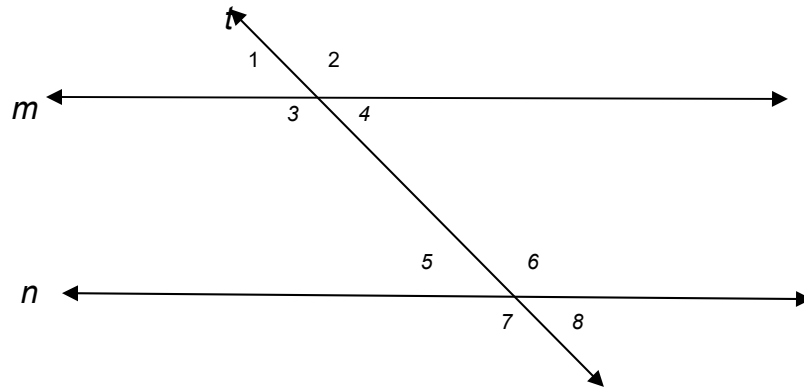


*Well, those were your thoughts and ideas about our lesson. Let’s start a new activity to further explore on the important key concepts about parallel and perpendicular lines. I guess you have it already in your previous Math, but just to recall, I want you to answer the next activity.*

**ACTIVITY 3. Name It! A Recall...**



We see parallel lines everywhere. Lines on a pad paper, railways, edges of a door or window, etc. suggest parallel lines. Click on this site: <http://www.mathwarehouse.com/geometry/angle/transveral-and-angles.php> to recall the different angles formed by parallel lines cut by a transversal. Then complete the table below using the given figure as your reference:



<b>Corresponding Angles</b>	<b>Alternate Interior Angles</b>	<b>Alternate Exterior Angles</b>	<b>Same Side Interior Angles</b>	<b>Same Side Exterior Angles</b>

**End of EXPLORE:**

You gave your initial ideas on naming angle pairs formed by two lines cut by a transversal line. What you will learn in the next sections will enable you to do the final project which involves integrating the key concepts of parallelism and perpendicularity of lines in model making of a cabinet book case. Let's now find out how these pairs of angles are related in terms of their measures by doing the first activity on investigating the relationship between the angles formed by parallel lines cut by a transversal and perpendicular lines.



**FIRM-UP**

Your goal in this section is to learn and understand key concepts of measurement of angles formed by parallel lines cut by a transversal and basic concepts about perpendicularity. And towards the end of this section, you will be encouraged to learn the different ways of proving deductively.

**ACTIVITY 4. Let's Investigate!**

Two parallel lines when cut by a transversal line form eight angles. This activity leads students investigate on how one angle is related to the other angle. Using this link: <http://www.mathwarehouse.com/geometry/angle/interactive-transveral-angles.php>

In this site, you have to note down the following:

- a. Identify the different pairs of angles formed by parallel lines cut by a transversal line.
- b. As you move the lines, observe which pairs of angles are congruent? Which pairs are supplementary?
- c. Make an inference based on your observations on how one angle is related to the other angle.  
What inferences can you make for each pair of angles?

*Generalization:*

Now think about the answers to the following questions. Write your answer in the answer box.

**PROCESS QUESTIONS:**

1. *What pair of angles is formed when two lines are cut by a transversal line?*
2. *What pair of angles has equal measures? What pair of angles are supplementary?*
3. *Can the measures of any pair of angles (supplementary or equal) guarantee the parallelism of lines? Support your answer.*
4. *How can the key concepts of parallel lines facilitate solving real-life problems using deductive reasoning?*

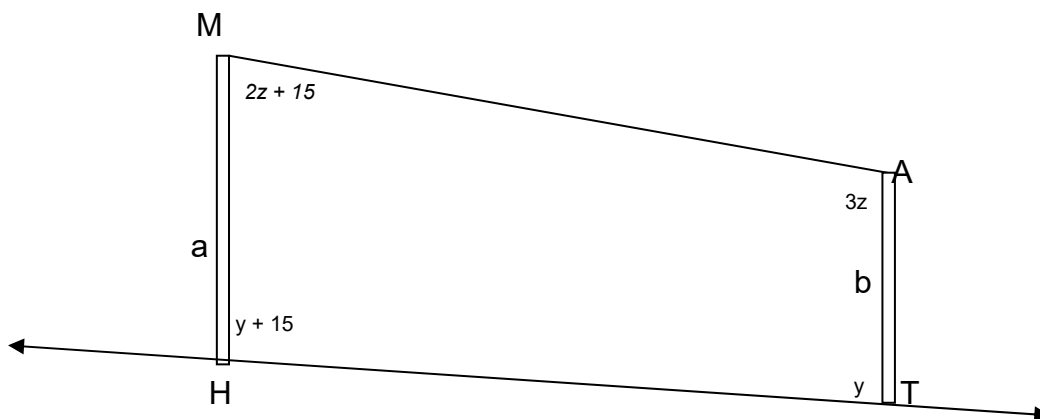
*Answer:*

**ACTIVITY 5.**

**Uncovering the Mystery of Parallel Lines Cut by a Transversal**

Study the problem situation below and answer succeeding questions:

*A zip line is a rope or a cable that you can ride down on a pulley. The pair of zip lines below goes from a 20-foot tall tower to a 15 foot tower 50 feet away in a slightly inclined ground as shown in the sketch.*





1. What kind of angle pairs are  $\angle M$  and  $\angle A$ ?  $\angle MHT$  and  $\angle ATH$ ?  
 \_\_\_\_\_  
 \_\_\_\_\_
2. Using the information stated in the figure, what are the measures of the four angles?

Solution:

Answers:

$$m \angle M = \underline{\hspace{2cm}}$$

$$m \angle A = \underline{\hspace{2cm}}$$

$$m \angle AHT = \underline{\hspace{2cm}}$$

$$m \angle ATH = \underline{\hspace{2cm}}$$

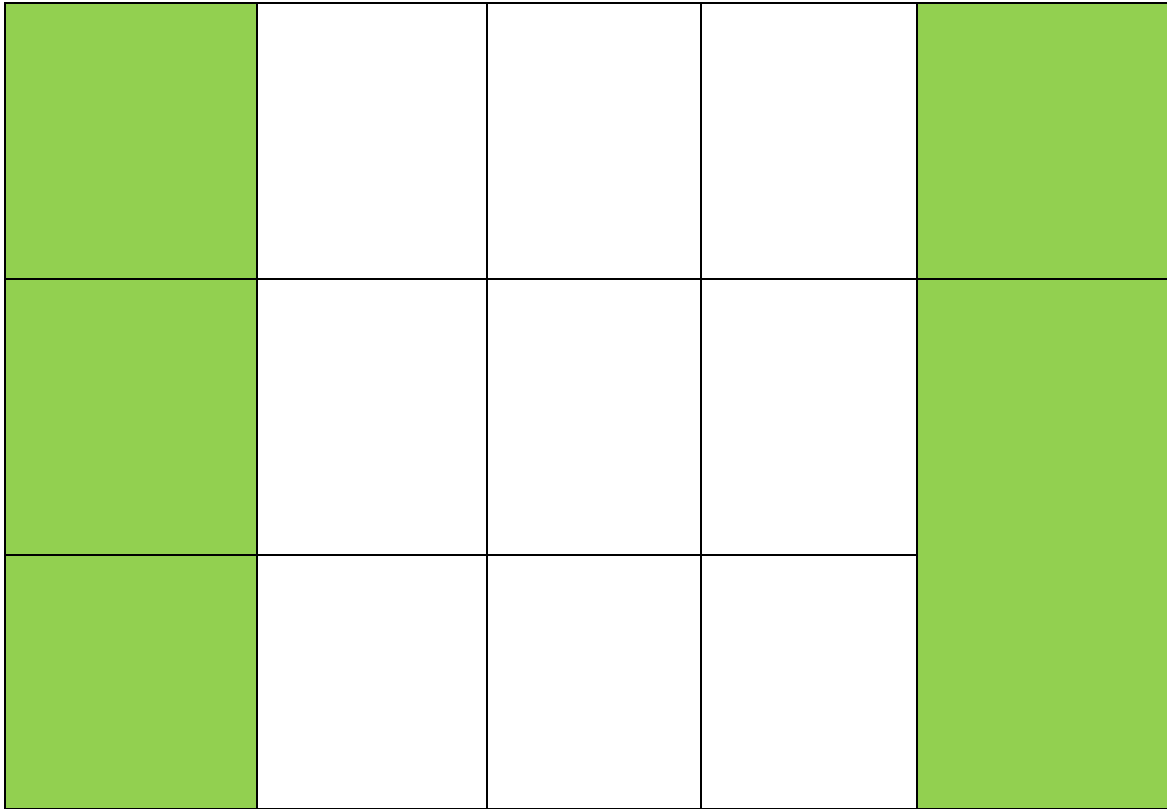
3. Are the two towers parallel? Why do you say so?  
 \_\_\_\_\_  
 \_\_\_\_\_
4. Is the zip line parallel to the ground? Why or why not? Why do you say so?  
 \_\_\_\_\_  
 \_\_\_\_\_

### **ACTIVITY 6. Generalization Table**

Fill in the second, third and fourth column of the generalization table below by stating your revised or new thoughts on the question, then save your answer.

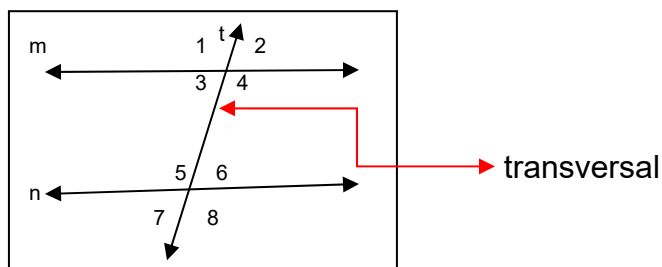
***“How can parallelism or perpendicularity of lines be established?”***

<i>My Initial Thoughts</i>	<i>My Findings and Corrections</i>	<i>Supporting Evidence</i>	<i>Qualifying Conditions</i>	<i>My Generalizations</i>



Discussion: **Parallelism**

- Two lines are parallel if and only if they are coplanar and do not intersect.  
(  $m \parallel n$  )



- A line that intersects two or more lines at different points is called a **transversal**.
  - The angles formed by the transversal with the two other lines can be identified as:
    - **exterior angles** ( $\angle 1$ ,  $\angle 2$ ,  $\angle 7$  and  $\angle 8$ )
    - **interior angles** ( $\angle 3$ ,  $\angle 4$ ,  $\angle 5$  and  $\angle 6$ ).



b. The pairs of angles formed by the transversal with the other two lines can be identified as:

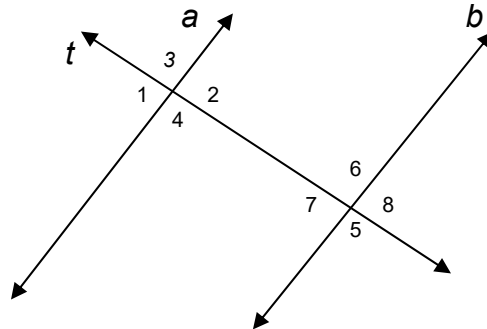
- **corresponding angles** ( $\angle 1$  and  $\angle 5$ ,  $\angle 2$  and  $\angle 6$ ,  $\angle 3$  and  $\angle 7$ ,  $\angle 4$  and  $\angle 8$ )
- **alternate-interior angles** ( $\angle 3$  and  $\angle 6$ ,  $\angle 4$  and  $\angle 5$ )
- **alternate-exterior angles** ( $\angle 1$  and  $\angle 8$ ,  $\angle 2$  and  $\angle 7$ )
- **interior angles on the same side of the transversal** ( $\angle 3$  and  $\angle 5$ ,  $\angle 4$  and  $\angle 6$ )
- **exterior angles on the same side of the transversal** ( $\angle 1$  and  $\angle 7$ ,  $\angle 2$  and  $\angle 8$ )

3. If two lines are cut by a transversal, then the two lines are parallel if:
- a. corresponding angles are congruent
  - b. alternate-interior angles are congruent
  - c. alternate-exterior angles are congruent
  - d. interior angles on the same side of the transversal are supplementary
  - e. exterior angles on the same side of the transversal are supplementary

**ACTIVITY 7. Quiz No. 1 (FORMATIVE ASSESSMENT)**

Study the figure and answer the following questions as accurate as you can.

1. The figure below shows  $a \parallel b$  with  $t$  as transversal.



Name: \_\_\_\_\_

- |   |       |       |
|---|-------|-------|
| 1. 2 pairs of corresponding angles                                | _____ | _____ |
| 2. 2 pairs of alternate-interior angles                           | _____ | _____ |
| 3. 2 pairs of alternate-exterior angles                           | _____ | _____ |
| 4. 2 pairs of interior angles on the same side of the transversal | _____ | _____ |
| 5. 2 pairs of exterior angles on the same side of the transversal | _____ | _____ |



**ACTIVITY 8. Am I Perpendicular? Let's Find Out....!**

On a plane, given any two distinct lines, the two lines either intersect or they are parallel. If two lines intersect, then it forms four angles of different measures. Consider the figures below to answer the questions given.

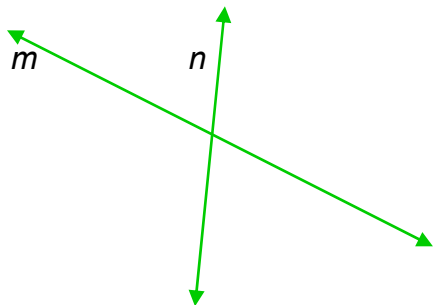


Figure 1

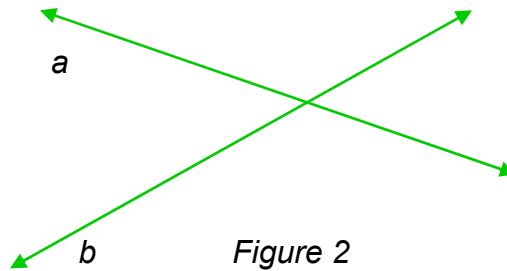


Figure 2

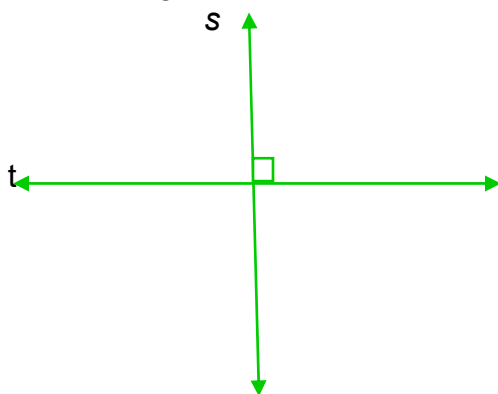


Figure 3

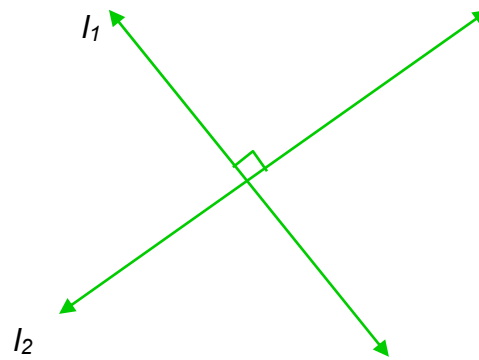


Figure 4

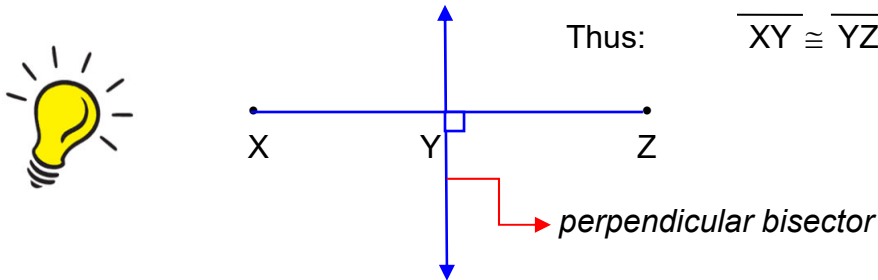


Process Questions:

