## MATHEMATICS 9

## MODULE 3: GEOMETRY PARALLELOGRAM AND TRIANGLE SIMILARITY

## MODULE INTRODUCTION AND FOCUS QUESTION(S):



Have you ever wondered how engineers make structural designs? How do artist choose shapes in their art pieces? In this module you will try find answers to these questions. You will acquire knowledge and skills needed to solve problems involving shapes and geometric relationships. You will learn how to name and classify quadrilaterals. You will also verify some of the properties of parallelograms, rectangles, rhombuses, squares, trapezoids and kites and use these properties to differentiate one from the other. It is hoped that you will acquire deep understanding of the lesson to enable you to determine the best way to solve problems involving quadrilaterals and triangle similarity.

A quadrilateral can be classified into many different forms, but in this learning module you will focus on the most important family of quadrilaterals-trapezoids, parallelograms and kites, along with their subshapes. You will explore uses of quadrilaterals in real life. Make sure to write down insights gained on how understanding of quadrilaterals can be effectively used in real life as you do the various learning tasks in this module.

To be able to succeed in this module you need to ensure that you have a good understanding of what polygons are, the relationship been sides and angles as well as concepts of parallel and perpendicular lines.
This module contains four lessons. Each lesson includes applications of the shapes to real life situations for better appreciation of the topics.
Remember to search for the answer to the following question: What is the best way to solve problems involving quadrilaterals and triangle inequality?

## MODULE LESSONS AND COVERAGE:

In this module, you will examine this question when you take the following lessons:

| Lesson No. | Title | You'll learn to... | Estimated Time |
| :---: | :---: | :---: | :---: |
| Lesson 1 | Parallelogram | Identify quadrilateral that are parallelogram, trapezoid and kite <br> Determines the conditions that guarantee a quadrilateral is a parallelogram. <br> Use properties to find measures of angles, sides and other quantities involving parallelograms <br> Prove theorems on different kinds of parallelograms. <br> Prove the Midline Theorem |  |
| Lesson 2 | Trapezoid and Kites | Prove theorems on trapezoids and kites <br> Describe special trapezoid and their properties <br> Demonstrate uses of quadrilaterals in real life. <br> Solve problems involving parallelograms, trapezoids and kites. |  |
| Lesson 3 | Triangle Similarity | Describes a proportion. <br> Applies the fundamental theorems of proportionality to solve problems involving proportions <br> Illustrates similarity of figures. |  |


|  |  | Proves the conditions for similarity of triangles <br> a. SAS Similarity Theorem <br> b. SSS Similarity Theorem <br> c. AA Similarity Theorem <br> d. Right Triangle Similarity Theorem |  |
| :---: | :---: | :---: | :---: |
| Lesson 4 | Special Right Triangle | Illustrates Special Right Triangle Theorem. <br> Applies the theorems to show that give triangles are similar. <br> Proves the Pythagorean Theorem. <br> Solves problems that involve triangles similarity and right triangles. |  |

## Concept Map of the Module

Here is a simple map of the above lessons you will cover:


## Expected Skills

To do well in this module, you need to remember and do the following:

1. Carefully read the module and do the activities neatly and accurately.
2. Break tasks into manageable parts.
3. Complete all activities even if you may not be asked to hand these in, but they will help you learn the material.
4. Keep copies of all accomplished activities. These are needed to assess your progress and for grading.
5. If you are having problems, do NOT wait to request help. The longer you wait the bigger the problem becomes!
6. Form study groups if possible.

## Module 3: Geometry, Parallelogram and Triangle Similarity

PRE-ASSESSMENT


Let's find out how much you already know about this module. Click on the letter that you think best answers the question. Please answer all items. After taking this short test, you will see your score. Take note of the items that you were not able to correctly answer and look for the right answer as you go through this module.

1. Which statement best differentiates squares from the rectangles?
A. Squares must have four $90^{\circ}$ angles, rectangles do not have all $90^{\circ}$ angles.
B. Squares have two sets of equal sides, rectangles have only one pair of equal sides.
C. Squares have four equal sides. Rectangles have two pairs of equal opposite sides.
D. Squares have the diagonals that bisect each other. Rectangles have diagonals that are perpendicular.
2. When comparing a trapezoid and a kite, one similarity is:
A. They both have congruent diagonals.
B. They both have at least one set of parallel sides.
C. They both have four congruent sides.
D. They both have four sides
3. Points $(2,1),(4,3)$ and $(3,4)$ are vertices of a quadrilateral. What should be the coordinates of the fourth point to form a parallelogram?
A. $(1,2)$
B. $(2,4)$
C. $(1,4)$
D. $(1,3)$
4. Suppose we are told two things about a quadrilateral: first, that it is aparallelogram, and second, that one of its interior angles measures $60^{\circ}$. The measure of the angle adjacent to the $60^{\circ}$ angle is
A. $60^{\circ}$
B. $90^{\circ}$
C. $120^{\circ}$
D. impossible to know without more information
5. $A B C D$ is a rhombus. How will you prove that its diagonals are perpendicular?
A. Show $\angle \mathrm{AXB} \cong \angle \mathrm{CXB}$, Since $\overline{A C}$ and $\overline{B D}$ intersect to form congruent adjacent angles, $\overline{A C} \perp \overline{B D}$.
B. Show $\triangle A X B \cong \triangle C X B$. Since congruent parts of congruent triangles are congruent, then, $\overline{A C} \perp \overline{B D}$.

