Lesson 4: Pythagorean Theorem and Special Right Triangles

In this lesson you will learn the following:

1. Proves the conditions for similarity of triangles involving Special Right Triangle Theorems
2. Applies the theorems to show that give triangles are similar
3. Proves the Pythagorean Theorem
4. Solves problems that involve triangles similarity and right triangles.

EXPLORE

You have just finished with the different theorems on similar triangles and polygons. In this lesson you will be dealing with theorems involving similarity theorems on special right triangles which are useful in analysis and solving problem involving geometric designs and figures.

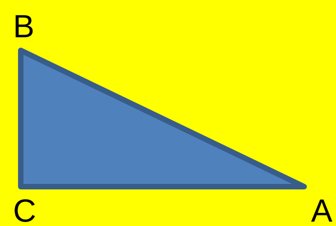
Before we discuss the main lesson, let's find out what you know about the topic. Bear in mind as you go through this module you are to answer the question:

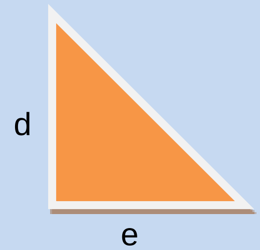
What is the best way to solve problems involving quadrilaterals and triangle similarity?

Answer the first column of the ARG by clicking on the AGREE or DISAGREE column. Click A if you agree with the statement and click B if you disagree. Please answer all items. As you go through this module, look for the best correct answer to the statements included in this guide.

<u>Activity 1d.</u> Agree or Disagree?

Anticipation Reaction Guide
Directions: Answer the first column of the ARG by clicking on the AGREE or DISAGREE column. Click A if you agree with the statement and click B if you disagree. Please answer all items.

Before Discussion		Statements	After Discussion	
Agree	Disagree		Agree	Disagree
A	B	1. The hypotenuse of a right triangle is the longest of all its three sides..	A	B
		2. In a right triangle, the measure of the hypotenuse is equal to the sum the measure of its legs.		
		3. The numbers 3, 4, and 5 represent a Pythagorean triple.		
		4. In an isosceles right triangle, the side opposite the right angle is $\sqrt{2}$ times as long as either of the legs.		
		5. In a 30-60-90 triangle, the side opposite the smallest angle is twice as long as the longest side.		
		6. In rt $\triangle$ BCA, the measure of $\overline{AC}$ is x $\sqrt{3}$ if $\overline{BC} = x$, $\angle A = 30^0$, $\angle B = 60^0$ *and* $\angle C = 90^0$. 		
		7. The Pythagorean theorem is applicable to any triangle.		
		8. A square mirror 7 ft on each side must be delivered through the doorway 3 ft x 6.5 ft. Can the mirror fit through the doorway?		

		9. Cathy, Luisa and Morgan are writing an equation to find the length of the third side of a right triangle given below. Only Luisa wrote the correct equation.  Cathy: $f + e = d$ Luisa : $f^2 = d^2 + e^2$ Morgan: $e^2 = d^2 – f^2$		
		 10. The support for a basketball goal forms a right triangle as shown. The length x of the horizontal portion of the support is approximately 2.98 ft.		

End of EXPLORE:

You have just finished answering a pre-assessment activity. What you will learn in the next sections will also enable you to do a final project which involves creating a model or structural design that will help you use materials efficiently or maximize the use of space. We will start by doing the next activity.

FIRM - UP

Your goal in this section is to have a good understanding of the Pythagorean Theorem and theorems involving special right triangles. The activity focus will be the formal proof of these theorems. Formative assessments on the relationships of the sides of the special right triangles will also be provided.

Start by performing the Activity 2d and learn how the theorems are derived applied.

Activity 2d. A Man Named PYTHAGORAS!

DESCRIPTION: In this activity you are to read an article about the works of a certain mathematician who is best remembered today because of his theorem which deals with the relationships among the sides of a right triangle. Take note of the words you have encountered by highlighting or underlining it then summarize in your own words what you have read.

Click the website below, read and answer the questions that follow.

http://www.themathlab.com/Algebra/lines%20and%20distances/pythagor.htm

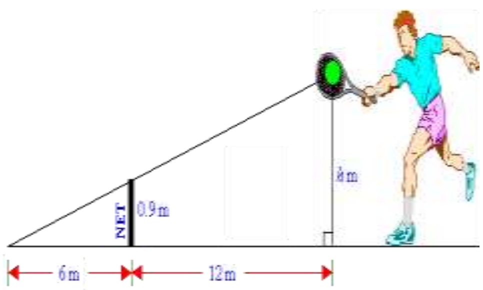
This website gives pertinent information about the life of Pythagoras and how he derived his Pythagorean Theorem. The following information comes from a wonderfully readable math history book by Julia E. Diggins called, **STRING, STRAIGHT-EDGE, & SHADOW, THE STORY OF GEOMETRY** .

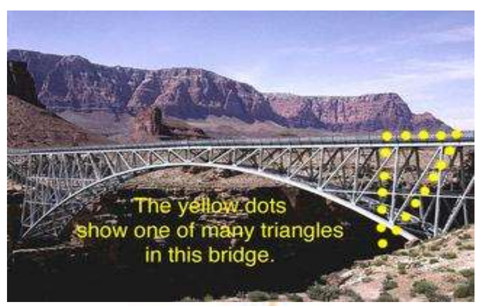
Questions to Answer:

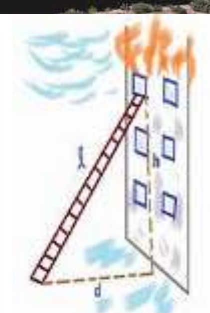
- What is the reading all about?
- Do you agree to the statement " Without Pythagoras, school may never have been invented nor much of what we know of mathematics"?
- How did the early engineers make use of the "rope - stretchers" in building structures like the Great Pyramid of Egypt?
- How did Pythagoras of Samos derived his remarkable theorem on right triangles?
- Do you think the concept or idea of quadrilaterals and triangles is the best way to ensure quality foundation of structures and designs?

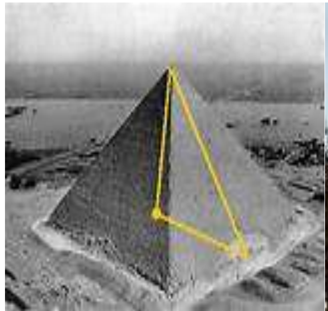
Write your answers here.

<u>Activity 3d</u>. Focus: Identifying Use of Pythagorean Theorem in Real Life
Below are some pictures of right triangles as applied in the real world. Can you identify them? Explain why such shape was used.









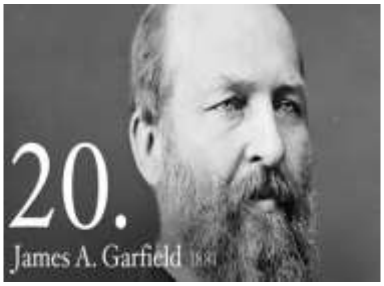
<u>Activity 4d.</u> Different Ways of Proving the Pythagorean Theorem
Description: Click the websites below and see how the proof is shown through illustrations, Algebra, paper cutting and others.

http://www.mathsisfun.com/pythagoras.html - This website gives the discussion of the derivation of Pythagorean Theorem using the concept of area.

http://www.brainingcamp.com/content/pythagorean-theorem/manipulative.php. This is an interactive site where the Pythagorean Theorem is proven using the concept of area. To show the proof, click the Show Proof icon under Action button and drag the parts of the violet and green squares to form a bigger square in the orange square.

http://www.mathisfun.com/geometry/pythgorean-theorem-proof.html - Algebra Proof of Pythagorean Theorem

http://www.watchknowlearn.org/Video.aspx?VideoID=54330&CategoryID=5370. This website shows video of the proof of Pythagorean Theorem by James Garfield the 20th century President of the United States. He derived the theorem using the concept of the area of the trapezoid.



Additional website showing some other ways of deriving the proof of Pythagorean Theorem. Click the website below and you will see the other proofs of the theorem.

http://www.cut-the-knot.org/pythagoras/index.shtml- This website gives the list of 100 proofs of the Pythagorean Theorem.

Questions to Answer:

1. How was the Pythagorean theorem derived in the different sites? Write your answers in the table below.

2. Is the theorem applicable to all kinds of triangles? What is its limitation?

3. Do the different proofs of the theorem lead to the same conclusion? Explain.

4. Which of the different proofs presented made you understand clearly? Discuss.

5. Do you think the theorem will help you understand and solve problems involving right triangles? Explain.

SUBMIT

<u>Activity 5d.</u> The Triangle and the Theorem

www.shodor.org/interactive/activities/SquaringTheTriangle/ - This applet allows users to explore right triangles and the Pythagorean Theorem. This can also be used to explore the angle measurements of the triangles. This activity will develop an understanding of the Pythagorean Theorem. It displays a right triangle with a square against each side. Each square has sides that are equal in length to the side of the triangle it is against. By adjusting the lengths of the sides of the triangle, the user can visually experience the Pythagorean Theorem.

Questions to Answer:

The questions below can be obtained by clicking the “Learner” icon and then click the Worksheet Squaring the triangle Exploration Questions.

1. What defines a right triangle?
2. What is the area of the square?
3. How are the angles and the sides opposite them related?
4. How are the blue squares related?
5. How are the two non-right angles related?
6. How are the sides related in a right triangle
7. If the triangle is NOT right, will the theorem still hold?
8. What generalization can you make based on the Pythagorean Theorem?

Write your answers here.