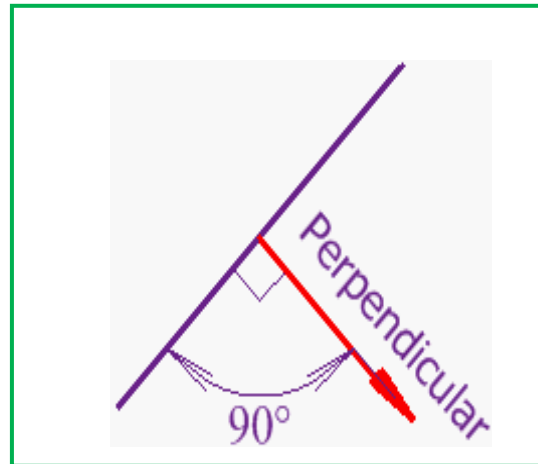
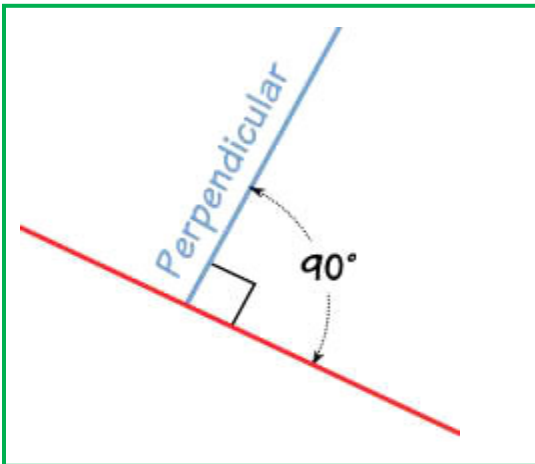
1. What is common in the four figures given above?  
\_\_\_\_\_
2. What makes figures 3 and 4 different from the other two figures?  
\_\_\_\_\_
3. What does this symbol  $\perp$  mean?  
\_\_\_\_\_
4. Of the four figures, which is/are perpendicular?  
\_\_\_\_\_
5. When is the lines said to be perpendicular?  
\_\_\_\_\_
6. How useful the knowledge on perpendicularity in real life? Cite an example in which perpendicularity is said to be of important in real life.  
\_\_\_\_\_  
\_\_\_\_\_

Discussion: **Perpendicularity**

Two lines that intersect to form a right angle are said to be **perpendicular**. This is not limited to lines only. Segments and rays can also be perpendicular. A **perpendicular bisector** of a segment is a line or a ray or another segment that is perpendicular to the segment and intersects the segment at its midpoint.



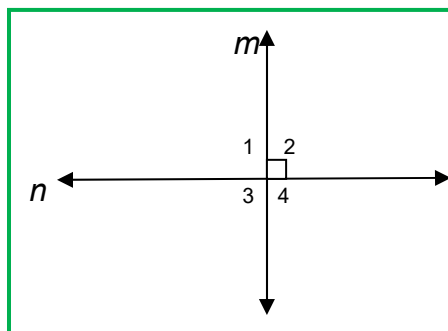
Samples of perpendiculars:



The small box drawn in the corner, means “right angle”. Whereas,  $\perp$  is a symbol use to indicate perpendicularity of lines.

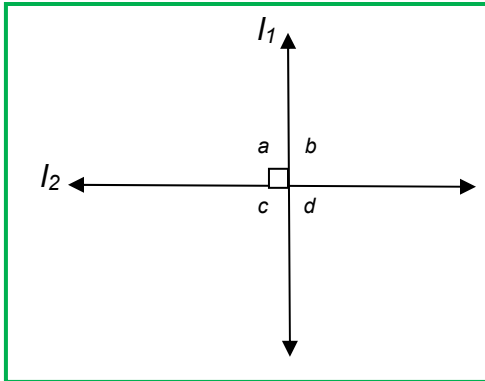
To prove that two lines are perpendicular, you must show that one of the following theorems is true:

1. If two lines are perpendicular, then they form four right angles.



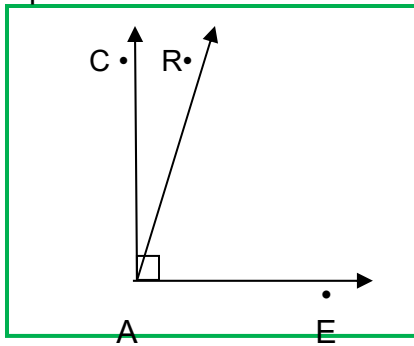
*If  $m \perp n$ , then we can conclude that  $\angle 1$ ,  $\angle 2$ ,  $\angle 3$  and  $\angle 4$  are right angles.*

2. If the angles in a linear pair are congruent, then the lines containing their sides are perpendicular.



*If  $\angle a$  and  $\angle b$  form a linear pair and  $\angle a \cong \angle b$ , then  $l_1 \perp l_2$ .*

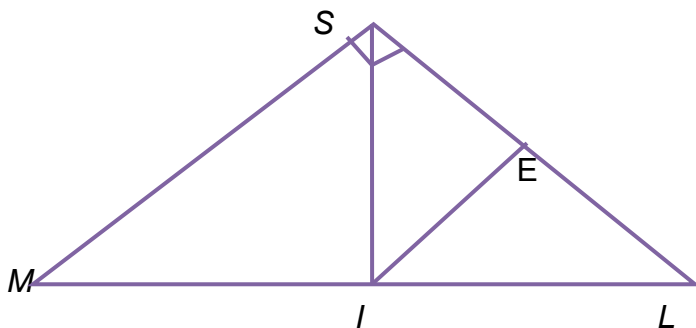
3. If two angles are adjacent and complementary, the non-common sides are perpendicular.



*If  $\angle CAR$  and  $\angle EAR$  are complementary and adjacent, then  $\overline{AC} \perp \overline{AE}$ .*

**ACTIVITY 9. Yes / No Card!**

Refer to the given figure and the conditions given, then shade the yes or no box.



$$\frac{\overline{MI} \cong \overline{IL}}{\overline{SE} \cong \overline{EL}}$$

1. Is  $\overline{ML} \perp \overline{IS}$ ?
2. Are  $\angle SIE$  and  $\angle LIE$  complementary angles?
3. Is  $\overline{EI}$  a perpendicular bisector of  $\overline{SL}$ ?

YES	NO
<input type="checkbox"/>	<input type="checkbox"/>
<input type="checkbox"/>	<input type="checkbox"/>
<input type="checkbox"/>	<input type="checkbox"/>

4. Do  $\angle$  MIS and  $\angle$  SIL form a linear pair?
5. Is the  $m\angle$  MIS =  $90^\circ$ ?

<input type="checkbox"/>	<input type="checkbox"/>
<input type="checkbox"/>	<input type="checkbox"/>

### END OF FIRM UP:

In this section, the discussion was about the key concepts on parallelism and perpendicularity. Relationships of the different angle pairs formed by parallel lines cut by a transversal were also given emphasis.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.



### DEEPEN

Your goal in this section is to take a closer look at some aspects of the topic. This section gives emphasis on proving through deductive reasoning involving conditions that guarantee parallelism or perpendicularity of lines. I hope that you are now ready to answer the exercises given in this section to intensify or make deeper application of what has been learned.

#### Discussion: *Writing Proofs / Proving*



In the previous discussions, you have solved a lot of equations and inequalities by applying the different properties of equality and inequality. To name some, you have the APE (addition property of equality), MPE (multiplication property of equality) and TPE (transitive property of equality). Now, you will use the same properties with some geometric definitions, postulates and theorems to write a complete proof.

One of the tools used in proving is reasoning, specifically deductive reasoning. **Deductive reasoning** is a type of logical reasoning that uses accepted facts to reason in a step-by-step manner until we arrive at the desired statement.

A **proof** is a logical argument in which each statement you make is backed up by a statement that is accepted as true.

**EXERCISE:** Identify whether the given statement is a deduction or a proof.

1. All segments such as  $\overline{AB}$ , have only one bisector.
2. Given that C is the midpoint of AB. Then by definition of a midpoint,  $AC \cong BC$ .
3. A student who attains a perfect score in the unit test will be exempted in the final examination. Ted attained a perfect score in the unit test. Thus Ted will be exempted in the final examination.
4.  $\overline{AB} \cong \overline{CD}$ . By definition of congruent segments,  $AB = CD$ .
5.  $\angle 1$  and  $\angle 6$  are corresponding angles and are congruent. Since if two lines m and n are cut by a transversal line t and corresponding angles 1 and 6 are congruent, then  $m \parallel n$ .

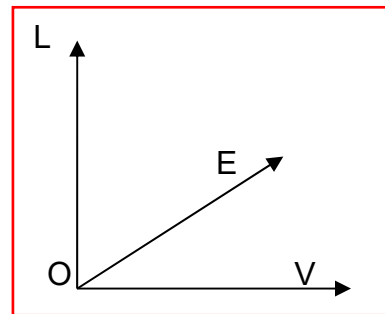
Proof can be written in three different ways:

**1. Paragraph Form/ Informal Proof:**

The **paragraph or informal proof** is the type of proof where you write a paragraph to explain why a conjecture for a given situation is true.

Given:  $\angle LOE$  and  $\angle EO V$   
are complementary

Prove:  $LO \perp OV$



Proof:

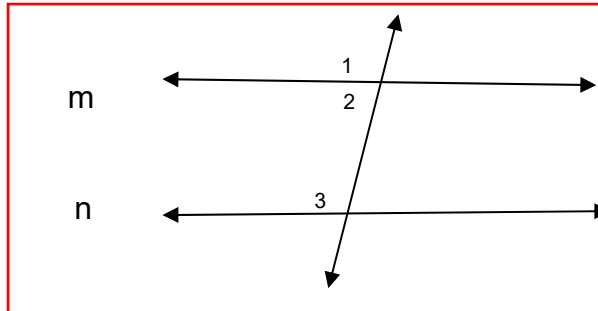
Since  $\angle LOE$  and  $\angle EO V$  are complementary, then  $m\angle LOE + m\angle EO V = 90^\circ$  by definition of complementary angles. Thus,  $m\angle LOE + m\angle EO V = m\angle LOV$  by angle addition postulate and  $m\angle LOV = 90^\circ$  by transitive property of equality. So,  $\angle LOV$  is a right angle by definition of right angles and therefore,  $LO \perp OV$  by definition of perpendicularity.

### 2. Two-Column Form/ Formal Proof:

**Two-column form/Formal Proof** is the most formal proof with statements and reasons. The first column is for the statements and the other column for the reason.

Given:  $m \parallel n$

Prove:  $\angle 2 \cong \angle 3$



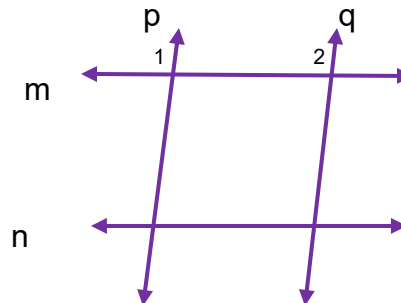
Statements	Reasons
1. $m \parallel n$	1. Given
2. $\angle 1 \cong \angle 3$	2. Corresponding angles are congruent.
3. $\angle 1 \cong \angle 2$	3. Vertical angles are congruent.
4. $\angle 2 \cong \angle 3$	4. Transitive Property of Congruence

### 3. Flowchart Form:

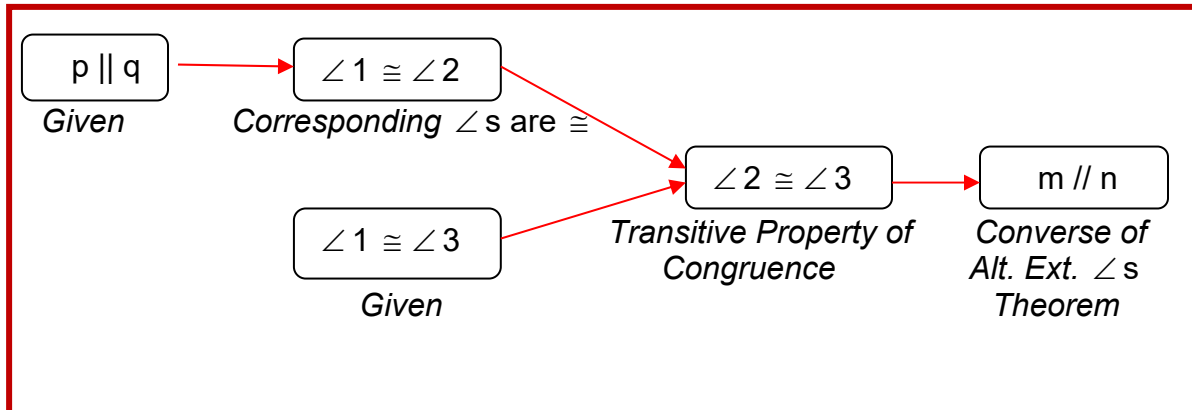
A **flowchart-proof** organizes a series of statements in a logical order, starting with the given statements. Each statement together with its reason is written in a box, and arrows are used to show how each statement lead to another. It can make ones logic visible and help others follow the reasoning.

Given:  $p \parallel q$  and  $\angle 1 \cong \angle 3$

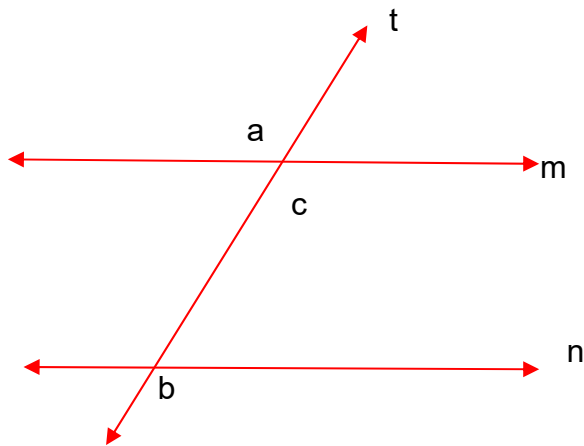
Prove:  $m \parallel n$







Given:  $m \angle c + m \angle b = 180$   
 Prove:  $m$  is not parallel to  $n$



Proof:

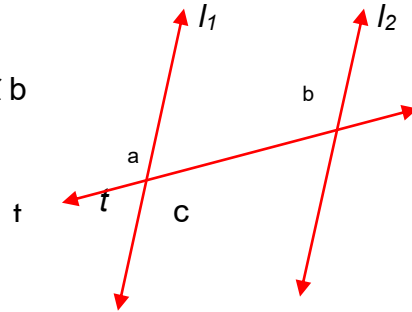
Statements	Reasons
1. $m \angle c + m \angle b = 180$	1. Given
2. $\angle a \cong \angle c$	2. Vertical angle theorem
3. $m \angle a = m \angle c$	3. Definition of congruent angles
4. $m \angle a + m \angle b = 180$	4. Substitution
5. $m$ is not parallel to $n$	5. Any two lines cut by a transversal line where alternate exterior angles are supplementary is not a guarantee that the two lines are always parallel.

**ACTIVITY 10. Complete Me!**

In relation to the discussion on writing proofs deductively, complete each proof below:

1. Given:  
 $t$  intersects  $l_1$  and  $l_2$  such that  $\angle a = \angle b$

Prove:  
 $l_1 \parallel l_2$

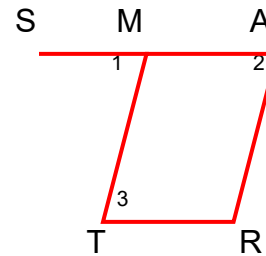


Proof:

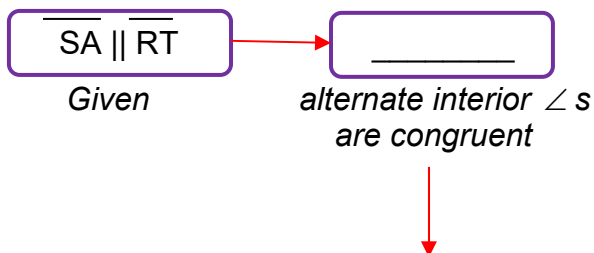
Statements	Reasons
1. $\angle a = \angle b$	1. _____
2. _____	2. Vertical $\angle$ s are equal
3. $\angle c = \angle b$	3. Substitution
4. $l_1 \parallel l_2$	4. _____

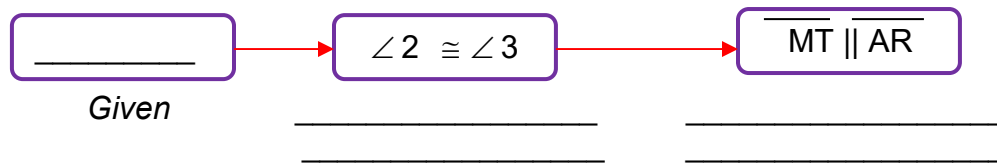
2. Given:  
 $\overline{SA} \parallel \overline{RT}$   
 $\angle 2 \cong \angle 3$

Prove:  
 $\overline{MT} \parallel \overline{AR}$



Proof:





### END OF DEEPEN

In this section, the discussion was about parallelism and perpendicularity of lines.