C. Show $\triangle B A D \cong \triangle B C D$. Since congruent parts of congruent triangles are congruent, then $\angle \mathrm{AXB} \cong \angle \mathrm{CXB}$, then $\overline{A C} \perp$ $\overline{B D}$.
D. Show $\triangle \mathrm{AXB} \cong \triangle C X D$. Since $\angle \mathrm{AXB} \cong \angle \mathrm{CXD}$, then $\overline{A C} \perp \overline{B D}$.
6. A carpenter accurately measure four boards to frame a door: two sides of 8 inches and a top and bottom of 40 inches. What else should a carpenter do to ensure that it will fit a rectangular door?
A. Set one side piece at right angle to the floor piece.
B. Ensure that the sides parallel.
C. Connect the opposite vertices with a pieces of woods of equal length forming diagonals.
D. Connect the two sides at their midpoints with a piece of wood.
7. Maria knows the following information about quadrilateral $B E S T: B T=E S$, $\overline{T S} / / \overline{B E}$, and $\angle \mathrm{T} \cong \angle S$. Maria concludes that BEST is an isosceles trapezoid. Why can't Maria make this conclusion?
A. $\triangle B E S T$ is a rectangle
B. $\square B E S T$ is a square
C. $\square$ There is not enough information
D. Length of $\overline{B T}$ is and $\overline{E S}$ not known.

8. A Mothers' Club is making a quilt consisting of squares with each side measuring 40 cm . The quilt has five rows and 6 columns and with cord edging. How many meters of cord should the Club buy for the quilt?
A. 880
B. 88
C. 9
D. 8

9. If you are to design a room in the attic of a Victorian style house which looks like an isosceles triangle in the front and back view whose ceiling is parallel to the floor, furniture and fixtures are also designed in such a way to maximize the space. The possible things which may happen includes the following;
10. The floor area is wider than the ceiling.
11. The ceiling is wider than the floor area.
12. The bed can be attached to the side wall.
13. The built-in cabinets on the side wall are rectangular prisms.
A. 1 only
B. 2 only
C. 1 and 3 only
D. 1 and 4 only
14. One liter of a certain paint can cover about 80 square feet. I want to paint a circle with a diameter of 28 ft . How many liters of paint will I buy?
A. 7
B. 8
C. 196
D. 616
15. What is the longest stick that can be placed inside a box with inside dimensions of 24 inches, 30inches, and 18 inches?
A. 38.4 inches
B. 30
C. $30 \sqrt{2}$ inches
D. Can no be determined

16. A triangular plot of land has boundary lines 45 meters, 60 meters, and 70 meters long. The 60 meter boundary line runs north-south. Is there a boundary line for the property that runs due east-west?
A. Yes. It's the 45 meters boundary.
B. Yes. It's the 70 meters boundary.
C. No. The plot is not a right triangle.
D. Cannot be determined.
17. Which of the following supports statement 3 in the proof?

Figure:


| Statements | Reasons |
| :--- | :--- |
| 1.Draw $C D \perp A B$. | 1. There is only one and only one line perpendicular <br> to a given line form an external point. |
| 2. CD is the altitude to AB. | 2. Definition of Altitude |
| $3 . h^{2}=m n$ <br> $a^{2}=m(m+n)$ <br> $b^{2}=n(m+n)$ | 3. |
| $4 \cdot a^{2}+b^{2}=m(m+n)+n(m+n)$ | 4. Addition Property of Equality |
| $5 \cdot a^{2}+b^{2}=(m+n)(m+n)$ | 5. Factoring |
| $6 \cdot m+n=c$ | 6.Definition of Betweeness |
| $7 \cdot a^{2}+b^{2}=(c)(c)$ <br> $a^{2}+b^{2}=c^{2}$ | 7.Addition Property of Equality <br> Simplifying |

A. Geometric Mean Theorem
B. SSS Similarity Theorem
C. SAS Similarity Theorem
D. AA Similarity Theorem
14. A new pipeline is being constructed to re-route the water flow around the exterior of the City Park. The plan showing the old pipeline and the new route is shown below. About how many extra miles will the water flow once the new route is established?
A. 24
B. 68
C. 92
D. 160

15. The dimensions of a rectangular doorway are 200 cm by 80 cm . Can a circular mirror with a diameter of 210 cm be carried through the doorway?
A. No. Diameter is longer than the length of the door.
B. Yes. The diameter is less than 200 cm by 80 cm .
C. Yes. Hold it along the diagonal of the door.
D. Additional information is needed to answer the question.
16. A baseball diamond is a square. The distance fromthe base to base is 90 ft . How far does the second baseman throw a ball to home plate?
A. $6 \sqrt{5} \mathrm{ft}$
B. 180 ft
C. $90 \sqrt{2} \mathrm{ft}$
D. $45 \sqrt{5} \mathrm{ft}$

17. Makee would like to have a new corner cabinet for his room. He is trying to figure out how to design it so that his TV which is 30 " high, 34 inches wide and 20 " deep would fit. He wants the new cabinet to be the same length on each side (along the two walls). How long should each side of the cabinet be?
A. 84 inches
B. 64 inches
C. 42.2 inches
D. 37 inches
18. Which triangles must be similar?
A. Two obtuse triangles
B. Two scalene triangles with congruent bases
C. Two right triangles
D. Two isosceles triangles with congruent
19. Which of the following best describes the triangles at the right?
A. Both are similar and congruent
B. Similar but not congruent.
C. Congruent but not similar
D. Neither similar nor congruent.

