## Lesson 3: Triangle Similarity

In this lesson you will learn the following:

1. Describe a proportion.
2. Illustrates similarity of figures.
3. Proves the conditions for similarity of triangles

SAS Similarity Theorem
SSS Similarity Theorem
AA Similarity Theorem
4. Right Triangle Similarity Theorem

## EXPLORE



You learned from lesson 2 the different concepts of quadrilaterals which are very essential in solving problems related to geometric figures.

In this lesson, you will learn the concepts of proportion and how to use it in many situations. You will also learn the concepts of triangle similarity and the different theorems related to these lessons which are useful in solving real world problems. You will also gather ideas to answer the question "What is the best way to solve problems involving triangle similarity?" These concepts will also help you visualize situations and create solutions to the problems that you encounter. Answers to the question above will also help you do your performance task.

In this section you need to analyze picture by answering different questions for you to discover important concepts. You will also do selfmonitoring activity as you fill up the map of conceptual change.

Let's us start the lesson by analyzing the pictures and answering the questions that follow.
A. Triangle Similarity

What is the best way to solve problems involving triangle similarity? Let's answer these questions by doing the activities below.

Activity 1c. Picture Analysis (Eliciting of prior Knowledge, Motivation, Hook) Observe the pictures below and answer the questions.

https://www.google.com.ph/\#q=SIMILAR+PICTURES

## (?) Questions to Answer:

1. Do you have a look alike? Why did you say that?
$\square$
2. Can figures be that similar? In what way?
$\square$
3. What would you consider to determine that two figures are similar?
$\square$
4. Why is it important to know two similar figures?

5. Focusing on the triangles, how would you know that two triangles are similar?
$\square$
6. What is the best way to solve problems involving triangle similarity?
$\square$

## SUBMIT

## CONCEPTUAL UNDERSTANDING CHECK

In the table below, write your answers on the initial part for the question what is the best way to solve problems involving triangle similarity?

| INITIAL ANSWER |
| :---: |
|  |
| REVISED ANSWER |
|  |
| FINAL ANSWER |

## SUBMIT

End of EXPLORE:
You just have tried to find out how mathematics can help you determine the best way to solve problems involving triangle similarity. Let us now strengthen that insight by doing the succeeding activities. What you will be learning in this section will help you perform well in your final performance task which will challenge you to use what you know to create a model and solve problems involving structures, space and aesthetic appeal.

Now move to the next activity to learn the knowledge and skills you need to be a good problem solver and respond to different situations accurately.

## FIRM - UP



Your goal in this section is to learn and understand key concepts of proportion which are important in solving problems involving triangle similarity. In this section there are activities which will help you discover and understand the different theorems and postulates which are useful tools in solving real life problem related to triangle similarity.

## Activity 2c. Situational Analysis

In the previous activity you are task to determine similar figures based from the given pictures. Now, let's see if you can be able to develop the concepts that you learned to respond to the situation below.


You are a newly hired employee of an organization who is working for the improvement of the environment. As your initial task, you need to estimate the number of trees of a 10 hectare forest near your place. This will be part of your company's report to plan for improvement. When you visit the forest you observe that trees are planted consistently which is about 20 meters from each other.

Questions to Answer:

1. What concept would you use to solve the problem above? How would you use it?
$\square$
2. What is the estimated number of trees of the 10 hectare forest?
$\square$
3. How is proportionality used in this situation?

## SUBMIT

In the previous activity you learned the importance of proportion in answering problems in real life. Now, you will improve your knowledge in proportion by answering the activities below.

## Activity 3c. Let's Consult the Expert

Directions: Click any of the videos below which explain the concepts of proportion with step by step procedure on how to solve problem related to the topic. After watching the video do the exercises below.
http://www.youtube.com/watch?v=D8dA4pE5hEY
http://www.youtube.com/watch?v=2d578xHNac8
http://www.youtube.com/watch?v=G8qy4f7GKzc
These sites contain videos which explain the concepts of proportion with step by step procedure on how to solve problem related to the topic.