What new realizations do you have about the topic? What new connections have you made for yourself?

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section



### TRANSFER

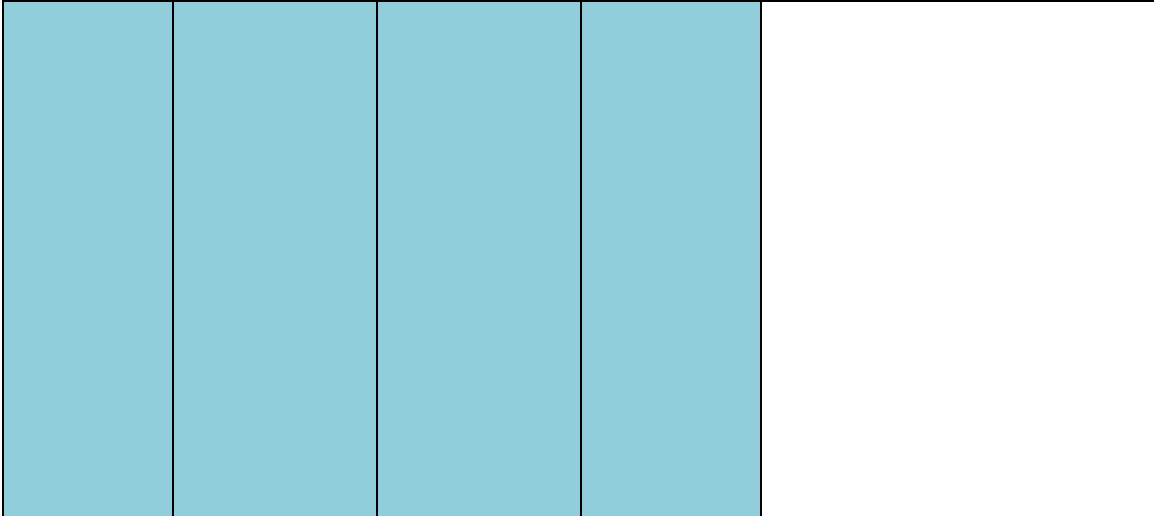
Your goal in this section is apply your learning in proving through deductive reasoning. You will be given a task which will demonstrate your understanding.

### ACTIVITY 11. Generalization Table

After a lot of exercises, it's now time for you to fill in the last column of the generalization table below by stating your conclusions or insights about parallelism and perpendicularity.

***“How can parallelism or perpendicularity of lines be established?”***

<i>My Initial Thoughts</i>	<i>My Findings and Corrections</i>	<i>Supporting Evidence</i>	<i>Qualifying Conditions</i>	<i>My Generalizations</i>
----------------------------	------------------------------------	----------------------------	------------------------------	---------------------------



**ACTIVITY 12. Prove It!**

It has been discussed in the previous section the different ways on proving through deductive reasoning. Many formulas in a form of theorems are applied in the world around us only after careful and often through deductive reasoning has established their validity and be used with confidence. Likewise, many circumstances in our lives make use of deductive reasoning. For example, we know that Filipinos are hospitable. We are Filipinos, so we consider ourselves hospitable. It is also a common knowledge that simple interest of money deposited in bank is computed by multiplying the amount deposited, the rate of interest and the number of years we put the money in bank. If you deposited Php 5,000.00 at 3% rate of interest in two years it earns a simple interest of Php 300.00.

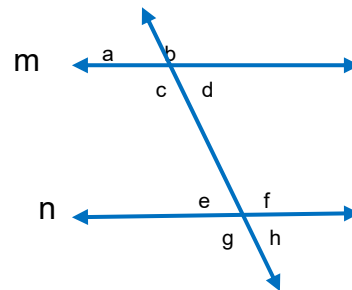
In the previous activity, you were asked to complete the proof of the given problem. In this activity, I want you to decide on your own so as to prove the given statements below:

1. Given:

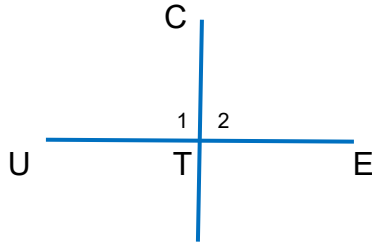
$$m \parallel n$$

Prove:

$\angle a$  and  $\angle g$  are supplementary



2. In the figure, if  $m\angle 1 = (3x + 15)^\circ$ ,  $m\angle 2 = (4x - 10)^\circ$  prove that  $\overline{CT}$  is perpendicular to  $\overline{UE}$  if  $x = 25^\circ$ .



Process Questions:

1. What are the three different ways of proving deductively?

2. Which of the three ways is the best? Why

3. How can one reason out deductively?

4. Why is there a need to study deductive reasoning? How is it related to real-life? Cite a situation where deductive reasoning is applied.



**END OF TRANSFER**

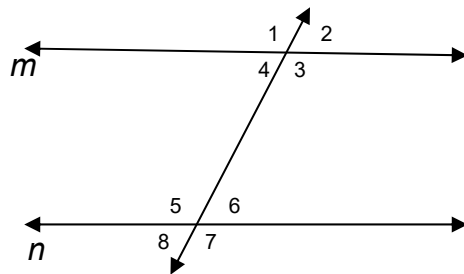
You have completed this lesson. Before you go to the next lesson, you have to answer the following lesson post-assessment.

## LESSON: POST-ASSESSMENT



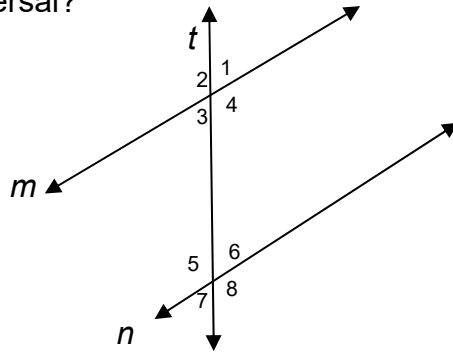
*It's now time to evaluate your learning. Click on the letter of the answer that you think best answers the questions. If you do well, you may move on to the next lesson. If your score is not at the expected level, you have to go back and take the module again.*

For numbers 1 – 4, refer to the figure below:



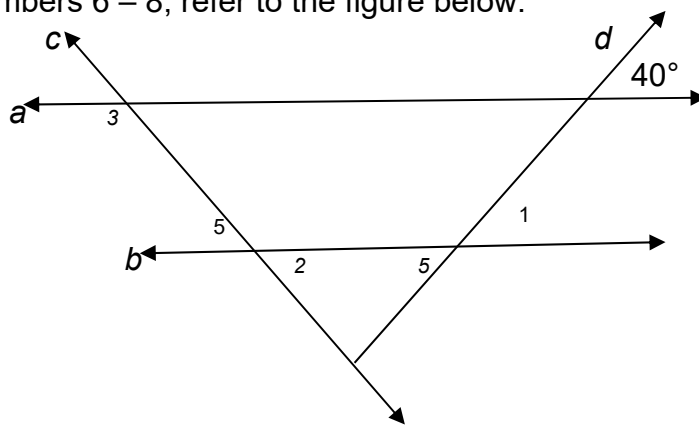
- What relationship exists between  $\angle 4$  and  $\angle 5$ , and  $\angle 3$  and  $\angle 6$ ?
  - supplementary angles
  - congruent angles
  - complementary angles
  - vertical angles
- Which of the following does not guarantee that  $m \parallel n$ ?
  - $\angle 1 \cong \angle 5$
  - $\angle 2 \cong \angle 7$
  - $\angle 3 \cong \angle 5$
  - $\angle 2 \cong \angle 8$
- Which of the following angles are congruent to  $\angle 5$ ?
  - $\angle 6$ ,  $\angle 7$  and  $\angle 8$
  - $\angle 1$ ,  $\angle 2$  and  $\angle 3$
  - $\angle 1$ ,  $\angle 3$  and  $\angle 7$
  - $\angle 3$ ,  $\angle 4$  and  $\angle 6$
- In the figure  $m \parallel n$ , if  $\angle 2 = (2x + 5)^\circ$  and  $\angle 8 = (3x - 20)^\circ$ , then what is  $m \angle 3$ ?
  - $115^\circ$
  - $120^\circ$
  - $125^\circ$
  - $130^\circ$

5. Which of the following conditions will guarantee that  $m \parallel n$  with  $t$  as the transversal?



- A.  $\angle 1$  is congruent to  $\angle 3$
- B.  $\angle 3$  and  $\angle 4$  are supplementary angles
- C.  $\angle 3$  and  $\angle 7$  are congruent angles
- D.  $\angle 1$  and  $\angle 7$  are supplementary angles

For numbers 6 – 8, refer to the figure below:

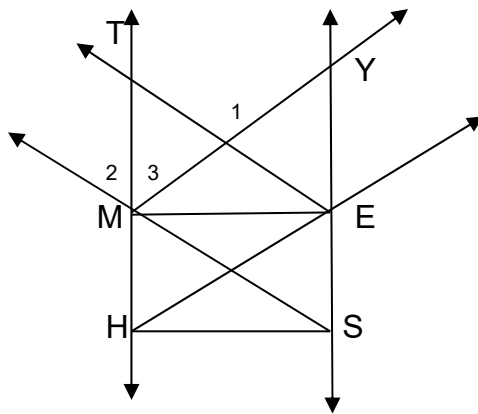


6. If  $a \parallel b$  with  $d$  as the transversal, then what is  $m \angle 1$ ?
- A.  $40^\circ$
  - B.  $80^\circ$
  - C.  $100^\circ$
  - D.  $120^\circ$



7. Given  $a \parallel b$  and  $c \perp d$ , find the  $m\angle 2$ .
- $40^\circ$
  - $50^\circ$
  - $60^\circ$
  - $80^\circ$
8. What relationship exists between  $\angle 2$  and  $\angle 3$ ?
- $\angle 5 \cong \angle 3$
  - $\angle 5$  and  $\angle 3$  are supplementary angles
  - $\angle 5$  and  $\angle 3$  form linear pair
  - $\angle 5$  and  $\angle 3$  are vertical angles

For numbers 9 and 10, refer to the figure below:



9. If  $\overline{ME} \perp \overline{TH}$  and  $\overline{ME} \perp \overline{YS}$ , then which are parallel?
- $\overline{MY} \parallel \overline{HE}$
  - $\overline{ME} \parallel \overline{HS}$
  - $\overline{TH} \parallel \overline{YS}$
  - $\overline{SM} \parallel \overline{ET}$
10. Given that  $\overline{SM} \parallel \overline{ET}$ ,  $m\angle 1 = 2x + 50$  and  $m\angle 2 + m\angle 3 = 2x + 105$ , solve for  $x$ .
- $25^\circ$
  - $35^\circ$
  - $45^\circ$
  - $55^\circ$



After having clear idea of what parallelism and perpendicularity are, let us investigate how these concepts are proved in the properties of quadrilaterals. These properties will be proved and applied in solving word problems and in proving other statements. The study of quadrilaterals is important since most of the things that can be seen around are in the shape of quadrilaterals like ceilings, tables, floors, windows, doors and other structural designs of the buildings. The following activity helps you familiarize yourself with the family of quadrilaterals.

## LESSON NO. 4.2 Parallelogram, Its Properties and Applications

### LESSON: PRE-ASSESSMENT

---



Let's find out how much you already know about this lesson. Click on the letter that you think best answers the question. Please answer all items. After taking this short test, you will see your score. Take note of the items that you were not able to correctly answer and look for the right answer as you go through this lesson.

1. Which of the following figures is a parallelogram?

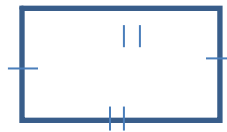
a.



b.



c.



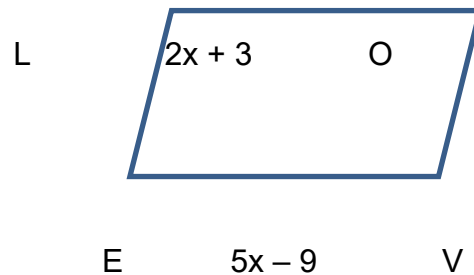
d.



2. Which of the following statements is true?

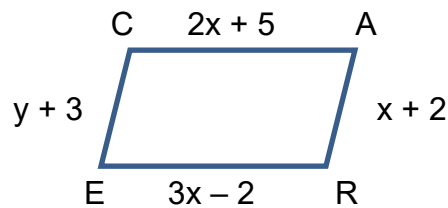
- Each diagonal of a rhombus bisects a pair of opposite angles. \*
- Diagonals of a rhombus are congruent.
- Diagonals of a rectangle are perpendicular to each other.
- Diagonals of a parallelogram are congruent and perpendicular to each other.

3. Which of the following information is enough to determine that a quadrilateral is a parallelogram?
- All four sides are congruent.
  - Diagonals are congruent.
  - The diagonals bisect each other. \*
  - Two pairs of consecutive angles are congruent.
4. Which values of  $x$  will make  $\square$  LOVE a parallelogram?



- 3 units
- 4 units
- 5 units
- 6 units

5. Which values of  $x$  and  $y$  will make  $\square$  CARE a parallelogram?



- $x = 3$  and  $y = 2$  Units
  - $x = 3$  and  $y = 4$  Units
  - $x = 7$  and  $y = 6$  Units
  - $x = 7$  and  $y = 12$  Units
6. Quadrilateral ABCD is a parallelogram. If  $m\angle A = x + 50$  and  $m\angle C = 2x + 30$ , what is  $m\angle A$ ?
- 20
  - 30
  - 40
  - 50

7. Quadrilateral ABCD is a parallelogram with diagonals  $\overline{AC}$  and  $\overline{BD}$  intersecting at E. If  $AE = x + y$ ,  $CE = 12$ ,  $DE = 2x - y$ , and  $BE = 9$ , what is  $y$ ?
- 5 Units
  - 6 Units
  - 7 Units
  - 8 Units
8. Which of the following information will make  $\blacksquare$  FAIR a parallelogram?
- $\overline{FA} \parallel \overline{RI}$ ,  $\overline{FR} \parallel \overline{AI}$
  - $\overline{FA} \parallel \overline{RI}$ ,  $\overline{FR} \cong \overline{AI}$
  - $\overline{FA} \cong \overline{RI}$ ,  $\overline{FR} \cong \overline{AI}$
  - $\overline{FA} \cong \overline{RI}$ ,  $\overline{FR} \parallel \overline{AI}$
9. The diagonals PR and QS of a rhombus intersect at O. If  $PO = 4x + 8$  and  $OR = 6x - 12$ . Find PO.
- 10
  - 18
  - 42
  - 48
10. In the parallelogram CARE,  $m\angle R = 2x - 10$  and  $m\angle C = 4(x - 20) + 6$ , what is  $m\angle E$ ?
- 32
  - 54
  - 92
  - 126



## EXPLORE

Let's start the module by doing the activities below. Always keep in mind this question:

**How can parallelism and perpendicularity of lines be established?**

### ACTIVITY 1. AGREE OR DISAGREE!

#### ANTICIPATION-REACTION GUIDE

Instruction: Read each statement under the column TOPIC and write **A** if you agree with the statement, otherwise write **D** in the first column.