20. $\overline{S M} \| \overline{E L}$. Which of the following guarantee that $\triangle$ SME is similar to $\triangle \mathrm{CDE}$ ?
A. SAS Similarity
B. SSS Similarity
C. AA Similarity
D. None of the above


## SUBMIT

Did you do well in the pretest? Are there items that you were not sure of your answers? You can go back to those items as you gain new knowledge and skills. Now, proceed to Lesson 1. Take time to list ideas and concepts in the lessons.

## Lesson 1: Quadrilaterals

In this lesson you will learn the following:

1. Describe the properties of quadrilaterals.
2. Determine the conditions that guarantee a quadrilateral is a parallelogram.
3. Use properties to find measures of angles, sides and other quantities involving parallelograms
4. Prove theorems on different kinds of parallelograms.
5. Demonstrate uses of quadrilaterals in real life.
6. Solve problems involving parallelograms.

## EXPLORE



What are quadrilaterals? How do we use them? Where do we see them in real life? Quadrilaterals are everywhere. We see them on signs, buildings, work of art, books, computers, floor designs and many more. A polygon with four sides is a quadrilateral. In this lesson you will know more about quadrilaterals. Start by doing Activity 1.

## Activity 1a: Map It

Arrange the boxes to form a concept map that will show the relations between and among the different shapes.


Your concept map here.

## SUBMIT

Questions to Answer:

1. What is the basis of your arrangement?
2. How can you differentiate a shape from the other?
3. How can we use the properties of quadrilaterals to create designs and solve problems?

| YOUR ANSWERS |  |  |  |
| :---: | :---: | :---: | :---: |
| SUBMIT <br>  |  |  |  |

In Activity 1, you made a concept map on your initial understanding of the different quadrilaterals. In the next activity you will see actual use of these shapes in real life. Take note of some questions that you might want to ask from your teacher especially on the reason why a shape is used in a particular situation.

## Activity 2a: Quadrilaterals Everywhere

Below are some pictures of quadrilaterals in real life. Can you identify them? Can you give possible reasons why the shape is such?


SUBMIT

You see why you need to have a good knowledge of quadrilaterals. Identifying these shapes and the understanding the properties will enable you to use appropriately these shapes.

Can you cite other examples of the uses of the quadrilaterals in real life?


## SUBMIT

In the next activity you will learn how to classify quadrilaterals. Doing so, will help you identify similarities and differences between and among quadrilaterals.

At the end of this module you are expected to make a model that will demonstrate the best solution to a problem. Challenge yourself to finish the module in not more than two weeks.

## Activity 3a. To Be or Not To be

Given a set of quadrilaterals, group them based on their common features by dragging them in the column for examples. Name and describe each group in terms of their identifying characteristics in 5 minutes.


| Type of <br> Quadrilaterals | Definition | Examples |
| :---: | :---: | :---: |
|  |  |  |
|  |  |  |
|  |  |  |



Questions to Answer:

1. What are quadrilaterals?
2. How are the different types of quadrilaterals named?
3. How are quadrilaterals denoted?
4. Are there other ways of grouping quadrilaterals? How?
5. What are the properties of each quadrilateral?
6. Why is there a need to know the properties of these quadrilaterals?

| YOUR ANSWERS |
| :--- |
|  |
|  |
|  |
|  |

## SUBMIT

Now, you have a clear idea of what quadrilaterals are. In the next activity, you will build up your knowledge and skills in parallelograms by mastering their properties.

## End of EXPLORE:



You just have classified quadrilaterals. Let us now strengthen that insight by doing the succeeding activities. What you will be learning in this section will help you perform well in your final performance task which will challenge you to use what you know to create a design that will help you use the materials efficiently or maximize the use of space.

## FIRM - UP



How do you know a shape is a quadrilateral? Why are quadrilaterals important? How are they used in real life? What is the best way to solve problems involving quadrilaterals? Discover the answers to these questions as you do the next tasks.

Your goal in this section is to have a good understanding of the properties of quadrilaterals. Properties of quadrilaterals will enable you to differentiate one from the other and use these shapes efficiently.

## Activity 4a. Show and Tell

Access http://www.mathsisfun.com/geometry/quadrilaterals-interactive.html. Click on angles and diagonals to answer the questions below.


Drag the dots, observe the measures of the angles and the length and positions of the diagonals. Use your discovery to answer the following questions.

## 2 Questions to Answer:

1. Given a quadrilateral $A B C D$, when are its side said to be opposite?

Consecutive?
2. What angle is opposite $\angle \mathrm{A} ? \angle \mathrm{~B} ? \angle \mathrm{C} ? \angle \mathrm{D}$ ?
3. Which are consecutive angles?
4. What is the sum of the interior angles of a quadrilateral? How did get this number?
5. Can an interior angle of a quadrilateral measure more than $180^{\circ}$ ? Draw a sample of this quadrilateral.

Your answers here.
$\square$

## SUBMIT

## Activity 5a. Try This Out

Use your answers to the questions above to find the measure of the indicated angles below. Drag the points to show the measures of the four angles. Are your answers correct?
a) $m \angle E$

b) What is $x$ ?