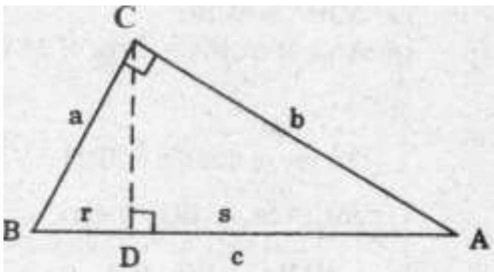
SUBMIT

<u>Activity 6d.</u> The Formal Proofs

DESCRIPTION: In this activity, the students will derive the mathematical proof of Pythagorean theorem by supplying the corresponding reason of the given statements.

Given: △ABC with ∠C a right angle and $\overline{CD}$ is the altitude of △ABC.

Figure:



Prove: $c^2 = a^2 + b^2$

Proof: Supply the corresponding reason for each of the given statement.

Statements	Reasons
1. $r + s = c$	1.Definition of Betweeness
2. $a^2 = cr$ and $b^2 = cs$	2. ?
3. $a^2 + b^2 = cr$	3. Addition Property of Equality
4. $a^2 + b^2 = c(r + s)$	4. ?
5. $a^2 + b^2 = c(c)$	5. Substitution
6. $a^2 + b^2 = c^2$	6. Product Rule on Exponent
7. $c^2 = a^2 + b^2$	7. ?

Questions to Answer:

1. How was the theorem proven? ____________________(insert a box for the students to answer)
2. How does it differ from the previous proof shown in Activity 6c? __________ (insert a box for the students to answer)
3. Does it lead to the same conclusion? Explain.

<u>Activity 7d.</u> Formal Proof of the Converse of Pythagorean Theorem
DESCRIPTION: In this activity, the students will derive the geometric proof of the converse of Pythagorean theorem by supplying the corresponding reason of the given statements.

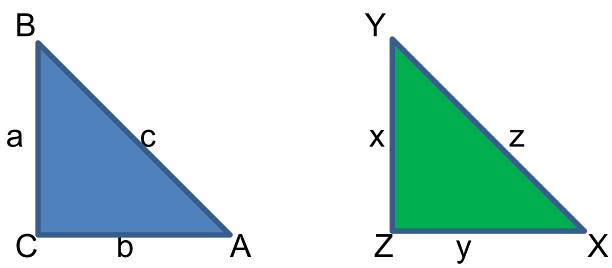
Proof of the Converse of Pythagorean Theorem

Converse of the Pythagorean Theorem

If in a triangle, the square of the length of one side is equal to the sum of the square of the lengths of the other two sides, then the triangle is a right triangle and the right angle is opposite the longest side.

Given : △ABC with $a^2 + b^2 = c^2$
Prove : △ ABC is a right triangle

Figure:



Proof:

Statements	Reasons
1. Let XYZ be a right triangle with legs of length x and y and hypotenuse of length z	1.By Construction
2. $x^2+y^2 = z^2$	2. ?
3. $a^2 + b^2 = c^2$	3. Given
4. $c^2 = z^2$ or $c = z$	4. Transitive Property of Equality
5. △ ABC ≅ △ XYZ	5. SSS Congruence Postulate
6. ∠C ≅ ∠Z	6. ?
7. m∠C ≅ m∠Z	7. Definition of congruent angles
8. m∠Z= 90	8. ?

9. m $\angle$C = 90	9. Transitive Property of Equality
10. $\angle$C is a right angle	10. Definition of a right angle
11. $\triangle$ ABC is a right triangle.	11. ?

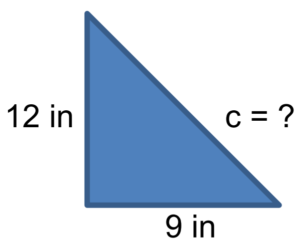
The Pythagorean Theorem describes the relationship between the lengths of the legs and the hypotenuse of any right triangle. You can use the Pythagorean Theorem to find the length of a side of a right triangle when you know the other two sides.

To understand the above theorems, study the given examples how the theorems are used or applied in solving problems involving right triangles.

<u>Activity 8d.</u> Looking for the Third Side!
Consider the problems below:

Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary.

1.



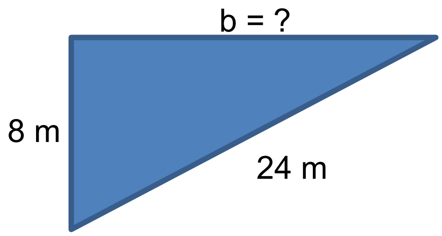
$a^2 + b^2 = c^2$	Pythagorean Theorem
$12^2 + 9^2 = c^2$	Substitution
$144 + 81 = c^2$	Evaluate $12^2 + 9^2$
$225 = c^2$	Add 81 and 144
$\pm\sqrt{225} = c$	Taking square root of both sides of the equation
$c = 15$ or -15	Simplify

The equation has two solutions, 15 and -15. However, the length of a side must be positive. So, the hypotenuse is 15 inches long.

Check:

$a^2 + b^2 = c^2$
$12^2 + 9^2 = 15^2$
$144 + 81 = 225$
$225 = 225$

2.



$a^2 + b^2 = c^2$	Pythagorean Theorem
$8^2 + b^2 = 24^2$	Substitution
$64 + b^2 = 576^2$	Evaluate $8^2 + 24^2$
$64 - 64 + b^2 = 576 - 64$	Subtracting 64 both sides of the equation
$b^2 = 512$	Simplify
$b = \pm\sqrt{512}$	Taking square root of both sides of the equation
or $b \approx 22.6$ m	

The length of side b is about 22.6 meters.

3. The measure of the three sides of a triangle are 5 inches, 12 inches, and 13 inches. Determine whether the triangle is a right triangle.

$a^2 + b^2 = c^2$	Pythagorean Theorem
$5^2 + 12^2 = 13^2$	Substitution
$25 + 144 = 169$	Evaluate $5^{2,}$ $12^2$and 13^2
$169 = 169$	Simplify

Since both sides of the equation are equal, then the triangle is a right triangle.

4. Geo claims that the following sides of triangle 4 m, 7 m , 5 m and 30 km, 122 km, 125 km determine a right triangle. Justify if his claim is TRUE.

$a^2 + b^2 = c^2$	Pythagorean Theorem
$4^2 + 5^2 \stackrel{?}{=} 7^2$	Substitution
$16 + 25 \stackrel{?}{=} 49$	Evaluate $4^{2,}$ 5^2 and 7^2
$41 \stackrel{?}{=} 49$	Simplify

Since the sum of the squares of two sides is not equal to the square of the longest side, then the sides do not form a right triangle. Therefore Geo's claim is not true.

$$a^2 + b^2 = c^2 \quad \text{Pythagorean Theorem}$$

$$30^2 + 122^2 \stackrel{?}{=} 125^2 \quad \text{Substitution}$$

$$900 + 14\,884 \stackrel{?}{=} 15\,625 \quad \text{Evaluate } 30^{2,}\ 122^2 \text{ and } 125^2$$

$$15\,784 \stackrel{?}{=} 15\,625 \quad \text{Simplify}$$

Since the sum of the squares of two sides is not equal to the square of the longest side, then the sides do not form a right triangle. Therefore Geo's claim is not true.

From the example problem in item 4 since the sides of a triangle do not determine a right triangle, what kind of triangle is formed?

From these examples it further leads to the theorem regarding the kind of triangle if the sum of the squares of two sides is greater or lesser than the square of the other side. This theorem is called the Pythagorean Inequality Theorems.

If the sum of the squares of the lengths of the shorter sides of a triangle is greater than the square of the length of the longest side then the triangle is acute.

If the sum of the squares of the lengths of the shorter sides of a triangle is less than the square of the length f the longest side then the triangle is obtuse.

For further acquisition and comprehension of the concepts regarding right triangles, and the visual and algebraic proofs of Pythagorean Theorem, visit the website below

Activity 9d.Pythagorean, A Second Look
Click the website below as a second look of the discussion of Pythagorean Theorem. This further discusses the theorem coupled with formative assessment.

www.brainingcamp.com/content/pythagorean-theorem/lesson.php.This website consists of the discussion on the visual and algebraic proofs of the Pythagorean Theorem.

Activity 4i- Test me in the Web
To check if you understand the presented lessons and examples , perform the exercises given below as formative assessment using Pythagorean Theorem

www.brainingcamp.com/content/pythagorean-theorem/questions.php. This website gives a 10-item questions involving facts and computations of sides of a right triangle using Pythagorean Theorem. To check if the given answers are correct, click the submit icon.

For the website given below, solve for items 1 to 21(odd numbers only).

hotmath.com/help/gt/genericprealg/section_8_5.html - This site contains 43 practice problems with solutions on Pythagorean Theorem and its converse.

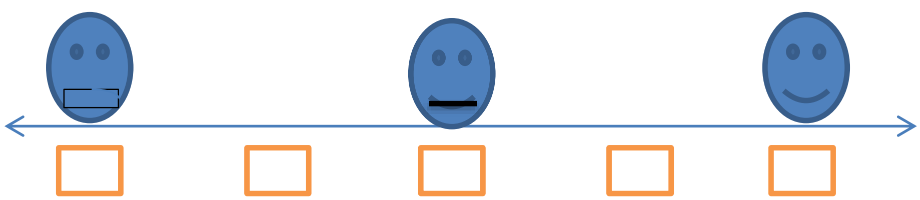
www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=1-57039-850-X&chapter=8&lesson=5 – This is a 5-item self-check quiz about Pythagorean Theorem

http://www.shodor.org/interactivate/activities/PythagoreanExplorer/ - Interactive website determining the 3rd side of a right triangle using Pythagorean Theorem. You can choose 3 levels of difficulty and check your answer by clicking the corresponding icon and determine your score.