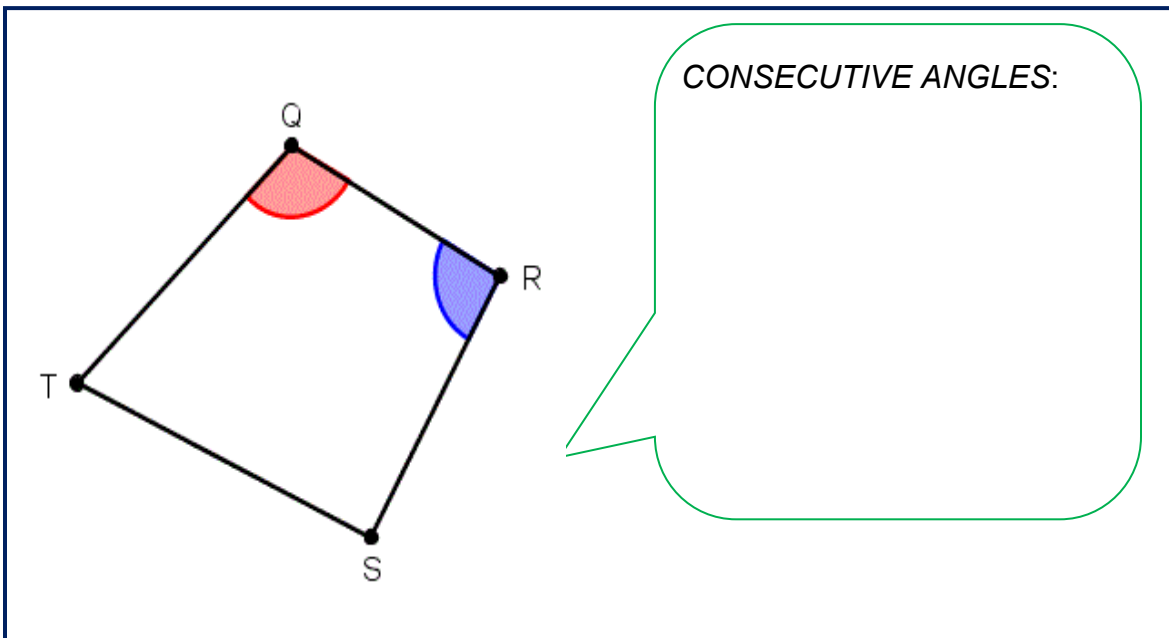
Response Before Lesson	TOPIC: Parallelism and Perpendicularity	Response After Lesson
	Lines that do not intersect are parallel lines.	
	Skew lines are coplanar.	
	Transversal is a line that intersects two or more lines.	
	Perpendicular lines are intersecting lines.	
	If two lines are parallel to a third line, then the two lines are parallel.	
	If two lines are perpendicular to the same line, then the two lines are parallel.	
	If one side of a quadrilateral is congruent to the opposite side, then the quadrilateral is a parallelogram.	
	Diagonals of a parallelogram bisect each other.	
	Diagonals of a parallelogram are congruent.	
	Diagonals of a parallelogram are perpendicular.	
	Opposite sides of a parallelogram are parallel.	
	Opposite angles of a parallelogram are congruent.	
	Consecutive angles of a parallelogram are congruent.	
	Squares are rectangles.	
	Squares are rhombi.	

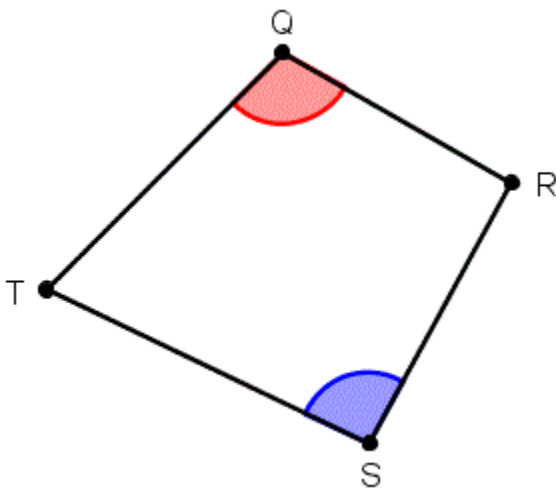
Process questions:

1. What comes in your mind while filling in the first column of the ARG?
2. How do you think parallelism and perpendicularity of lines be established?

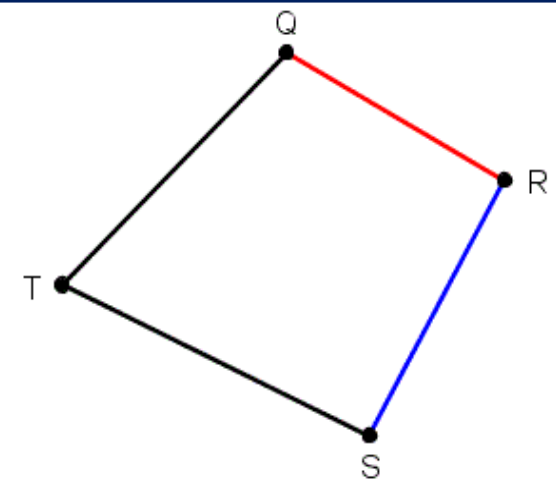
### **ACTIVITY 2. Quadrilateral Terminologies (A Recall)**

Before we start with an in-depth study of quadrilaterals, let's first recall the different terminologies to facilitate better understanding of specific relationships among sides, angles and segments. Using the illustrations provided specifically the red and blue markings, define or describe each term by writing your answer on the answer box provided.

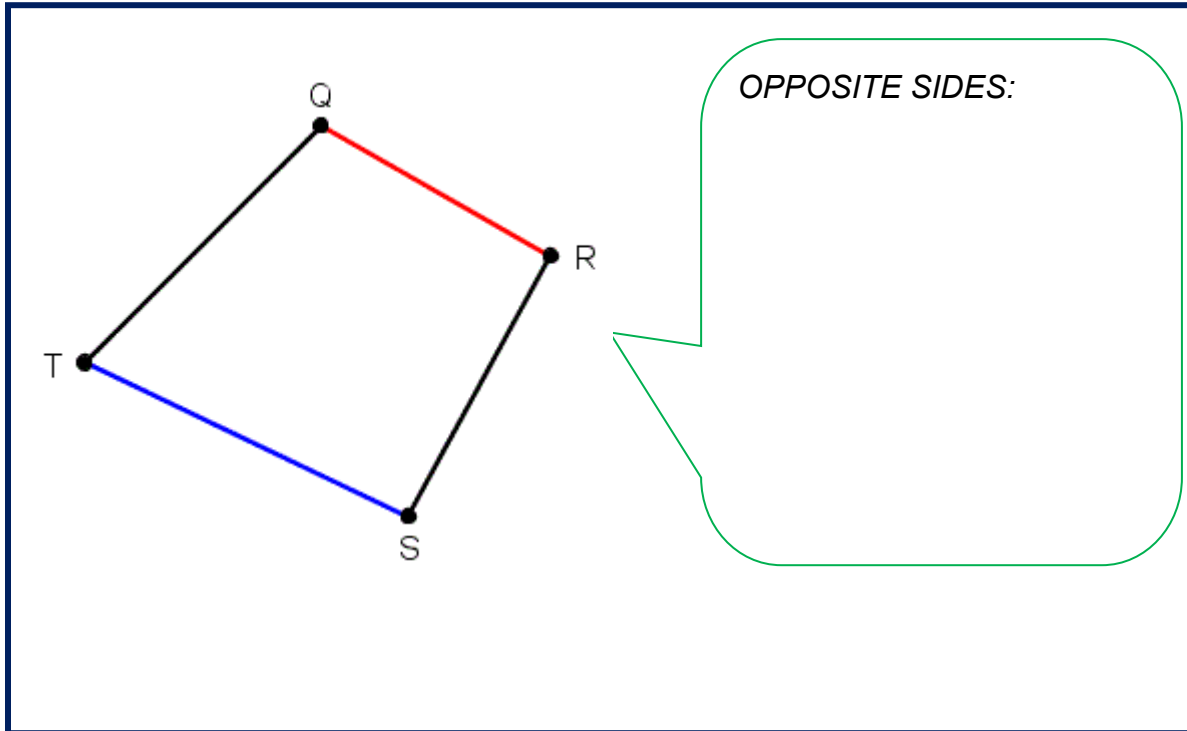




*OPPOSITE ANGLES:*

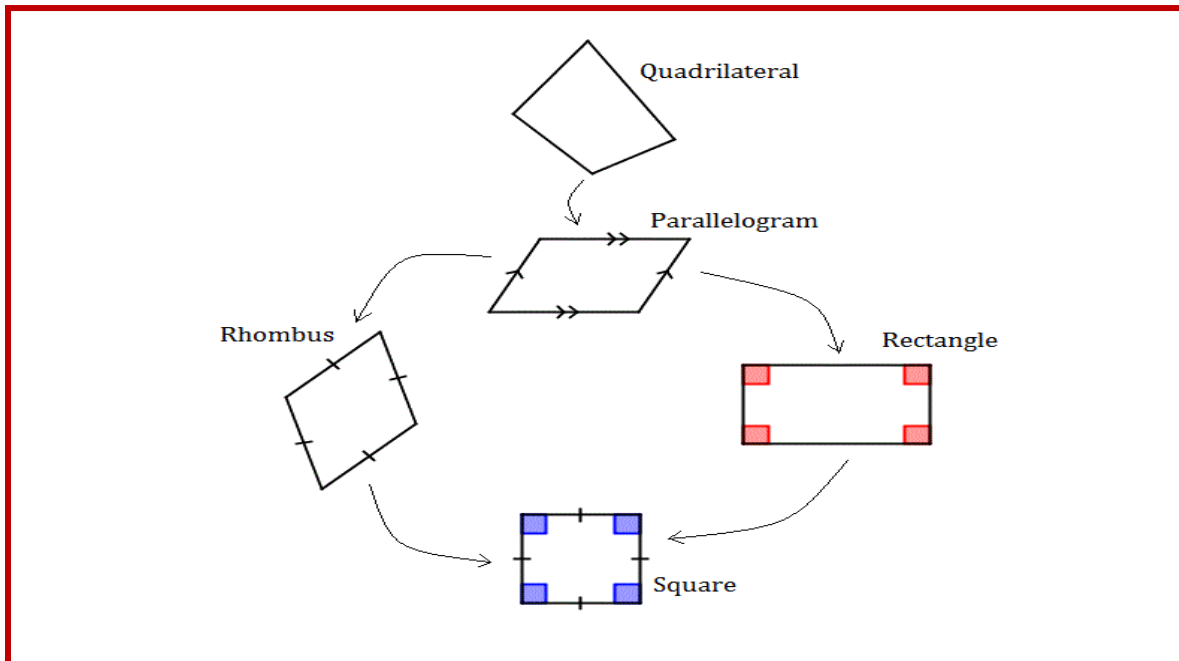


*CONSECUTIVE SIDES:*



**ACTIVITY 3. Hierarchy of Quadrilaterals**

Look at the diagram of quadrilaterals below, after which complete the table that follows by placing a check mark (✓) on the column corresponding to the appropriate properties listed.





PROPERTIES	Parallelogram	Rectangle	Rhombus	Square
Opposite sides are congruent.				
Opposite angles are congruent.				
The sum of consecutive angles is $180^\circ$				
Diagonals are congruent.				
Diagonals are perpendicular.				
Diagonals bisect each other.				

*Description: Given the illustrations of seven quadrilaterals , fill in the following table with the letter of the figure that you think has the characteristics described by each row. A figure can have two or more characteristics.*

Figures:

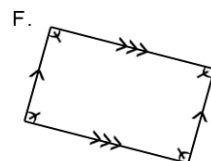
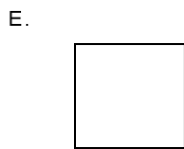
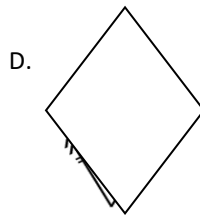
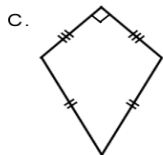
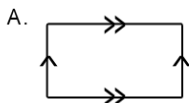


Table:

Description	Letter of the given figure/s
Has 4 Sides	
Has Exactly 1 Pair of Parallel Sides	
Has 2 Pairs of Consecutive congruent sides	
Has 4 Right Angles	
Has 4 Right Angles and 4 congruent sides	
Has 4 Congruent Sides	
Has Exactly 2 Pairs of Parallel Sides	

**Process Questions:**

1. What discovery did you have based on the activity?

2. Are the key concepts of parallelism and perpendicularity made sense?  
How can these be established?

**END OF EXPLORE:**

You just tried finding out properties of quadrilaterals. Let's now find out or investigate quadrilaterals that are parallelograms by doing the next part.



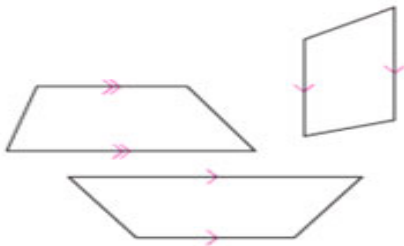
**FIRM-UP**

Your goal in this section is to learn and understand key concepts of parallelograms and discovering its properties.

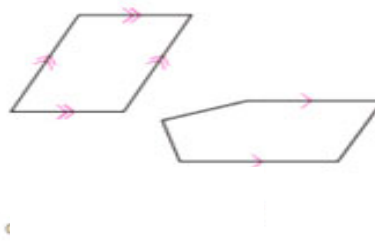
**ACTIVITY 4. Investigating Parallelograms**

**Description:** Investigating quadrilaterals. In the table below, write a good definition of each boldfaced term and then click on "save". Discuss your definitions with others in your group. Agree on a common set of definitions for your class and add them to your definitions' list. In your notebook, draw and label a figure to illustrate each definition.

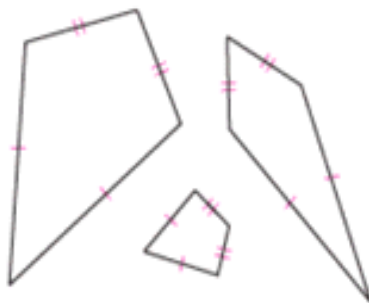
Trapezoids



Not Trapezoids



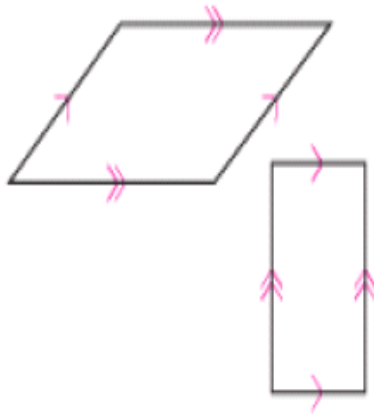
Kites



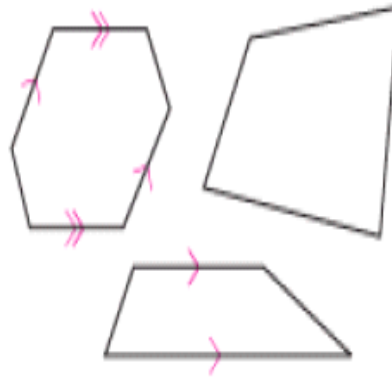
Non Kites



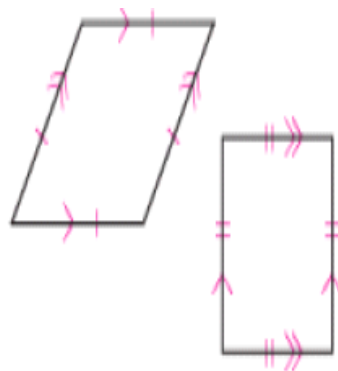
Parallelograms



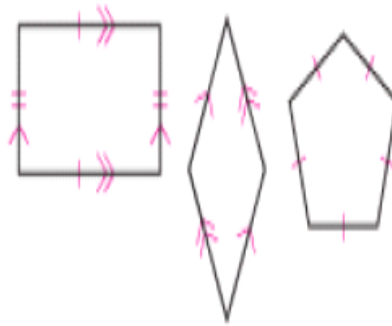
Not Parallelograms



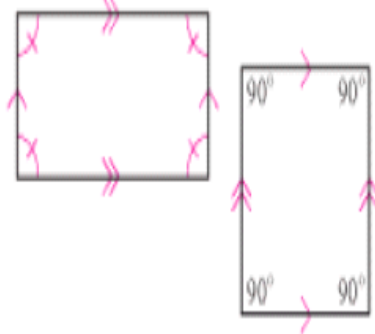
Rhombuses



Not Rhombuses



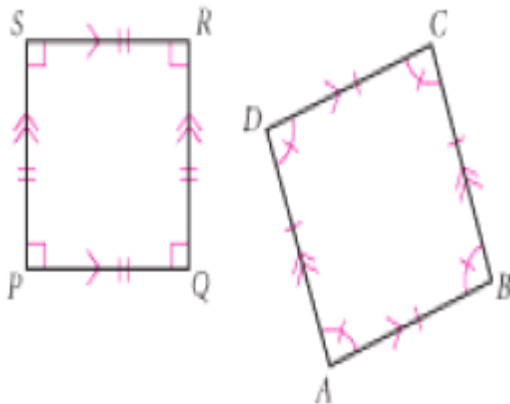
Rectangles



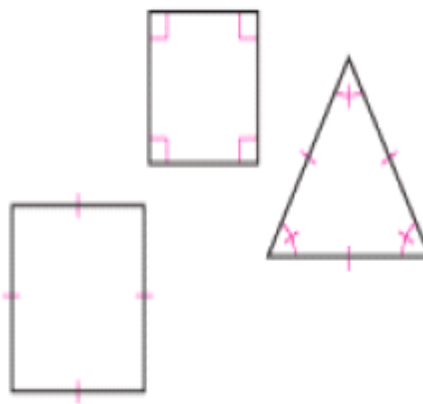
Not Rectangles



Squares



Not Squares



As you learned in the investigation, a figure that looks like a square is not a square unless it has the proper markings. Keep this in mind as you work on any exercises.