Now summarize the properties of a quadrilateral by completing the phrase" polygon is a quadrilateral if ..." considering the following:

|  | A polygon is a quadrilateral if ... |
| :--- | :--- |
| Sides |  |
| vertices |  |
| Interior angles |  |

## SUBMIT

## Activity 6a. Check your Understanding

Do self-assessment by accessing the website
http://www.learnalberta.ca/content/mejhm/index.html?l=0\&ID1=AB.MATH.JR.SHAP\&ID
2=AB.MATH.JR.SHAP.SHAP\&lesson=html/video interactives/classifications/classificati onsInteractive.html


Click on new and answer the items. Click check when you are done.
Did you do well in the quiz? Are there questions that you would like to ask? Post it in the Discussion Forum.

Before you proceed to the next activity do a self- assessment. This help you identify areas that you are doing well and areas that you need to work harder.

## Activity 7a. Shaping Up Review



Your answers here.

## SUBMIT

From the previous activity, you tried to classify quadrilaterals into three groups: parallelogram, trapezoid and kite. How can you differentiate one from the other? In what way are they similar? How can knowledge on the properties of these shapes help you determine their best use?

### 1.2 Parallelograms

In this section you will firm up your knowledge on quadrilaterals by identifying special quadrilaterals known as parallelograms. Questions that you need to answer in this section are: When is a quadrilateral called a parallelogram? How can you find measures of angles, sides and other quantities involving parallelograms? How are parallelograms used in real life? Write down your answers to these questions as you do the next activities.

## Properties of Parallelograms

In the next activity you will focus on quadrilaterals which are parallelograms. Here you will put to use your knowledge on parallel lines and their transversals to identify properties of parallelograms.

## Activity 8a. Draw and Discover



Use the diagram to help you follow the instructions below. You are expected to identify properties of quadrilateral formed by parallel lines in terms of the relationships of its sides, angles and diagonals.


(?)Questions to Answer:

1. Draw two horizontal lines passing through points $A$ and $B$ and name them $\ell_{1}$ and $\ell_{2}$.
2. Draw a transversal $t_{1}$ passing through point A .
3. Draw a transversal $t_{2}$ passing through B parallel to the first transversal. How do you know the transversals are parallel?
4. Label the other intersections C and D .
5. Measure the length of the two opposite sides AC and DB. What can you say about the two opposite sides? Do the same for sides AD and CB. What theorem on parallel lines supports your answer?
6. Now, cut the parallelogram and duplicate by tracing the cut out figure.
7. Using a straight edge connect the two opposite vertices A and B and cut along this line.
8. Superimpose the two triangles such that corresponding parts coincides. Does one triangle completely cover the other triangle? Are the corresponding sides congruent? Are the corresponding angles congruent? Are the two triangles congruent?
9. Do the same for the duplicate parallelogram connecting CD and cutting along this diagonal.
By superimposing the cut out triangles, can you say that the triangles formed are congruent? What conclusion can you make about the diagonals of the parallelogram?
10. Using the cut out figures show that diagonals of a parallelogram bisect each other. Describe your process.
11. With the help of the same cut out figures answer the question: Are the diagonals of a parallelogram equal?
12. When is a quadrilateral said to be a parallelogram? Summarize your findings by completing the table below:

Write your answers here...

SUBMIT

|  | If a quadrilateral is a parallelogram, <br> then... | EXAMPLE |
| :--- | :--- | :--- |
| SIDES |  |  |
| ANGLES |  |  |
|  |  |  |
| DIAGONALS |  |  |

## SUBMIT

Now check your findings by going back to the interactive site: http://www.mathsisfun.com/geometry/quadrilaterals-interactive.html. Click on parallelograms and angles. Drag the points to verify your findings. Click on diagonals. Drag points and verify your findings. Is there a need to revise your initial findings. Go back to your summary table and finalize your answers. Can you draw other shapes of parallelograms?

Now that you know the properties of parallelograms, put it to use by answering the next activity.

## Activity 9a. Parallelograms Challenge

Use the properties of parallelograms to answer the following:

1. Find the measures of the other angles of parallelogram CDEB.

2. In the figure below, find $m \angle A, m \angle C, m \angle D, C D$, and $A D$.

3. Find $m \angle M O E, m \angle N O E$, and $m \angle M Y O$.

$\mathrm{m} \angle \mathrm{MON}=120^{\circ}$

Write your answers here...

## SUBMIT

Were you able to answer all the items? Are there questions that you would like to ask? Post it in the Discussion Forum.

Now rate your progress in understanding of the lesson based on your performance in Activities 6.

| Ineed to shine | Leveling up! | Good job! | Excellent! |
| :--- | :--- | :--- | :--- |
| my star! |  |  |  |

You have learned in the previous section that parallelograms are special type of quadrilaterals. Are there special types of parallelograms? How do you differentiate one from the other? You will continue to build up your understanding of parallelograms in the succeeding activities.

## END OF FIRM - UP

In this section you have learned that parallelograms are quadrilaterals. What make them special quadrilaterals are their unique properties: they have two pairs of opposite sides that are parallel, opposite angles are congruent and consecutive angles are supplementary.

In the next section you will deepen your knowledge on parallelograms and study special parallelograms. When are parallelograms said to be special?