After you have browsed and answered the given questions in the different sites, what do you think is the best way to solve problems involving triangle similarity specifically Pythagorean Theorem?

Activity 10d. **Rate Yourself!**

How confident are you about using the Pythagorean Theorem? Check the box that applies.



Not Confident Confident Very Confident

Activity 11d. **Are We the Threesome Numbers?**
Description: In this activity you are to determine whether the given three numbers represent a Pythagorean triple.

Three numbers can be called a Pythagorean triple if $a^2 + b^2 = c^2$ where a, b and c are positive integers.

Given a set of three numbers, show that these represent a Pythagorean triple.

1. Fill in the table with the correct value and tell whether the set represents a Pythagorean triple.

	A	b	C	a^2	b^2	c^2	a^2+b^2	Is $a^2 + b^2 = c^2$?
i.	3	4	5					
ii.	6	8	10					
iii.	5	12	13					
iv.	7	24	25					
v.	10	24	26					
vi.	9	12	15					

2. Explain why i, iii and iv above are more important than the others?
3. Observe the following triples.
   3, 4, 5
   5, 12, 13
   7, 24, 25

   What can be said about the smallest number in the triple?
   What is unique with the difference between the other number?

4. What are the other two numbers in a triple if the smallest is 9? 11?13?

5. Do you think these set of numbers will guide and help solve problems involving right triangles? Explain.

Answer:

1. Fill in the table with the correct value and tell whether the set represents a Pythagorean triple.

	A	b	C	a^2	b^2	c^2	a^2+b^2	Is $a^2 + b^2 = c^2$?
i.	3	4	5	9	16	25	25	Yes
ii.	6	8	10	36	64	100	100	Yes
iii.	5	12	13	25	144	169	169	Yes
iv.	7	24	25	49	576	625	625	Yes
v.	10	24	26	100	576	676	676	Yes
vi.	9	12	15	81	144	225	225	Yes

2. Explain why i, iii and iv above are more important than the others? ii and vi are multiple of i; v is a multiple of iii. i, iii and iv are important because they are not multiples of the other triples.
3. Observe the following triples.
   3, 4, 5
   5, 12, 13
   7, 24, 25

What can be said about the smallest number in the triple? The smallest number in the triple is an odd number.
What is unique with the difference between the other number? The difference between the other number is 1.

4. What are the other two numbers in a triple if the smallest is 9? 11?13? 9, 40, 41; 11, 60, 61; 13, 84, 85

Activity 12d. Are We Really the TRIO?
Click the website below and answer the given questions
www.math.com/school/subject3/practice/S3U3L4/S3U3L4Pract.html - This website gives 8 sample sets of numbers and determine whether each set represents the sides of a right triangle.

Activity 13d. Finding My Threesome
We have learned that if three numbers satisfy the Pythagorean Theorem, they are called Pythagorean triples. The numbers a, b, and c are a Pythagorean triple if,

- $a = m^2 – n^2$
- $b = 2mn$
- $c = m^2 + n^2$

Where m and n are relatively prime positive integers and m>n.

Example: Choose m = 5 and n = 2.

$a = m^2 – n^2$	$b = 2mn$	$c = m^2 + n^2$	Check: $a^2 + b^2 = c^2$
$= 5^2 – 2^2$	$= 2(5)(2)$	$= 5^2 + 2^2$	$20^2 + 21^2 = 29^2$
= 25 – 4	**= 20**	**= 25 + 4**	**400 + 441 = 841**
= 21		= 29	841 = 841

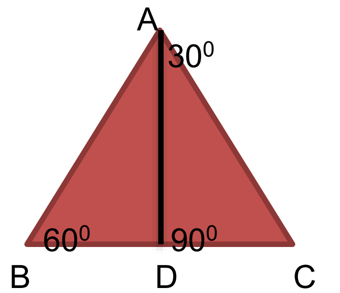
Activity 14d. Pythagorean Triples

Use the following values of m and n to find Pythagorean triples.

1. m = 3 and n = 2
2. m = 4 and n = 1
3. m = 5 and n = 3
4. m = 6 and n = 5
5. m = 10 and n = 7
6. m = 8 and n = 5

Source:
www.ed.gov.nl.ca/edu/k12/curriculum/documents/mathematics/gr8/pythagorean_triples.pdf

One of the theorems in right triangle is the 30-60-90 Triangle Theorem. Read the text below before performing Activity 4i to derive the 30-60-90 TriangleTheorem .

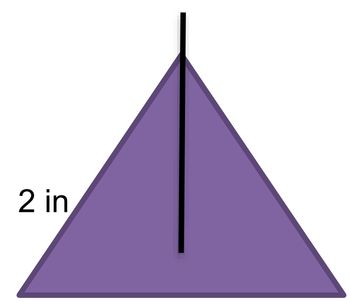


An equilateral triangle has three equal sides and three equal angles. Because the sum of the measures of the angles in a triangle is 180°, the measure of each angle in an equilateral triangle is 60°. If you draw a median from vertex A to side $\overline{BC}$, the median bisects the angle A. The median of an equilateral triangle separates it into two 30-60-90 triangles. In the figure above AD is the median.

<u>Activity 15d.</u> Hands-On Geometry

C

2 in
Materials Needed : compass, protractor and ruler



A D

B

Procedures:

Step 1. Construct an equilateral triangle with sides 2 inches long. Label its vertices A, B, and C.

Step 2. Find the midpoint of $\overline{AB}$ and label it D. Draw $\overline{AD}$, a median.

Step 3. Use the protractor to measure $\angle ACD\ \angle A\ and\ \angle CDA$. Use the ruler to measure $\overline{AD}$ and use the Pythagorean Theorem to find $\overline{CD}$. Write the answers for the measure of sides in simplest form. Use the table below to write the needed data.

Side AC	$\angle ACD$	$\angle A$	$\angle CDA$	AD	CD
2 in					
4 in					
3 in					

Step 4. Suppose the length of a side of an equilateral triangle is 10 inches. What values would you expect for AC(hypotenuse), AD (shorter leg), and CD (longer leg)?

Step 5. From the activity, what can be deduced regarding the relationships between the hypotenuse , the length of the shorter leg and the length of a longer leg in a 30-60-90 triangle?

Answers:

Side AC	$\angle ACD$	$\angle A$	$\angle CDA$	AD	CD
2 in	30^0	60^0	90^0	1 in	$\sqrt{3}$ in
4 in	30^0	60^0	90^0	2 in	$2\sqrt{3}$ in
3 in	30^0	60^0	90^0	1.5 in	$1.5\sqrt{3}$ in

If the measure of the side of an equilateral triangle is 10 inches, the hypotenuse is 10 inches, the shorter leg is 5 inches and the longer leg is $5\sqrt{3}$ inches.

In a 30-60-90 triangle, the hypotenuse is twice the length of the shorter leg, and the longer leg is $\sqrt{3}$ times the length of the shorter leg.

Remember that the shorter leg is always opposite the 30^0 angle, and the longer leg is opposite the 60^0 angle.

This further leads to the 30-60-90 Triangle Theorem, which states that in a 30-60-90 triangle, the side opposite the 30^0 angle is half as long as the hypotenuse and the side opposite the 60^0 angle is $\sqrt{3}$ times as long as the side opposite the 30^0 angle.

<u>Activity 16d.</u> Geometric Proof of the 30-60-90 Triangle Theorem

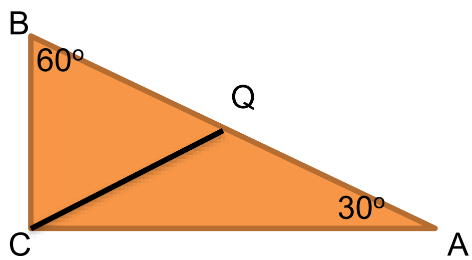
After deriving the theorem, you are to derive the same theorem using the geometric proof. You will be provided with statements and its corresponding reasons as you go through its proof.

Given: Right △ ABC with m∠A = 30, m∠B = 60 and m∠C = 90.