Solve the proportion by determining the value of the variable.

| GIVEN | SOLUTION AND REASON | ANSWER |
| :--- | :--- | :--- | :--- |
| EX. $\frac{5}{10}=\frac{6}{X}$ | $\begin{array}{l}\frac{5}{10}=\frac{6}{X} \\ \text { proportion } \\ 5(\mathrm{X})=6(10) \\ 5 X=60 \quad \text { write original } \\ \mathrm{X}=12\end{array}$ | 12 |
| 1. $\frac{5}{10}=\frac{x}{16}$ | multiply product property |  |
| divide each by 5 |  |  |$]$

Questions to Answer:

1. What did you learn about proportion?
$\square$
2. How do we solve proportion?
$\square$
3. Why should we learn proportion?
$\square$

## SUBMIT

Now that you have enough exercises in solving proportion, you are ready for a short quiz. However, if you feel you are not yet ready for the quiz, you may try the practice quiz below. Click the interactive website and take the sample quiz. http://www.softschools.com/quizzes/math/proportion word problems/quiz3766.html This site contains interactive quiz about proportion.

## Activity 4c. QUIZ

Directions: Read each statement below and decide whether the statement is true or false. Tick $(\sqrt{ })$ the column of your answer.

| Are these statements true or false? | TRUE | FALSE |
| :--- | :--- | :--- |
| 1) $3: 5$ and $12: 20$ are equal ratios. |  |  |
| 2) $\frac{4}{5}=\frac{16}{18}$ |  |  |
| 3) 6 for 0.85 m is longer than 8 for 1 m |  |  |
| 4) 7 km in 10 minutes $=3.5 \mathrm{~km}$ in 5 minutes |  |  |
| 5) 5 cm in 2 seconds is faster than 9 km in an hour |  |  |
| 6) If $\frac{x}{15}=\frac{12}{36}$, then $\mathrm{x}=3$. |  |  |
| 7) If cross products are equal, the ratios are equal. |  |  |
| 8) $\frac{3}{5}=\frac{6}{10}$, so $3 \times 6=5 \times 10$. |  |  |

After learning proportion, you are about to learn the concepts of triangle similarity which will help you answer other problems in geometry. To start your journey in triangle similarity, do the next activity to discover important concepts of the lesson.

## Activity 5c. LET'S DISCOVER!

Directions: Follow the procedure below and answer the questions.
Materials: Ruler and protractor
Reflecting Question: What can you conclude about two triangles if you know two pairs of corresponding angles are congruent?
Step 1: Draw $\triangle$ ABC so that $m \angle A=40^{\circ}$ and $m \angle B=50^{\circ}$.
Step 2: Draw $\triangle$ DEF so that $m \angle D=40^{\circ}$ and $m \angle E=50^{\circ}$ and $\triangle \mathrm{DEF}$ is not congruent to $\triangle \mathrm{ABC}$.
Step 3: Calculate $\angle C$ and $\angle F$ using the Triangle Sum Theorem. Use a protractor to verify that your results are true.
Step 4: Measure and record the side lengths of both triangles. Use a ruler.
Step 5: Repeat steps 1 to 4 , use different angle measures.


Questions to Answer:

1. What can you conclude about two triangles if you know two pairs of corresponding angles are congruent?
$\square$
2. Are the triangles similar? Explain your reasoning.
$\square$
3. Make a conjecture about two triangles with two pairs of congruent corresponding angles.
$\square$

## Submit

The things that you learned from this activity will have an important role in the next activity. So you better remember your conclusion, to help you remember it you may use this codes $\mathbf{A}$-for angles and $\mathbf{S}$ - for sides.

## Activity 6c. PROVE IT!

Directions: Analyze the figures below and then complete the table to complete the proofs for triangle similarity

| Given: $\frac{R S}{J K}=\frac{S T}{K L}=\frac{T R}{L J}$ |  |
| :--- | :--- |
| Locate P on $\overline{R S}$ so that $\mathrm{PS}=\mathrm{JK}$. |  |
| Draw $\overline{P Q}$ so that $\overline{P Q} \\| \overline{R T}$ |  |
| Prove: $\Delta \mathrm{RST} \sim \Delta \mathrm{JKL}$ | Reason |
| Statement | 1. AA Similarity Postulate |
| 1. $\Delta \mathrm{RST} \sim \Delta \mathrm{PSQ}$ | 2. |
| 2. $\frac{R S}{P S}=\frac{S T}{S Q}=\frac{T R}{Q P}$ | 3. SSS Congruence Postulate |
| 3. $\mathrm{PS}=\mathrm{JK}, \mathrm{SQ}=\mathrm{KL}$ and QP $=$ | 4. |
| 4. $\Delta \mathrm{PSQ} \cong \Delta \mathrm{JKL}$ | 5. AA Similarity Postulate |
| 5. |  |