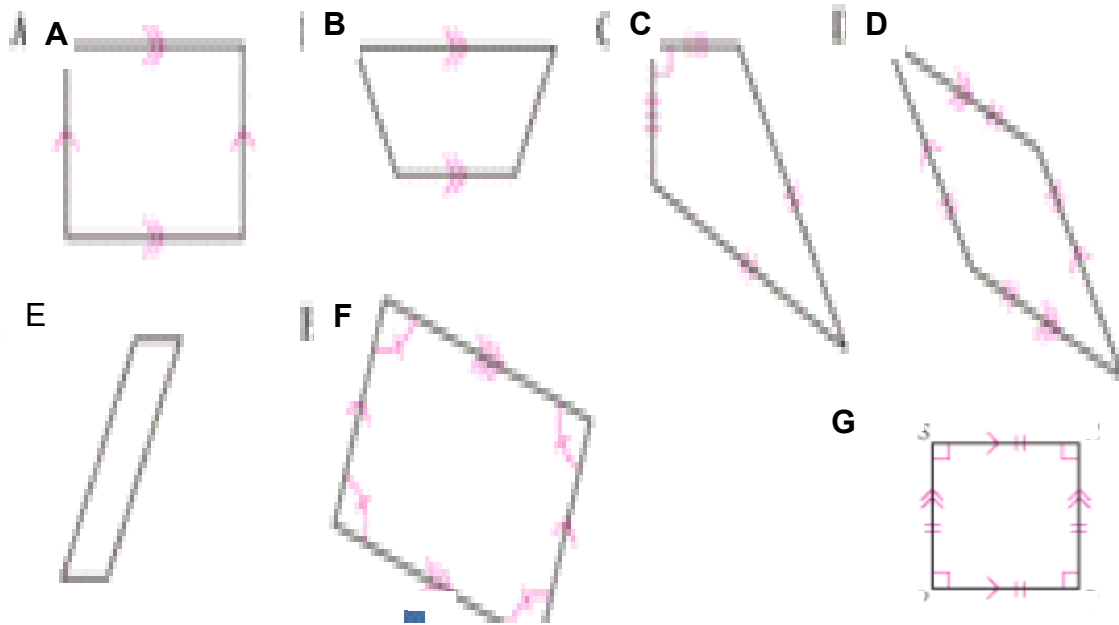
- Definitions:**
- Parallelogram**
  - Rectangle**
  - Rhombus**
  - Square**
  - Trapezoid**
  - Kite**



To further check the validity of your definitions, answer the following activities.  
(Match It and Draw It)

**ACTIVITY 5. Match It!**

Match the term on the left with its figure on the right. Write the letter of your answer in the answer box that follows. A term maybe matched in one or more figures.



Answers:

1. Trapezoid \_\_\_\_\_
2. Rhombus \_\_\_\_\_
3. Rectangle \_\_\_\_\_
4. Kite \_\_\_\_\_
5. Squares \_\_\_\_\_
6. Parallelogram \_\_\_\_\_

**ACTIVITY 6. Draw It!**

---

Sketch and label the figure in the answer box below. Put appropriate mark/s in the figures.

1. Trapezoid  $ZOID$  with  $ZO \parallel ID$
2. Kite  $BENF$  with  $BE = EN$
3. Rhombus  $EQUL$  with diagonals  $EU$  and  $QL$  intersecting at  $A$
4. Rectangle  $RGHT$  with diagonals  $RH$  and  $GT$  intersecting at  $I$

Answer Box:

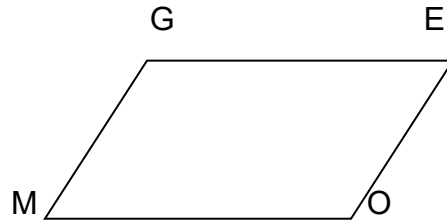
Process Questions:

1. What guarantees parallelism of lines?
2. What guarantees congruency of lines?
3. What guarantees perpendicularity of lines?
4. How are quadrilaterals named?
5. How do you distinguish one quadrilateral from the other?



**ACTIVITY 7. Discovering the Properties of Parallelograms**

To investigate on properties of parallelograms regarding the sides, angles and diagonals refer to the given illustration below and the guide questions Provided.



1.  $\overline{GE} \parallel \overline{MO}$  and  $\overline{EO}$  is the transversal. What can you conclude about  $\angle E$  and  $\angle O$ ?

\_\_\_\_\_

If  $\overline{GM}$  is the transversal, what can you conclude about  $\angle G$  and  $\angle M$ ?

\_\_\_\_\_

2. Draw  $\overline{ME}$ . What do you call this segment in terms of the parallelogram?

\_\_\_\_\_

3. If  $\overline{GE} \parallel \overline{MO}$ , what pairs of angles are congruent? Why?

\_\_\_\_\_

4. Can you find two triangles that are congruent? What is the proper correspondence? What congruence postulate was used?

\_\_\_\_\_

5. What corresponding parts are congruent?

\_\_\_\_\_

6. If  $\overline{GM} \cong \overline{EO}$ , what are these sides with respect to the parallelogram? How about  $\overline{GE}$  and  $\overline{MO}$ ?  $\angle G$  and  $\angle O$ ?

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

7. Draw  $\overline{GO}$  intersecting  $\overline{EM}$  at  $Y$ . Compare the length of  $\overline{GY}$  to  $\overline{YO}$  and  $\overline{MY}$  to  $\overline{YE}$ . How are these lengths related?

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Summarize your observations by writing down the properties of parallelogram that you've discovered by completing the table below:

	If a quadrilateral is a parallelogram, then ...	Illustration with proper markings
Sides		
Angles		
Diagonals		

You may visit this link to check your conclusions:

<http://www.regentsprep.org/Regents/math/geometry/GP9/JavaParallel.htm>

Process Questions:

1. What should you keep in mind while doing this exploration activity?

2. Are these observations true to all parallelograms?

3. Are these observations can be proved deductively?



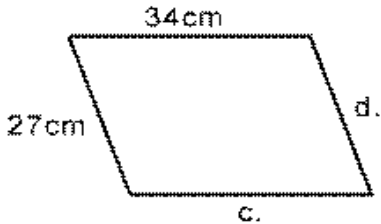
**Process Questions:**

1. What should you keep in mind while doing this exploration activity?
2. What makes these parallelograms special?
3. Are these observations true to all special parallelograms (rectangle, square, & rhombus)?
4. Are these observations can be proved deductively?
5. Are these findings helpful in establishing parallelism and perpendicularity of lines?

**ACTIVITY 9. Find It!**

Each figure below is a parallelogram. Use your observations in the previous activity to answer the items below.

1.



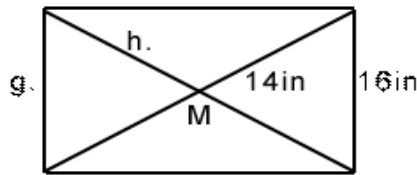
c =  
d =

2.



a =  
b =

3.

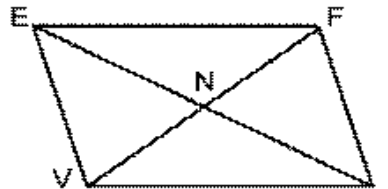


g =  
h =

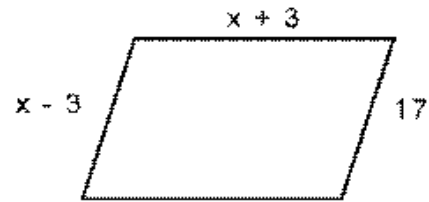
4.

VF = 36m  
EF = 24m  
EI = 42m

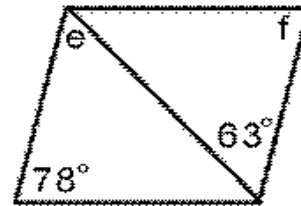
What is the perimeter of  $\triangle NVI$ ?



5. What is the perimeter?



6.



**ACTIVITY 10. Check It**

Direction: In the table below, place a check mark (/) in the boxes if you think the quadrilateral listed along the top row has the properties listed in the left column.

	Parallelogram	Rectangle	Rhombus	Square
Opposite sides are $\cong$ .				
Opposite $\angle$ s are $\cong$ .				
Consecutive $\angle$ s sum = $180^\circ$ .				
Diagonals are $\cong$ .				
Diagonals are $\perp$ .				
Diagonals bisect each other.				

**PROCESS QUESTIONS:**

1. What property/properties is/are common to rectangles, rhombi and squares?

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2. What makes a rectangle different from a rhombus? A rectangle from a square? A rhombus from a square?

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For additional information regarding properties of rectangles, squares and rhombuses, kindly visit these links:

[http://www.wyzant.com/Help/Math/Geometry/Quadrilaterals/Rectangles\\_Rhombuses\\_Squares.aspx](http://www.wyzant.com/Help/Math/Geometry/Quadrilaterals/Rectangles_Rhombuses_Squares.aspx)

<http://www.onlinemathlearning.com/quadrilaterals.html>

After visiting the given link, try answering the practice exercises on the link given below to further enrich your knowledge and skills regarding properties of parallelogram.

<http://www.onlinemathlearning.com/quadrilateral-properties.html>

**End of FIRM UP:**

In this section, the discussion was about properties of parallelograms. Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion?

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.



**DEEPEN**

Your goal in this section is to take a closer look at some aspects of the topic. This section gives emphasis on proving through deductive reasoning involving properties of parallelogram. I hope that you are now ready to answer the exercises given in this section to intensify or make deeper application of what has been learned.

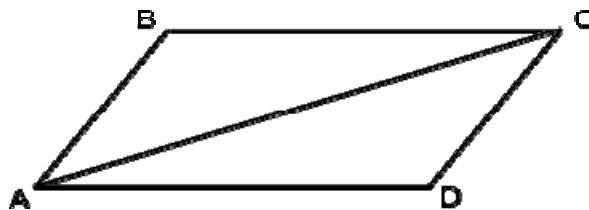
Discussion: **Proving Parallelograms**

Below are examples on proving properties of parallelogram:

Property: A diagonal of a parallelogram forms two congruent triangles.

**Given:** Parallelogram ABCD with diagonal AC

**Prove:**  $\triangle ABC \cong \triangle CDA$





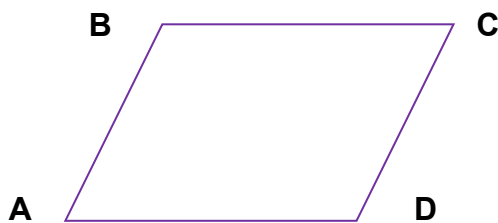
**Proof:**

<b>Statements</b>	<b>Reasons</b>
1. Parallelogram ABCD with diagonal AC.	Given
2. $AB \parallel DC$ and $BC \parallel AD$	Definition of a parallelogram
3. $\angle BAC \cong \angle DCA$	Parallel-Alternate Interior Angle Postulate
4. $\overline{AC} \cong \overline{CA}$	Reflexive Property
5. $\angle BCA \cong \angle DAC$	Parallel-Alternate Interior Angle Postulate
6. $\triangle ABC \cong \triangle CDA$	ASA Postulate

Property: In a parallelogram, any two consecutive angles are supplementary.

**Given: Parallelogram ABCD**

**Prove:  $\angle A$  and  $\angle B$  are supplementary**

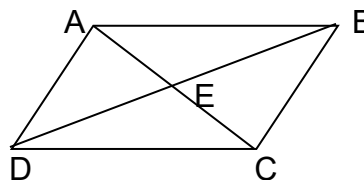


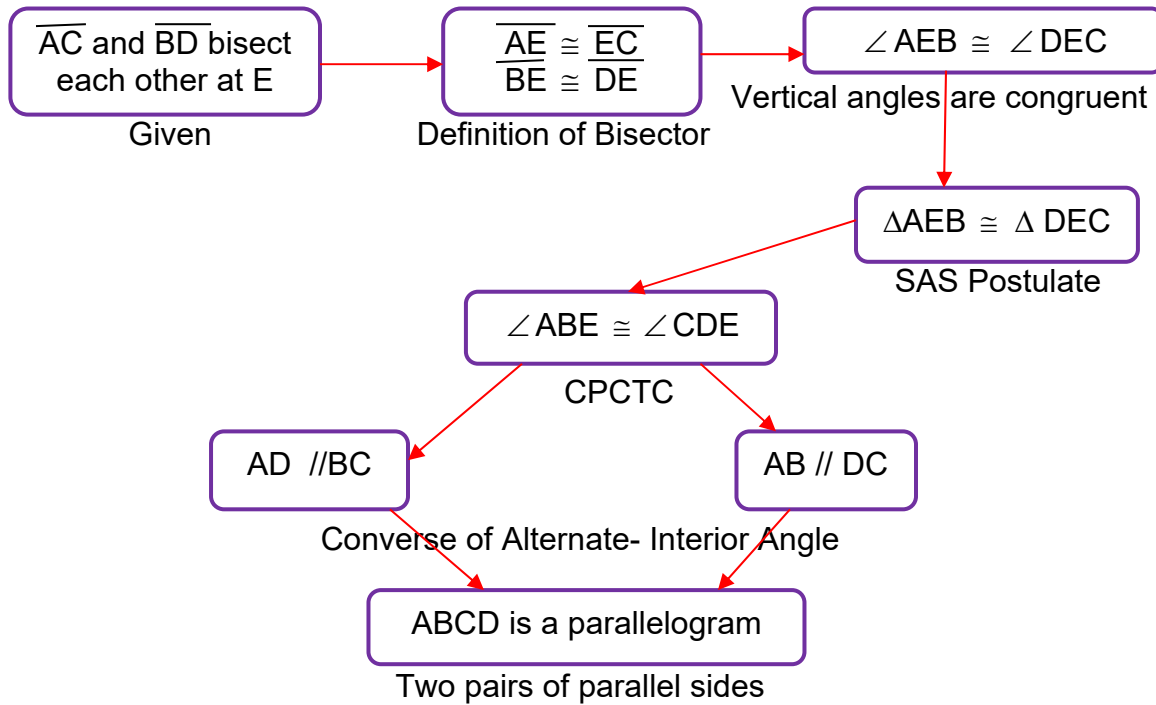
**Proof:**

<b>Statements</b>	<b>Reasons</b>
1. Parallelogram ABCD	Given
2. $BC \parallel AD$	Definition of a parallelogram
3. $\angle A$ and $\angle B$ are supplementary	Parallel- Interior Angles-Same Side Theorem

Given:  $\overline{AC}$  and  $\overline{BD}$  bisect each other at E.

