2017

LEARNING MODULE

Mathematics G10 | Q1.2

Polynomial Function

PEAC National Secretariat
Units 2505-2507, 25th Floor Philippine AXA Life Centre
1286 Sen. Gil Puyat Avenue corner Tindalo St., Makati City
Tel No.: (02) 8-570-7322
Fax No.: (02) 8-511-7695

NOTICE TO THE SCHOOLS

This learning module (LM) was developed by the Private Education Assistance Committee under the GASTPE Program of the Department of Education. The learning modules were written by the PEAC Junior High School (JHS) Trainers and were used as exemplars either as a sample for presentation or for workshop purposes in the JHS In-Service Training (INSET) program for teachers in private schools.

The LM is designed for online learning and can also be used for blended learning and remote learning modalities. The year indicated on the cover of this LM refers to the year when the LM was used as an exemplar in the JHS INSET and the year it was written or revised. For instance, 2017 means the LM was written in SY 2016-2017 and was used in the 2017 Summer JHS INSET. The quarter indicated on the cover refers to the quarter of the current curriculum guide at the time the LM was written. The most recently revised LMs were in 2018 and 2019.

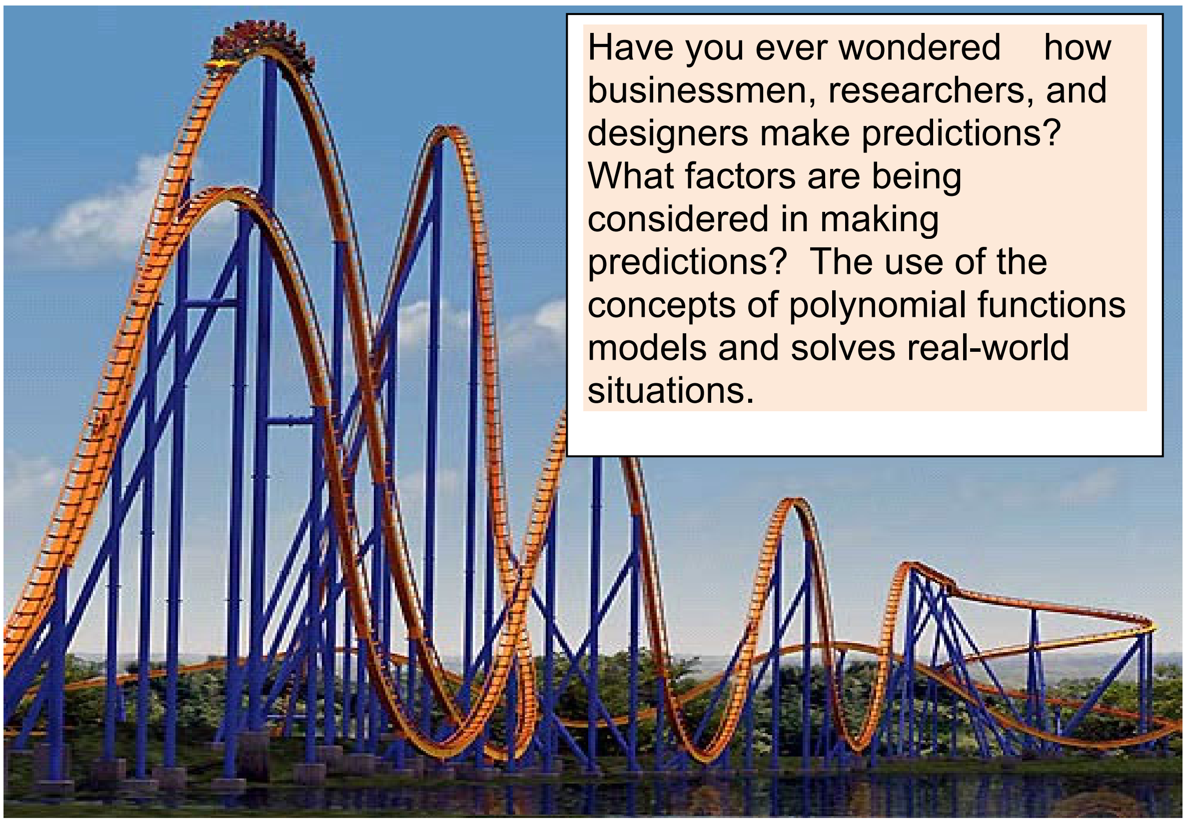
The LM is also designed such that it encourages independent and self-regulated learning among the students and develops their 21st century skills. It is written in such a way that the teacher is communicating directly to the learner. Participants in the JHS INSET are trained how to unpack the standards and competencies from the K-12 curriculum guides to identify desired results and design standards-based assessment and instruction. Hence, the teachers are trained how to write their own standards-based learning plan.

The parts or stages of this LM include Explore, Firm Up, Deepen and Transfer. It is possible that some links or online resources in some parts of this LM may no longer be available, thus, teachers are urged to provide alternative learning resources or reading materials they deem fit for their students which are aligned with the standards and competencies. Teachers are encouraged to write their own standards-based learning plan or learning module with respect to attainment of their school’s vision and mission.

The learning modules developed by PEAC are aligned with the K to 12 Basic Education Curriculum of the Department of Education. Public school teachers may also download and use the learning modules.

Module 1.2: Polynomial Function

☑ INTRODUCTION AND FOCUS QUESTION(S):



Have you ever wondered how businessmen, researchers, and designers make predictions? What factors are being considered in making predictions? The use of the concepts of polynomial functions models and solves real-world situations.

This module seeks to find the answer to the question: ***How do values of one variable behave in terms of the other in the polynomial function that models a real- world situation?***

☑ MODULE LESSONS AND COVERAGE:

In this module, you will examine this question when you study the following lessons:

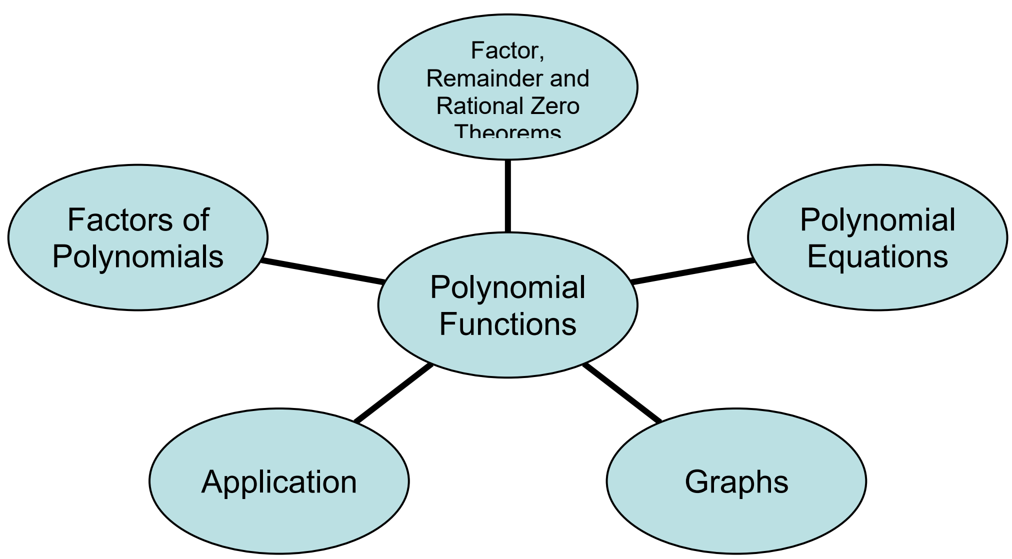
Lesson 1: Remainder and Factor Theorem
Lesson 2: Polynomial Equations
Lesson 3: Polynomial Functions

In these lessons, you will learn the following:

Lesson 1	Remainder and Factor Theorem 1. Performs division of polynomials using long division and synthetic division – S 2. Proves the Remainder Theorem and the Factor Theorem – S 3. Factors polynomials - K
Lesson 2	Polynomial Equations 1. Illustrates polynomial equations - K 2. Proves rational root theorem - S 3. Solves polynomial equations - S 4. Solves problems involving polynomials and polynomial equations. – S
Lesson 3	Polynomial Functions 1. Illustrates polynomial functions – K 2. Graphs polynomial functions – S 3. Solves problems involving polynomial functions - S

☑ **MODULE MAP:**

Here is a simple map of the above lessons you will cover:



☑ **EXPECTED SKILLS:**

To do well in this module, you need to remember and do the following:

1. *Define terms that are unfamiliar to you.*
2. *Explore websites that would be of great help for your better understanding of the lessons.*
3. *Take down notes of the important concepts in your journal.*
4. *Perform and complete the exercises provided.*
5. *Collaborate with your teacher and peers.*

MODULE: PRE-ASSESSMENT

Let's find out how much you already know about this module. Click on the letter that you think best answers the question. Please answer all items. After taking this short test, you will see your score. Take note of the items that you were not able to correctly answer and look for the right answer as you go through this module.

1. Which of the following is not a solution to the cubic equation below?

   $x^3 + x^2 - 4x - 4 = 0$

   A. -2
   B. -1
   C. 1
   D. 2

2. What is the remainder when $x^3 - 5x^2 - x + 5$ is divided by $x + 2$?

   A. -21
   B. -5
   C. 5
   D. 21

3. Which of the following equations is a polynomial equation?

   A. $f(x) = x^{-3} + 3x^2 + 5x - 1$
   B. $f(x) = \frac{2}{x} + 4x^2 - 5$
   C. $f(x) = 3x^4 - x^3 + x^2 + 5x - 4$
   D. $f(x) = \sqrt{x} + 5x - 3$

4. Given a polynomial and one of its factors, what are the remaining factors of the polynomial?

   $2x^3 + 9x^2 + 7x - 6; x + 2$

   A. $(2x - 1)(x - 3)$
   B. $(2x - 1)(x + 3)$
   C. $(2x + 1)(x - 3)$
   D. $(2x + 1)(x + 3)$

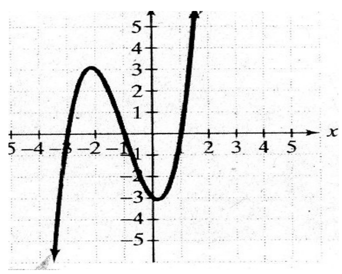
5. Which of the following values is a root of $f(x) = x^4 - 13x^2 + 36$?

   A. 1
   B. -1
   C. 4
   D. -3

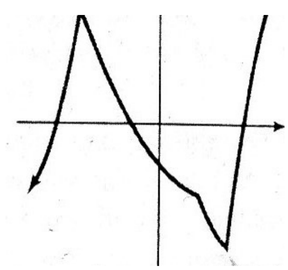
6. What are the roots of $x^3 + 6x^2 + 11x + 6 = 0$?
   A. 1,2 and 3
   B. 1, 2, and 6
   C. -1, -2 , and -3
   D. 1, 2 and -3

7. Which of the following illustrates the the graph of polynomial function?

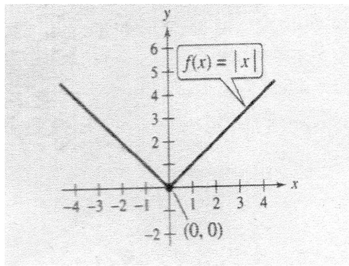
A.



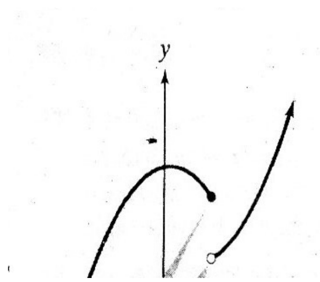
B.



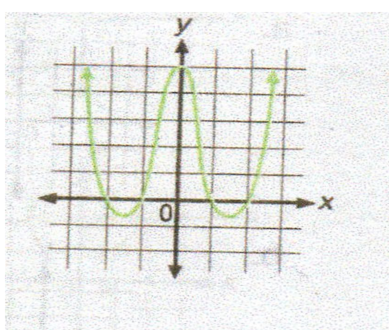
C.



D.



8. Which function is repesented by the graph?



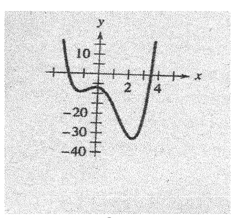
A. $f(x) = 2x + 1$
B. $f(x) = 2x^3 + 4x^2 – x + 4$
C. $f(x) = x^4 – 5x^2 + 4$
D. $f(x) = x^5 + 2x^4$

9. A specific car's economy in miles per gallon can be approximated by $f(x) = 0.00000056x^4 - 0.000018x^3 - 0.016\,x^2 + 1.38x - 0.38$, where x represents the car's speed in miles per hour. What is the fuel economy when the car is travelling 40 miles per hour?
    A. 27.25 miles per gallon
    B. 28.5 miles per gallon
    C. 29.5 miles per gallon
    D. 30 miles per gallon

10. If the volume of the box is represented by the expression $(x^3 + 4x^2 + x - 6)\ cm^3$ and its width by (x - 1) cm, what binomials can be used to represent the other two dimensions?
    A. (x – 2) (x+3)
    B. (x – 2) (x-3)
    C. (x+2) (x+3)
    D. (x-2) (x-3)

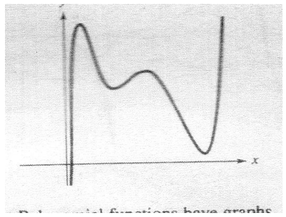
11. An air conditioning manufacturer determines that the profit for producing x air condition units per day is P(x) = -.006x + 0.15x – 0.05x – 1.8x. What is the meaning of zeros in this situation?
    A. Number of air condition units sold
    B. Profit earned in a day
    C. Turning points of the graph of P(x)
    D. Break even points

12. The graph of one of the following functions is shown below. Identify the function shown in the graph and explain why each of the others is not the correct function. You can use graphing utility to verify your result.