## DEEPEN

Now you know that a quadrilateral can be a parallelogram. In the next activity, you will put to test what you know about parallelograms. Doing this activity will deepen your knowledge on parallelograms and help you solve problems involving quadrilateral. After this section answer the question: What is the best way to solve problems involving quadrilaterals? Then compare your answer to your previous answer. Do you see an improvement to your answer?

## Classifying Parallelograms

## Activity 10. What Makes Them Special?



Study the special quadrilaterals above. Like markings denote congruent sides or angles and arrows denote parallel sides. When is a parallelogram a rectangle? A square? A rhombus? Summarize your answers in the table below.

| Special <br> Parallelogram | Definition | Draw an illustrative example. <br> If ABCD is a ... |
| :--- | :---: | :---: |
| Rectangle |  |  |
| Square |  |  |
| Rhombus |  |  |

## SUBMIT

Look at your definitions of the three quadrilaterals. Will you be able to compare and contrast a rectangle from the square? from a rhombus? Are there other ways of representing these parallelograms?

The rectangle, rhombus, and square have a few other special properties. First, remember that these figures are all parallelograms; therefore, they possess the same properties as any parallelogram. However, because these figures are special parallelograms, they also have additional properties.

What are the properties of special quadrilaterals? How can you use these properties to differentiate one from the other? How can these properties help you identify the best way to solve problems involving quadrilaterals? You will discover these special properties by doing an investigation.

### 1.2.1 Rectangles

In this section you will deepen your knowledge on special parallelogram called rectangle. What makes it special? How is it different from the other types of parallelograms? Discover answers to these questions by doing the next activity.

## Activity 11a. When Am I a Rectangle?

Read and answer the questions below about rectangles.

A rectangle is a parallelogram with equal angles What is the full meaning of this definition?
If the rectangle is to be equiangular, what is
 the measure of each angle?
Draw the diagonals. What relationship exist between the diagonals?

Summarize your answer in the table below.

|  | If a parallelogram a rectangle, then... | EXAMPLE |
| :--- | :--- | :---: |
| SIDES |  |  |
| ANGLES |  |  |
|  |  |  |
| DIAGONALS |  |  |
|  |  | SUBMIT |
|  |  |  |

Verify your answer by accessing the same website. This time, click rectangles and angles. Click on diagonals. Observe. Do the measures of the interior angles change when you increase or decrease the length of the rectangle?

Now let us go back to our previous activity about quadrilateral let us see if you have now a better understanding.

## Lesson 1.1 Quadrilaterals

What are quadrilaterals? How do we use them? Where do we see them in real life?
Quadrilaterals are everywhere. We see them on signs, buildings, work of art, books, computers, floor designs and many more. A polygon with four sides is a quadrilateral. In this lesson you will know more about quadrilaterals. Start by doing the next Activity.

Activity 12a. Map It
Arrange the boxes to form a concept map that will show the relations between and among the different shapes.


Your concept map here.
$\square$

Questions to Answer:

1. What is the basis of your arrangement?
2. How can you differentiate a shape from the other?
3. How can we use the properties of quadriaterals to create designs and solve problems?

Write your answers here....

## SUBMIT

Proving that the Diagonals of Rectangles are Congruent

## Activity 13a. Are we Congruent?

Given: Rectangle FLAG.
Prove: $\overline{F A} \cong \overline{L G}$


Do this by supplying the reasons for each of the given statement.

| We know that |  |
| :--- | :--- |
| $\mathrm{FL}=\mathrm{GA}$ |  |
| $\angle \mathrm{LFG} \cong \angle \mathrm{FGA}$ |  |
| $\mathrm{FG}=\mathrm{FG}$ |  |
| $\triangle \mathrm{GFL} \cong \triangle \mathrm{GAL}$ |  |
| $\mathrm{GL}=\mathrm{FA}$ |  |

## SUBMIT

What did you realize about the diagonals of a rectangle? Write it down.
Diagonals of rectangles are $\qquad$ .

## SUBMIT

Now probe deeper into the properties of rectangles. You have discovered that diagonals of rectangles are congruent. Now prove that the diagonals bisect each other.

## Proving Properties of Rectangles

## Activity 14a. Mutually Bisecting Diagonals?

$A B C D$ is a rectangle. Prove the diagonals of a rectangle bisect each other.


| We know that | Because... |
| :--- | :--- |
| $A B / / D C$ |  |
| $\angle B A C \cong \angle A C D$ |  |
| $\angle B D C \cong \angle A B D$ |  |
| $\triangle A O B \cong \triangle D O C$ |  |
| $A O=O C$ |  |
| $D O=O B$ |  |
| Conclusion: |  |
|  | SUBMIT |

Now generalize the properties of a rectangle by completing the table below.

|  | A parallelogram is a rectangle if $\ldots$ |
| :--- | :--- |
| Sides |  |
| Interior angles |  |
| diagonals |  |
|  |  |

Now that you have been writing proofs for the different properties of parallelograms, reflect on your experiences of learning to write proofs.

## Activity 15a. Auto-math-ography

What does it mean to write a mathematical proof? Write in few sentences your explanation.

Your explanation here


## SUBMIT

Now use the properties to answer the exercises below.

## Activity 16a. Rectangles Challenge



1. Given: $P A=18, m \angle R Y A=35^{\circ}$. Find: $R Y, P R, P O, m \angle R P A, m \angle P O Y$
2. In figure below, find $\mathrm{BD}, A O, O C, D O$, and $O B$. Given: $\mathrm{AC}=16$


Your answers here.