Prove: ABCD is a parallelogram.



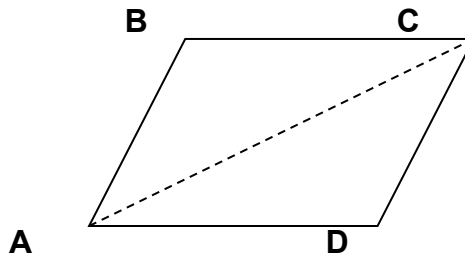


You have learned properties of parallelograms and proved some of these deductively. The following proof is one of the conditions guaranteeing that a quadrilateral is a parallelogram. Fill in the missing part of the proof.

**ACTIVITY 11. Complete My Proof!**

Given: Quadrilateral ABCD with  $\overline{AB} \parallel \overline{CD}$  and  $\overline{BC} \parallel \overline{DA}$

Prove: ABCD is a parallelogram



**Proof:**

<b>Statements</b>	<b>Reasons</b>
1.	Given
2. Draw $\overline{AC}$	
3.	Reflexive Property
4. $\triangle ABC \cong \triangle DCA$	
5.	CPCTC
6. $\overline{BC} \parallel \overline{DA}$	
7.	CPCTC
8. $\overline{AB} \parallel \overline{DC}$	
	Definition of parallelogram

Process Questions:

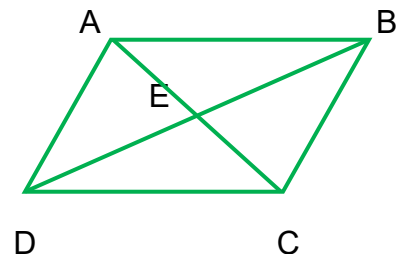
1. What guarantee/s that a quadrilateral is a parallelogram?
2. How deductive reasoning does affects your thinking skills?

**ACTIVITY 12. Prove Some More ...Okay!**

To strengthen your skill in proving deductively, kindly provide a complete proof for the given problem below. You may use a two-column poof or a flowchart.

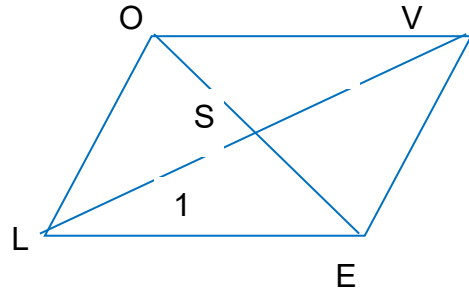
1. Given:  
 $\overline{AC} \perp \overline{BD}$

Prove: ABCD is a parallelogram



2. Given:  
Parallelogram LOVE with diagonals  $\overline{OE}$  and  $\overline{LV}$

Prove:  $\overline{OE}$  and  $\overline{LV}$  bisect each other.



The following websites will expose you more on some proofs and conditions needed to prove that a quadrilateral is a parallelogram:

<http://www.youtube.com/watch?v=9fxmt-WU86U>

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is parallelogram.

<http://www.youtube.com/watch?v=9fxmt-WU86U>

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is parallelogram.

**ACTIVITY 13. Take the Challenge....**

To strengthen your skill in proving deductively, kindly provide a complete proof and illustration to the given problem below. The use of flowchart is highly encouraged.

Draw  $\square$  KITE and diagonal  $\overline{IE}$ . Bisect  $\angle K$  and  $\angle T$ , with the bisectors meeting  $\overline{IE}$  at points A and O, respectively. Draw  $\overline{OK}$  and  $\overline{AT}$ . What can you conclude about quadrilateral TAKO?

ILLUSTRATION:

PROOF:

Process Question:




1. How does deductive reasoning affects your thinking skills?

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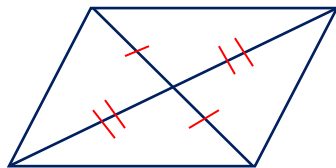


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**ACTIVITY 14. Quiz No. 2 (FORMATIVE ASSESSMENT)**

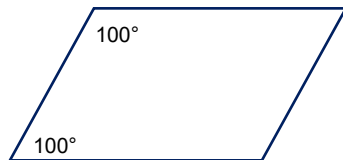
I. Study the markings on the given figures and shade  if it is a parallelogram and  if it is not. Your answer is  state the definition or theorem that justifies your answer.

1.



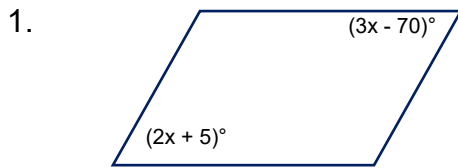

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2.

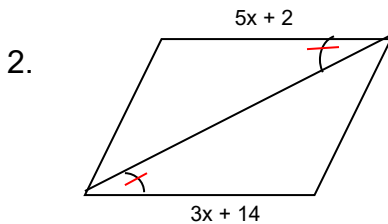



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II. At what value of  $x$  will make each quadrilateral a parallelogram?





Solution:



Solution:

III. Show a complete proof:

1. Given:  NICE,  $NI \parallel CE$ ,  $NI \cong CE$   
Prove:  NICE is a parallelogram

Proof:

Fill up the last column of the anticipation reaction guide. Compare your answers with your answer in the earlier part of the module and check if there is an improvement in your score.

**ACTIVITY 15. A R G! (A revisit )**

**ANTICIPATION-REACTION GUIDE**

Instruction: Respond to each statement below. In the last column, write **A** if you agree with the statement and write **D** if you disagree.

Response Before Lesson	TOPIC: Parallelism and Perpendicularity	Response After Lesson
	Lines that do not intersect are parallel lines.	
	Skew lines are coplanar.	
	Transversal is a line that intersects two or more lines.	
	Perpendicular lines are intersecting lines.	
	If two lines are parallel to a third line, then the two lines are parallel.	
	If two lines are perpendicular to the same line, then the two lines are parallel.	

	If one side of a quadrilateral is congruent to the opposite side, then the quadrilateral is a parallelogram.	
	Diagonals of a parallelogram bisect each other.	
	Diagonals of a parallelogram are congruent.	
	Diagonals of a parallelogram are perpendicular.	
	Opposite sides of a parallelogram are parallel.	
	Opposite angles of a parallelogram are congruent.	
	Consecutive angles of a parallelogram are congruent.	
	Squares are rectangles.	
	Squares are rhombi.	

Process Question:

1. How are the parallelism and perpendicularity of lines established in different properties of parallelograms?

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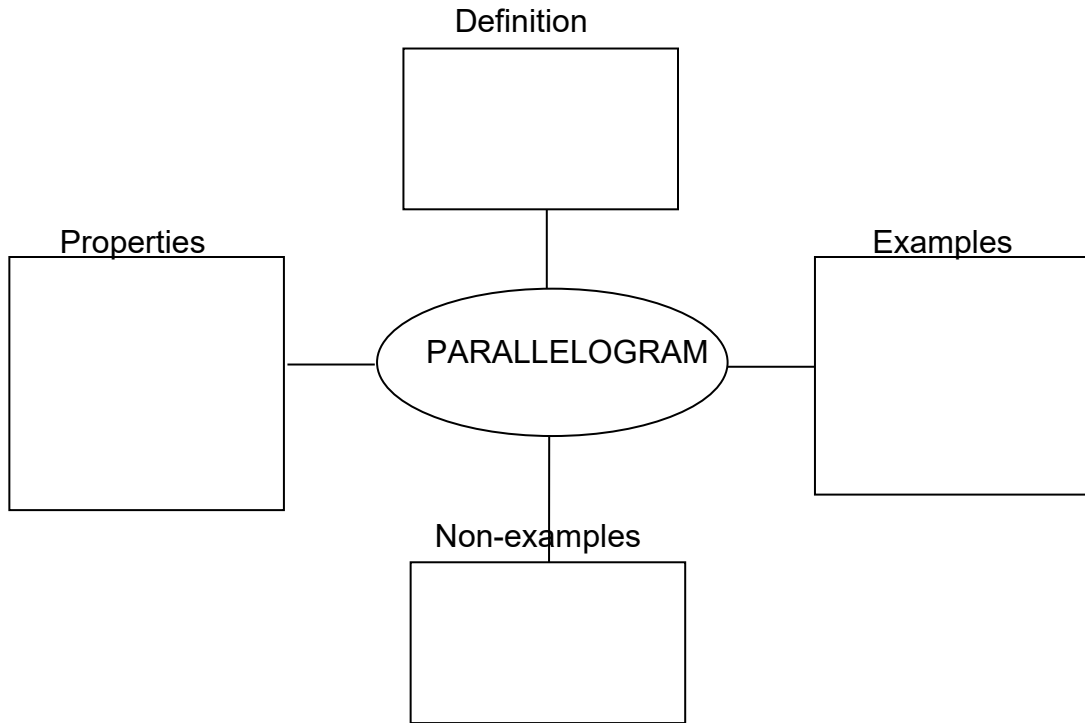
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**ACTIVITY 16. CONCEPT MAPPING:**

Reference:

<http://www.glencoe.com/sec/teachingtoday/downloads/pdf/ReadingWritingMathClass.pdf>

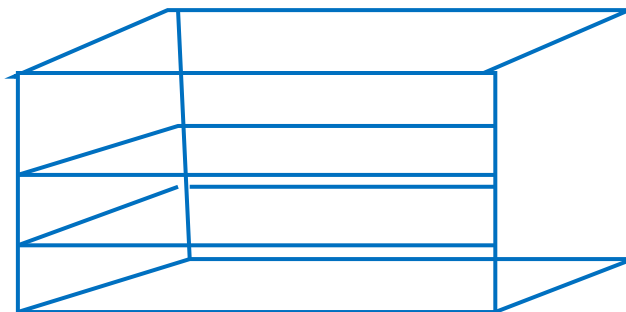
Summarize the important concepts about parallelogram by completing the concept map below. You may do it in wise mapping by clicking this site: [www.wisemapping.com](http://www.wisemapping.com)



**ACTIVITY 17. Design It!**

You are working in a furniture shop as designer. One day your immediate supervisor asked you to make a design of a wooden shoe rack for a new client who is a well-known artist in the film industry. In as much as you don't want to disappoint your boss, you immediately think of the design and try to research on the different designs available on internet.

Below is your design:





Process Questions:

1. Based on you design, how will you ensure that the divisions of the shoe rack are parallel? Describe the different ways to guarantee that the divisions are parallel.

A large, empty rectangular box with a red border, intended for the student's answer to question 1. The box has a small red tab-like shape at the bottom right corner.

2. Why is there a need to ensure parallelism on the divisions? What would happen if the divisions are not parallel?

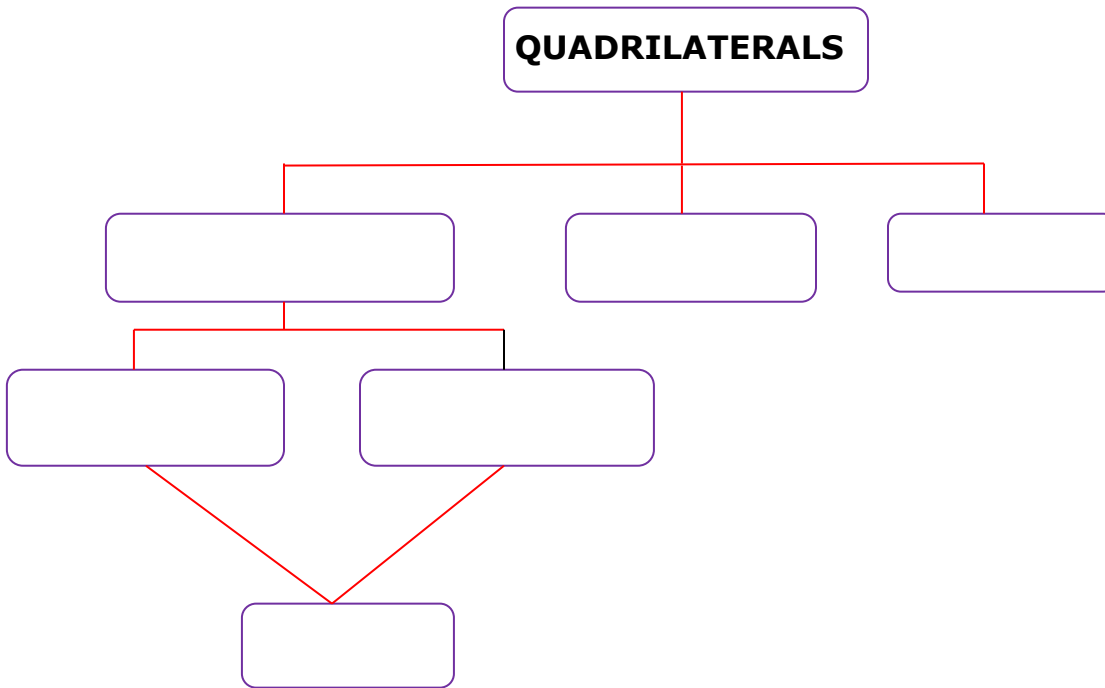
A large, empty rectangular box with a red border, intended for the student's answer to question 2. The box has a small red tab-like shape at the bottom right corner.

3. How should the sides be positioned in relation to the base of the shoe rack? Does positioning of the sides in relation to the base matters?

A large, empty rectangular box with a red border, intended for the student's answer to question 3. The box has a small red tab-like shape at the bottom right corner.

**ACTIVITY 18. Graphic Organizer**

Illustrate the family of quadrilaterals by filling in the concept map below.



**ACTIVITY 19. SUMMATIVE TEST: Long Test**

**A. LEVEL 1!** Determine whether the statement is ALWAYS TRUE (AT), SOMETIMES TRUE (ST) OR NEVER TRUE (NT).

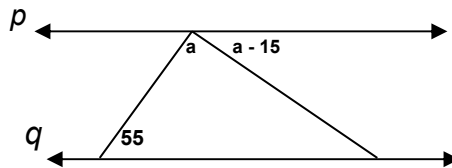
- \_\_\_\_\_ 1. Parallelograms have diagonals that bisect each other.
- \_\_\_\_\_ 2. A parallelogram with diagonals that are congruent is a square.
- \_\_\_\_\_ 3. A parallelogram has diagonals that are perpendicular.
- \_\_\_\_\_ 4. The adjacent sides of a parallelogram are congruent.
- \_\_\_\_\_ 5. A rhombus has congruent sides and congruent angles.
- \_\_\_\_\_ 6. The diagonals of a square are perpendicular bisectors of each other.
- \_\_\_\_\_ 7. If two lines are perpendicular to the same line, then those lines are perpendicular.
- \_\_\_\_\_ 8. A transversal intersects two or more other lines at a single point.
- \_\_\_\_\_ 9. Lines that do not intersect are parallel.

\_\_\_\_\_ 10. Intersecting lines that form  $90^\circ$ -angle are perpendicular lines.

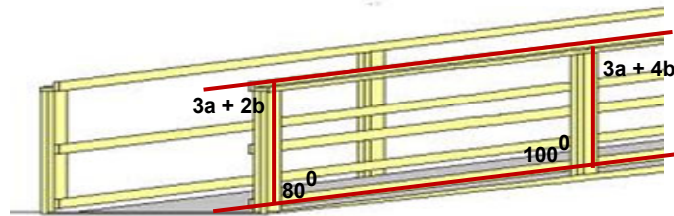
**B. LEVEL UP!** Answer the following:

1. Draw two lines and a transversal such that  $\angle 1$  and  $\angle 3$  are corresponding angles,  $\angle 1$  and  $\angle 2$  are alternate interior angles and  $\angle 3$  and  $\angle 4$  are alternate exterior angles. What type of angle pair is  $\angle 2$  and  $\angle 4$ ?