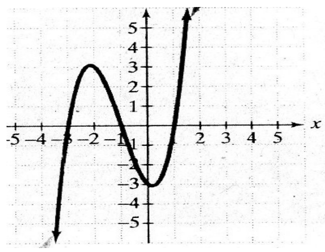


    A. $f(x) = x^2(x + 2)(x - 3.5)$
    B. $g(x) = (x + 2)(x - 3.5)$
    C. $h(x) = (x + 2)(x - 3.5)(x^2 + 1)$
    D. $k(x) = (x + 1)(x + 2)(x - 3.5)$

13. Which of the following CANNOT be a graph of polynomial function?
    A.

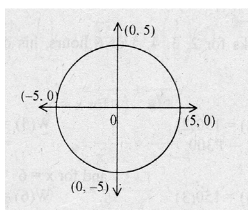


B.

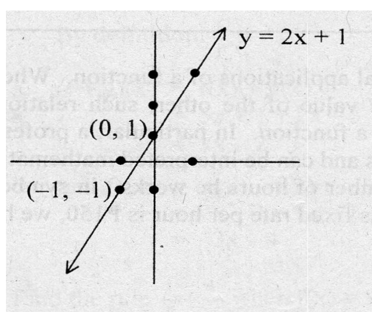


C.

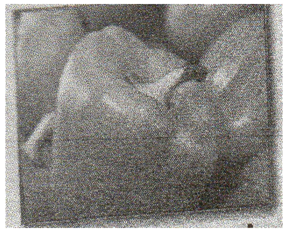
*



D.



14. A bulk fruit and vegetable storage bin with dimension 2 feet by 3 feet by 4 feet needs to be increased in size to hold five times as much food as the current bin.
(Assume each dimension is increased by the same amount.). What is the function that represents the volume V of the new bin? What are the dimensions of the new bin?



A. $V(x) = x^3 + 9x^2 + 26x – 96$
   Dimensions: 4 ft x 5 ft x 6 ft

B. $V(x) = x^3 - 9x^2 + 26x – 96$
   Dimensions: 4 ft x 6 ft x 7 ft

C. $V(x) = x^3 + 9x^2 - 26x – 96$
   Dimensions: 5 ft x 6 ft x 7 ft

D. $V(x) = x^3 - 9x^2 - 26x – 96$
   Dimensions: 3 ft x 5 ft x 7 ft

15. The function $C(x) = 2.46x^3 – 22.37x^2 + 53.81x + 548.24$ can be used to approximate the number in thousands, of international college students studying in the Philippines x years since 2000. Using the model, how many international college students can be expected to study in the Philippines in 2015?
    A. 565 thousand
    B.1124 thousand
    C. 665 thousand
    D. 98 thousand

16. The width of a rectangular container is 2 meters less than the length and the height is 1 meter less that the length. How would you determine the dimensions of the container if its volume is 60 m^3 ?
    A. Determine the polynomial function for the volume and compute one real zero to represent the length of the container. *
    B. Use guess and check in finding the length of the container
    C. Determine the polynomial function for the volume and compute one real zero to represent the height of the container
    D. Determine the polynomial function for the volume and compute one real zero to represent the width of the container

17. The function $g(x) = 1.384x^3 – 0.003x^2 + 0.28x + 1.365$ can be used to model the average price of a gallon of gasoline in a given year if x is the number of years since 2000. What is the easier way to determine the average price of gasoline in year 2016?
    A. Find the zeros of g(x).

B. Evaluate g(2016)
C. Find the factors of g(x)
D. Evaluate g(16)

18. The water district manager wants to know the extent of use of water in your barangay for some research purposes. As a water district officer you are tasked to predict water consumption of your barangay for the next two years and present a mathematical equation for the data.

Which of the following standards should the report be evaluated?

A. Organization of data and justification of recommendation
B. Accuracy of computation, organization of data and justification of recommendation
C. Representation of output and organization of data
D. Representation of output

19. You are a packaging designer of a certain company and you were tasked to create a tool for evaluating the product. The product is a box which will be used for storing the canned goods that will yield a maximum volume. Which of the following criteria should be considered the least as part of the rubric. for the performance task?

A. Justification/Application of concept.
B. Practicality of the Product
C. Accuracy of Computations
D. Delivery of Report

20. Which of the following product will best apply the concept of a cubic polynomial equations/functions?

A. Volume of a box
B. Surface area of a box
C. Minimum cost for creating a box
D. Maximum profit

SUBMIT

Lesson 1: Factor and Remainder Theorems

Lesson Pre-assessment:

Let's find out how much you have already know about this lesson. Click on the letter that you think best answers the question. Please answer all items. After taking this short test, you will see your score. Take note of the items that you were not able to correctly answer and look for the right answer as you go through this lesson.

1. Which of the following expressions is a polynomial?
   A. $2x^2 - \frac{5}{x}$
   B. $3x^3 + 2x^2 - 5x + 6$
   C. $5x^2 - 4x^{-2} + 3$
   D. $3x^2 + 5x^{1/2} - 2x$

2. What is the remainder when $x^6 + 4x^5 - 3x^2 + x - 2$ is divided by $x - 1$?
   A. -1
   B. 1
   C. 2
   D. 3

3. Which of the following is a factor of $x^3 + 8x^2 + 19x + 12$?
   A. x - 3
   B. x - 4
   C. x + 4
   D. x 1

4. If x- 1 and x + 5 are the two factors of $x^3 + 8x^2 - x - 5$, what is the other factor?
   A. x + 1
   B. x – 5
   C. x + 4
   D. x + 3

5. What is the remainder when $x^4 + 2x^3 - 3x^2 + 5x + 8$ is divided by x + 1?
   A. -1
   B. 0
   C. 9
   D. 16

6. What are the factors of $2x^3 + 3x^2 - 8x + 12$?
   A. $(x + 2)(x + 2)(2x - 3)$
   B. $(x + 2)(x - 2)(2x - 3)$
   C. $(x - 2)(x + 2)(2x - 3)$
   D. $(x - 2)(x + 2)(2x + 3)$

7. If the volume of the box is represented by the expression $(x^3 - x^2 - 10x - 8)$ cm^3 and its width by $(x+2)$ cm, what binomials can be used to represent the other two dimensions?
   A. $(x-4)(x+1)$
   B. $(x+4)(x+1)$
   C. $(x+4)(x-1)$
   D. $(x-4)(x-1)$

8. If 2 and -1 are factors of $2x^4 - 3x^3 + ax^2 + bx + 2$, what are the values of a and b?
   A. $a = 4; b = -3$
   B. $a = -4; b = 3$
   C. $a = 4; b = 3$
   D. $a = 4; b = -3$

9. $x^3 + px^2 + qx + 6$ gives a remainder of 8 when divided by $x + 1$, and a remainder of -4 when divided by $x - 2$. Find the values of p and q.
   A. $p = -2; q = -5$
   B. $p = -3; q = -5$
   C. $p = -2; q = -4$
   D. $p = 2; q = 5$

10. If the volume of the box is represented by the expression $(x^3 + 4x^2 + x - 6)$ cm^3 and its width by $(x + 2)$ cm, what binomials can be used to represent the other two dimensions?
    A. $(x + 3)$ and $(x - 1)$
    B. $(x + 3)$ and $(x + 1)$
    C. $(x - 3)$ and $(x - 1)$
    D. $(x - 3)$ and $(x + 1)$

EXPLORE

In the preceding grades, you were introduced to the properties of two simple functions, namely, the linear function and the quadratic function. These two functions belong to the family of polynomial functions, which you shall work within this unit. You will get to know more about polynomials and polynomial functions, and some of their applications.

ACTIVITY 1.

Read an article from http://www.purplemath.com/modules/polydefs.htm

Do the following activities.

a. Is it a Polynomial?

Fill in the 2nd and 3rd columns of the table below.

Expression	YES, NO	Reason
$4x^2 y + 8xy + 5$		
$5d^2 + 11d – 9$		
$\frac{3}{x}$ 2		
7		
$\frac{1}{2} x^3$		
$8\sqrt{x}$ - 5		

b. Complete the Frayer Model using the word POLYNOMIALS.

Definition	**Facts/ Characteristics**
Examples	**Non-examples**

POLYNOMIALS

Process Questions:

1. How did you determine the examples and non- examples of a polynomial?
2. How does the polynomial differ from other algebraic expressions?
3. **What makes an expression a polynomial?**

Click **SAVE** if you have completed the table above and the Frayer Model. Then

You have just tried describing a polynomial. In our next activity, your prior knowledge on polynomials and polynomial functions will be elicited.

ACTIVITY 2. AGREE OR DISAGREE! (ANTICIPATION-REACTION GUIDE)

Read each statement in the column TOPIC and write **A** if you agree with the Statement, otherwise write **D** in the first column.

Response Before Lesson	TOPIC: Polynomial Functions	Response After Lesson
	1. $\frac{(2x-5)}{3}$ is a polynomial	
	2. $\sqrt{2x-5}$ is a polynomial	
	3. The degree of $x^3 - 6x^2 + 8x + 5$ is 3	
	4.The leading coefficient of the polynomial $x - 11 + x^4 - x^2$ is -1.	
	5. In synthetic division, the last number in the last row of the process represents the quotient.	
	6. If $f(x) = x^4 - 5x^2 - 7x + 6$, $f(3) = 21$	
	7. Either the Remainder Theorem or Synthetic Division can be used to find the remainder when a polynomial in x is divided by x – r, where r is a constant.	
	8. $x - 2$ is a factor of $x^4 - 16$.	
	9. $x + 3$ is a factor of $P(x)$ if $P(-3) = 0$.	
	10. If $x^3 - 6x^2 + 8x + 5$ is divided by $x - 2$, the remainder is 6.	
	11. If $3x^{14} - 2x^{12} + 3$ is divided by $x + 1$, the remainder is 3.	
	12. If one of the factors of $hx^3 + 3x^2 - 2hx + 1$ is $x+1$, what is the value of h?	

Process questions:

1. What comes in your mind while filling in the first column of the ARG?
2. How do values of one variable behave in terms of the other in a polynomial function that models a real-world situation?

ACTIVITY 3. Complete Me!

Complete the two tables below.

Table 1:

Polynomial	Degree	Number of terms	Classification According To Degree	Classification Accdg. To Number of Terms
-10	Zero	one	constant	monomial
$2x + 1$	One	two	linear	binomial
$x^2 -5x -3$	Two	three	quadratic	trinomial
$2x^3 + 5x$	?	?	?	?
$X^6 + 4x^3 -5$	?	?	?	?

Table 2:

Polynomial	Leading Term	Leading Coefficient	Number of Terms	Degree
$2x + 5$	$2x$	2	two	one
$3 -5x + 4x^2 -2x^3$	$-2x^3$	-2	?	?
$4x^5 -2x^3 + x$		4	?	?
$(2x - 3)^2$			three	
$(x^2 -2x) - (x^4 + 5x^2)$	$-x^4$	?	?	four

A polynomial in one variable x is an algebraic expression formed by the sum of an expressions of the form ax^n, where a represents a constant and n is a nonnegative integer. These expressions are the terms of the polynomial. There are no negative exponents and no variables in the denominator for polynomials.

In general, a polynomial is an algebraic expressions of the form $a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0$ where the coefficients $a_0, a_1, \ldots, a_n$ are real numbers. The terms are usually arranged such that the first term contains the variable with the highest exponent and the exponents decrease with each succeeding term. In this case, the first term is called the leading term of the polynomial, and its numerical coefficient is called the leading coefficient. The exponent of x in the leading coefficient is called the degree of the polynomial. The polynomial may be named according to their degree or to the number of terms they contain. Complete the two tables below.

ACTIVITY 4. Dividing Polynomials - A Review

In your earlier grade, you learned how to divide with polynomials. Using the skills that you have learned:

divide $x^2 - 9x - 10$ by $x + 1$;

Long division for polynomials works in much the same way division with plain numbers are done.