## Activity 17a. My Golden Rectangle

The Golden Rectangle is proposed to be the most aesthetically pleasing of all possible rectangles.


This is the reason why it has been used extensively in art and architecture. The most prominent and well known uses of the Golden Rectangle in art were created by the great Italian artist, inventor, and mathematician, Leonardo da Vinci.

In da Vinci's "Mona Lisa" Golden Rectangle frames central elements in the composition. If you draw a rectangle whose base extends from the woman's right wrist and extend the rectangle vertically until it reaches the very top of her head, you will have a Golden Rectangle.

Then, if you draw squares inside this Golden Rectangle you will discover that the edges of these new squares come to all the important focal points of the woman: her chin, her eye, her nose, and the upturned corner of her mysterious mouth.

Questions to Answer:

1. Why is it called a Golden Rectangle?
2. How can one draw a Golden Rectangle?
3. What are the other uses of Golden Rectangles in real life?
4. Make your own presentation using the golden rectangle. As soon as done share it with your teacher.

Your answers here.
$\square$

## SUBMIT

Go to the website: http://www.youtube.com/watch?v=suiDK61jAc8 (The Golden Ratio). Or go to: http://goldenratiorocks.wordpress.com/golden-ratio-real-life-examples/ You will see more examples on the uses of golden rectangles and watch a video.

My Golden Rectangle

$\square$
When this activity is done, set an appointment with your teacher for a chat. Make sure that you did all the task and have answered questions embedded in each activity. Your teachers will ask you questions related to what you have studied.

## Activity 18a. Let's Chat

Answer the questions of your teachers to the best you can. Don't hesitate to ask your own questions if you have any at the end of your chat.

In the next activity you will deepen your understanding on the properties of squares.

## Activity 19a. The Power of the Square

The Power of the Center explains why most painting prefers squarer shape. As we move from the center things lose importance. Also, as we'll see, a squarer format is consistent with rest, repose, dignity, and timelessness; things that artists often want their paintings to convey.

But the square format has one property that the rectangular does not have; it gives a scene stillness and serenity, a calm and dignity associated with the round format. This makes it ideal for a subjects such as a Madonna.

Other than art the square is also used in architecture. One of the most famous structure the made use of squares is the Petronas Twin Tower of Malaysia. The design of the tower is composed of an 8-point star formed by intersecting squares. This is a common characteristic of a Muslim architecture.


Questions to Answer:

1. Where else in real life do we usually find squares?
2. Cite an instance where square is a better option than the other parallelograms.
3. What are the properties of the squares that make them so useful?

Your answers here.
$\square$

## SUBMIT

Activity 20a. Closer Look at the Square
Make three different sizes squares. Investigate the relationships of the sides and the angles.


Questions to Answer:

1. Fold the biggest square so that the opposite sides coincide. Are the opposite sides equal? Do the same for the other squares. Is your observation the same for the smaller squares?
2. Draw the diagonals and measure. What can you say about the measures of the diagonals?
3. Fold the squares so that the four vertices coincide. Are the angles equal? What are the measures of the angles? What theorem supports your answer.
4. What kind of quadrilateral is a square? Based on this fact what are the other properties of a square?
5. Summarize your findings by completing the table below.

|  | If a parallelogram a square , then... | EXAMPLE |
| :--- | :--- | :--- |
| SIDES |  |  |
| ANGLES |  |  |
|  |  |  |
| DIAGONALS |  |  |
|  |  |  |

## SUBMIT

Now check your findings by going back to the interactive site:
http://www.mathsisfun.com/geometry/quadrilaterals-interactive.html. Click on squares and angles. Drag the points to verify your findings. Click on diagonals. Observe. Are your findings consistent with what you have observe?


Now you know the special properties of the squares. These are the properties that makes them very useful. Challenge yourself to use these knowledge by doing the next activity.

## Activity 21a. Designing with a Square

Look at ads, magazines, brochures, logos, and other printed projects and try to find as many different examples of square shapes. Study the designs. Identify examples of square shapes that convey the attributes of honesty, stability, equality, comfort, or familiarity. Which designs convey rigidity or uniformity?

Design your own presentation using a square. What is the most important feature of your design? Use the resources of voki.com to explain your output. When done share your design by publishing it in a social media. Tag your teacher for the evaluation of your output.

Your Output here
$\square$
Now you know two of the very special parallelograms: rectangles and squares. The third type is the rhombus. Discover its properties by doing the next activity.

### 1.2.3 Rhombus

A rhombus is a special kind of parallelogram. Many architectural structures and art pieces make use of rhombi. Knowing about the special properties of a rhombus is important to identifying and using these special parallelograms. Access the following sites to see examples.