2. What value of  $a$  will make lines  $p$  and  $q$  parallel?

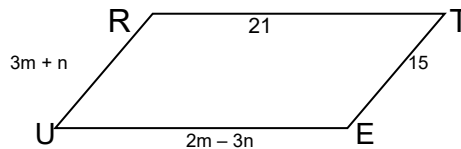


The railing of a wheel chair ramp is parallel to the ramp. Find the value of  $a$  and  $b$  in the diagram.



[http://t2.gstatic.com/images?q=tbn:ANd9GcQ7AC8KOpnReRVmCMYqZR7gpNrpFXtPacXAhfpFYML8kkGg\\_gr](http://t2.gstatic.com/images?q=tbn:ANd9GcQ7AC8KOpnReRVmCMYqZR7gpNrpFXtPacXAhfpFYML8kkGg_gr)

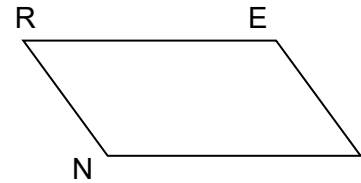
3. What values of  $m$  and  $n$  make quadrilateral TRUE a parallelogram?



- C. THE HIGHEST LEVEL!** Present a proof (in any way you want) for the following problem.

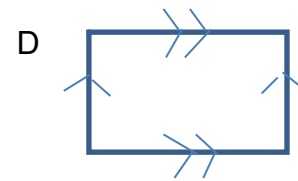
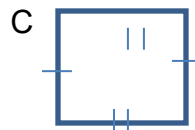
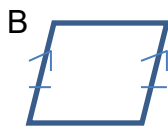
Given:  $\overline{RE} \parallel \overline{IN}$   
 $\overline{RE} \cong \overline{IN}$

Prove: REIN is a parallelogram

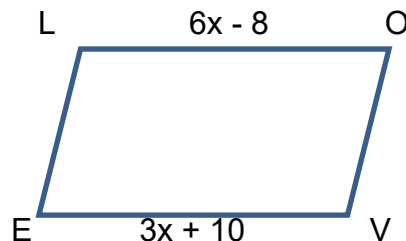


- D. MULTIPLE CHOICE.** Write the letter of the answer.

1. Which of the following figures is a parallelogram?

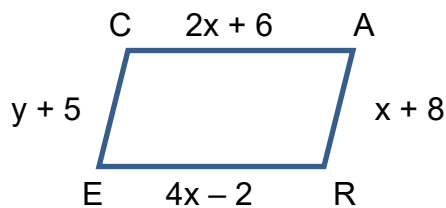


2. Which of the following statements is true?  
 A. Diagonals of a rhombus are congruent.  
 B. Diagonals of a rectangle are perpendicular to each other.  
 C. Diagonals of a parallelogram are congruent and perpendicular to each other.  
 D. In a parallelogram, any two opposite angles are congruent.
3. Which of the following information is enough to determine that a quadrilateral is a parallelogram?  
 A. The diagonals bisect each other.  
 B. Diagonals are congruent.  
 C. Two pairs of consecutive angles are congruent.  
 D. All four sides are congruent.
4. Which values of  $x$  will make  $\square$  LOVE a parallelogram?



- A. 5 units      B. 6 units      C. 7 units      D. 8 units

5. Which values of  $x$  and  $y$  will make  $\blacksquare$  CARE a parallelogram?



- A.  $x = 2$  and  $y = 5$  units  
 B.  $x = 2$  and  $y = 3$  units  
 C.  $x = 3$  and  $y = 6$  units  
 D.  $x = 3$  and  $y = 5$  units
6. Quadrilateral ABCD is a parallelogram. If  $m\angle A = 3x + 45$  and  $m\angle C = 2x + 80$ , what is  $m\angle A$ ?
- A. 120  
 B. 130  
 C. 140  
 D. 150
7. Quadrilateral ABCD is a parallelogram with diagonals  $\overline{AC}$  and  $\overline{BD}$  intersecting at E. If  $AE = 2x + y$ ,  $CE = 24$ ,  $DE = 2x - y$ , and  $BE = 14$ , what is  $y$ ?
- A. 5 units  
 B. 6 units  
 C. 7 units  
 D. 8 units
8. Which of the following information will make  $\blacksquare$  FAIR a parallelogram?
- A.  $\overline{FA} \parallel \overline{RI}$ ,  $\overline{FR} \cong \overline{AI}$   
 B.  $\overline{FA} \cong \overline{RI}$ ,  $\overline{FA} \parallel \overline{RI}$   
 C.  $\overline{FA} \cong \overline{RI}$ ,  $\overline{FR} \parallel \overline{AI}$   
 D.  $\overline{FA} \parallel \overline{RI}$ ,  $\overline{FR} \parallel \overline{AI}$
9. The diagonals PR and QS of a rhombus intersect at O. If  $PO = 5x + 8$  and  $OR = 2x + 24$ . Find PO.
- A. 8  
 B. 40  
 C. 48

- D. 50
10. In the parallelogram CORE,  $m\angle R = 3x - 12$  and  $m\angle C = 7(x - 21) + 7$ , what is  $m\angle E$ ?
- A. 32
  - B. 84
  - C. 96
  - D. 102

**End of DEEPEN:**

In this section, the discussion was about parallelism, perpendicularity and parallelograms

What new realizations do you have about the topic? What new connections have you made for yourself?

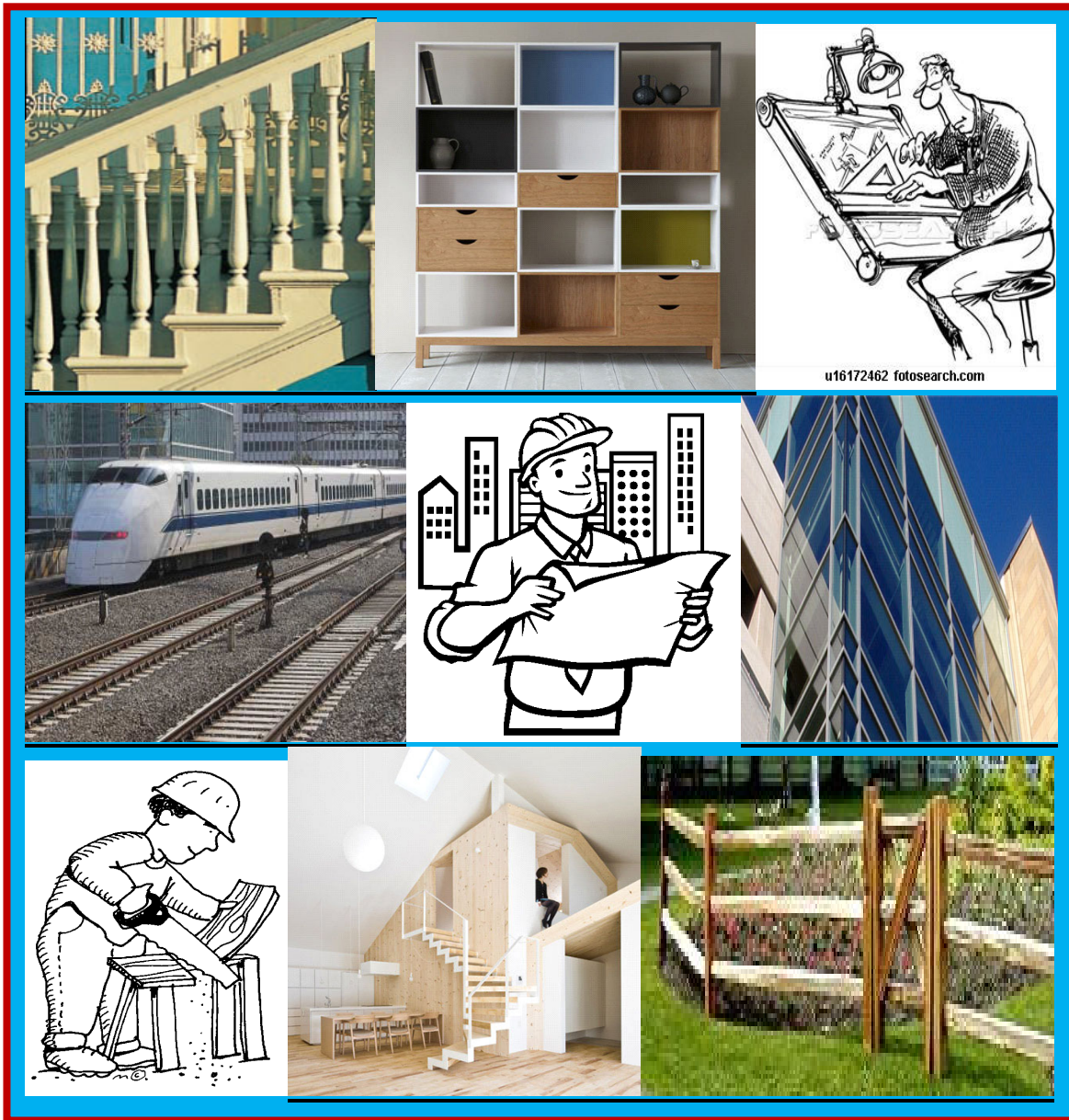
Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.



**TRANSFER**

Your goal in this section is apply your learning to real life situations. You will be given a practical task which will demonstrate your understanding.

**ACTIVITY 20. Picture Analysis**



The pictures above illustrate the concepts of parallelism and perpendicularity in real-life situations. Professionals such as engineers and architects see how useful parallelism and perpendicularity in their works. Even ordinary workers such as carpenters also apply parallelism and perpendicularity in their works. Buildings, furniture, fence and railroads are the common structures that apply these concepts.

PROCESS QUESTIONS:

1. How do you think an engineer, architect or carpenter ensures the application of parallelism and perpendicularity in their works?

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2. How useful it is to ensure that there is parallelism or perpendicularity in their work/product?

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3. What do you think would happen if they fail to ensure parallelism or perpendicularity in the structures shown above?

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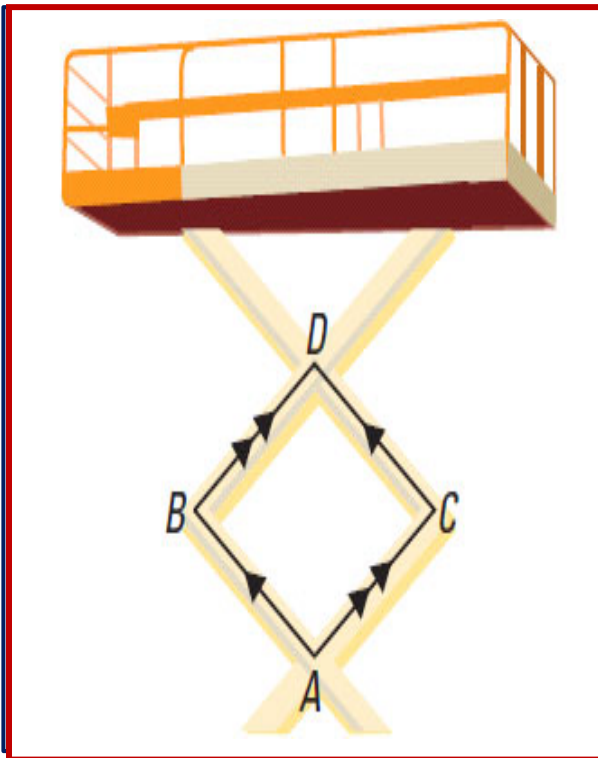
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**ACTIVITY 21. Situational Analysis**



<http://acr.keypress.com/KeyPressPortalV3.0/Viewer/Lesson.htm>





<http://www.nexuslearning.net/books/ml-geometry/Chapter6/ML%20Geometry%20->

[2%20Properties%20of%20Parallelograms.pdf](#)

### **ACTIVITY 22. Designers Forum!**

Scenario:

The Student Council of a school had a fund raising activity in order to put up a book case or shelf for the Student Council Office. You are a carpenter who is tasked to create a model of a book case/shelf using Euclidean tools: compass and a straightedge and present it to the council adviser. Your output will be evaluated according to the following criteria: stability, accuracy, creativity and mathematical reasoning.

G –Putting up a book case/shelf for the student council office

R – You are a carpenter who uses Euclidean tools (compass and straight edge)

A –The council Adviser

S –The student Council of the school is having a fund raising project for Council’s Office book case/shelf

P –Model of a book case/shelf

S – Stability, Accuracy, Creativity, and Mathematical Reasoning

**ACTIVITY 23. Lesson Closure – Reflection Organizer**

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You have accomplished the task successfully. You learned concepts in this unit which are very important for the next units. To end this unit meaningfully and to welcome the next unit, I want you to accomplish the next activity.

In this unit I learned about

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These concepts can be used in

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I understand that

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These are important because

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I can use the concepts of parallelism and perpendicularity in my life by

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**End of TRANSFER:**

In this section, your task was to create a model of a book case or shelf.

How did you find the performance task? How did the task help you see the real world use of the topic?

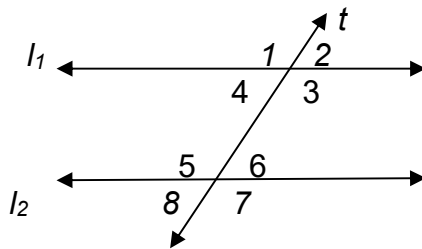
You have completed this lesson. Before you go to the next lesson, you have to answer the following post-assessment.

**POST-ASSESSMENT:**

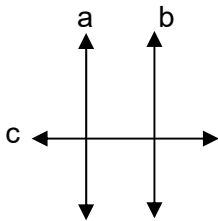
It's now time to evaluate your learning. Click on the letter of the answer that you think best answers the question. Your score will only appear after you answer all items. If you do well, you may move on to the next module. If your score is not at the expected level, you have to go back and take the module again.

**Do It Now....**

- (A) 1. Using the figure below, if  $l_1 \parallel l_2$  and  $t$  is a transversal, then which of the following is true about the measures of  $\angle 4$  and  $\angle 6$  ?



- A. The sum of the measures of  $\angle 4$  and  $\angle 6$  is  $180^\circ$ .  
 B. The measure of  $\angle 4$  is equal to the measure of  $\angle 6$ .  
 C. The measure of  $\angle 4$  is greater than the  $\angle 6$ .  
 D. The measure of  $\angle 4$  is less than the measure of  $\angle 6$ .
- (A) 2. Which of the following statements is *true*?
- A. A rhombus is a square.  
 B. A diagonal divides a square into two isosceles right triangles.  
 C. A diagonal divides a square into two congruent equilateral triangles.  
 D. A rectangle is a square.
- (A) 3. What theorem proves the following?



If  $a \perp c$  and  $b \perp c$ , then  $a \parallel b$ .

- A. Given a line and a point on the line, there is only one line through the given point that is perpendicular to the given line.  
 B. In a plane, if two lines are perpendicular to the same line, then the two lines are parallel.  
 C. Two lines are parallel if they do not intersect.

D. Two lines are perpendicular if they intersect at right angles.

(A) 4. Lines  $m$  and  $n$  are cut by transversal  $q$ . At what value of  $x$  will make  $m \parallel n$ , given that  $\angle 1$  and  $\angle 4$  are corresponding angles and  $m\angle 1 = 5x - 11$  and  $m\angle 4 = 3x + 5$ ?

- A. 6
- B. 8
- C. 10
- D. 12

(A) 5.  $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$  at point E. If  $m\angle BEC = 2x + 3$ , then what is the value of  $x$ ?

- A. 43.5
- B. 55
- C. 77.5
- D. 90

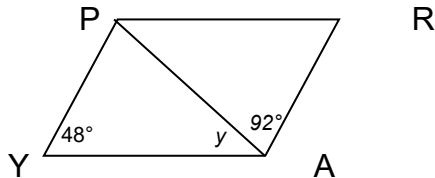
(A) 6.  $\square$ ALYS is a parallelogram. If  $m\angle A$  is twice the measure of  $m\angle L$ , find the measure of  $\angle Y$ .

- A.  $60^\circ$
- B.  $90^\circ$
- C.  $120^\circ$
- D.  $150^\circ$

(A) 7.  $\angle 1$  and  $\angle 2$  are non-adjacent angles on the same side of a transversal, one is exterior and the other is interior. If  $m\angle 1 = 2x + 25$  and  $m\angle 2 = 3x + 15$ , find the  $m\angle 2$ .