First, I set up the division: For the moment, I'll ignore the other terms and look just at the leading *x* of the divisor and the leading x^2 of the dividend.	$x+1\overline{)x^2 \quad -9x-10}$
If I divide the leading x^2 inside by the leading *x* in front, what would I get? I'd get an *x*. So I'll put an *x* on top:	$\begin{array}{r} x \quad\quad\quad\quad \\ x+1\overline{)x^2 \quad -9x-10} \end{array}$
Now I'll take that *x*, and multiply it through the divisor, *x* + 1. First, I multiply the *x* (on top) by the *x* (on the "side"), and carry the x^2 underneath:	$\begin{array}{l} \quad\quad\quad x \\ x+1\overline{)x^2 \quad -9x-10} \\ \quad\quad\; x^2 \end{array}$
Then I'll multiply the *x* (on top) by the 1 (on the "side"), and carry the 1*x* underneath:	$\begin{array}{l} \quad\quad\quad x \\ x+1\overline{)x^2 \quad -9x-10} \\ \quad\quad\; x^2 \quad +1x \end{array}$
Then I'll draw the "equals" bar, so I can do the subtraction. To subtract the polynomials, I *change all the signs* in the second line...	$\begin{array}{l} \quad\quad\quad x \\ x+1\overline{)x^2 \quad -9x-10} \\ \quad\; \underline{-x^2 \quad \mp 1x \quad\quad\;} \end{array}$

...and then I add down. The first term (the x^2) will cancel out:	$\begin{array}{rl} & x \\ x+1 \,\big) & \overline{x^2 - 9x - 10} \\ & \underline{-x^2 \overset{-}{+} 1x} \\ & \quad -10x \end{array}$
I need to remember to carry down that last term, the "subtract ten", from the dividend:	$\begin{array}{rl} & x \\ x+1 \,\big) & \overline{x^2 - 9x - 10} \\ & \underline{-x^2 \overset{-}{+} 1x} \\ & \quad -10x - 10 \end{array}$
Now I look at the *x* from the divisor and the new leading term, the –10*x*, in the bottom line of the division. If I divide the –10*x* by the *x*, I would end up with a –10, so I'll put that on top:	$\begin{array}{rl} & x - 10 \\ x+1 \,\big) & \overline{x^2 - 9x - 10} \\ & \underline{-x^2 \overset{-}{+} 1x} \\ & \quad -10x - 10 \end{array}$
Now I'll multiply the –10 (on top) by the leading *x*(on the "side"), and carry the –10*x* to the bottom:	$\begin{array}{rl} & x - 10 \\ x+1 \,\big) & \overline{x^2 - 9x - 10} \\ & \underline{-x^2 \overset{-}{+} 1x} \\ & \quad -10x - 10 \\ & \quad -10x \end{array}$
...and I'll multiply the –10 (on top) by the 1 (on the "side"), and carry the –10 to the bottom:	$\begin{array}{rl} & x - 10 \\ x+1 \,\big) & \overline{x^2 - 9x - 10} \\ & \underline{-x^2 \overset{-}{+} 1x} \\ & \quad -10x - 10 \\ & \quad -10x - 10 \end{array}$
I draw the equals bar, and *change the signs* on all the terms in the bottom row:	$\begin{array}{rl} & x - 10 \\ x+1 \,\big) & \overline{x^2 - 9x - 10} \\ & \underline{-x^2 \overset{-}{+} 1x} \\ & \quad -10x - 10 \\ & \quad \underline{+10x + 10} \end{array}$

<table>
<tr>
<td>Then I add down:</td>
<td>

$$\begin{array}{r}
x - 10 \\
x+1\overline{)\,x^2 - 9x - 10} \\
-x^2 \overset{-}{+} 1x \\ \hline
-10x - 10 \\
+10x + 10 \\ \hline
0
\end{array}$$

</td>
</tr>
</table>

Answer:

Dividend	Divisor	Quotient	Degree of Dividend	Degree of Quotient
?	?	?	?	?

Divide $3x^3 - 5x^2 + 10x - 3$ by $3x + 1$;

$$\begin{array}{r}
x^2 - 2x + 4 \\
3x+1\overline{)\,3x^3 - 5x^2 + 10x - 3} \\
-3x^3 \overset{-}{+} 1x^2 \qquad\qquad \\ \hline
-6x^2 + 10x - 3 \\
+6x^2 + 2x \quad \\ \hline
12x - 3 \\
-12x \overset{-}{+} 4 \\ \hline
-7
\end{array}$$

The solution to this division is $x^2 - 2x + 4 - \dfrac{7}{3x + 7}$.

Dividend	Divisor	Quotient	Degree of Dividend	Degree of Quotient
?	?	?	?	?

Try This:
1. Divide $x^2 – 6x + 8$ by $x – 2$
2. Divide $2x^3 + 3x^2 - 6x - 8$ by $x – 4$
3. Divide $4c^3 + 4c^2 + 5c - 3$ by $2c + 1$
4. Divide $9x^2 -21x + 4x^4 – 9$ by $2x – 3$
5. Divide $3x^4 + 4 – 10x + 7x^3$ by $3x - 2$

Process Questions:

1. What did you notice about the degrees of the dividend and of the divisor?
2. What did you notice about the degree of the dividend and the degree of the quotient?
3. How do these degrees of the dividend and the divisor affect the degree of the quotient?

ACTIVITY 5. Reflection Log:

1. What difficulties did you encounter while learning the concept of dividing polynomials?
2. How did you overcome these difficulties?

End of Explore

You gave your initial ideas about polynomials, and you recall the long process of dividing polynomials. What you will learn in the next section will enable you to identify ways that help facilitate one's understanding of new concepts.

FIRM-UP

Your goal in this section is to learn and understand key concepts of the Remainder and Factor theorems, and Factoring Polynomials. And towards the end of this section you will be encouraged to answer problems applying these concepts.

LONG DIVISION VS SYNTHETIC DIVISION

Synthetic division is a shortcut method for dividing two polynomials which can be used in place of the standard long division algorithm. This method reduces the dividend and divisor polynomials into a set of numeric values. After these values are processed, the resulting set of numeric outputs is used to construct the polynomial quotient and the polynomial remainder.

In a long division, if x – 1 is a factor, then it will divide out evenly; that is, if we divide $x^2 + 5x + 6$ by $x - 1$, we would get a zero remainder. Let's check:

$$\begin{array}{r l}
 & x + 6 \\ \cline{2-2}
x - 1 & \big) x^2 + 5x + 6 \\
 & \underline{x^2 - x} \\
 & \phantom{x^2 + {}} 6x + 6 \\
 & \phantom{x^2 + {}} \underline{6x - 6} \\
 & \phantom{x^2 + 5x + {}} 12
\end{array}$$

As expected (since we know that $x - 1$ is not a factor), we got a non-zero remainder. What does this look like in synthetic division?

First, write the coefficients ONLY inside an upside-down division symbol:

$$\begin{array}{|ccc} 1 & 5 & 6 \\ & & \\ \hline \end{array}$$

Make sure you leave room inside, underneath the row of coefficients, to write another row of numbers later.

Put the test zero, $x = 1$, at the left:

$$\begin{array}{c|ccc} 1 & 1 & 5 & 6 \\ & & & \\ \cline{2-4} \end{array}$$

Take the first number inside, representing the leading coefficient, and carry it down, unchanged, to below the division symbol:

$$\begin{array}{r|rrr} 1 & 1 & 5 & 6 \\ & & & \\ \hline & 1 & & \end{array}$$

Multiply this carry-down value by the test zero, and carry the result up into the next column:

$$\begin{array}{r|rrr} 1 & 1 & 5 & 6 \\ & & 1 & \\ \hline & 1 & & \end{array}$$

Add down the column:

$$\begin{array}{r|rrr} 1 & 1 & 5 & 6 \\ & & 1 & \\ \hline & 1 & 6 & \end{array}$$

Multiply the previous carry-down value by the test zero, and carry the new result up into the last column:

$$\begin{array}{r|rrr} 1 & 1 & 5 & 6 \\ & & 1 & 6 \\ \hline & 1 & 6 & \end{array}$$

Add down the column:

$$\begin{array}{r|rrr} 1 & 1 & 5 & 6 \\ & & 1 & 6 \\ \hline & 1 & 6 & 12 \end{array}$$

This last carry-down value is the remainder.

Comparing, you can see that we got the same result from the synthetic division, the same quotient (namely, 1x + 6) and the same remainder at the end (namely, 12), as when we did the long division:

$$\begin{array}{r|l} & x + 6 \\ \hline x-1 & x^2 + 5x + 6 \\ & \underline{x^2 - x} \\ & \qquad 6x + 6 \\ & \qquad \underline{6x - 6} \\ & \qquad\qquad 12 \end{array}$$

$$\begin{array}{r|rrr} 1 & 1 & 5 & 6 \\ & & 1 & 6 \\ \hline & 1 & 6 & 12 \end{array}$$

The results are formatted differently, but you should recognize that each format provided us with the result, being a quotient of x + 6, and a remainder of 12.

$$\begin{array}{r|rrr} 1 & 1 & 5 & 6 \\ & & 1 & 6 \\ \hline & 1 & 6 & 12 \end{array}$$

Try another one.

Complete the indicated division. $(2x^4 - 3x^3 - 5x^2 + 3x + 8) \div (x - 2)$

$$\begin{array}{r|rrrrr} 2 & 2 & -3 & -5 & 3 & 8 \\ & & & & & \\ \hline & & & & & \end{array}$$

For this first exercise, I will display the entire synthetic-division process step-by-step.

First, carry down the "2" that indicates the leading coefficient:

$$\begin{array}{r|rrrrr} 2 & 2 & -3 & -5 & 3 & 8 \\ & \downarrow & & & & \\ \hline & 2 & & & & \end{array}$$

Multiply by the number on the left, and carry the result into the next column:

$$\begin{array}{r|rrrrr} 2 & 2 & -3 & -5 & 3 & 8 \\ & & 4 & & & \\ \hline & 2 & & & & \end{array}$$

Add down the column:

$$\begin{array}{r|rrrrr} 2 & 2 & -3 & -5 & 3 & 8 \\ & & 4 & & & \\ \hline & 2 & 1 & & & \end{array}$$

Multiply by the number on the left, and carry the result into the next column:

$$\begin{array}{r|rrrrr} 2 & 2 & -3 & -5 & 3 & 8 \\ & & 4 & 2 & & \\ \hline & 2 & 1 & & & \end{array}$$

Add down the column:

$$\begin{array}{r|rrrrr} 2 & 2 & -3 & -5 & 3 & 8 \\ & & 4 & 2 & & \\ \hline & 2 & 1 & -3 & & \end{array}$$

Multiply by the number on the left, and carry the result into the next column:

$$\begin{array}{r|rrrrr} 2 & 2 & -3 & -5 & 3 & 8 \\ & & 4 & 2 & -6 & \\ \hline & 2 & 1 & -3 & & \end{array}$$

Add down the column:

$$\begin{array}{r|rrrrr} 2 & 2 & -3 & -5 & 3 & 8 \\ & & 4 & 2 & -6 & \\ \hline & 2 & 1 & -3 & -3 & \end{array}$$

Multiply by the number on the left, and carry the result into the next column:

$$\begin{array}{r|rrrrr} 2 & 2 & -3 & -5 & 3 & 8 \\ & & 4 & 2 & -6 & -6 \\ \hline & 2 & 1 & -3 & -3 & \end{array}$$

Add down the column for the remainder:

$$\begin{array}{r|rrrrr} 2 & 2 & -3 & -5 & 3 & 8 \\ & & 4 & 2 & -6 & -6 \\ \hline & 2 & 1 & -3 & -3 & 2 \end{array}$$

The completed division is:

$$\begin{array}{r|rrrrr} 2 & 2 & -3 & -5 & 3 & 8 \\ & & 4 & 2 & -6 & -6 \\ \hline & 2 & 1 & -3 & -3 & 2 \end{array}$$

These are the numerical coefficients of the terms in the quotient or also known as the depressed polynomial and the last number which is 2 is the remainder. →

The depressed polynomial is
$2x^3 + x^2 -3x – 3$ Remainder 2
The quotient is:

$$\boxed{2x^3 + x^2 -3x – 3 + 2/(x – 2)}$$

Process Questions:

1. What is the quotient when $2x^4 – 3x^3 – 5x^2 + 3x + 8$ is divided by x – 2?
2. How would you compare the degree of the dividend and the degree of the quotient when the divisor is linear of degree one?
3. What is the remainder when $2x^4 – 3x^3 – 5x^2 + 3x + 8$ is divided by x – 2?
4. What is the degree of the quotient when $2x^4 – 3x^3 – 5x^2 + 3x + 8$ is divided by
   x – 2 ?
5. What is the advantage of using the synthetic division than using the long division?

For more examples of division of polynomial using synthetic division, you may visit the following websites.

https://www.youtube.com/watch?v=nefo9cUo-wg
https://www.youtube.com/watch?v=u0ep4v_bweQ
These websites show examples of division of polynomials using synthetic division.

ACTIVITY 6. Divide using Synthetic Division:

1. $(x^3 - 12x^2 - x + 8) \div (x - 2)$
2. $(x^4 - 6x^2 + 7x - 12) \div (x + 3)$
3. $(2x^3 - 7x^2 - 3x - 5) \div (x + 1)$
4. $(3x^3 - 5x^2 - 17x - 12) \div (x - 4)$
5. $(4c^3 + 4c^2 + 5c - 3) \div (2c + 1)$

ACTIVITY 7. WHAT'S THE REMAINDER?

Review of evaluating polynomials:
If $P(x) = x^3 + 3x^2 - 5x - 4$, P(2) means replacing x by 2 in the given P(x). $P(2) = (2)^3 + 3(2)^2 - 5(2) - 4 = 8 + 12 - 10 - 4 = 6$. Hence, P(2) = 6.

Complete the table below:

Divide P(x) by (x-r)			Quotient	Remainder	Value of P(r)
P(x)	x-r	r			
$X^4 - 3x^2 - 5x + 20$	x + 5	-5			
$3x^3 - 5x^2 - 17x - 12$	x - 4	4			
$x^3 - 12x^2 - x + 8$	x – 2				
$2x^3 - 7x^2 - 3x - 5$	x + 1				

Process Questions:

1. What method did you use in finding the quotient and the remainder?
2. What did you notice about the values of the remainder and the corresponding values of P(r)?
3. What is the other way of finding the remainder when P(x) is divided by x – r?
4. How would you express the remainder when (x – r) divides P(x) in terms of r?
5. What is the remainder when (x – 1) divides ($x^3 + 1$) ?