## (2) Questions to Answer:

1. How did you prove triangle similarity?
$\square$
2. What theorem have you discovered?
3. Why do we need to prove theorems of similar triangles?
$\square$
4. What is the best way to solve problems involving triangle similarity?


## Submit

Now that you learned how to show the proof of similar triangles step by step you are now ready to do it independently. In the next activity you will complete the table to prove another theorem involving similar triangles.

## Activity 7c. Do it alone!

Directions: Analyze the figures below and then complete the table to complete the proofs of another theorem related to similar triangles.

| Given: $\angle \mathrm{A} \cong \angle \mathrm{D}, \frac{A B}{D E}=\frac{A C}{D F}$ |
| :--- | :--- |
| Prove: $\triangle \mathrm{ABC} \sim \triangle \mathrm{DEF}$ |

## Submit

You learned how to prove similar triangles through guided exercises and independent practice. In the next activity you will use your observation to learn another theorem about similar triangles.

## Activity 8c. Watch and Learn!

Directions: Observe the picture below and answer the questions to discover similar right triangles, and then answer the questions below?

STEP 7


Draw a diagomeal Drav a diagonal on your rectangular piece of paper to form two congruent right triangles.


Cut and label triangles Cut the rectangle into the three right triangles that you drew. Label the angles and color the triangles as shown.

STEP2


Draw an altitude Fold the paper to make an altitude to the hypotenuse of one of the triangles.


Arrange the triangles Arrange the triangles so $\angle 1, \angle 4$, and $\angle 7$ are on top of each other as shown.

Questions to Answer:

1. How are the two smaller right triangles related to the large triangle?
$\square$
2. Explain how you would show that the green triangle is similar to the red triangle.
$\square$
3. Explain how you would show that the red triangle is similar to the blue triangle.
4. Why is it important to prove theorems involving similar triangles?
$\square$
5. What is the best way to solve problems involving triangle similarity?


You learned different ways on proving theorems involving similar triangles through guided proving, independent proving and using your observation. Now to deepen your understanding about these concepts, you will do another activity with the use of technology.

## Activity 9c. Meet the Digital Native

Directions: Click the website below and watch the video presentation on how to prove theorems related to similar triangles. Answer the questions below.
http://www.youtube.com/watch?v=EbN tDggldA
This video contains detailed discussion about the proving of similar triangles (AA, SAS, SSS).
http://www.youtube.com/watch?v=QCyvxYLFSfU
This video contains detailed discussion about the proving of similar triangles (Right Triangle Similarity Theorem.)

Questions to Answer:

1. What new things did you learn from the video?
$\square$
2. Did you encounter any inconsistency between the concepts and procedures you learned from the video and the previous activities?
$\square$
3. How are similar triangles solved?
$\square$
4. What is the best way to solve problems involving triangle similarity?
$\square$

## Activity 10c. Let's Use It!

Directions: Read the situation below and follow the procedure carefully then answer what is asked.

Note: Your answers for this activity will be submitted in a soft copy. Do this activity in MS Word and then send the file to your teacher. To do that just go to student dashboard - click message - attach file - and send it to your teacher.
a. Find a door or object of similar height that can be easily measured later to verify your results. Extend a 12 inch ruler in front of your body so that it is vertical and parallel to the door. Adjust your distance from the door or object so that your line of sight causes the ends of the ruler to correspond with the top and bottom of the door or object. See diagram below.

$\triangle \mathrm{ABE} \sim \triangle \mathrm{ACD}$ by the AA Postulate.
b. In the diagram above, BE is parallel to CD . Write a similarity statement using triangle $\triangle \mathrm{ABE}$ and another triangle in the diagram. Justify your statement with a postulate or theorem.
c. In the diagram above, $A P$ is proportional to $A Q$. According to the similarity statement you wrote in part $b, \mathrm{BE}$ is proportional to which other segment length?
d. Write a proportion in terms segment length that will allow you to find the height of the door by indirect measure.
e. Have your partner measure the following lengths to the nearest quarter inch. Record the lengths in the diagram above.