Prove : a. BC = $\frac{1}{2}AB$

b. AC = $\sqrt{3}BC$

Figure:



Proof:

Statements	Reasons
1. Right △ ABC with m∠A = 30, m∠B = 60 and m∠C = 90	1. Given
2. Let Q be the midpoint of AB	2. Every segment has exactly one midpoint.
3. Construct, $\overline{CQ}$ the median to the hypotenuse.	3. The Line Postulate/Definition of median of a triangle.
4. CQ = $\frac{1}{2}AB$	4. The Median Theorem
5. AB = BQ + AQ	5. Definition of Betweeness
6. BQ = AQ	6. Definition of median of a triangle
7. AB = AQ + AQ	7. Substitution(5 and 6)
8. AB = 2AQ	8. Combining Similar Terms
9. CQ = $\frac{1}{2}(2AQ)$	9. Substitution(4 and 8)
10.CQ = AQ	10. Multiplicative Inverse
11. CQ = BQ	11. Transitive Property of Equality(6 and s10)
12. $\overline{CQ} \cong \overline{AQ}; \overline{CQ} \cong \overline{BQ}$	12. Definition of Congruent Segments
13. ∠B ≅ ∠BCQ	13. Isosceles Triangle Theorem
14. m∠B = m∠BCQ	14. Definition of Congruent Angles
15. m∠BCQ = 60	15. Transitive Property of Equality(1

	and 14)
16. $m\angle B + m\angle BCQ + m\angle BQC = 180$	16. The sum of the measures of the angles of a triangle is equal to 180.
1. $60 + 60 + m\angle BQC = 180$	17.Substitution(1 and 15)
18. $m\angle BQC = 60$	18. Simplification/APE
19. $\triangle$ BCQ is equiangular and therefore equilateral.	19. Definition of equiangular triangle.
20. BC = CQ	20. Definition of equilateral triangle.
21. $BC = \frac{1}{2}AB$	21. Transitive Property of Equality(8, 10 and 20)

To prove that $AC = \sqrt{3}BC$, we simply apply the Pythagorean Theorem.

$$c^2 = a^2 + b^2$$

$$AB^2 = \left(\frac{1}{2}AB\right)^2 + AC^2$$

$$AB^2 = \left(\frac{AB^2}{4}\right) + AC^2$$

$$\frac{3}{4}AB^2 = AC^2$$

$$\frac{\sqrt{3}}{2}AB = AC$$

$$\frac{\sqrt{3}}{2}2(BC) = AC$$

$$\sqrt{3}BC = AC$$

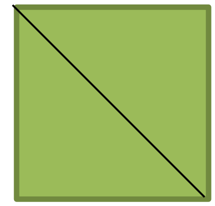
$$AC = \sqrt{3}BC$$

This further states that in a 30-60-90 triangle, the side opposite the 30^0 angle is half as long as the hypotenuse and the side opposite the 60^0 angle is $\sqrt{3}$ times as long as the side opposite the 30^0 angle..

Another Right Triangle Theorem is the 45-45-90 Triangle Theorem, perform Activity 4k to derive the theorem and see how the theorem was formally proved geometrically.

The 45-45-90 Triangle Theorem or the Isosceles Right Triangle Theorem

<u>Activity 17d.</u> Hands-On Geometry



Materials Needed; Ruler and Protractor

Procedures:

Step 1. Draw a square with sides 4 centimeters long. Label its vertices A, B, C, and D.

Step 2. Draw the diagonal $\overline{AC}$.

Step 3. Using a protractor and Pythagorean theorem measure $\angle CAB\ \angle ACB\ and\ \overline{AC}$ respectively. Express $\overline{AC}$ in simplest form. Use the table below to write the needed data.

Step 4. Repeat the steps 1 – 3 for squares using 6 cm long and 8 cm long.

Side of a square	m$\angle CAB$	m $\angle ACB$	m$\overline{AC}$
4 cm			
6 cm			
8 cm			

Step 5. Make a conjecture about the length of the diagonal of a square with sides 7 cm long.

Step 6. From the activity, what can be deduced regarding the relationship of the hypotenuse and the length of a leg in a 45-45-90 triangle?

Answer:

Side of a square	m$\angle CAB$	m $\angle ACB$	m$\overline{AC}$
4 cm	45^0	45^0	$4\sqrt{2}$
6 cm	45^0	45^0	$6\sqrt{2}$
8 cm	45^0	45^0	$8\sqrt{2}$

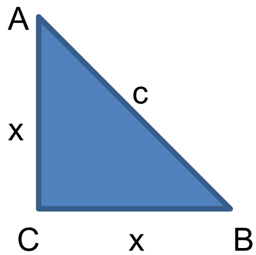
If a square has side 7 cm long, then the diagonal is $7\sqrt{2}$ cm. In a 45-45-90 triangle, the hypotenuse is $\sqrt{2}$ times the length of a leg.

This further leads to the 45-45-90 Triangle Theorem which states that in a 45-45-90 Triangle, an isosceles right triangle, the hypotenuse is $\sqrt{2}$ times as long as either of the legs.

Activity 18d.Geometric Proof of the theorem

Given : △ABC is an isosceles triangle with AC = BC = x, AB = c and m∠C = 90.

Figure:



Prove: c = $x\sqrt{2}$

Proof:

Statements	Reasons
1. △ABC is an isosceles triangle, with AC = BC = x, AB = c and m∠C = 90.	1. Given
2. $c^2 = x^2 + x^2$	2. Pythagorean Theorem
3. $c^2 = 2x^2$	3. Simplification/Combining similar terms
4. c = $x\sqrt{2}$	4. Extracting Square root of both sides of the Equation

From the given proof it further states that in a 45-45-90 Triangle or in an isosceles right triangle, the hypotenuse is $\sqrt{2}$ times as long as either of the legs.

You have just presented with the different theorems on right triangles, to further understand the concepts and its derivation, visit the websites below

<u>Activity 19d.</u> The Web

In this activity, you are to see how the theorems were discussed and derived

http://www.ixl.com/math/algebra-1/special-right-triangles – interactive exercises about special right triangle.

http://www.regentsprep.org/regents/math/algtrig/ATT2/PracSpecial.htm – interactive website on Special Right Triangle.

http://www.mrperezonlinemathtutor.com/G/3_3_Using_30_60_90_and_45_45_90_ratios.html – Discussion on Special Right Triangle.

http://exchange.smarttech.com/search.html?q=%20special%20right%20triangles

Questions to Answer:

1. What are the websites all about?
2. How the website facilitates the concept formation?
3. From the given websites, what are the characteristics a 45-45-90 and 30-60-90 triangles?
4. Compare and contrast the 30-60-90 Triangle Theorem and the 45-45-90 Triangle Theorem.

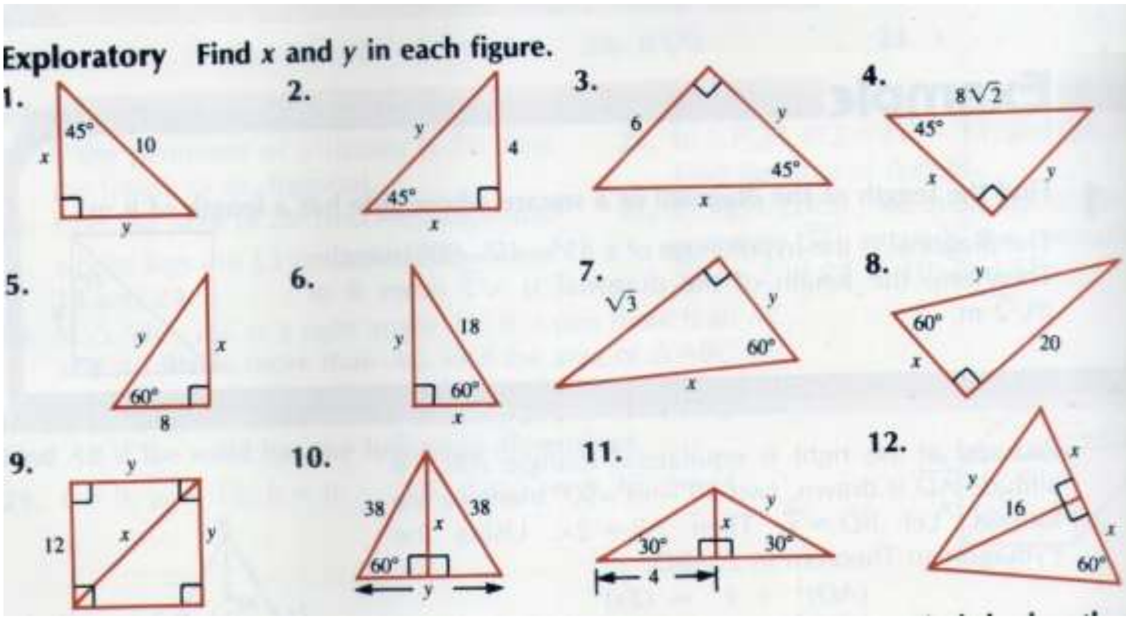
Write your answers here….

SUBMIT

After looking at the examples and browsing the websites, find out if you can perform the exercises by answering the following problems given in the next Activity.

<u>Activity 20d.</u> Solve Me

In this activity you will be solving problems involving right triangles and use the concept of special right triangle theorems. After solving the given exercises, answer the questions given at the end of the activity.



2. Use Figure M to find each measure.

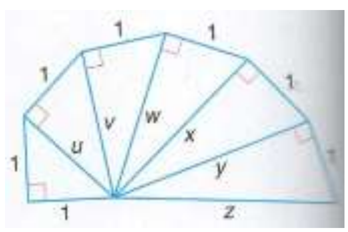
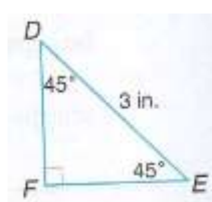


Figure M

a. u d. x
b. v e. y
c. w f. z



3. Norman says that the length of a leg of a △DEF is $3\sqrt{2}$ inches. Dale says the length of a leg is $\frac{3\sqrt{2}}{2}$ inches. Who is correct? Explain your answer.

4. Draw and label a 30-60-90 triangle in which the sides are 5 inches, 10 inches and
$5\angle 3$ inches.

5. www.shodor.org/interactivate/activities/PythagoreanExplorer/ - This is an interactive site solving for the third side of a right triangle using Pythagorean theorem. It has 3 levels of difficulty to choose from. Click the icons below the figure to use its application.

6. http://www.themathlab.com/Algebra/pythagorean%20theorem%20intro%20to%20trig/pythagtest.htm - This is a treasure hunt website. It contains 9 problems to be solved to answer the puzzle. The answers are given in the box for you to choose from. After you have answered correctly all the problems, the password will be used to claim your treasure.