Write in the box below your formulated generalization about the remainder when P(x) is divided by x – r.

The website below shows examples of long division of polynomials and states the remainder theorem. Illustrates the use of the remainder theorem showing that the remainder in both processes remains the same.
https://www.youtube.com/watch?v=3LXB2FIR2WU

For mastery of the use of the remainder theorem, visit an Interactive website below for your practice.
http://interactive.onlinemathlearning.com/remainder_theorem.php?action=generate&numProblems=10

ACTIVITY 8. IS IT A FACTOR?

A. Fill up the last two columns and answer the process questions below.

DIVIDEND P(x)	=	QUOTIENT	TIMES	DIVISOR (x – r)	PLUS	REMAINDER (R)	P(r)
$X^2 + 3x - 18$	=	x + 6	•	x - 3	+	?	?
$2x^3 – 3x^2 –8x+ 12$	=	$2x^2 – 7x + 6$	•	x + 2	+	?	?
$3x^3 + 4x^2 - 5x - 2$	=	$3x^2 + 7x + 2$	•	x - 1	+	?	?

B. Tell whether the second expression is a factor of the first expression:

	YES	NO
1. $x^2 – 3x + 2$; x + 2		
2. $2x^3 + 17x^2 + 23x – 42$; x – 1		

3. $x^3 - x^2 - 10x - 8$; $x + 2$		
4. $x^4 + 2x^3 + 2x^2 - 3$; $x - 2$		
5. $6x^3 - 25x^2 + 2x + 8$; $2x + 1$		

Process Questions:

1. What are the values of P(r)?
2. How would you compare the values of P (r) and the values of the remainder
   or R?
3. If a polynomial is divided by a first degree binomial and the remainder is 0, what does this tell you about the relationship between the binomial and the polynomial?
4. When can you say that x – r is a factor of P(x)?
5. What generalization can you formulate based on your findings?
6. Does this mean that you have to evaluate p(r) to determine if x – r is a factor of p(x)?
7. What short cut can you use to test if (x – r) is a factor of p(x)?

The website below explains the factor theorem.
http://www.purplemath.com/modules/factrthm.htm
This website explains the remainder and factor theorems.
https://www.youtube.com/watch?v=_IPqCaspZOs
This website describes and gives examples of the factor theorem
https://www.youtube.com/watch?v=P-RvhBqBPOA

ACTIVITY 9. FACTORING POLYNOMIALS!

	PROBLEM 1	PROBLEM 2	PROBLEM 3
How do we factor polynomials?	What are the factors of $3x^2$-10x -8 ?	What are the factors of $x^3 + 3x^2 - x - 3$?	What are the factors of x^4 - 16?
	The polynomial is a ______________ trinomial.	The polynomial is of degree ______ so we first use the factor theorem using __________.	The polynomial is of degree _____ and it is a __________ of two _________. It can be factored using factoring _________ or using the ------------------ -theorem.

Complete this statement: The process of factoring polynomials depends on the _______________ and _______________ of the polynomial.

PROCESS QUESTIONS:

1. Was there only one way of factoring polynomials? If yes, explain. If not, why are there many ways to factor polynomials? How will you know which way to use? Explain.

   The generalization you formulated in the earlier activity is called the Factor Theorem. It can be used to determine whether a binomial x-r is a factor of a polynomial P(x). It can also be used to determine all the factors of a polynomial.

Do the following:

1. Show that $x – 3$ is a factor of $x^3 -4x^2 + x + 6$ using the factor theorem.
2. Use synthetic division in dividing $x^3 -4x^2 + x + 6$ by $x – 3$. What is the depressed polynomial? Is the depressed polynomial factorable? What are its factors? What are the factors of $x^3 -4x^2 + x + 6$?
3. Check your answer by multiplying out these factors.
4. Show that $x – 2$ is a factor of $x^3 – 7x^2 + 4x + 12$. Then find the remaining factors of the polynomial.

Process Questions:

1. What are the different ways of factoring polynomials?
2. Is there any best way of factoring polynomials? Why?
3. Why is there a need to study the factors of polynomials? How is it used to real-life? Cite a situation where it is applied..
4. How are the complete factors of a polynomial determined?

   This website below explains how to find factors of polynomials.
   https://www.youtube.com/watch?v=xJvrhlqwCr0

   For more practice with factoring polynomials, visit the website below.
   https://www.youtube.com/watch?v=xJvrhlqwCr0

The following activity is a real-life situation where you can make use of the concepts of the factors of polynomials and polynomial equations. Each part of the modelling process is provided to guide you as you answer the given problem.

ACTIVITY 10. Best Fit

PROBLEM: An appliance store has limited data on the profit P(x) in thousands, for selling x appliances during a month period. If x denotes the months in a year and x = o corresponds to January, find the third degree polynomial function that best fits or that can model the information in the following table.

x (Month)	January	February	April	June	November
P(x) Profit	0	36	40	0	0

FORMULATE: (Write the concepts you need to answer the problem.)

COMPUTE: (Write your complete solution.)

INTERPRET: (Write a paragraph about the interpretation of your answer.)

VALIDATE: (Provide one more example to validate your answer.)

REPORT:

Non-Online: Submit your written report.

Online: Send your written report using the student dash board.

ACTIVITY 11. Quiz (Formative Assessment)

A. Find the remainder when the first polynomial is divided by the second polynomial.

1. $x^3 - 5x^2 + 17x - 12$; $x + 5$
2. $3x^3 - 28x^2 + 58x + 52$; $x + 2$
3. $3x^4 - 8x^3 - 10x - 5$; $x + 3$
4. $5x^6 - 5x^5 - 5x^4 + 5x^3 + 5x^2 + 5x - 5$; $x + 1$
5. $x^5 + x^3 - x^2 - 1$; $2x - 1$

B. Determine if the second polynomial is a factor of the first polynomial.

1. $x^3 - 2x + 1$; $x - 1$
2. $4x^3 - x^2 - 6x + 16$; $x - 2$
3. $x^4 + 2x^2 - 7x - 9$; $x - 2$
4. $2x^4 + 11x^3 + 21x^2 + 17x + 5$; $x + 1$
5. $x^5 - x^4 - 7x^3 + 7x^2 - 12x - 24$; $x - 3$

END OF FIRM UP

In this section, the discussion was about the key concepts on Remainder and Factor Theorems, and Factors of Polynomials. The use of Synthetic Division was also given emphasis.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.

DEEPEN

ACTIVITY 12. What's Up in a Roller Coaster?

The polynomial function $f(x) = 0.001x^3 - 0.12x^2 + 3.6x + 10$ models the path of a portion of the track of a roller coaster. Find the height of the track for x = 20 meters.

You can use the function to estimate the height of the track by evaluating the function for x = 0, 20, 40 and 60 meters.

Use synthetic substitution to divide $0.001x^3 - 0.12x^2 + 3.6x + 10$ by $x - 20$.

20	0.001	-0.12	3.6	10
		0.02	-2.0	24
	0.001	-0.10	1.2	34

If x = 20 meters, the height of the track is 34 meters.
Using the remainder theorem, f(20) = 0.001(20) -0.12(20)+36(20)+10.
Validate if f(20) = 34 meters.

ACTIVITY 13. What's the Future Sales?

A company's sales, in millions of pesos, of consumer electronics can be modelled by $S(x) = -1.2x^3 + 18x^2 + 26.4x + 678$, where x is the number of years since 2005.

Complete the table and answer the questions below.

YEAR	$S(x) = -1.2x^3 + 18x^2 + 26.4x + 678$	SALES
2005	$S(x) = -1.2(1)^3+18(1)^2+26.4(1)+678$	721.20
2006	$S(x) = -1.2(2)^3+18(2)^2+26.4(2)+678$	668.40
2007	$S(x) = -1.2(3)^3+18(3)^2+26.4(3)+678$	886.8
2017		
2020		

1. What theorem is used in computing the sales in the first three columns of the table above?
2. What other method can you use to estimate the sales?
3. How would you interpret the result?
2. Do you think this model is useful in estimating future sales? Validate your answer.

ACTIVITY 14. Customer Service Alert!

PROBLEM: A technical help service of a computer software company found that the following functions approximate the number of calls received at any one time. It t represents the time (in hours) since the service opened at 8:00 am., how many calls are expected at noon?

DAY	Polynomial function
Saturday	$C(t) = -0.0052t^4 + 2t^3 + 24t$
Sunday	$C(t) = -0.00625t^4 + t^3 + 16t$
Monday	$C(t) = -0.008t^4 + 3t^3 + 10t$

FORMULATE: (Write the concepts you need to answer the problem.)

There are 4 hours from 8:00 am to 12 noon. Use t = 4.

COMPUTE: (Write your complete solution.)

For Saturday, $C(4) = -0.0052(4)^4 + 2(4)^3 + 24(4) = 223$
For Sunday, $C(4) = -0.00625(4)^4 + (4)^3 + 16(4) = 127$
For Monday, $C(4) = -0.008(4)^4 + 3(4)^3 + 10(4) = 230$

INTERPRET: (Write a paragraph about the interpretation of your answer.)

From 8:00 am to 12:00 noon, 223 calls are expected on a Saturday; 127 calls are expected on a Sunday and 230 calls are expected on a Monday. The computation is based on the function found by the technical help service of a computer software company that approximates the number of calls received at any one time.

VALIDATE: (Provide one more example to validate your answer.)

Using the same functions in the given problem, how many calls are expected at 10:00 am? In this case t = 2 hrs.
For Saturday, $C(4) = -0.0052(2)^4 + 2(2)^3 + 24(2) = 64$
For Sunday, $C(4) = -0.00625(2)^4 + (2)^3 + 16(2) = 40$
For Monday, $C(4) = -0.008(2)^4 + 3(2)^3 + 10(2) = 44$

Process Questions:

1. How many calls are expected at noon for each day?
2. How many calls are expected at 10:00 am for each day? Will this validate the
   answers to the number of calls at noon for each day? Explain your answer?
3. How did you come up with your answers?
4. Which of the three days has the most number of calls at noon time?
5. Which of the three days has the least number of calls at noon time?
6. If you are a customer service manager, would you still be interested to know the mathematical models for other days? Why?
7. What do you think is the use or implication of the results of these mathematical models to customer service manager?

Use the modelling process in doing activity # 15.

ACTIVITY 15. Find Out…

a. The number of college students from Korea who study in the Philippines can be modelled by the function $s(x) = 0.02x^4 - 0.52x^3 + 4.03x^2 + 0.09x + 77.54$, where x is the number of years since 1993 and s(x) is the number of students in thousands. How many Korean students will study in the Philippines in 2018?

How many years are there from 1993 to 2018? You can use this function to estimate the number of Korean students studying in the Philippines in 2018 by evaluating the function for $x = 25$. You may use synthetic division to divide $s(x)$ by $x - 25$. Hence, by 2018, there will be about 2,286 Korean College students studying in the Philippines.

b. The function $c(x) = 2.64x^3 - 22.37x^2 + 53.81x + 548.24$ can be used to approximate the number, in thousands, of Asian college students studying in the Philippines x years since 2000. How many Asian college students can be expected to study in the Philippines in 2015?

Process Questions:
1. What value of x will you use to evaluate $c(x)$? Why?
2. What method will you use to evaluate $c(x)$?
3. Which method did you find easier to use and why?
4. Will this value practically answer: "How many Asian college students can be expected to study in the Philippines in 2015?"

c. Find the value of k so that $(x-4)$ is a factor of $x^3-x^2 + kx -8$.

Process Questions:
1. Will $p(4) = 0$? Why?
2. What is the value of k when $p(4)=0$?

d. If $(x+2)$ is one of the factors of $x^3 - x^2 -10x -8$, what are the remaining factors?

Process Questions:
1. What is the quotient when $x^3 - x^2 -10x -8$ is divided by $(x + 2)$?
2. $x^3 - x^2 -10x -8 = (x+2)$ (?   )
3. How will you factor your answer in item 2?
4. What are the complete factors of $x^3 - x^2 -10x -8$?
5. What method did you use?

e. What is the value of k so that $(x^3 +4x^2+x + k)\div(x+2)$ give a remainder of 3?

Process Questions:
1. What is the value of your r?
2. What is the value of $p(r)$? Why?
3. What is the value of k if $p(-2) = 3$?

f. What are the values of a and b if $(x + 1)$ and $(x + 2)$ are factors of $x^4 + ax^3+ 6x^2 + bx -27$?

Process Questions:
1. What are the two values of r? What is your $p(x)$?
2. What is the resulting equation using $p(-1)=0$?

3. What is the resulting equation using p(-2)=0?
4. How will you solve the values of a and b in the system of equations?
5. How will you check your answer?

g. A specific car's fuel economy in miles per gallon can be approximated by $f(x) = 0.00000056x^4 - 0.000018x^3 + 1.38x - 0.38$, where x represents the car's speed in miles per hour. Determine the fuel economy when the car is traveling 40, 50, and 60 miles per hour.