1. http://www.youtube.com/watch?v=i2a4B4M5L1M

Watch how a rhombus can be used to make a flexible paper structure.
2. http://www.youtube.com/watch?v=S-nNib5HzUA

Gives another type of flexible paper structure.
3. http://www.youtube.com/watch?v=p9xKxEV1FkY

Demonstrate how to make an Origami Fireworks making use of rhombi shapes.
4. http://www.youtube.com/watch?v=knMEBSXM6WU

Demonstrate how to make use of rhombi to make an origami flexiball.
5. http://rhombusspace.blogspot.com/

Gives examples of art pieces using rhombi.
6. http://www.ysjournal.com/article.asp?issn=0974-

6102;year=2009;volume=2;issue=7;spage=35;epage=46;aulast=Khair
Gives examples of the uses of rhombi in architecture
7. www.photoxpress.com/photos-skyscraper-lozenge-rhombus-4723361

Gives photos depicting how rhombi are used in architecture


British Museum has glass, tessellated roof.
What makes rhombus very useful as a shape? What makes it flexible? Know more about the rhombus in the next activities. Take note of these questions as you do the activities.

Your answers here.

## SUBMIT

Activity 22a. Rhombus under Scrutiny.


## Access:

http://www.mathsisfun.com/geometry/quadrilateralsinteractive.html. Click on rhombus and drag the points. Observe. Are there changes in the lengths of sides? Click on angles and drag.

Observe what happens to the measures of the angles. Click on diagonals. Drag and Observe.


0
Questions to Answer:

1. What relationships exist among the consecutive sides? The opposite sides?
2. What relationships exist between the opposite angles? Consecutive angles?
3. What relationship exist between diagonals?
4. Summarize your findings below.

|  | If a parallelogram is a rhombus, <br> then... | EXAMPLE |
| :--- | :--- | :--- |
| SIDES |  |  |
| ANGLES |  |  |
|  |  |  |
| DIAGONALS |  |  |

## SUBMIT

What meaningful insight did you gain from the activity? Write this insight in your Learning Log. Have you discovered what makes rhombi flexible? The next activity will deepen your understanding of rhombi. You will investigate on the relationship between diagonals.

## Activity 23a. Always at the Right!

Given: $A B C D$ is a rhombus. Show that its diagonals $A C$ and $B D$ are perpendicular.


| We know that... | Because... |
| :--- | :--- |
| 1. $A C$ bisect $B D$ |  |
| 2. $A B=B C, A D=D C$ |  |
| 3. $A C \perp B D$ |  |

## SUBMIT

You have established that diagonals of a rhombus are perpendicular to each other. You will not stop here. Investigate some more. This time on the relationship of diagonals and angles.

## Activity 24a. Great Angle Dividers

Show that Perpendiculars of a rhombus bisect the angles.


| We know that... | Because... |
| :---: | :---: |
| 1. $\triangle \mathrm{BCD} \cong \triangle \mathrm{BAD}$ |  |
| $\begin{aligned} & \text { 2. } \angle \mathrm{CBD} \cong \angle \mathrm{ABD}, \\ & \angle \mathrm{CDB} \cong \angle \mathrm{ADB} \end{aligned}$ |  |
| 3. BD bisect $\angle \mathrm{ABC}$ and $\angle A D C$ |  |
| 4. $\triangle \mathrm{ABC} \cong \triangle \mathrm{ADC}$ |  |
| $\begin{aligned} & \text { 5. } \angle \mathrm{CBD} \cong \angle \mathrm{ABD}, \\ & \angle \mathrm{CDB} \cong \angle \mathrm{ADB} \end{aligned}$ |  |
| Thus: |  |

## SUBMIT

Now that you have investigated on the properties of the rhombi, use your knowledge by answering the next activity.

## Activity 25a. The Rhombus Challenge

Task: Given $A B C D$ is a rhombus. Find:
a. $m \angle D C E$
b. $m \angle B A E$
c. $m \angle E A D$
d. $m \angle E C D$
e. $m \angle E B A$

f. $m \angle E B C$

| We know that | Because... |
| :--- | :--- |
| 1. $\mathrm{m} \angle \mathrm{BEA}=$ |  |
| 2. $\mathrm{m} \angle \mathrm{DCE}=$ |  |
| 3. $\mathrm{m} \angle \mathrm{DAB}=\mathrm{m} \angle \mathrm{DCB}$ | Opposite angles of quadrilaterals are congruent |
| 4. $\mathrm{m} \angle \mathrm{BAE}=$ |  |
| $5 . \mathrm{m} \angle \mathrm{EAD}=$ |  |
| 6. $\mathrm{m} \angle \mathrm{DAB}+\mathrm{m} \angle \mathrm{ABC}=180^{\circ}$ | Consecutive angles of a parallelogram are supplementary |
| 7. $\mathrm{m} \angle \mathrm{ABC}=$ |  |
| 8. $\mathrm{m} \angle \mathrm{ABE}=$ |  |
| 10. $\mathrm{m} \angle \mathrm{ADC}=$ |  |
| 11. $\mathrm{m} \angle \mathrm{ADE}=$ |  |

Proving relationships between sides and angles is a good exercise to sharpen your reasoning skills which is a very important skill in problem solving. In the next activity you will investigate on some real life uses of a rhombus. Write down ideas as you do the activity.

## Activity 26a. Beautiful and Flexible Rhombi

The Penrose rhombuses are a pair of rhombuses with equal sides but different angles.


Penrose tiling at Mitchell Institute for Fundamental Physics and Astronomy

Where else in real life are rhombi used? Create a list. Now create your own design using rhombi only. What kind of rhombi did you use in your design? Share your output at the discussion forum. Make sure that you attach your output or picture of your output. Output could be an art piece, an origami or structural design.

You can use tessellation tools for tessellation output. Access the following sites for practice.
http://www.shodor.org/interactivate/activities/Tessellate/
You can start by selecting shape then try changing corners and edges then tessellate.