Questions to Answer:

1. How did you find the exercises?
2. Were you able to solve correctly the problems?
3. Were there difficulties encountered in performing the activity?
4. What is the best way to solve problems involving triangle similarity specifically Pythagorean Theorem and theorems on Special Right Triangles?

Write your answers here….

SUBMIT

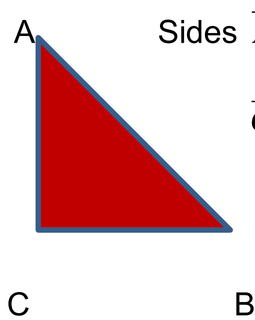
<u>Activity 21d.</u> Fill Me

To check if you understood the lesson, fill up the table below to check your understanding of the lesson about the Pythagorean Theorem.

Draw a Right Triangle and label the three sides.	Describe the sides: Legs: Hypotenuse:
Write the Theorem	The Formula:

Pythagorean Theorem

Answer:

Draw a Right Triangle and Label the Three Sides. Sides $\overline{AB}$ or c, $\overline{AC}$ or b, $\overline{CB}$ or a 	***Describe the Sides***: Legs: These are sides opposite the acute angles of a right triangle. The sides that include the right angle. In the figure the legs are $\overline{AC}$ or c and $\overline{CB}$ or a. Hypotenuse: This is the

Pythagorean Theorem	side opposite the right angle. It is considered the longest side of a right triangle. In the figure , $\overline{AB}$ or c is the hypotenuse.
Write the Theorem In any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.	***The Formula***: $\overline{AB}^2 = \overline{CB}^2 + \overline{AC}^2$ or $c^2 = a^2 + b^2$

END OF FIRM - UP

In this section, the discussion was about the Pythagorean Theorem and the Special Right Triangle theorems on how were these proven and used in solving problems

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision? What new learning goal should you now try to achieve?

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.

DEEPEN

Your goal in this section is to take a closer look at some aspects of Pythagorean Theorem and Special Right Triangle Theorems. Real- life problems involving these theorems will be given in this section and will be asked to solve related problems.

Activity 22d. **The Crayon and the Panel**

A thorough discussion was given on solving triangles using the Pythagorean theorems and special right triangle. In this activity, real world problems will be presented. You will guided in solving these problems by answering the questions provided.

Consider the two problems given below:

Problem No. 1. A company makes crayons that ”do not roll off tables” by shaping them as triangular prisms with equilateral bases. Sixteen of these crayons fit into a box shaped like a triangular prism that is 1 ½ inches wide. The crayons stand on end in the box and the base of the box is equilateral. What are the dimensions of each crayon?

You will be guided to solve the problems by answering the questions given below.

Understand:

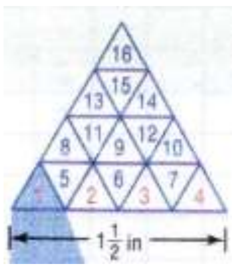
- What are the given information and data of the problem?
- What are the required data ?
- How many crayons will fit into the box?
- What shape will the box be?

Plan: What are the things you need to consider to solve for the problems?

- Is sketching of the possible placement or position of the crayons needed? If so, draw a sketch so that 16 crayons be accommodated in the box shaped like a triangular prism.
- How is the width of each crayon be determined?
- What theorem will be used to solve for its altitude?

Solve:

- What should be the sketch drawing to show that the total number of crayons will fill into the box?

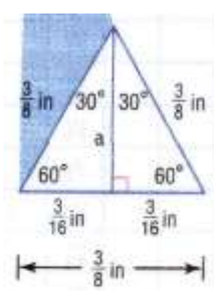


- If the width of the box is 1 ½ inches, so what is the width of one crayon? Answer 1 ½ ÷ 4 or 3/8 inch.
- Draw an equilateral triangle representing one crayon. Its altitude forms the longer leg of two 30-60-90 triangles. Using the theorem, find the approximate length of altitude a.

Answer: longer leg length = shorter leg $\bullet \sqrt{3}$

$$a = \frac{3}{16} \bullet \sqrt{3}$$

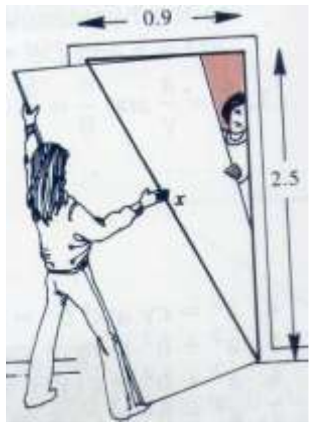
$$a \approx 0.3 \text{ inch}$$



- Each crayon is $\frac{3}{8}$ or about 0.4 inch by about 0.3 inch

Check: Find the length of the box using the 30-60-90 Triangle Theorem. Then divide by four, since the box is four crayons high. The result is a crayon height of about 0.3 inch.

2. The walls in the ABC Recreation Center are being covered with wall paneling. The doorway is 0.9 m wide and 2.5 m high. What is the widest rectangular panel that can be taken through the doorway?



Understand:

- What are the given data of the problem?
- What are the required data?

Plan:

- What is your plan in solving the given problem?
- How would you solve the problem?

The widest panel would be taken through diagonally, as shown in the figure. Let x be the measure of the panel.

Solve:

- What theorem will be used to solve the problem? Answer: Using Pythagorean theorem to find x, $(0.9)^2 + (2.5)^2 = x^2$. Solving for x, x ≈2.66 m. A width of 2.66 m would be a tight fit. To allow extra clearance, a narrower panel could be chosen.

Check: To check that the obtained value is correct, let x = 2.66 m and substitute it to the formula using Pythagorean Theorem. To accommodate the rectangular panel enter the door, the value of x must be less than 2.66 m.

You have just presented how the problems were solved using the concept of Pythagorean Theorem and the Theorems on Special Right Triangles. Summarize how the process was made in the Plan, Understand, Solve and Check stages based on the two problems presented.

In the next activity, solve the given problems by giving the necessary information/data in the Plan, Understand, Solve and Check Stages.

Activity 23d. The Baseball and the Ladder.
It is time to look at another situation where the Pythagorean Theorem and Special Right Triangle Theorems can be used to model real life problems. Use the websites below and solve the given problems. In solving the problems, use the table below and fill in the Plan, Understand, Solve and Check stages.

www.pbs.org/wgbh/nova/proof/puzzle/baseball.html - The Pythagorean theorem and Baseball. This website gives a sample problem on the application of Pythagorean Theorem in baseball.

www.pbs.org/wgbh/nova/proof/puzzle/ladder.html - The Pythagorean Theorem and ladders. This website gives a sample problem on the application of Pythagorean theorem.

Problem	
Plan	
Understand	
Solve	
Check	

Problem	
Plan	
Understand	
Solve	
Check	

SUBMIT

Activity 24d. COMPLETE ME!
Given the data/information , compose a problem and show the needed data/information for each stage.

Data/Information	Problem	Understand	Solve	Check	Final Answer
1. A rectangular picture frame Ratio of the length to the width to be 3:1					

<table>
<tr><td>Diagonal of the
frame is 12</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>2. Three pieces of
wood 65 cm, 72 cm,
and 97 cm long</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>3. a 15-feet ladder
m $\angle A = 60$
m $\angle A = 45$</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>4.

</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>5.

</td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

SUBMIT

Questions to Answer:

1. How did you find the activity?

2. Did you experience difficulty in framing a problem and solving it? Explain.

3. What generalization can you make about the Pythagorean Theorem and Special Right triangle Theorems as given in the problems above?

   Write you answer in the box.

4. How can we frame or compose and solve real-life problems involving the theorems discussed?

5. What is the best way to solve problems involving Pythagorean Theorems and Special Right triangle Theorems?

SUBMIT

Activity 25d. **Rate Yourself**.

Please check the appropriate box and finish the statement.

- [ ] I do understand how to apply the Pythagorean Theorem and Special Right Triangle Theorems because ________________.

- [ ] I still have questions about how to apply the Pythagorean Theorem and Special Right Triangle Theorems because ______________

- [ ]

END OF DEEPEN

... this section, the discussion was about the real life application of the Pythagorean theorem and theorems on Special Right triangles. Let us revisit the problems you composed earlier. If there will be changes in your answers, what would it be? Make your revision and give necessary justification and report it through this link. http://www.voki.com/create.php

Present your revised answer and justification in this link by creating your own video.

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.

TRANSFER

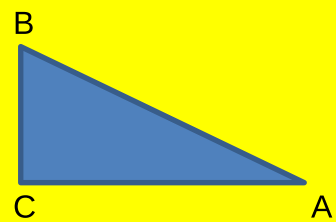
Your goal in this section is apply your learning to real life situations. You will be given a practical task which will demonstrate your understanding.

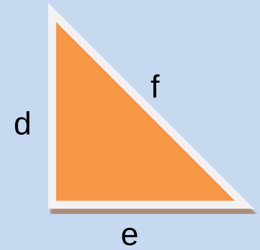
To help you correct your previous knowledge with new information, answer the ARG.

What new realizations do you have about the topic? What new connections have you made for yourself? What helped you make these connections? Go back to the ARG Chart and accomplish the After Discussion Column.

<u>Activity 26d.</u> Agree or Disagree, Revisited! Anticipation Reaction Guide

Direction: Answer the last column of the ARG by clicking on the AGREE or DISAGREE column. Click A if you agree with the statement and click B if you disagree. Please answer all items. Since this ARG was given in the Explore part, if there are changes in the final answers explain the changes/correction made.