Process Questions:

1. What is f(40)?, f(50)?, and f(60)?
2. Which car speed maximizes fuel consumption?
3. Can you suggest some ways or tips on how to save fuel?
4. What do you think is the best way to save fuel? Why?
5. How did the modelling process help you answer the different problems?
6. **How do values of one variable behave in terms of the other in the polynomial function that models a real- world situation?**

To enhance your skill on the application of polynomial function, visit
The interactive practice drills in the website below.
https://braingenie.ck12.org/skills/106914

ACTIVITY 16. Summative Quiz

A. Find the value of k so that (x+2) is a factor of the following polynomials:

1. $kx^3 - 6x^2 + 7x - 10$
2. $2x^2 - kx - 24$
3. $2kx^5 + 4kx^3 - 8k - 16$
4. $3kx^2 + 8x + 1 - k$
5. $(1+k)^2x^3 + 5kx + 2k^2$

B. Find the value of k so that the remainder is R when f(x) is divided by (x – r):

1. $f(x) = 2x^2 + 3kx + 4$, R = 10, r = 3
2. $f(x) = 2kx^3 + kx + 5$, R = -6 r = -1
3. $f(x) = kx^3 + (1 - k)x - 9$, R = 7, r = -1/2
4. $f(x) = x^3 + kx^2 - 4x - 4k$, R = -3, r = -3
5. $f(x) = kx^3 + x^2 - 18x - 10$, R = -1, r = -1/2

C. Determine the values of a and b so that (x + 1) and (x – 2) are factors of f(x).

1. $ax^3 - 2bx^2 + 3bx - 4a$
2. $ax^3 - bx^2 + 3x - 12$
3. $2bx^4 + 4ax^2 - 16x + 12$
4. $2x^4 + ax^3 - 13x^2 + bx + 12$
5. $x^3 + ax^2 - x + b$

D. Extend your understanding by solving the following problems.

1. Find the values of k if (3x – 2) is a factor of $f(x) = 24x^2 - 42 - 6k$.
2. What is the value of p in $f(x) = 4px - 2x + x - 12$ if the remainder When f(x) is divided by (x – 3) is -4?
3. Find the values of p and q in $f(x) = 2x^4 + px^3 - 11x^2 + qx + 12$ if (x+3) and (x – 2) are both factors of f(x).
4. Find the values of m and n in $f(x) = mx^4 - 10nx^2 - 14x + 20$ if (x + 1) and (x-2) are both factors of f(x).
5. Find the values of a, b, and c if (x-1), (x+2), and (x-2) are factors of $f(x) = 2ax^4 - bx^3 - cx - 16$.

E. Solve the following problems following the modelling cycle.

1. A company's sales, in millions of peso, of electrical appliances can be modelled by $S(x) = -1.3x^3 + 16x^2 + 25.4x + 576$, where x is the number of years since 2005. Use synthetic substitution to estimate the sales for 2018 and 2020. Do you think this model is useful in estimating future sales? Why?

2. Because of the combustion of fossil fuels, the concentration of carbon dioxide in the atmosphere is increasing. Research indicates that this will result in a greenhouse effect that will change the average global surface temperature. Assuming a vigorous expansion of coal use, the future amount A(t) of atmospheric carbon dioxide concentration be approximated (in parts per million) by

   $A(t) = \frac{21}{2400}t^3 + \frac{1}{20}t^2 + \frac{7}{6}t + 340$, where t is in years, t = 0 corresponds to 1980 and $0 \leq t \leq 60$.

   Questions/answer:

   a. Estimate the year when the carbon dioxide concentration will be 400.

   b. Approximate the amount A(t) of atmospheric carbon dioxide concentration (in parts per million) in the year 2020. Is this result considered health friendly? Why?

   c. In your own simple way, how can you help solve the greenhouse effect problem that we are facing?

3. A box is to be constructed by cutting out equal squares form the corners of a square piece of cardboard and turning up the sides.
   a. Write a function V(x) for the volume of the box.
   b. For what value of x will the volume of the box equal 1152 cubic centimeters?

   c. What will be the volume of the box if x = 6 centimeters?

4. A rectangular box for a new product is designed in such a way that theThree dimensions always have a particular relationship defined by the variable x. The volume of the box can be written as $6x^3 + 31x^2 + 53x + 30$, and the height is always x + 2. What are the length and width of the box?

5. A motorboat travelling against waves accelerates from a resting position. Suppose the speed of the boat in feet per second is given by the function f(t) = -0.04t + 0.8t + 0.5t – t, where t is the time in seconds.
   a. Find the speed of the boat in 1, 2, and 3 seconds.

   b. It takes 6 seconds for the boat to travel between two buoys while it is accelerating. Use synthetic substitution to find f(6) and explain what this means.

F. Solve it!

PROBLEM: An aspirin tablet in the shape of a right circular cylinder has height 1/3 centimeter and radius ½ centimeter. The manufacturer also wishes to market the aspirin in capsule form. The capsule is to be 3/2 centimeters long, in the shape of a right circular cylinder with hemisphere attached to both ends.

a. Can you find the formula for the volume of the capsule?
b. What do you think is the radius of the capsule so that its volume is equal to that of a tablet?

FORMULATE: (Write the concepts you need to answer the problem.)

COMPUTE: (Write your complete solution.)

INTERPRET: (Write a paragraph about the interpretation of your answer.)

VALIDATE: (Provide one more example to validate your answer.)

REPORT:

Non-Online: Submit your written report.
Online: Send your written report using the student dash board.

Now think about the answers to the following questions. Write your answer in the answer box.

1. In getting the remainder when a polynomial p(x) is divided by a linear expression (x-r), you may use either the remainder theorem or synthetic division. Which method did you find easier to use, and why?

2. How do values of one variable behave in terms of the other in the polynomial function that models a real- world situation?

Answer Box:

ACTIVITY 17. ANTICIPATION-REACTION GUIDE: A REVISIT!

Read each statement in the column TOPIC and write **A** if you agree with the Statement, otherwise write **D** in the third column.

Response Before Lesson	TOPIC: Polynomial Functions	Response After Lesson
	1. $\frac{(2x-5)}{3}$ is a polynomial	
	2. $\sqrt{2x-5}$ is a polynomial	
	3. The degree of $x^3 - 6x^2 + 8x + 5$ is 3	
	4. The leading coefficient of the polynomial $x - 11 + x^4 - x^2$ is -1.	
	5. In synthetic division, the last number in the last row of the process represents the quotient.	
	6. If $f(x) = x^4 - 5x^2 - 7x + 6$, $f(3) = 21$	
	7. Either the Remainder Theorem or Synthetic Division can be used to find the	

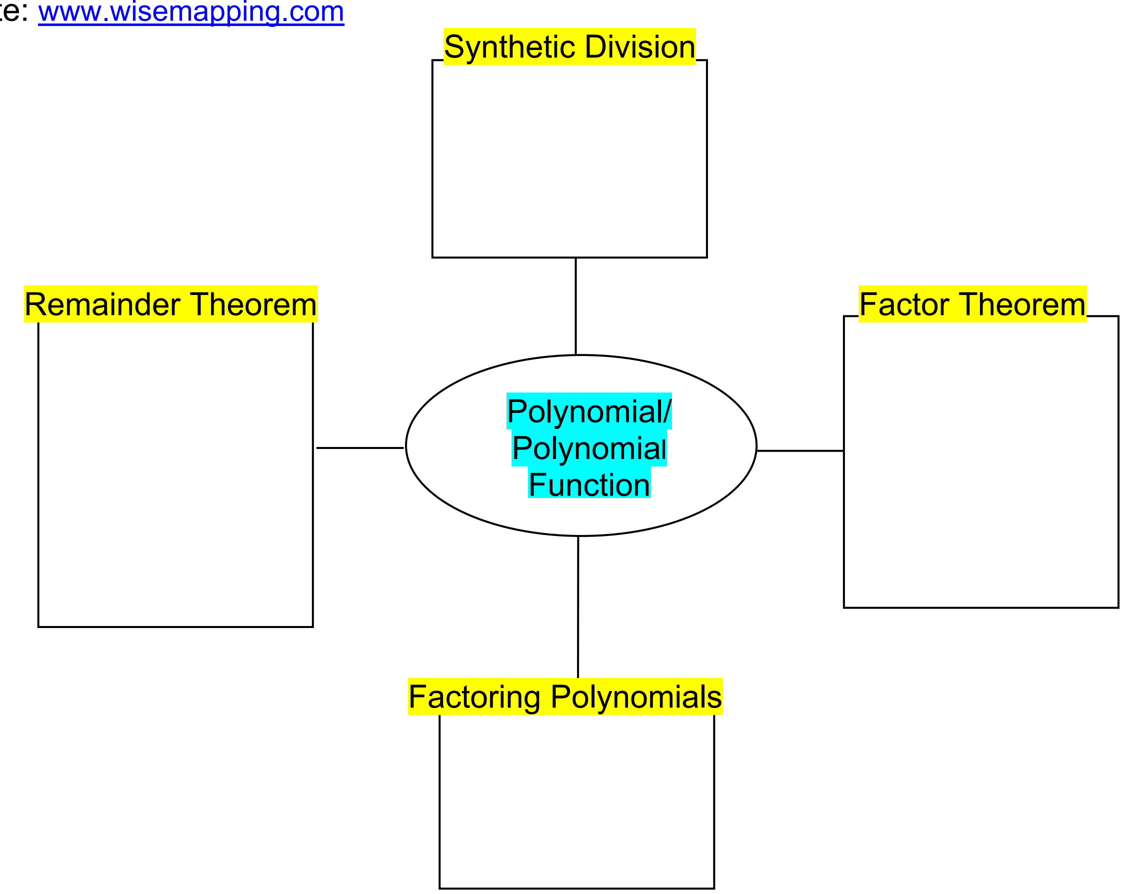
	remainder when a polynomial in x is divided by x – r, where r is a constant.	
	8. $x - 2$ is a factor of $x^4 - 16$.	
	9. $x + 3$ is a factor of $P(x)$ if $P(-3) = 0$.	
	10. If $x^3 - 6x^2 + 8x + 5$ is divided by $x - 2$, the remainder is 6.	
	11. If $3x^{14} - 2x^{12} + 3$ is divided by $x + 1$, the remainder is 3.	
	12. If one of the factors of $hx^3 + 3x^2 - 2hx + 1$ is $x+1$, what is the value of h?	

SUBMIT

ACTIVITY 18. CONCEPT MAPPING

Reference: http://www.glencoe.com/sec/teachingtoday/downloads/pdf/ReadingWritingMathClass.pdf)

Summarize the important concepts about properties of polynomial function by completing the concept map below. You may do it in wise mapping by clicking this site: www.wisemapping.com



END OF DEEPEN

In this section, the discussion was about the deeper use of the basic properties of polynomial functions. What new realizations do you have about the topic? What new connections have you made for yourself? Now that you have enhanced your understanding of the topic, you are now ready to do the tasks in the next section.

TRANSFER

Your goal in this section is to apply your learning on the basic properties of polynomial functions to real life situation. You will be given a practical task which will demonstrate your understanding of the concepts.

ACTIVITY 19. Performance Task

1. Use synthetic substitution to find p(2) for the polynomial
   $X^6 – 4^4 + 3x^2 -10$

2. If $f(x) = 3x^3 -6x^2 + x – 11$, find f(3).

3. Students individually or in groups of 3 members perform the following steps:
   a. Consider the polynomial $30x^3 – 48x^2 + 1758x + 10\ 000$ as the annual representation of DVD sales in million pesos where x is the number of years since 2010.

   b. Use the polynomial in step a to estimate the year where the DVD sales will be about Php 9 billion.

   c. Considering the mathematical model in a for the DVD sales, would you consider this business feasible? Why?

4. Submit your group or individual output.

RUBRIC for Scaffold level 1				
CRITERIA	EXCELLENT 4	SATISFACTORY 3	PROGRESSING 2	NEEDS IMPROVEMENT 1

Accuracy of Computation	Shows correct computation and solution with appropriate explanation for the problem.	Shows correct computation and solution	Some solutions show incorrect computation.	Shows incorrect computation and solution.
Presentation of Output	Written output is well organized, detailed and complete	Written output is well organized.	Written output has missing parts somewhat disorganized.	Written output is disorganized.
Mathematical model justification	The recommendation shows sophisticated understanding of the relevant ideas and processes.	The recommendation shows solid understanding of the relevant ideas and processes.	The recommendation shows somewhat limited understanding of the relevant ideas and processes.	The recommendation shows erroneous understanding of the relevant ideas and processes.

ACTIVITY 20. Lesson Closure – Reflection Organizer

You have accomplished the task- estimating a year when sales reached a specified amount using a mathematical model, successfully. How did you find the performance task? How did the task help you see the real world use of the topic?

You learned concepts in this lesson which are very important for the next lesson. To end this meaningfully and to welcome the next lesson, I want you to accomplish the next activity.

In this lesson, I learned about…

____
____
____
____
____
____

These concepts can be used in…

____
____
____
____
____
____

I understand that…

____
____
____
____
____
____

One aspect of the lesson I find most difficult…., the reason of my difficulty is…, I plan to remedy this by…

____
____
____
____
____
____

I can use these concepts in my life by…

____
____
____
____
____
____
____
____

END OF TRANSFER

You have completed the lesson. You have just learned the two theorems, the Factor and the Remainder Theorems, together with the Synthetic Division. These concepts will help you in the next part of the module. Before you go to the next lesson, you have to answer the following lesson post-assessment.