1. The distance your eye is from the ruler, AP.
2. The distance the ruler is from the door, $P Q$.
3. Add the previous lengths to find $A Q$.
f. Substitute the measures from part $e$ into the proportion you wrote in part $d$ and solve for CD, the height of the door.

©
Questions to Answer:

1. What did you learn from this activity?
$\square$
2. How would you relate your personal daily experiences to the situation that you encountered?
$\square$
3. Can you site any advantages of learning this lesson? Discuss.
$\square$
4. What is the best way to solve problems involving triangle similarity?


Note: It's time to consolidate your answers now and send it to your teacher. If you have questions or things to clarify do not hesitate to type your questions to the discussion board.

You encountered problem about triangle similarity but that is not the only situation where similar triangles are used. There are many situations where you can use the theorems and postulates about similar triangles. To help you master the skills in solving problems related to this topic, complete the next activity.
You may encounter some difficulties in doing the next worksheet to help you do it; you may click the website below.
http://www.youtube.com/watch?v=PXBFDBmBPOI
This website contains video which explains the step by step procedure in solving problem related to similar triangles.

## Activity 11c. Let's Practice

Directions: Find the value of the variables. Write your answers on the space provided for below. You don't have to include your solution in fact if you can do it mentally, the better. (Note: Lines that appear parallel are parallel.)

2.

5.

8.

11.

3.

6.

12.


Write your answers here.

| 1. | 2. | 3. |
| :--- | :--- | :--- |
| 4. | 5. | 6. |
| 7. | 8. | 9. |
| 10. | 11. | 12. |

Questions to Answer:

1. What concept did you use to answer the worksheet above?
$\square$
2. How did you use it?
$\square$
3. Why is it important to learn many concepts in solving problems related to similar triangles?
$\square$
4. What is the best way to solve problems involving triangle similarity?


## Submit

You encountered a lot of concepts related to triangle similarity. Now it's time to pause for a while and reflect to your learning process by doing the 3-2-1.

What are the 3 most important things you learned?

What are the 2 things you are not sure about?

What is 1 thing that you want to clarify immediately?

Your answer in the last part may also send to your teacher for immediate response through the discussion forum. To do that, go to student dash board - message - conversation.

## Activity 12c. Interactive Quiz

Directions: Click the website below and answer the interactive quiz. You may try this as many as you can.
http://www.regentsprep.org/regents/math/geometry/MultipleChoiceReviewG/Tria ngles.htm
This website contains interactive quiz about triangle similarity. This may be used as practice exercises to develop more the knowledge, process and analysis of the students for them to answer more complicated problems. After answering the 20 questions answers will appear.
http://www.classzone.com/etest/viewTestPractice.htm?testld=4545
This website contains interactive quiz about triangle similarity. This contains 5 items multiple choice more on word problems which you may encounter in your quiz. After answering each item you may click the feedback button for the answer and solution. This is helpful for to check your work immediately.


Questions to Answer:

1. What new things did you learn from the interactive quizzes?
$\square$
2. What item or lesson appears confusing for you?
$\square$
3. How do the questions help you develop your critical thinking and problem solving skills?
$\qquad$
4. What is the best way to solve problems involving triangle similarity?
$\square$

After doing a lot of activities - proving the theorems, analysing situations, watching videos, interactive quizzes, etc., you are now ready for your quiz on problem solving about triangle similarity. This quiz will challenge you connect and apply the things that you learned about triangle similarity.