Before Discussion		Statements	After Discussion	
Agree	Disagree		Agree	Disagree
A	B	1. The hypotenuse of a right triangle is the longest of all its three sides..	A	B
		2. In a right triangle, the measure of the hypotenuse is equal to the sum the measure of its legs.		
		3. The numbers 3, 4, and 5 represent a Pythagorean triple.		
		4. In an isosceles right triangle, the side opposite the right angle is $\sqrt{2}$ times as long as either of the legs.		
		5. In a 30-60-90 triangle, the side opposite the smallest angle is twice as long as the longest side.		
		6.In rt. △ BCA, the measure of $\overline{AC}$ is x$\sqrt{3}$ if $\overline{BC} = x,$ $\angle A = 30^0$, $\angle B = 60^0$ *and* $\angle C = 90^0$. 		
		7. The Pythagorean theorem is applicable to any triangle.		
		8. **A square mirror 7 ft on each side must be delivered through the doorway 3 ft x 6.5 ft. Can the mirror fit through the doorway?**		

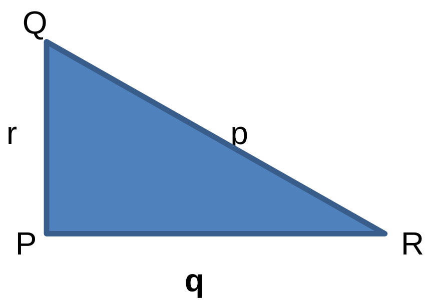
		9. Cathy, Luisa and Morgan are writing an equation to find the length of the third side of a right triangle given below. Only Luisa wrote the correct equation.  Cathy: $f + e = d$ Luisa : $f^2 = d^2 + e^2$ Morgan: $e^2 = d^2 - f^2$		
		 10. The support for a basketball goal forms a right triangle as shown. The length x of the horizontal portion of the support is approximately 2.98 ft.		

To assess if you have mastered the knowledge and skills necessary in doing your performance task, answer the Quiz below.

<u>Activity 27d.</u> QUIZ

Direction: Read each question below, choose the correct answer. Write the letter of your answer on the box provided below.

1. Which of the following equations you could use to find the length of one of the sides of the right triangle?

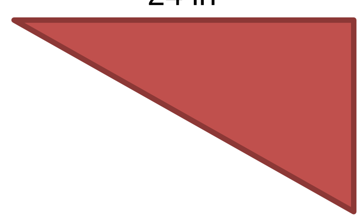


A. $r^2 = p^2 + q^2$ **C.** $p^2 = r^2 + q^2$
B. $p^2 = r^2 - q^2$ D. $q^2 - r^2 = p^2$

2. In item number 1, if q= 12 cm and r = 16 cm, what is the length of p?

A. 20 cm C. 10.58 cm
B. 25 cm D. 16.25 cm

3. What is the perimeter of triangle ABC?
B



A. 26 in C. 34 in

B. 60 in D. 68 in

C

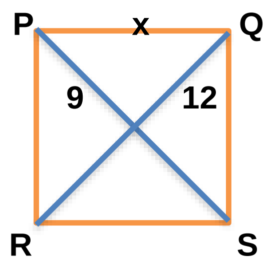
Which of the following is NOT a Pythagorean triple?

A. 3-4-5 B. 12-35-37 C. 3-5-7 D. 6-8-10

4. Why 7-14-16 determines an obtuse triangle?

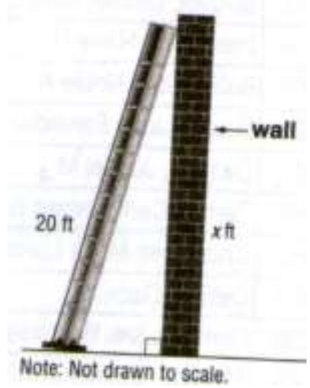
A. Because $7^2 + 14^2 > 16^2$ C. Because $7^2 + 14^2 < 16^2$
B. Because $7^2 + 14^2 = 16^2$ D. Because $7^2 + 14^2 \neq 16^2$

5. In square PQRS, what is the value of x?



A. 12 units B. 15 units C. 18 units D. 20 units

6. According to you company's safety regulations, the distance from the base of a ladder to a wall that it leans against should be at least one fourth of the ladder's total length. You are given a 20-foot ladder to place against a wall at a job site. If you follow the company's safety regulations, what is the maximum distance x up the wall the ladder will reach, to the nearest tenth?

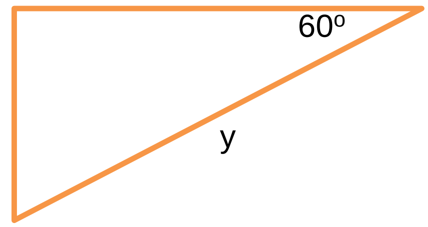


A. 12 ft B. 19.4 ft C. 20.6 ft D. 30.6 ft

7. What is the length ℓ of the hypotenuse of a 45-45-90 triangle if the leg length is 6 centimeters?

A. $\ell = 12\sqrt{2}$ cm B. $\ell = 3\sqrt{2}$ cm C. $\ell = 6\sqrt{2}$ cm D. $\ell = \sqrt{2}$ cm

8. What is the value of y in the given right triangle XYZ?



A. $10\sqrt{3}$ m B. $5\sqrt{3}$ m C. $15\sqrt{3}$ m D. $20\sqrt{3}$ m

9. A square mirror 7 ft on each side must be delivered through the doorway 3 ft x 6.5 ft. Can the mirror fit through the doorway?

A. No, because the door has a maximum of length of 6.5 ft.
B. No, because the measure of the side of the square mirror is more than the length of the door.
C. Yes, because the given dimensions of the door and the mirror represent a Pythagorean triple.
D. Yes, because the dimension of the square mirror can be entered through the doorway in slant position.

WRITE YOUR ANSWERS HERE:

1.	**2.**	**3.**	**4.**	**5.**
6.	**7.**	**8.**	**9.**	**10.**

SUBMIT

Now that you have a deeper understanding of the topic, you are ready to do the performance task of this unit

<u>Activity 28d.</u> Performance Task: Architect in Action

As Architect you have been hired by a new couple to design their dream house. They want to have four bedrooms, a living room, a garage, a stock room , a prayer room, a music room and a library. Each room must be a quadrilateral whose shape is different from the others. The floor area must not be more than 90 square meters. Each part of the house must be tiled using not more than three different quadrilaterals.

The roof must also be composed of quadrilaterals. The house must be elegant but the design must be such that the owner will be able to maximize access to all areas. The house will stand on the lot 12 by 14 meters.

Draw your design and label all dimensions. Then make a model of the house that will fit in a box that is 2 feet by 1.5 feet by 1.5 feet. A final write-up must be presented to owner of house. You also must inform him of the amount of unused area available for landscaping. In addition, you will do an oral presentation to the panel while displaying your design.

Final product:

Your product will be evaluated based on the following criteria: mathematical concept, accuracy of computation, organization of report, presentation of output, quality of output, fluency of presentation, and effort.

Rubric for the Performance Task

	4 Excellent	3 Satisfactory	2 Developing	1 Beginning
Mathe-matical Concept	Demonstrates a thorough understanding of topic and use it appropriately to design and to construct a miniature of the house.	Demonstrates adequate understanding of the concepts and uses it to design and construct a miniature of the house.	Demonstrates incomplete understanding and has some misconceptions	Shows lack of understanding and has severe misconceptions
Accuracy of computation	All computations are correct and are logically presented	The computations are correct.	Generally, most of the computations are not correct	Errors in computations are serious
Organi-zation of the report	Highly organized, flows smoothly, logical and easy to understand.	Adequately organized, sentence flow is generally smooth and logical.	Somewhat cluttered. Flow is not consistently smooth, appears disjointed.	Illogical and obscure. No logical connections of ideas. Difficult to determine the meaning.
Presentation of Output	Ability to verbally present work intelligently, clearly and succinctly.	Ability to verbally present work clearly and succinctly.	Ability to verbally present work adequately.	Poor verbal skills and little participation in class discussions, collaborations, peer reviews and juries, often with unrelated or inappropriate comments or remarks; poor

				team playing skills.
Quality of Output	The output was beautiful and patiently done. Appearance is appealing and shows craftsmanship.	The output was good and adequate. It shows craftsmanship.	The output shows minimal skill, carelessly done; adequate interpretation of the assignment, but lacking finish.	The output was poor, carelessly done, and inadequate.
Fluency of presentation	Fluent, confident and thoroughly explained each point by providing support that contains rich, vivid and powerful detail .	Generally fluent, confident and clearly explained the proposal.	Somewhat hesitant, less confident and failed to explain significant number of points	Hesitant, not confident. Explanation is missing.
Effort	*Completed on time, no modification needed. Effort exerted was beyond the requirement of the task*	*Completed yet more could have been done*	*Project was completed but needs improvement and finishing touches*	*Work was not completed adequately*

To reflect on the learning process you may now answer the reflection log.

Reflection Log- Answer the following questions.

Questions to Answer:

1. How did you find the performance task?
2. What are the important factors did you consider which contributed to the success of the Performance Task?
3. To what extent is your knowledge, skills and understanding of quadrilaterals and triangle similarity have helped you accomplish the task?
4. How did the task help you see the real world use of the topic? In what other real life situations can you apply the learning you've gained in this module?
5. What is the best way to solve problems involving quadrilaterals and triangle similarity?

Write your answers here….

SUBMIT

To summarize your understanding, try to complete the synthesis journal.

Fill in the SYNTHESIS JOURNAL by completing the statements.