LESSON: POST-ASSESSMENT

It's now time to evaluate your learning. Click on the letter of the answer that you think best answers the questions. If you do well, you may move on to the next lesson. If your score is not at the expected level, you have to go back and take the module again.

1. Which of the following expressions is a polynomial?
   A. $2x^2 - \frac{5}{x}$
   B. $3x^3 + 2x^2 – 5x + 6$
   C. $5x^2 – 4x^{-2} + 3$
   D. $3x^2 + 5x^{1/2} – 2x$

2. What is the remainder when $x^6 + 4x^5 – 3x^2 + x – 2$ is divided by $x – 1$?
   A. -1
   B. 1
   C. 2
   D. 3

3. Which of the following is a factor of $x^3 + 8x^2 + 19x + 12$?
   A. x - 3
   B. x - 4
   C. x + 4
   D. x 1

4. If x- 1 and x + 5 are the two factors of $x^3 + 8x^2 – x – 5$, what is the other factor?
   A. x + 1
   B. x – 5
   C. x + 4
   D. x + 3

5. What is the remainder when $x^4 + 2x^3 - 3x^2 + 5x + 8$ is divided by x + 1?
   A. -1*
   B. 0
   C. 9
   D. 16

6. What are the factors of $2x^3 + 3x^2 – 8x + 12$?
   A. (x + 2) (x + 2) (2x – 3)
   B. (x + 2) (x - 2) (2x – 3)
   C. (x - 2) (x + 2) (2x – 3)

D. (x - 2) (x + 2) (2x + 3)

7. If the volume of the box is represented by the expression ($x^3 – x^2 - 10x - 8$) cm^3 and its width by (x+2) cm, what binomials can be used to represent the other two dimensions?
   A. (x-4)(x+1)
   B. (x+4)(x+1)
   C. (x+4)(x-1)
   D. ((x-4)(x-1)

8. If 2 and -1 are factors of $2x^4 - 3x^3 + ax^2 + bx + 2$, what are the values of a and b?
   A. a = 4; b = -3
   B. a = -4; b = 3
   C. a = 4; b = 3
   D. a = 4; b = -3

9. $x^3 + px^2 + qx + 6$ gives a remainder of 8 when divided by x + 1, and a remainder of -4 when divided by x – 2. Find the values of p and q.
   A. p = -2; q = -5
   B. p = -3; q = -5
   C. p = -2; q = -4
   D. p = 2; q = 5

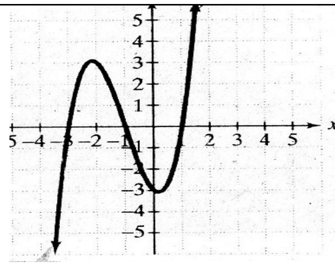
10. If the volume of the box is represented by the expression ($x^3 + 4x^2 + x - 6$) cm^3 and its width by (x + 2) cm, what binomials can be used to represent the other two dimensions?
   A. (x + 3) and (x - 1)
   B. (x + 3) and (x + 1)
   C. (x - 3) and (x - 1)
   D. (x - 3) and (x + 1)

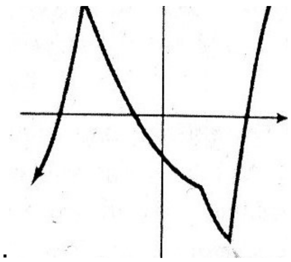
You have completed this lesson. Before you proceed to the next module, you have to answer the following post assessment.

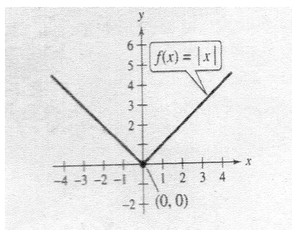
It's now time to evaluate your learning. Click on the letter of the answer that you think best answers the question. Your score will only appear after you answer all items. If you do well, you may move on to the next module. If your score in not at the expected level, you have to go back and take the module again.

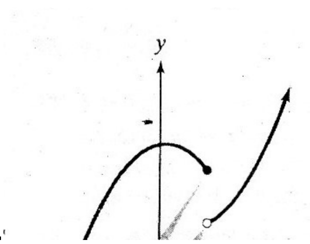
MODULE POST ASSESSMENT

1.	Which of the following is not a solution to the cubic equation below? $x^3 + 3x^2 – 4x - 12 = 0$ A. -3 B. -2 C. 2 D. 3	
2.	What is the remainder when $x^3 – 5x^2 – x + 5$ is divided by x - 2 ? A. -10 B. – 9 C. 9 D. 10	
3.	Which of the following is a polynomial equation? A. $3x^4 - x^3 + 2x – 7 = 0$ B. $\frac{4}{x} + 2x^3 - 4x^2 + 5 = 0$ C. $(3x – 4)^{-2} = 0$ D. $x^{1/2} – 2x + 8$	
4.	Given a polynomial and one of its factors, what are the remaining factors of the polynomial? $x^3 + 4x^2 - 4x – 16$; x + 2 A. (x – 2) (x – 4) B. (x – 2) (x + 4) C. (x + 2) (x + 4) D. (x + 2) (x - 4)	
5.	What are the zeros of the function $f(x) = x^3 - 4x^2 - 19x - 14$? A. -1, -2, 7 B. -1, 2, -7 C. -1, 2, 14 D. -2, 7, -14	
6.	One zero of $f(x) = x^3 - 7x^2 - 6x + 72$ is 4. What is the factored form of $x^3 - 7x^2 - 6x + 72$? A. (x-6)(x+3)(x+4) B. (x-6)(x+3)(x-4) C. (x+6)(x+3)(x-4) D. (x+12)(x-1)(x-4)	
7.	Which of the following illustrates the the graph of polynomial function?	

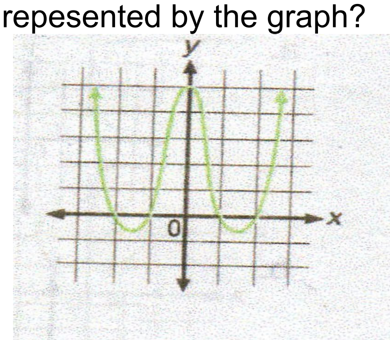
A. 

B. 

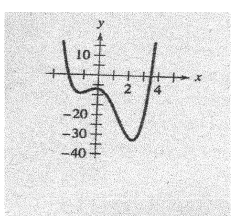
C. 

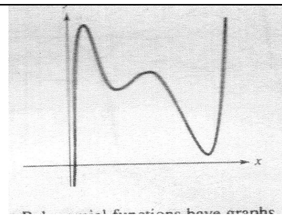
D. 

8. Which function is repesented by the graph?

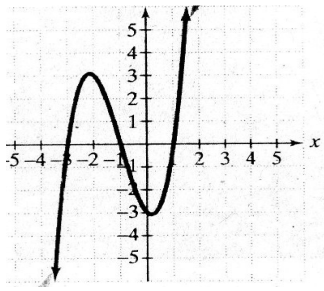


A. $f(x) = 2x + 1$
B. $f(x) = 2x^3 + 4x^2 - x + 4$
C. $f(x) = x^4 - 5x^2 + 4$
D. $f(x) = x^5 + 2x^4$

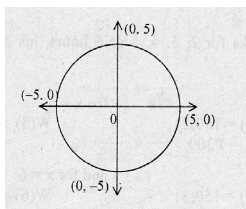
9. A specific car's economy in miles per gallon can be approximated by $f(x) = 0.00000056x^4 – 0.000018x^3 – 0.016x^2 + 1.38x – 0.38$, where x represents the car's speed in miles per hour. What is the fuel economy when the car is travelling 40 miles per hour? A. 27.25 miles per gallon B. 28.5 miles per gallon C. 29.5 miles per gallon D. 30 miles per gallon
10. The profit in hundreds of peso for selling c calculators per day can be modelled by $f(x) = -0.005c^4 + 0.25c^3 + 0.01c^2 – 2.5c + 100$. What is the meaning of zeros in this situation? A. profit on sales B. calculators sold C. break even sales D. sales shortages
11. The amount of money a certain foundation took in from 2006 to 2013 can be modeled by $M(x) = -2.03x^3 + 50.1x^2 -214x+4020$. What is the significance of each zero in the context of the situation? A. The amount of money taken by the foundation. B. The years where the foundation did not take any amount of money. * C. The years where the foundation took a of money. D. The years where the foundation have the most income.
12. The graph of one of the following functions is shown below. Identify the function shown in the graph and explain why each of the others is not the correct function. You can use graphing utility to verify your result.  A. $f(x) = x^2(x + 2)(x – 3.5)$ B. $g(x) = (x + 2)(x – 3.5)$ C. $h(x) = (x + 2)(x – 3.5)(x^2 + 1)$ D. $k(x) = (x + 1)(x + 2)(x – 3.5)$
13. Which of the following CANNOT be a graph of polynomial function? A.



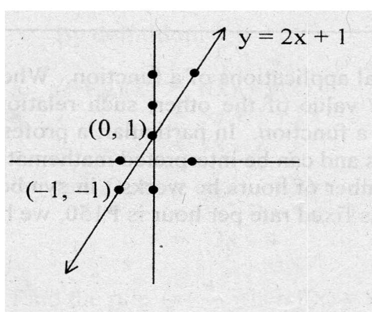
B.

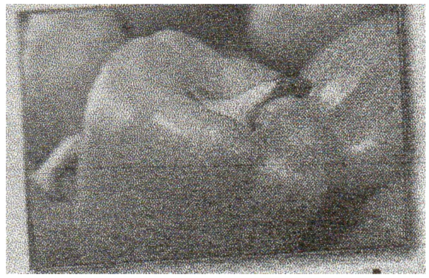


C.



D.





14. A bulk fruit and vegetable storage bin with dimension 2 feet by 3 feet by 4 feet needs to be increased in size to hold five times as much food as the current bin.
(Assume each dimension is increased by the same amount.). What is the function that represents the volume V of the new bin? What are the dimensions of the new bin?
    A. $V(x) = x^3 + 9x^2 + 26x - 96$
       Dimensions: 4 ft x 5 ft x 6 ft

    B. $V(x) = x^3 - 9x^2 + 26x - 96$
       Dimensions: 4 ft x 6 ft x 7 ft

    C. $V(x) = x^3 + 9x^2 - 26x - 96$
       Dimensions: 5 ft x 6 ft x 7 ft

    D. $V(x) = x^3 - 9x^2 - 26x - 96$
       Dimensions: 3 ft x 5 ft x 7 ft

15. A computer store determines that the profit for producing x computer units per day is $P(x) = x - 7x + 8x + 12$. What possible number of computer units sold in a day for a break even?
    A. 1 and 2
    B. 2 and 3
    C. 3 and 4
    D. 5 and 6

16. The length of the rectangular tank is 3 ft more than its width and the height is 5 ft more than its width.
How would you determine the dimensions of the tank if its volume is 100 ft^3?
    A. Determine the polynomial function for the volume and compute one real zero to represent the width of the tank
    B. Use guess and check in finding the width of the tank
    C. Determine the polynomial function for the volume and compute one real zero to represent the height of the tank
    D. Determine the polynomial function for the volume and compute one real zero to represent the length of the tank

17. The function $g(x) = 1.284x^3 - 0.004x^2 + 0.27x + 1.263$ can be used to model the average price of a gallon of diesel in a given year if x is the number of years since 2010. What is the easier way to determine the average price of diesel in year 2017?
    A. Find the zeros of g(x).
    B. Evaluate g(2017)
    C. Find the factors of g(x)
    D. Evaluate g(7)

18. The water refilling station owner wants to know the volume of water dispensed in the station in a day. As a station in charge you are tasked to predict water dispensed in the station for the next thirty days and present a mathematical equation for the data.
    Which of the following standards should the report be evaluated?
    A. Organization of data and justification of recommendation
    B. Accuracy of computation, organization of data and justification of recommendation
    C. Representation of output and organization of data
    D. Representation of output

19. You are a packaging designer of a certain company and you were tasked to create a tool for evaluating the product. The product is a box which will be used for storing the canned goods that will yield a maximum volume. Which of the following criteria should be considered the least as part of the rubric for rating the performance task?
    A. Justification/Application of concept.
    B. Practicality of the Product
    C. Accuracy of Computations
    D. Delivery of Report

20. Which of the following product will best apply the concept of a cubic polynomial equations/functions?
    A. Volume of a box
    B. Surface area of a box
    C. Minimum cost for creating a box
    D. Maximum profit

GLOSSARY OF TERMS USED IN THIS LESSON

Polynomial function – is a function defined by an equation of the form $f(x) = a_n x^n + a_{n-1} xn-1 + \ldots + a_1 x + a_0.$ The highest exponent is the degree and a_n is the leading coefficient of the polynomial.

Synthetic Division – a short form of division using only the coefficient of p(x) and he value of r.

Factors of Polynomials - these are polynomial expressions multiplied to get a polynomial product

Remainder Theorem – If a polynomial function f(x) is divided by x-r, where r is a real number, the remainder is f(r).

Factor Theorem – (x – r) is a factor of f(x) if if and only if f(r) = 0.

REFERENCES AND WEBSITES LINKS USE IN THIS LESSON

References:

Ruivivar, Leonor A. (2011) Advanced Algebra,Trigonometry and Statistics:Sibs Publishing House, Inc.