## Your Output here

Now rate your progress in understanding of the lesson based on your performance in activities 1 to 22 .

| Ineed to shine |
| :--- | :--- | :--- | :--- |
| my star! | Leveling up! $\quad$ Good job! $\quad$ Excellent!

Now that you have knowledge quadrilaterals, do the next activity to ensure that you acquire skills in the construction of the following quadrilaterals.

## Activity 27a. Measure your Progress?

Access the website below and take the quiz. How well did you perform?
http://library.thinkquest.org/20991/textonly/quizzes/geo/q6/test.html
Now build up your skills in construction by doing the next activity
Activity 28a. (Level 1 Scaffold). Let's Construct
(Your skills acquired in this activity will help you later in the making of a three dimensional model.)

Use one of the web 2.0 tools to construct the following quadrilaterals.
a. Geometry Skecthpad
b. GeoGebra

1. General Parallelograms
a. One angle measures $43^{\circ}$
b. Shorter side is 2 inches and one angle measures $105^{\circ}$.
2. Rectangles
a. shorter side is 1 inch and longer side is 3 inches.
b. longer side is 3 inches diameter is 4 inches.
3. Square
a. side is 3 inches
4. Rhombus
a. smallest angle is $50^{\circ}$
b. one side measure 2 inches and the other measures 5 inches.

How did you fare in this activity? How can you apply the skills that you have gained in construction in problem solving involving quadrilaterals?

## Your Answers Here

## SUBMIT

It is expected that at this point you have gained deep understanding of quadrilaterals and their applications. Express this understanding by revising your original concept map to reflect your new realizations. It is expected that this time your map will be more comprehensive and will reflect the relationships between and among shapes being considered.

## Activity 29a. Do Is See A Bigger Picture Now?

Go back to your concept map. Revised based on what you have learned about parallelograms and make it more comprehensive.

## Your Answer Here

## SUBMIT

Questions to Answer:

1. What changes did you make?
2. What misconceptions were you able to correct?
3. How can your knowledge of parallelograms help in identifying the best way to solve problems involving efficiency of the use of materials and space?


## SUBMIT

## Activity 30a. Let's Sum It Up!

In the table below summarize what you have learned about parallelograms.

| Parallelograms | Properties | Where best to use it? |
| :--- | :--- | :--- |
| Rectangles |  |  |
| Squares |  |  |
| Rhombi |  |  |

Given a problem involving quadrilaterals, how can your knowledge of its properties help determine the best shape to use? What other factors would you consider to identify the best solution to the problem?

## Your answer here.

$\square$

Post your answers in http://whiteboard.com

## SUBMIT

In this section you were able to deepen your knowledge on parallelograms by classifying them into three subsets: rectangles, squares and rhombi. You have learned to construct this figures using web 2.0 technologies. In the next section, you will use your knowledge and skills gained in many situations in life to better appreciate what you are learning.

## TRANSFER



In the previous sections, you gain understanding on quadrilaterals and its sub-set parallelograms. In the next activity describe insights you have gained from the various activities. Use the questions below to guide you in your reflection.

## Activity 31a. Journal Writing

- How can you tell one quadrilateral from the other?
- How are quadrilaterals used in real life?
- How can models be used to show solutions to problems involving quadrilaterals?


## Your Answer Here

## SUBMIT

Should you have questions related to the questions above, click Discussion Forum and post your question

Now sum up what you have learned about quadrilaterals. Should you have questions, post these questions in the Discussion Forum. In the next activity you will challenge yourself to use what you have learn to solve the problem.

## Activity 32a. Tiling Challenge

You are task to study the number and size of tiles needed for the floor of the receiving room. The room is a square with an area of 81 square meters. The whole area must be divided into 9 congruent squares, the middle square must be divided again into 9 congruent squares and the middle square must be divided again into nine congruent squares. What is the side of the smallest middle square in the pattern? Can you use the same pattern for other number of
 squares? Show your solution.

Your solution here.

What guided you in answering this challenge? How did you identify the best solution?
In the next activity use the insight that you have gained to identify the best way to solve the problem below.

## Activity 33a. Architect's Square Parquet Floor



After making a parquet floor in an office building, the carpenters had left-over pieces of wood in the shape of right triangles with sides of 1,2 , and 5 . The architect would like to use these pieces for a parquet floor in his own house. He wants to know: can he make a perfect square from 20 of these triangles? If so, what will it look like?

Questions to Answer:

1. How did you solve the problem? What properties of quadrilaterals did you use?
2. What other Math concepts were useful in solving the problem?
3. Is there another way of solving the problem? Which of the process do you think is the best way to solve the problem? Justify.


## SUBMIT

Congratulations! You have were able to finish studying the section on parallelograms. You are now ready to study the two other subsets of quadrilaterals: trapezoids and kites. The knowledge and skills that you will gain will certainly help you acquire confidence in the use of quadrilaterals to solve problems or create designs.

## END OF TRANSFER:

In this section, your task was to make different quadrilaterals with the use of web 2.0 with different conditions.

How did you find the transfer task? How did the task help you see the real world use of the topic?

You have completed this lesson. But you have three more lessons before you finish this module. You need to learn more about trapezoid and kites, triangle similarity, and special right triangles to complete what you need in doing your performance task.