The lesson was about ________________. One key idea was __________. This is important because _____________. Another key idea was _________. This is also important because_________.
I was able to think that the best way solve problems involving Pythagorean Theorem and Special Triangle Theorem is ____________________________. This further leads me to develop an essential understanding that ____________ _____________________.

Write your statements inside the box.

SUBMIT

If there are some clarifications, write your questions and email it to your teacher or post it in the discussion forum.

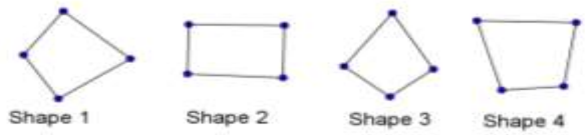
END OF TRANSFER:

In this section, your task was to make a scaled model of a house.

How did you find the performance task? How did the task help you see the real world use of the topic?

You have completed this module. But before you end answer the post assessment.

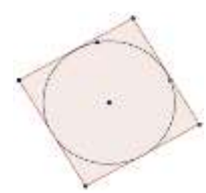
1. Which of the following statements about the shapes below is true?



A. Shapes 1 and 3 are kites if their diagonals intersect at right angles.
B. Shapes 2 and 4 are trapezoids if they have at two pairs of parallel sides.
C. All four shapes are parallelograms if they four sides and one pair of parallel sides.
D. Shapes 1,2 and 3 are parallelograms if they have two pairs of side with equal length.

2. Nora wants to enclose a circular garden with a square fence, as shown at the right.
If the circumference of the circular garden is about 20 meters, which of the following is the approximate length of fencing needed in meters?

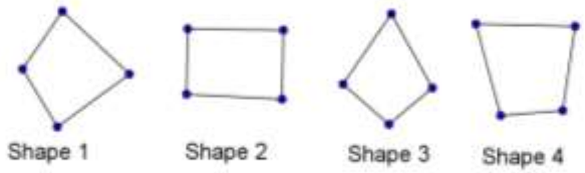
A. 6.4m B. 16m C. 25.5m D. 80π



3. I inherited a magic antique circular table. It is big enough to accommodate six people for dinner. The table is divided in the middle so that leaves can be added to make the table bigger, thus creating a rectangle with two semi-circular ends. Unfortunately the leaves for making the table bigger have been lost. I asked a carpenter to make for new leaves composed of three differently shaped quadrilaterals and a triangle.


Which set of quadrilaterals would be the best choice?
A. {rectangle, square, parallelogram}
B. {trapezoid, rhombus, square}
C. {kite, square, parallelogram}
D. {rectangle, trapezoid, parallelogram}

4. Which of the following statements about the shapes below is true?



A. Shapes 1 and 3 are kites if their diagonals intersect at right angles.
B. Shapes 2 and 4 are trapezoids if they have at two pairs of parallel sides.
C. All four shapes are parallelograms if they four sides and one pair of parallel sides.
D. Shapes 1,2 and 3 are parallelograms if they have two pairs of side with equal length.

5. If you are to design a room in the attic of a Victorian style house which looks like an isosceles triangle in the front and back view, furniture and fixtures are also designed in such a way to maximize the space. The possible thing/s which may happen includes the following;
   1. The floor area is wider than the ceiling.
   2. The ceiling is wider than the floor area.
   3. The bed can be attached to the side wall.
   4. The cabinets on the side wall are rectangular prisms.

A. 1 only
B. 2 only
C. 3 only
D. 4 only

6. In the isosceles trapezoid ABCD, the legs are
A. AB & DC
B. AD & BC
C. AB & BC
D. AD & DC

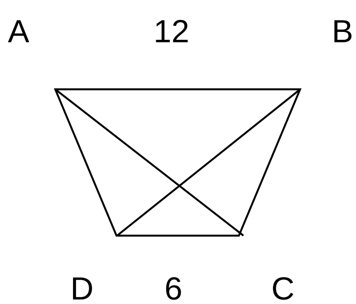


Figure 1

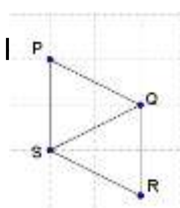
7. In figure 1, if the m∠ADC = 96^0, then the m∠ABC is
A. 84^0
B. 96^0
C. 168^0
D. 192^0

8. AC & BD are diagonals of the isosceles trapezoid ABCD, which of the following statements is TRUE;

I. AC and BD are congruent.
II. ∆ADC & ∆BCD are congruent.
III. ∆ ABC & ∆BAD are congruent
A. I only
B. II only
C. III only
D. I, II & III

9. To provide more space to enhance creativity and balance, in designing a 3-layer round cake the most appropriate diameter of the cake should be
A. 6, 9, 12
B. 7, 9, 12
C. 5, 7, 10
D. 5, 8, 10

10. Given quadrilateral PQRS, PQ ≅ RS, and I
Prove: ∠Q ≅ ∠S



Statement	Reason
1. PQ ≅ RS	Given
2. PQ||RS	Given
3. ∠PRS ≅ ∠QRP	Alternate Interior Angles Theorem
4. PR = PR	Reflexive Property
5. ∆PQR ≅∆RSP	SAS
6. ∠Q ≅ ∠S	?

A. SSS Postulate
B. ASA Postulate
C. Definition of Congruent Angles
D. CPCTC

11. In a proportion if $\frac{a}{b}=\frac{c}{d}$ then which of the following statement is not true?

A. $\frac{b}{a}=\frac{d}{c}$ C. $\frac{a+b}{b}=\frac{c+d}{d}$

B. $\frac{a}{c}=\frac{b}{d}$ D. $\frac{a+d}{b}=\frac{b+c}{d}$

12. If photo 2 is a reduction of photo 1, what can you conclude about the relationships of sides and angles?
A. Corresponding sides and corresponding angles are congruent.
B. Corresponding sides and corresponding angles are not congruent.
C. Corresponding sides and corresponding angles are proportionate.
D. Corresponding sides and corresponding angles are not proportionate.

13. You are required by your teacher to create a map from Manila to Bicol which is about 400 km away from each other. Which of the following scales would you consider such that your map would fit to a short type writing paper?
A. 1 in to 33 km C. 1 mm to 1250 m
B. 1 cm to 14 km D. 1 cm 1200 m

14. Given the figure below with triangles ABG, ACF and ADE and BG, CF and DE parallel with one another, which postulate or theorem would you use to show that the three triangles that make up the racecar window net are similar? Justify your answer.

A. SAS -Because the corresponding two sides and the included angle are congruent.
B. SAS -Because the corresponding two sides are proportionate and the included angle is congruent.
C. SSS –Because the three corresponding sides are proportionate.
D. AA –Because there are at least two angles for each triangle are congruent.

15. A flagpole casts a shadow that is 50 feet long. At the same time, you who are 64 inches tall cast a shadow that is 40 inches long. How tall is the flagpole to the nearest foot?

    A. 12 feet   C. 80 feet
    B. 40 feet   D. 140 feet

16. In a right triangle, what is TRUE about the hypotenuse?
    A. It is always the longest side.
    B. It is opposite the acute angle.
    C. It is always greater than the sum of the other two sides.
    D. The hypotenuse is always equal to the sum of the other two sides.

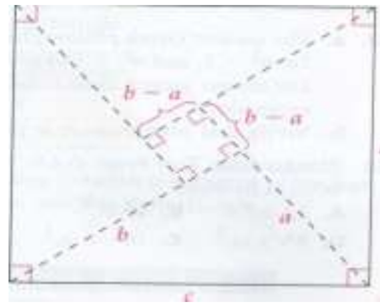
17. Which of the following statements is NOT TRUE?
    A. The altitude to the hypotenuse is the geometric mean between the segments into which it separates the hypotenuse.
    B. The leg of the right triangle is the geometric mean of the hypotenuse and the segment of the hypotenuse adjacent to the leg.
    C. The geometric mean is the product of the hypotenuse and the two legs of a right triangle.
    D. The geometric mean is the square root of the product of the hypotenuse and the segment of the hypotenuse adjacent to the leg.

18. What is the geometric mean of 4 and 18?
    A. 72
    B. $\sqrt{72}$
    C. $6\sqrt{7}$
    D. $6\sqrt{2}$

19. Which of the following lengths of the sides of a triangle determine a right triangle?
    A. 7, 9, 12
    B. 8, 15, 17
    C. 4, 5, 7
    D. All of the above

20. Show the proof that c2 = a2 + b2. What is the correct equation to prove the theorem?

Let m^2 = area of the big square
n^2 = area of the small square
P^2 = area of the right triangle



A. $m^2 = n^2 + p^2$
B. $m^2 = ½ n^2 + 4 p^2$
C. $m^2 = n^2 + 4p^2$
D. $m^2 = ½ n^2 + 2p^2$

GLOSSARY OF TERMS USED IN THIS MODULE:

Parallelogram
A parallelogram is a quadrilateral with opposite sides parallel (and therefore opposite angles equal).

Quadrilateral
A polygon with four sides.

Rectangle
A parallelogram whose angles are all right angles.

Rhombus
Plural rhombi or rhombuses, is a simple quadrilateral whose four sides have the same length. Another name is equilateral quadrilateral, since equilateral means that all of its sides are equal in length.

Square
A rectangle with four equal sides.

Base angles
The angles between a base and its adjacent side.

Bases
The parallel sides of a trapezoid.

Scalene trapezoid
A trapezoid with no congruent sides.

Isosceles trapezoid
A trapezoid with a pair of congruent legs.