Ogena, Ester, Diaz, Rosemarievic Fortes, Erminda, Balagtas, Marilyn. (2013) Our Math: Mc Graw Hill and Vibal

Onsones, Rigor B., Ocampo, Shirlee R. Tresvalles, Regina M.(2013). Math Ideas and Life Applications. Abiva Publishing House, Inc.

Bernabe, Julieta G.,et al.(2013) Our World of Math. Vibal Publishing House, Inc.

Oronce, Orlando A. and Mendoza, Marilyn O. (2013). *E-Math: Intermediate Algebra*. Quezon City: Rex Book Store, Inc.

Larson, Ron, et al (2012) Advanced Algebra with Trigonometry. Anvil Publishing, Inc.

Websites:

https://www.youtube.com/watch?v=znsUfxzpiGE
https://www.youtube.com/watch?v=KTvQXspWhbM
These show examples of long division for polynomials.

https://www.youtube.com/watch?v=nefo9cUo-wg
https://www.youtube.com/watch?v=u0ep4v_bweQ
These websites show more examples of division of polynomial using synthetic division.

https://braingenie.ck12.org/skills/106914
This websites gives interactive practice drills on the application of polynomial function

https://www.youtube.com/watch?v=3LXB2FIR2WU
This website shows example of long division of polynomials and states the remainder theorem. Illustrates the use of the remainder theorem showing that the remainder in both processes remains the same.

http://www.purplemath.com/modules/factrthm.htm
This website explains the factor theorem

https://www.youtube.com/watch?v=P-RvhBqBPOA
This website describes and give examples of the factor theorem.

https://www.youtube.com/watch?v=_IPqCaspZOs
This website explains the remainder and factor theorems.

https://www.youtube.com/watch?v=xJvrhlqwCr0
This website explains how to find factors of polynomials.

https://www.youtube.com/watch?v=xJvrhlqwCr0
This website includes exercises with factoring polynomials

http://interactive.onlinemathlearning.com/remainder_theorem.php?action=generate&numProblems=10
This is an Interactive website on remainder theorem

Lesson 2: Polynomial Equation/Function

LESSON: PRE-ASSESSMENT

Let's find out how much you have already know about this lesson. Click on the letter that you think best answers the question. Please answer all items. After taking this short test, you will see your score. Take note of the items that you were not able to correctly answer and look for the right answer as you go through this lesson.

1. What is the leading coefficient of $f(x) = x^3 + 8x^2 + 19x + 12$?
   A. 1
   B. 3
   C. 8
   D. 12

2. Which of the following is a possible zero of $F(x) = 2x^3 -1x^2 - 13x – 6$?
   E. -3/2
   F. 1/4
   G. 2/3
   H. -5

3. Which of the following is a zero of the function $f(x) = 2x^3 -1x^2 - 13x – 6$?
   A. 2
   B. 1/2
   C. -1/2
   D. -3

4. What are the zeros of $f(x) = x^3 + 8x^2 –x – 5$?
   A. -5, -1, and 1
   B. -5, and 1 of multiplicity 2
   C. 5, -1 and 1
   D. 5 and -1 of multiplicity 2

5. What is the constant term of $f(x) = x^4 + 2x^3 - 3x^2 +5x + 8$?
   A. 1
   B. 4
   C. 5
   D. 8

6. What are the roots of $2x^3 + 3x^2 - 8x + 12 = 0$?
   A. -3/2, 3 and 2
   B. -2, -3/2 and 2
   C. 2, -3/2 and 4
   D. ½, -3/2 and 2

7. What is the possible number of positive zeros of $f(x) = x^3 - 2x^2 - x + 2$?
   A. 1
   B. 0 or 2
   C. 1 or 3
   D. 3

8. What is the number of negative zeros of $f(x) = x^3 - 2x^2 - x + 2$?
   A. 1
   B. 2
   C. 3
   D 4

9. What are the possible rational zeros for $f(x) = x^3 - 7x - 6$?
   A. ± 1, ± 2, ± 4, ± 6
   B. ± 1, ± 2, ± 3, ± 6
   C. ± 1, ± 5, ± 4, ± 6
   D. ± 1, ± 2, ± 4, ± 7

10. A portion of the path of a certain roller coaster can be modelled by $F(t) = t^3 - 6t^2 + 11t - 6$ where t represents the time in seconds and f(t) represents the height of the roller coaster. Determine the three times at which the roller coaster is at ground level.
    A. 1, 2, and 3 seconds
    B. 2, 3, and 4 seconds
    C. 1, 3, and 4 seconds
    D. 2, 3, and 5 seconds

EXPLORE

In the previous lesson, you have learned about the remainder and factor theorems. In this lesson, you will explore further the concepts of polynomial equation/function.

As you go through the lesson, keep in mind the question: ***How do values of one variable behave in terms of the other in the polynomial function that models a real- world situation?***

Let's start the module by doing the activity below and then answer the question given.

ACTIVITY 1. Maximizing Volume

Suppose you have a 8 inches by 8 inches square piece of paper. From the corners, you first cut four small squares that are equal in size, then fold up the sides and tape the corners. In this way you have made a tray. There are many trays that can be made in this fashion depending on the size of the square cutouts. What size of squares should you cut out from the four corners of the paper to make the tray hold as much water as possible? (Assume that the tray does not leak.)

Complete the table below to organize the different resulting measures that you get from every length of a side of a square cutout.

Length of a side of a square cutouts (in)		Length of base(bottom) of a tray (in)	Perimeter of the base of the tray	Area of base of the tray (sq. in.)	Height of tray (in)	Volume of tray (cu. in.)
0.5						
1.0		6	24	36	1	36
1.5						
2.0						
2.5						
3.0						
3.5						
4.0						

Process Questions:

1. What is the largest possible perimeter you can get?
2. What is the largest possible area you can get?
3. What is the largest possible volume you can get?
4. What is the length of a side of the square cutouts that gives the largest volume of a tray?
5. How did you compute the perimeter of the square base?
6. How did you compute the area of the square base?
7. How did you compute the volume of a tray?

Your answers in questions 5, 6 and 7 are polynomial functions of first, second and third degree respectively.

ACTIVITY 2. What is a Polynomial Equation?

Complete the Frayer Model using the word POLYNOMIAL EQUATION/FUNCTION.

Definition	**Facts/Characteristics**
Examples	**Non-examples**

POLYNOMIAL EQUATION

Process Questions:

1. How did you determine the examples and non- examples of a polynomial equation/function?
2. How does the polynomial equation/function differ from other equations/functions?
3. **What makes an equation a polynomial equation/function?**

Click **SAVE** if you have completed the Frayer Model. Then post your answer to the Process Questions in the Discussion Forum.

You have just tried describing a polynomial equation/function. In our next activity, your prior knowledge on polynomial equation/function will be elicited.

ACTIVITY 3. AGREE OR DISAGREE! (ANTICIPATION-REACTION GUIDE)

Read each statement in the column TOPIC and write **A** if you agree with the Statement, Otherwise write **D** in the first column.

Response Before Lesson	TOPIC: Polynomial Equations	Response After Lesson
	1. The degree of a polynomial function is the power of x in the leading term.	
	2. All polynomial functions of degree greater than 2 have at least one negative real root.	
	3. One zero of $f(x) = x^3 - 7x^2 - 6x + 72$ is 4 the factored form of the expression $x^3 - 7x^2 - 6x + 72$ is $(x-6)(x+3)(x-4)$.	
	4. If all the possible zeros of a polynomial function are integers, then the leading coefficient of the function is 1 or -1.	
	5. If p(x) is a polynomial function with integral coefficients, a leading coefficient of 1, and a nonzero constant term, then any rational zeros of P(x) must be factors of the constant term.	
	6. In the function $f(x) = 3x^3 -2x^2 + 8x + 5$, the leading coefficient is 3.	
	7. If f(c) = 0, then c is a zero of f(x).	
	8. A polynomial function is of the form $F(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$ where a_n $a_{n-1},\dots,a_2,a_1,a_0$ are real numbers, $a \neq 0$, and n is a non-negative integer.	
	9. In the polynomial function f(x) where all the coefficients are integers and $a_n \neq 0$, if p/q in lowest terms is a rati0nal zero, then P is a factor of a_0 and q is a r and q is then factor of a_n.	

	10. 5 is a root of the polynomial function defined by $6x^3 - 25x^2 - 31x + 30 = 0$.	
	11. If $f(x) = 5x - 3x + 2$, $f(3) = 376$.	
	12. The polynomial function whose zeros are -4, -2 and 3 is $f(x) = x^3 + 3x^2 - 10x - 24$.	
	13. The possible number of positive zeros of $F(x) = x^3 - x^2 + 4x - 4$ is ether 3 r 1.	
	14. The possible rational zeros of $f(x) = x^3 - 4x^2 + x + 6$ are $\pm 1, \pm 2, \pm 3, \pm 6$.	
	15. The zeros of a polynomial function can be rational, irrational, or imaginary numbers.	

Process questions:

3. What comes in your mind while filling in the first column of the ARG?
4. **How do values of one variable behave in terms of the other in the polynomial function that models a real- world situation?**

END OF EXPLORE

You gave your initial ideas about polynomial function/equation.

What you will learn in the next section will enable you to discover ways of finding the rational zeros of the function.

FIRM-UP

Your goal in this section is to learn and understand key concepts of the Rational Zero Theorem and the Descartes' Rule of Signs. And towards the end of this section, you will be encouraged to answer problems applying these concepts.

ACTIVITY 4. FIND OUT!

Given: $f(x) = (x-2)(x+3)(x+1)$

1. What are values of x that makes $f(x) = 0$?
2. Compute $f(2)$, $f(-3)$ and $f(-1)$. What values did you get?
3. Are these values 2, -3, and -1 zeros of $f(x)$? Why?
4. What does the factor theorem tells you?

You have learned previously that if r is a zero of the polynomial f(x), then x-r is a factor of f(x). This factor theorem has implications for polynomial functions. That is, if x-r is a factor of f(x), then f(r) = 0. In other words, r is a solution (or root or zero) of the function f. The real zeros of a polynomial function are either rational or irrational. (A **rational number** is one that can be written as a quotient of two integers a and b, $b \neq 0$.) The function above has three rational zeros. On the other hand, the function $f(x)=(x^2 – 2)(x - 1)$ has one rational zero: 1 and two irrational zeros, $\pm\sqrt{2}$. To find the zeros of a quadratic function, you may use the quadratic formula, completing the square or factoring.

Usually it is not practical to test all possible zeros of a polynomial function using synthetic substitution. Now you will explore how the possible rational zeros as well as the rational zeros of a polynomial functions are obtained by doing the next activity.

ACTIVITY 5. Rational Zeros

Complete the table below and answer the questions that follow.

Polynomial Function	Factors of f(x)	Degree of f(x)	Zeros of f(x), r	Leading Coeffi-cient, a_n	Factors of the Leading coefficient, p	Constant Term, a_0	Factors of the Constant Term, q
a. $f(x)=x^3 +4x^2 +x -6$	(x+3)(x+2)(x-1)						
b. $f(x)= 2x^3 +3x^2 -11x-6$	(x+3)(2x+1)(x-2)						
c. $f(x) =5x^3 +2x^2 +45x + 18$							
d. $f(x) = 16x^4 - 1$							
e. $f(x) = (x+2)(x^2 +3x + 2)$							

Questions:

1. What is the leading coefficient in a? What is its constant term?
2. In a, how are the zeros related to the factors of the constant term?
3. What is the leading coefficient in b? What is its constant term?
4. How are the rational zeros related to the factors of the leading coefficient and the factors of the constant term?
5. What are the zeros of the function in c?
6. Are all zeros of any polynomial functions always rational numbers? Why?
7. Is it possible for r to occur more than once?
8. How is the degree of the function related to its number of zeros?

9. How are the rational zeros of a polynomial function obtained?

Your findings in the previous activity leads to the Rational Zero Theorem: Given a polynomial function defined by $F(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$ with integer coefficients and where n is a non-negative integer. If $\frac{p}{q}$ is a rational number in its simplest form and a zero of f(x), then p and q are factors of a_0 and a_n, respectively. Or if a polynomial function, written in descending order of the exponents, has integer coefficients, then any rational zero must be of the form ± *p*/ *q*, where *p* is a factor of the constant term and *q* is a factor of the leading coefficient.

RATIONAL ZERO THEOREM: PROOF!

Since the rational number p/q is in the lowest terms, p and q are relatively prime, that is, they do not have a common factor greater than 1.

1. Substitute p/q in the polynomial equation .

   $$a_{n\,(p/q)}{}^{n} + a_{n-1} (p/q)^{n-1} + \dots + a_2 (p/q)^2 + a_{1(p/q)} + a_0 = 0$$

2. Multiply both sides by q^n .

   $$a_n p^n + a_{n-1} p^{n-1}q + \dots + a_2 p^2q^{n-2} + a_1pq^{n-1} + a_0 q^n = 0$$

3. Add - $a_n p^n$ to both sides.

   $$a_{n-1} p^{n-1}q + \dots + a_2 p^2q^{n-2} + a_1pq^{n-1} + a_0 q^n = - a_n p^n$$

4. Divide both sides by q.

   $$a_{n-1} p^{n-1} + \dots + a_2 p^2q^{n-1} + a_1pq^{n-2} + a_0 q^{n-1} = \frac{- a_n p^n}{q}$$

5. $\frac{-a_n p^n}{q}$ must be an integer since this is a result of sum, product and integral powers of integers.