## Activity 13c. Quiz (Problem Solving)

Directions: Read and analyse each problem carefully. Complete the table below by writing the theorem or postulate appropriate for each problem, your reason or justification, solution and final answer.


| Reason/ Justification |  | Final Answer |
| :--- | :--- | :--- | :--- |


| Reason/ Justification | Solution | Final Answer |
| :--- | :---: | :---: |
|  |  |  |
|  |  |  |
|  |  | SUBMIT |


| Problem \# 5 | Illustration | Theorem/ Postulate |
| :---: | :---: | :---: |
| 5. Find the height of the giraffe in the diagram below. |  |  |
| Reason/ Justification | Solution | Final Answer |
|  |  |  |
|  |  | SUBMIT |
| Problem \# 6 | Illustration | Theorem/ Postulate |
| 6. A tourist on the observation deck of a building looks east, facing another building 320 ft high and two blocks from the first building. Her view is 400 ft above street level. Her car is parked five blocks east of the second building. If no other buildings are seen |  |  |


| in her line of sight, can <br> she see her car? <br> Justify. |  |  |
| :--- | :--- | :--- |
|  |  |  |
| Reason/ Justification |  | Solution |
|  |  |  |

To help you summarize and remember important concepts that you learned about triangle similarity, try to complete the graphic organizer below.

Activity 14c. Do the Map
Directions: Observe the diagram below and complete the missing parts.


## END OF FIRM - UP

In this section, the discussion was about proportion and how similar triangles are solved with the use of postulates and theorems.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision? What new learning goal should you now try to achieve?

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.

## DEEPEN

Your goal in this section is to take a closer look at some aspects of the topic. With the activities that you have accomplished, do you think you are now ready for more challenging situations? Can you assess now what is the best way to solve problems involving triangle similarity?

To test your analysis and understanding of the concepts let's see if you can be able to identify mistakes and be able to correct it with justification. This activity will also help you check your understanding and possible misconceptions you absorb from the previous activities.

Activity 15c. Watch Your Error
Analyze each situation carefully and write your answer, solution and justification.

| SITUATION | ANSWER / SOLUTION / <br> JUSTIFICATION |
| :--- | :--- |
| $\frac{4}{6}=\frac{5}{x}$ to find the value of $x$ in the <br> figure. Explain why this proportion is <br> incorrect and write a correct <br> proportion. |  |

3. Two persons leave points $A$ and $B$ at the same time. They intend to meet at point $C$ at the same time. The person who leaves point $A$ walks at a speed of 3 miles per hour. How fast must the person who leaves point B walk?


A student who attempted to solve this problem claims that you need to know the length of $\overline{A C}$ to solve the problem. Describe and correct the error that the student made.
4. A man standing in his backyard measured the lengths of the shadows cast by him and a tree. What theorem/postulate would help him find the height of a tree? Write a proportion showing how he could find the height of a tree.

A student who tried to answer this problem said that the best way to use is AA Similarity Postulate. Do you agree with this answer? Show your justification.
5. You can measure the width of the lake below using a surveying technique, as shown in the diagram.


Your classmate said that SAS Similarity Theorem can be used to solve this problem since you are looking for a missing side and you have two given sides which are proportional.
Justify if the answer of your classmate is right or wrong. Show your solution.

## SUBMIT

Before you continue the learning process, it will be better if you stop and reflect. After taking different activities, what happened to your initial answers? You may now answer the R part of you IRF worksheet.

## CONCEPTUAL UNDERSTANDING CHECK

In the table below, write your answers on the revised answer for the question what is the best way to solve problems involving triangle similarity?

| INITIAL ANSWER |
| :---: |
|  |
| REVISED ANSWER |
|  |
| FINAL ANSWER |

## SUBMIT

To deepen your understanding about the concepts of similar triangles, you will observe two different simple experiments and then answer the questions.

## Activity 16c. Experiment

Directions: Study the two situations below and analyze how to do each process. Answer the questions below in a paragraph form using the box provided after the two situations.

4. What is the best way to solve problems involving similar triangles?
4. What is the best way to solve problems involving similar triangles?

## Situation 3: Measure Measure

Suppose you want to use the shadow method to measure the height of a building with the use of a shorter post. You make the following measurements. Materials: meter stick

## Given:

Look for any post which can be measured with the use of a meter stick. Consider the illustration below.


## Questions:

1. What concept would you use to solve the given problem? Justify your answer.
2. Why is it important to know how to measure things indirectly?
3. What are the advantages of estimation?
4. What is the best way to solve problems involving similar triangles?