- D.  $28^\circ$
- E.  $56^\circ$
- F.  $81^\circ$
- G.  $99^\circ$

(A) 8.  $\square$ PRAY is a parallelogram.

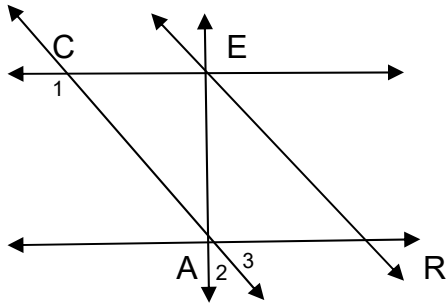


What is the measure of  $\angle y$ ?

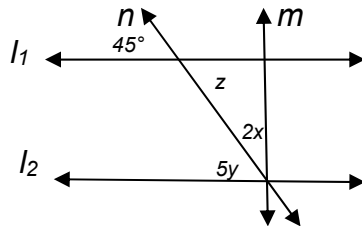
- E.  $40^\circ$
- F.  $48^\circ$
- G.  $92^\circ$
- H.  $94^\circ$



- (M) 9. In the figure below,  $AR \parallel CE$ ,  $CA \parallel RE$  and  $EA \perp RA$ . If  $m\angle 1 = 110^\circ$ , then what is the measure of  $\angle ERA$ ?



- A.  $10^\circ$   
B.  $20^\circ$   
C.  $70^\circ$   
D.  $180^\circ$
- (M) 10. Which of the following statements is not sufficient to prove that a quadrilateral is a parallelogram?
- A. The diagonals are perpendicular.  
B. The diagonals bisect each other.  
C. Pair of opposite angles is congruent.  
D. Pair of opposite sides is congruent and parallel.
- (M) 11. In  $\square RICH$ ,  $\overline{RI} = 4x - 7$  cm,  $\overline{IC} = 5x - 8$  cm,  $\overline{CH} = 3x + 2$  cm and  $\overline{RH} = 2x + 1$  cm. What value of  $x$  will make RICH a parallelogram?
- A. 3  
B. 5  
C. 7  
D. 9
- (M) 12. In the figure below, if  $l_1$  and  $l_2$  are cut by transversals  $m$  and  $n$ , then what value of  $y$  will make  $l_1 \parallel l_2$  and  $m \perp l_2$ ?



- A. 3  
B. 6  
C. 9  
D. 12
- (M) 13. All of the following figures illustrate parallel lines except:

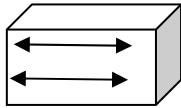


Figure 1

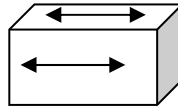


Figure 2



Figure 3

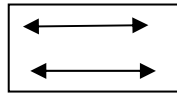
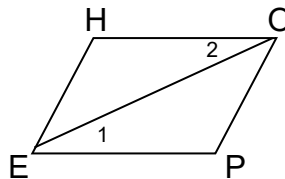


Figure 4

- A. Figure 1
- B. Figure 2
- C. Figure 3
- D. Figure 4

(M) 14. Choose the correct reason for the last statement to complete the two-column proof.

Given:  
 $HO = EP$   
 $\angle 1 = \angle 2$



Prove: HOPE is a parallelogram.

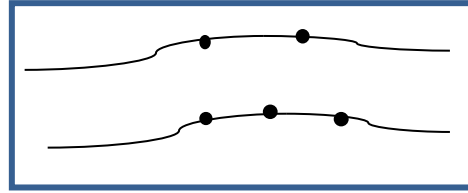
Statements	Reasons
1. $HO = EP$ and $\angle 1 = \angle 2$	1. Given
2. $HO \parallel EP$	2. If 2 lines cut by a transversal form equal alternate interior angles then the 2 lines are parallel.
3. HOPE is a parallelogram.	3. If a quadrilateral ____

- A. has a pair of opposite sides that are equal and parallel, then it is a parallelogram.
- B. has a pair of equal interior angles, then the quadrilateral is a parallelogram.
- C. has a pair of equal opposite angles, then the quadrilateral is a parallelogram.
- D. has a diagonal that divides the quadrilateral into two congruent triangles, then the quadrilateral is a parallelogram.

- (T) 15. A contractor tacked one end of a string to each vertical edge of a window. He then handed a protractor to his apprentice to find out if the vertical edges are parallel. What should the apprentice do?
- A. Measure the angles formed by the string and the vertical edge on both ends.
  - B. Measure the length of the string and the edge of the window.
  - C. Measure the length of the string and the horizontal edge of the window.
  - D. Measure the diagonal of the window and the angle formed by the edges of the window.
- (T) 16. How would one construct a rhombus by using a protractor and a ruler or a double-edged straightedge?
- A. Draw two intersecting segments and connect their endpoints.
  - B. Draw two perpendicular segments and connect their endpoints.
  - C. Draw two bisecting segments and connect their endpoints.
  - D. Draw two perpendicular and bisecting segments and connect their endpoints.
- (T) 17. As a design expert, a certain furniture shop invited you to conduct a mini-seminar on topic entitled: “*Ensuring Stability of Furniture.*” This seminar aims to orient the workers of the furniture shop on how they will ensure the stability of their product. Which one should you give emphasis on your talk?
- A. accuracy of measures, parallelism and perpendicularity of parts
  - B. attractive colors and accuracy of measures
  - C. parallelism of parts and quality of materials
  - D. perpendicularity of parts and quality of materials
- (T) 18. You are tasked to sketch a plan of a mall parking lot. Which of the following should you include in the plan in order to maximize the use of the area?
- A. landscaping designs
  - B. use of parallel lines
  - C. entrance art design
  - D. use of different shapes
- (T) 19. Michael is repairing a wooden clothes stand in which the legs are damaged. Which action should he consider?
- A. Check if the clothes stand is high enough for the lengthy garments.
  - B. Check if the legs of the clothes stand are parallel to one another.
  - C. Check if the distance between legs is greater than the length of the base.
  - D. Check if the length of the base is the same as the length of the legs.



(T) 20. An engineer is tasked to submit a design of a two lane bridge in one of the barangay of General Santos City. The length of the bridge affects the entire construction cost. Considering the sketch below, which of the following drawings would he make?



- A. Option A shows a bridge with two piers. A diagonal green line connects the top pier to the bottom pier, forming a triangle with the deck.
- B. Option B shows a bridge with two piers. A diagonal green line connects the top pier to the bottom pier, forming a triangle with the deck.
- C. Option C shows a bridge with two piers. A diagonal green line connects the top pier to the bottom pier, forming a triangle with the deck.
- D. Option D shows a bridge with two piers. A vertical green line connects the top pier to the bottom pier, forming a rectangle with the deck. A red asterisk is located to the right of the bridge.

## **GLOSSARY OF TERMS USED IN THIS LESSON:**

**Adjacent Sides** - Two sides with a common endpoint.

**Alternate Exterior Angles** - These are non-adjacent exterior angles that lie on opposite sides of the transversal.

**Alternate Interior Angles** - These are non-adjacent interior angles that lie on opposite sides of the transversal.

**Consecutive Angles** - Two angles whose vertices are the endpoints of a side.

**Consecutive Vertices** - These are the vertices at the ends of a side.

**Corresponding Angles** - These are non-adjacent angles that lie on the same side of the transversal, one interior angle and one exterior angle.

**Deductive Reasoning** - A type of logical reasoning that uses accepted facts to reason in a step-by-step manner until we arrive at the desired statement.

**Flowchart-Proof** - It organizes a series of statements in a logical order, starting with the given statements. Each statement together with its reason is written in a box, and arrows are used to show how each statement lead to another. It can make ones logic visible and help others follow the reasoning.

**Kite** - It is a quadrilateral with two pairs of adjacent sides congruent and no opposite sides congruent.

**Opposite Angles** - Two angles which do not have a common side.

**Opposite Sides** - These are the two sides that do not have a common endpoint.

**Paragraph or Informal Proof** - It is the type of proof where you write a paragraph to explain why a conjecture for a given situation is true.

**Parallel lines** - Parallel lines are coplanar lines that do not intersect.

**Parallelogram** - It is a quadrilateral with both pairs of sides parallel and congruent.

**Perpendicular Bisector** - It is a line or a ray or another segment that is perpendicular to the segment and intersects the segment at its midpoint.

**Perpendicular lines** - These are lines that intersect at  $90^{\circ}$ - angle.

**Proof** - A logical argument in which each statement you make is backed up by a statement that is accepted as true.

**Rectangle** - It is a parallelogram with four right angles.

**Rhombus** - It is a parallelogram with four congruent sides.

**Same-Side Interior Angles** - These are consecutive interior angles that line on the same side of the transversal.

**Same-Side Exterior Angles** - These are consecutive exterior angles that line on the same side of the transversal.

**Skew Lines** - Skew lines are non-coplanar lines that do not intersect.

**Square** - It is a parallelogram with four congruent sides and four right angles.

**Transversal** - It is a line that intersects two coplanar lines at two different points.

**Trapezoid** - It is a quadrilateral with exactly one pair of parallel sides.

**Two-Column Form/Formal Proof** - It is the most formal proof with statements and reasons. The first column is for the statements and the other column for the reason.

## **POSTULATES OR THEOREMS ON PROVING LINES PARALLEL:**

**CACP Postulate** - Given two lines cut by a transversal, if corresponding angles are congruent, then the two lines are parallel.

**AICP Theorem** - Given two lines cut by a transversal, if alternate-interior angles are congruent, then the lines are parallel.

**AECP Theorem** - If two lines are cut by a transversal such that the alternate-exterior angles are congruent, then the lines are parallel.

**SSIAS Theorem** - Given two lines cut by a transversal, if same side interior angles are supplementary, then the lines are parallel.

**SSEAS Theorem** - If two lines are cut by a transversal so that exterior angles on the same side of the transversal are supplementary, then the lines are parallel.

**The Three Parallel Lines Theorem** - In a plane, if two lines are both parallel to a third lines, then they are parallel.

The Perpendicular to a Third Line Theorem - If two coplanar lines are perpendicular to a third line, then they are parallel to each other.

### **THEOREMS ON PROVING LINES PERPENDICULAR:**

If two lines are perpendicular, then they form four right angles.

If the angles in a linear pair are congruent, then the lines containing their sides are perpendicular.

In a plane, through a point on a given line there is one and only one line perpendicular to the given line.

In a plane, a segment has a unique perpendicular bisector.

If two angles are adjacent and complementary, the non-common sides are perpendicular.

In a plane, if the non-common sides of adjacent angles are perpendicular then the angles are complementary.

### **PROPERTIES OF A PARALLELOGRAM:**

In a parallelogram,

1. Both pairs of opposite sides are parallel.
2. A diagonal divides it into two congruent triangles.
3. Both pairs of opposite sides are equal.
4. Both pairs of opposite angles are equal.
5. Any two consecutive angles are supplementary.
6. The diagonals bisect each other.

## REFERENCES AND WEBSITE LINKS USED IN THIS LESSON:

### References:

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- Chua, Simon L., Hipolito, Samuel G., et'al (2006). *Soaring 21<sup>st</sup> Century Mathematics: Exploring Geometry*. Quezon City: Phoenix Publishing House, Inc.,
- Coronel, Iluminada C. and Coronel, Antonio C. (2002). *Geometry*. Makati City: The Bookmark, Inc.
- Fisico, Misael Jose S., Sia, Lucy O., et'al (1995). *21<sup>st</sup> Century Mathematics: First Year*. Quezon City: Phoenix Publishing House, Inc.,
- Oronce, Orlando A. and Mendoza, Marilyn O. (2013). *E-Math: Intermediate Algebra*. Quezon City: Rex Book Store, Inc.

### Websites:

<http://t0.gstatic.com/images>

This site provides pictures of fence.

<http://oiangledlineswaves.jpg>

<http://brainden.com/images/cafe-wall.jpg>

<http://fiftysix.jpg>

These sites provide the optical illusions.

<http://www.mathwarehouse.com/geometry/angle/transveral-and-angles.php>

This site is an interactive site that may be used to review the different angle pairs formed by parallel lines cut by a transversal.

<http://www.mathwarehouse.com/geometry/angle/interactive-transveral-angles.php>

This site helps students find the relationships of the different angles formed by parallel lines cut by a transversal.

<http://www.youtube.com/watch?v=AE3Pqhlvqw0&feature=related>

This site provides an educational video discussion on parallel lines.

<http://www.youtube.com/watch?v=VA92EWf9SRI&feature=relmfu>

This site provides an educational video presentation about parallel lines.

<http://www.regentsprep.org/Regents/math/geometry/GP8/PracParallel.htm>

This site provides an interactive quiz which allows the students to practice solving problems on parallel lines cut by a transversal.

<http://www.mathsisfun.com/perpendicular-parallel.html>

This site provides illustrations on perpendicular lines.

<http://www.brighthubeducation.com/lesson-plans-grades-1-2/97443-parallel-perpendicular-and-intersecting-lines/>

This site provides a brief discussion about perpendicular lines.

<http://www.flvs.net/areas/student-services/EOC/Documents/Geometry%20Practice%20Test%20with%20Answers.pdf>

This site provides exercises involving parallel and perpendicular lines.

[http://www.redmond.k12.or.us/14552011718214563/lib/14552011718214563/Lesson\\_4.7.pdf](http://www.redmond.k12.or.us/14552011718214563/lib/14552011718214563/Lesson_4.7.pdf)

[http://www.cpm.org/pdfs/skillBuilders/GC/GC\\_Extra\\_Practice\\_Section12.pdf](http://www.cpm.org/pdfs/skillBuilders/GC/GC_Extra_Practice_Section12.pdf)

These sites provide a discussion on how to make a flowchart in proving through deductive reasoning.

<http://www.nexuslearning.net/books/ml-geometry/Chapter6>

This site provides exercises in proving parallelograms.

[www.nexuslearning.net/.../ML%20Geometry](http://www.nexuslearning.net/.../ML%20Geometry)

This site provides discussions and exercises about parallel lines and parallelograms.

[http://www.wyzant.com/Help/Math/Geometry/Quadrilaterals/Rectangles\\_Rhombuses\\_Squares.aspx](http://www.wyzant.com/Help/Math/Geometry/Quadrilaterals/Rectangles_Rhombuses_Squares.aspx)

This site provides the hierarchy of quadrilaterals with discussion on properties of parallelogram specifically squares, rectangles and rhombuses with illustrative examples on computations and an application of the different properties.

<http://www.regentsprep.org/Regents/math/geometry/GP9/JavaParallel.htm>

This site provides an interactive activity in exploring the properties of parallelogram with regards to the sides, angles and diagonals.

<http://www.onlinemathlearning.com/quadrilaterals.html>

This website contains the lesson on the different [quadrilaterals](#) and their corresponding properties. Read this for more information and examples.

<http://www.onlinemathlearning.com/quadrilaterals.html>

<http://www.onlinemathlearning.com/quadrilateral-properties.html>

These sites provide online exercises to strengthen students' skills in solving and proving.

<http://www.glencoe.com/sec/teachingtoday/downloads/pdf/ReadingWritingMathClass.pdf>

This site provides the concept map form.

[www.wisemapping.com](http://www.wisemapping.com)

This site helps the student make a concept map using this program.

<http://acr.keypress.com/KeyPressPortalV3.0/Viewer/Lesson.htm>

This site provides the problem given in the situational analysis part.

<http://thoughtsaboveall.blogspot.com>

This site provides the picture of different engineers.

<http://hockleyhouseandgarden.com>

This site provides pictures of carpenters.

<http://t3.gstatic.com/images>

<http://shoaibnzm1.blogspot.com>

<http://t2.gstatic.com/images>

These sites provide pictures of furniture.

<http://tighebond.com>

This site provides the picture of a building in the picture analysis part.

<http://westsideobserver.com>

This site provides the picture of a railroad in the picture analysis.

<http://trendir.com>

This site provides the picture of the inside part of a house in the picture analysis.

<http://www.nexuslearning.net/books/ml-geometry/Chapter6/ML%20Geometry%206-2%20Properties%20of%20Parallelograms.pdf>

This site is the source of the problem in the situational analysis part regarding scissors lift.