Kite
A quadrilateral with exactly 2 pairs of congruent adjacent sides
legs of the trapezoid – the non-parallel sides of a trapezoid
midline/midsegment – a segment joining the midpoints of the legs of a trapezoid.

Perpendicular bisector
A line/segment which divides a segment into equal parts and form right angles.

Perpendicular lines
Lines which intersect and form right angles.

Trapezium (US)
A quadrilateral with no parallel sides.

Trapezoid (US)
A quadrilateral with exactly one pair of opposite sides parallel.

Similar Triangles
Similar triangles are triangles that have the same shape but possibly different size. In particular, corresponding angles are congruent, and corresponding sides are in proportion.

AA Similarity Postulate
The angle-angle (AA) similarity test says that if two triangles have corresponding angles that are congruent, then the triangles are similar. Because the sum of the angles in a triangle must be 180°, we really only need to know that two pairs of corresponding angles are congruent to know the triangles are similar.

SAS Similarity Theorem
The side-angle-side (SAS) similarity test says that if two triangles have two pairs of sides that are proportional and the included angles are congruent, then the triangles are similar.

SSS Similarity Theorem
The side-side-side (SSS) similarity test says that if two triangles have all three pairs of sides in proportion, the triangles must be similar.

Right Triangle Similarity
When you drop an altitude from the right angle of a right triangle, the length of the altitude becomes a geometric mean. This occurs because you end up with similar triangles which have proportional sides and the altitude is the long leg of 1 triangle and the short leg of the other similar triangle.

Geometric Mean
The geometric mean between two positive numbers a,b is the number x such that x = $\sqrt{ab}$.

Proportion

A proportion is an equation written in the form $\frac{a}{b} = \frac{c}{d}$ stating that two ratios are equivalent.

In other words, two sets of numbers are proportional if one set is a constant times the other

REFERENCES AND WEBSITE LINKS USED IN THIS LESSON:

A. BOOKS AND OTHER PRINTED MATERIALS:

Henderson, David W. and Daina Taimina. Experiencing Geometry. Third Edition. Pearson Education, Inc., Prentice Hall, New Jersey, 2005

Bass, Laurie E., et al. Geometry. Pearson Education South Asia Pte Limited. Pearson Prentice Hall. 2005.

Boyd, Cindy J., et al.Geometry.Oklahoma Edition. McGraw-Hill Companies, Inc. 2005.

Oronce, Orlando A. & Mendoza, Marilyn O. E- math Geometry. Manila. Rex Book Store, Inc. 2010.

Geometry Based on the 2002 BEC: 2002, The Bookmark, Inc.; Sr. Iluminada C. Coronel, Antonio C. Coronel

ALGEBRA 1 Equations, Graphs ,Applications: 2004, mcDouggal Little; Ron. Larson, Laurie Boswell, Timothy Kanold, Lee Stiff

GEOMETRY; 2005; The McGraw-Hill Companies, Inc.; Cindy Boyd, Jerry Cummins, Carol Malloy, John Carter, Alfinio Flores
GEOMETRY; 2006 by Pearson Education, Inc; Laurie Bass, Art Johnson
Integrated Mathematics Course 2; 1997, by Glecoe/McGraw-Hill; Douglas Bumby, Richard Klutch, Donald Collins, Elden Egbers

GEOMETRY 1980. Academe Publshing House; Myrna A. Cabansay

GEOMETRY, Tools for a Changing World; 1998, Prentice Hall; Laurie Bass, Basia Hall, Art Johnson, Dorothy Wood
GEOMETRY, Concepts and Applications; 2005 by The McGraw-Hill Companies, Inc; Jerry Cummins, Tim Kanold, Margaret Kenny, Carol Maloy, Yvonne Mojica

DYNAMIC MATH GEOMETRY; 2009, Phoenix Publishing House, Inc.; Edna Zuela, Luis Allan Melosantos

NEXT CENTURY MATHEMATICS Geometry; 2008 by Phoenix Publishing House, Inc; Fernando Orines, Jesus Mercado, Josephine Suzara

Mathematics 8; 2006 by Nealson; Bernard Beales, Maria Bodiam, Doug Duff, et. Al.

B. WEBSITES:

http://www.mathsisfun.com/geometry/quadrilaterals-interactive.html.
An interactive resource to study properties of quadrilaterals.

http://www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.SHAP&ID2=AB.MATH.JR.SHAP.SHAP&lesson=html/video_interactives/classifications/classificationsInteractive.html
This interactive mathematics resource explores the properties of triangles, quadrilaterals and regular polygons and allows students to classify shapes based on their properties. The resource includes print activities, solutions, learning strategies, and a shape guessing game.

http://www.onlinemathlearning.com/properties-of-polygons.html
This site contains video lessons on the properties of trapezoids and kites.

http://www.youtube.com/watch?v=suiDK61jAc8
Gives a video on the Golden Triangle in real life.

http://goldenratiorocks.wordpress.com/golden-ratio-real-life-examples/
Gives examples o0f the use of Golden Ratio in real life.

http://www.youtube.com/watch?v=i2a4B4M5L1M
Watch how a rhombus can be used to make a flexible paper structure.

http://www.youtube.com/watch?v=S-nNib5HzUA
Gives another type of flexible paper structure.

http://www.youtube.com/watch?v=p9xKxEV1FkY
Demonstrate how to make an Origami Fireworks making use of rhombi shapes.

http://www.youtube.com/watch?v=knMEBSXM6WU
Demonstrate how to make use of rhombi to make an origami flexiball.

http://rhombusspace.blogspot.com/
Gives examples of art pieces using rhombi.

http://www.ysjournal.com/article.asp?issn=0974-6102;year=2009;volume=2;issue=7;spage=35;epage=46;aulast=Khair
Gives examples of the uses of rhombi in architecture

www.photoxpress.com/photos-skyscraper-lozenge-rhombus-4723361
Gives photos depicting how rhombi are used in architecture

http://www.shodor.org/interactivate/activities/Tessellate/
An interactive site for tessellation. Option for shape, color and changing corners and edges are available,

http://www.ixl.com/math/geometry/properties-of-trapezoids
This site contains interactive exercises about trapezoids and their theorems.

http://www.mathopolis.com/questions/q.php?id=621&site=1&ref=/quadrilaterals.html&qs=621_622_623_624_763_764_2128_2129_3230_3231
This site contains a quiz about quadrilaterals.

http://ph.images.search.yahoo.com/search/images;_ylt=A2oKiavkUe5SZRsAAji0Rwx.?p=real-life+applications+of+trapezoids+and+kites&ei=utf-8&iscqry=&fr=sfp
This site contains pictures of real-life applications of trapezoids and kites.

http://www.google.com/url?sa=i&rct=j&q=designs%20using%20different%20triangles%20and%20quadrilaterals&source=images&cd=&cad=rja&docid=ZHtUpKb7CtSd8M&tbnid=_4BboANCoJ0G_M:&ved=0CAMQjhw&url=http%3A%2F%2Fwww.mathpuzzle.com%2FAug52001.htm&ei=bXHNUrCeBsyxrgeVk4HoBA&psig=AFQjCNHLO5aKfHFDuC4OQ-24M5oWKkAA9Q&ust=1389277514813593
This site contains images of quadrilaterals particularly kites and trapezoid.

http://www.google.com/url?sa=i&rct=j&q=pictures%20of%20beams%20of%20hanging%20bridge&source=images&cd=&docid=&tbnid=&ved=0CAMQjhw&url=&ei=Hm3NUpDSL4qJrAeW8IBQ&psig=AFQjCNHQ-BrH9gtvfkQPYCgllpXcyCtf-Q&ust=1389280206982361
This site contains different ways of arranging trapezoid and quadrilateral in an architectural design.

http://www.onlinemathlearning.com/properties-of-polygons.html
This site contains video lessons on the properties of trapezoids and kites.

http://www.ixl.com/math/geometry/properties-of-trapezoids
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http://www.mathopolis.com/questions/q.php?id=621&site=1&ref=/quadrilaterals.html&qs=621_622_623_624_763_764_2128_2129_3230_3231
This site contains a quiz about quadrilaterals.

http://ph.images.search.yahoo.com/search/images;_ylt=A2oKiavkUe5SZRsAAji0Rwx.?p=real-life+applications+of+trapezoids+and+kites&ei=utf-8&iscqry=&fr=sfp
This site contains pictures of real-life applications of trapezoids and kites.

https://www.google.com.ph/#q=SIMILAR+PICTURES
This site contains picture of similarity.

http://www.youtube.com/watch?v=D8dA4pE5hEY
http://www.youtube.com/watch?v=2d578xHNqc8
http://www.youtube.com/watch?v=G8qy4f7GKzc
These sites contain videos which explain the concepts of proportion with step by step procedure on how to solve problem related to the topic.

http://www.softschools.com/quizzes/math/proportion_word_problems/quiz3766.html
This site contains interactive quiz about proportion.

http://www.youtube.com/watch?v=EbN_tDggldA
This video contains detailed discussion about the proving of similar triangles (AA, SAS, SSS).

http://www.youtube.com/watch?v=QCyvxYLFSfU
This video contains detailed discussion about the proving of similar triangles (Right Triangle Similarity Theorem.)

http://www.youtube.com/watch?v=PXBFDBmBP0I
This website contains video which explains the step by step procedure in solving problem related to similar triangles.

http://www.regentsprep.org/regents/math/geometry/MultipleChoiceReviewG/Triangles.htm
This website contains interactive quiz about triangle similarity.

http://www.classzone.com/etest/viewTestPractice.htm?testId=4545
This website contains interactive quiz about triangle similarity.