Activity 17c. Experiment - My Observation, Analysis and Generalization
$\qquad$
$\qquad$
$\qquad$
$\longrightarrow$

Now that you have observed how to do an experiment using the concepts of triangle similarity, it's time for you to try it in your own situation. Try to observe around you, what are the things which are impossible for you to measure but now with the use of stick, mirror and shadow will be possible? How are the things that you learned in this lesson become useful in your daily life?
In the next activity you will improve your imagination and appreciation in the beauty of mathematics particularly triangle similarity. You may also explore the beauty of technology through different applications under web 2.0.

## Activity 18c. THINK, REFLECT AND DISCOVER

Directions: Using at least three different theorems or postulates that you learned, you will create a situation in the form of experiment similar to the previous activity. You may follow the procedure below.

| Theorem / Postulate 1 | Theorem / Postulate 2 | Theorem / Postulate 3 |
| :--- | :--- | :--- |
| Goal: | Goal: | Goal: |
| Situation: | Situation: | Situation: |
| Materials: | Materials: | Materials: |
| Illustration: | Illustration: | Illustration: |
| Conclusion: | Conclusion: | Conclusion: |
| Justification: | Justification: | Justification: |

Note: Write your work in a short type writing paper for each situation. To help you create a better illustration of the problem, you may use geometersketchpad. To do that, just download geometersketchpad and you may use it for free. You may submit your work online or face to face. To do the online submission, go to student dash board - message - attached file - send.

For your explanation and justification you may do it face to face or you may try another web 2.0 present.me. Here you may record your explanation, justification and generalization of the lessons. In your presentation, do not forget to answer the questions, what is the best way to solve problems involving triangle similarity?

## END OF DEEPEN

In this section, the discussion was about triangle similarity.
What new realizations do you have about the topic? What new connections have you made for yourself? What helped you make these connections?

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.

## TRANSFER



Your goal in this section is apply your learning to real life situations. You will be given a practical task which will demonstrate your understanding.

To help you summarize everything that you learned in this lesson, complete the concept map in the next page.

Activity 19c. Concept Map

SIMILAR TRIANGLES

TRIANGLE SIMILARITY THEOREMS


GENERALIZATION
What is the best way to solve problems involving triangle similarity?

After completing the concept map, you may have enough knowledge, skills and understanding to do your transfer task. To assess what you learned and understand you will apply the concepts of the lesson in an actual situation by doing the transfer task of this lesson Scaffold 3.

## Activity 20c. SCAFFOLD 3

You are a newly hired designer of a company who develops condominiums and housing projects. The company is presently preparing for a bidding to develop a condominium with a floor area of 70 square meters. You are tasked by your superior to make a two dimensional design of all the faces of a condominium. It is important for you to show the scale and consider the different quadrilaterals and similar triangles to make your design appealing. You need also to solicit ideas from others to improve your work. You may post your work to any social network to solicit comments for improvement or you may present it to possible clients to get suggestions before you present your work to the higher officers of the company for approval.

Note: To make your design appealing and accurate you may use the geogebra. To do it, download geogebra then you may use it for free.

For your presentation and explanation, you may use voki.com. This will help you record your presentation in the most exciting way.

After doing your transfer task (Scaffold 3) it's time again to reflect on the learning process to check if there are ideas which you need to change, to revise or improve. You may now complete you IRF worksheet by writing your ideas on the F part.

## CONCEPTUAL UNDERSTANDING CHECK

In the table below, write your answers on the final answer for the question what is the best way to solve problems involving triangle similarity?

| INITIAL ANSWER |
| :---: |
|  |
| REVISED ANSWER |
|  |
| FINAL ANSWER |

## SUBMIT

To complete the learning process, reflect again and complete the table below. This will also check if you have absorbed some misunderstanding which need to be corrected.

Let's Reflect!


## SUBMIT

To summarize what you learned, you may complete the synthesis journal below.
Synthesis Journal
The lesson was on $\qquad$ . One key idea was $\qquad$ . This is important because
$\qquad$ . Another key idea was
$\qquad$ . This is also important because lesson $\qquad$
$\qquad$

## SUBMIT

## END OF TRANSFER:

In this section, your task was to make a two dimensional design of a condominium.

How did you find the task? How did the task help you see the real world use of the topic?

You have completed this lesson. But you have one more lesson before you finish this module. You need to learn more about triangles to complete what you need in doing your performance task.