6. Since p and q are relatively prime, therefore a_n must be a factor of q.

7. Similarly, if $-a_0q^n$ are added to each side of

   $a_n p^n + a_{n-1} p^{n-1}q + \ldots + a_2 p^2q^{n-2} + a_1pq^{n-1} + a_0 q^n = 0$ and divide the resulting equation by p, the obtained equation is

   $$a_np^{n-1} + a_{n-1}p^{n-2}q + a_{n-2}p^{n-1}q^2 + \ldots + a_2pq^{n-2} + a_1q^{n-1} = \frac{a_0p^n}{q}$$

8. $\frac{a_0p^n}{q}$ is an integer which is a result of adding, multiplying, and applying the laws of exponents.

9. Since p and q are relatively prime, therefore, a_0 must be a factor of q.

This theorem can help you choose some possible zeros to test. Each possible zero must be checked to find out if it is actually a zero of the polynomial function.

A polynomial equation of degree $n \geq 1$ has exactly n zeros and every polynomial equation has at least one zero, real or imaginary.

Example 1

Find all the possible rational zeros of

$f(x) = 2x^3 + 3x^2 - 8x + 3$

According to the rational zero theorem, any rational zero must have a factor of 3 in the numerator and a factor of 2 in the denominator.

p: factors of $3 = \pm 1, \pm 3$

q: factors of $2 = \pm 1, \pm 2$

The possibilities of *p*/ *q*, in simplest form, are $\pm 1,\ \pm\frac{1}{2},\ \pm 3,\ \pm\frac{3}{2}$

These values can be tested by using direct substitution or by using synthetic division and finding the remainder. Synthetic division is the better method because if a zero is found, the polynomial can be written in factored form and, if possible, can be factored further, using more traditional methods.

Example 2

Find rational zeros of $f(x) = 2x^3 + 3x^2 - 8x + 3$ by using synthetic division.

p/q	2	3	-8	3	
1	2	5	-3	0	1 is a zero
-1	2	1	-9	12	
1/2	2	4	-6	0	½ is a zero
-1/2	2	2	-9	15/2	
3	2	9	19	60	
-3	2	-3	1	0	-3 is a zero
3/2	2	6	1	9/2	
-3/2	2	0	-8	15	

The zeros of $f(x) = 2x^3 + 3x^2 - 8x + 3$ are 1, $\frac{1}{2}$, and –3. This means

$f(1) = 0$, $f\left(\frac{1}{2}\right) = 0$ and, $f(-3) = 0$

The zeros could have been found without doing so much synthetic division. From the first line of the chart, 1 is seen to be a zero. This allows $f(x)$ to be written in factored form using the synthetic division result.
$f(x) = 2x^3 + 3x^2 - 8x + 3 = (x - 1)(2x^2 + 5x - 3)$
But $2x^2 + 5x - 3$ can be further factored into $(2x - 1)(x + 3)$ using the more traditional methods of factoring.

$2x^2 + 5x - 3 = (x - 1)(2x - 1)(x + 3)$

From this completely factored form, the zeros are quickly recognized. Zeros will occur when

$x - 1 = 0$	$2x - 1 = 0$	$x + 3 = 0$
$x = 1$	$x = \frac{1}{2}$	$x = -3$

ACTIVITY 6. Reflection Log

1. What difficulties did you encounter while finding the rational zeros of the polynomial function?
2. How did you overcome these difficulties?

Writing a Polynomial Function:

Problem: What is a polynomial function of least degree with integral coefficients which zeros include -1 and 5-i?

a. Is 5 + i also a zero of the function? Why?
b. The factors of the polynomial are (x + 1), x – (5 –i), x – (5 + i).
c. Write the polynomial function as the product of its factors.
   f(x) = (x + 1)[x -(5 – i)][x - (5 + i)]
d. Multiply the factors to find the polynomial function.

$$
\begin{aligned}
f(x) &= (x + 1)[x -(5 - i)][x - (5 + i)] & \\
&= (x + 1)\,[(x-5) + i][(x-5) - i] & \textit{Regroup terms} \\
&= (x + 1)[(x - 5)^2 - i^2\,] & \textit{Difference of squares} \\
&= (x + 1)[x^2 - 10x + 25) - (-1)] & \textit{Square terms} \\
&= (x + 1)(x^2 -10x + 26) & \textit{Simplify} \\
&= x^3 - 10x^2 + 26x + x^2 -10x + 26 & \textit{Multiply} \\
&= x^2 - 9x^2 + 16x + 26 & \textit{Combine like terms}
\end{aligned}
$$

e. Because there are 3 zeros, the degree of he polynomial function must be 3, so $f(x) = x^2 - 9x^2 + 16x + 26$ is a polynomial function of least degree with integral coefficients and zeros of -1, 5 – i, and 5 + i.

ACTIVITY 7. Write a Polynomial function

Do these problems to find out.

1. Write a polynomial function of least degree with integral coefficients having zeros that include 4, -1, and 6.
2. Write a polynomial function of least degree with integral coefficients having zeros that include 3, -1, 1, and 2.

3. Write a polynomial function of least degree with integral coefficients having zeros that include -1 and 1 + 2i.

Since the Rational Zero Theorem provides only the possible rational zeros, see how the Descartes' Rule of Signs help you find the actual zeros of the polynomial functions. Recall the following: That every polynomial equation with complete coefficients and positive degrees n has exactly n complex roots; If a polynomial equation with real coefficients has a + bi as a root (a and b are real numbers, $b \neq 0$), then a – bi is also a root.

If the terms of a polynomial function are written in decreasing order according to the powers of x (ignoring missing terms), each pair of successive coefficients with *opposite signs* is called a **variation of sign.**

The polynomial $2x^4 - x^3 - 14x^2 + 19x - 6$ contains the three indicated variations of sign and 4 complex roots or zeros. $2x^3 + 11x^2 + 17x + 6$ contains no variation of sign. It has three complex roots.

Complete the tables below and answer the given process questions.

f(x)	Number of Variations of Sign in f(x)	f(-x)	Number of Variations of Sign in f(-x)
$f(x) = x^3 + 2x^2 - 5x - 6$			
$f(x) = 2x^3 + 3x^2 + 17x - 12$			
$f(x) = 2x^4 + 3x^3 - 12x^2 - 7x + 6$			
$f(x) = 2x^5 - 3x^4 - 21x^3 + 29x^2 + 27x - 18$			
$f(x) = x^3 - x^2 + 4x - 4$			

F(x)	**Xeros of f(x)**	**Number of Positive Zeros**	**Number of Negative Zeros**
$f(x) = x^3 + 2x^2 - 5x - 6$	**-1, 2, -3**		

$f(x) = 2x^3 + 3x^2 + 17x - 12$	**½, -3, -4**		
$f(x) = 2x^4 + 3x^3 – 12x^2 – 7x + 6$	**-1, 2, ½, -3**		
$f(x) = 2x^5 – 3x^4 – 21x^3 + 29x^2 + 27x - 18$	**-1, 2, 1/3, -3, 3**		
$f(x) = x^3 – x^2 + 4x - 4$	**1, 2i, -2i**		

Process Questions:

1. How did you determine the number of variations of sign in f(x) and in f(-x) of the different polynomial functions?
2. How would you compare the number of positive zeros with the number of variations of sign in f(x)?
3. How would you compare the number of negative zeros with the number of variations of sign in f(-x)?

The relation you discovered in the previous activity is a part of the Descartes' Rule of Signs as summarized below.

DESCARTES' RULE OF SIGNS

Let f(x) be a polynomial function with real coefficients and a nonzero constant term and arranged in descending powers of the variable.

a. The number of positive zeros of f(x) is the number of variations in signs of f(-x) or is less than this number by an even number.

b. The number of negative zeros of f(x) is the number of variations in signs of f(x) or is less than this number by an even number.

Consider the polynomial function $f(x) = x^4 – 2x^3 – 2x^2 + 5x – 2$. We say that there is a variation in signs if the signs of the two consecutive terms differ. The signs of the terms are **+ - - + -.** Since there are three variations in signs in f(x) the number of positive zeros of f(x) is either 3 or 1.

Now, $f(-x) = (-x)^4 – 2(-x)^3 – 2(-x)^2 + 5(-x) – 2$
$= x^4 + 2x^3 – 2^2 –5x – 2.$

The signs of the terms of f(-x) are **+ + - - -.** Since there is only one variation of sign in f(-x), the number of negative zeros of f(x) is 1.

For more examples and practice, visit the websites below.

http://regentsprep.org/Regents/math/algtrig/ATE13/HighPractice.htm
http://www.mathsisfun.com/algebra/polynomials-solving.html
http://www.mathopolis.com/questions/q.php?id=2277&site=1&ref=/algebra/polynomials-solving.html&qs=466_467_468_469

The websites above illustrate interactive practices on solving polynomial equations.

ACTIVITY 8. Finding dimensions…

x - 1

x - 2

x

PROBLEM: The width of a rectangular container is 2 meter less than the length and the height is 1 meter less than the length. Find the dimensions of the container if its volume is 60 m^3?

FORMULATE: (Write the concepts you need to answer the problem.)

COMPUTE: (Write your complete solution.)

INTERPRET: (Write a paragraph about the interpretation of your answer.)

VALIDATE: (Provide one more example to validate your answer.)

REPORT:

Non-Online: Submit your written report.

Online: Send your written report using the student dash board.

ACTIVITY 9. ZEROS OF POLYNOMIAL FUNCTIONS

SUM IT UP!

Put together in the table below your answers to the essential question that is asked for each problem.

	PROBLEM 1	PROBLEM 2	PROBLEM 3
ESSENTIAL QUESTION: 5. How do values of one variable behave in terms of the other in the poly-nomial function that models a real-world situation?	The compartment for the computer is a rectangular prism and will be 1400 cubic inches. The compartment will be 15 longer than it is wide and the height will be 9 inches greater than the width. Find the dimensions of the computer compartment.	The length of a rectangular tank is 2 ft more than its width and the height is 3ft more than its width. If the volume of the tank is 280 ft^3, what are its dimensions?	An open box with a volume of 48 ft^3 can be made by cutting a square of the same size from each corner of a piece of metal 8 ft wide and 10 ft long on a side and folding up the edges. What is the length of a side of a square that is cut from each corner?
	How dimensions are solved in the polynomial function that models a real-world situation?	How dimensions are solved in the polynomial function that models a real-world situation?	How dimensions are solved in the polynomial function that models a real-world situation?
	Solution of a polynomial function that models real-world situation depends on ________________.	Solution of a polynomial function that models real-world situation depends on ________________.	Solution of a polynomial function that models real-world situation depends on ________________.

PROCESS QUESTIONS:

1. What do variable in each problem represents?
2. How are polynomial functions formulated?
3. How are these polynomial functions solved?

4. Was there only one way to solve a polynomial function? If yes, explain. If not, why are there many ways to solve a polynomial function that models real-world situation? How will you know which way to use? Explain.

ACTIVITY 10. _AGREE OR DISAGREE! (ANTICIPATION-REACTION GUIDE) A REVISIT !

Read each statement in the column TOPIC and write **A** if you agree with the Statement, Otherwise write **D** in the third column.

Response Before Lesson	TOPIC: Polynomial Equations	Response After Lesson
	1. The degree of a polynomial function is the power of x in the leading term.	
	2. All polynomial functions of degree greater than 2 have at least one negative real root.	
	3. One zero of $f(x) = x^3 - 7x^2 - 6x + 72$ is 4 the factored form of the expression $x^3 - 7x^2 - 6x + 72$ is $(x-6)(x+3)(x-4)$.	
	4. If all the possible zeros of a polynomial function are integers, then the leading coefficient of the function is 1 or -1.	
	5. If p(x) is a polynomial function with integral coefficients, a leading coefficient of 1, and a nonzero constant term, then any rational zeros of P(x) must be factors of the constant term.	
	6. In the function $f(x) = 3x^3 -2x^2 + 8x + 5$, the leading coefficient is 3.	
	7. If f(c) = 0, then c is a zero of f(x).	
	8. A polynomial function is of the form $F(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$ where a_n $a_{n-1},\ldots,a_2,a_1,a_0$ are real numbers, $a \neq 0$, and n is a non-negative integer.	
	9. In the polynomial function f(x) where all the coefficients are integers and $a_n \neq 0$, if p/q in lowest terms is a rati0nal zero, then P is a factor of a_0 and q is a r and q is then factor of a_n.	
	10. 5 is a root of the polynomial function defined by $6x^3 -25x^2 - 31x + 30 = 0$.	
	11. If $f(x) = 5x - 3x + 2$, $f(3) = 376$.	
	12. The polynomial function whose zeros are -4, -2 and 3 is $f(x) = x^3 +3x^2 -10x - 24$.	

	13.The possible number of positive zeros of f(x) = $x^3 - x^2 + 4x - 4$ is ether 3 r 1.	
	14. The possible rational zeros of f(x)= $x^3 -4x^2 + x + 6$ are $\pm 1, \pm 2, \pm 3, \pm 6$.	
	15. The zeros of a polynomial function can be rational, irrational, or imaginary numbers.	

PROCESS QUESTIONS:

1. Was there any difference in your answer/s between before and after lesson?
2. **How do values of one variable behave in terms of the other in the polynomial function that models a real- world situation?**

ACTIVITY 11. Quiz (Formative Assessment)

A. Determine the number of roots of each polynomial equation.

1. $x^3 + 3x^2 - 10x - 24 = 0$
2. $56 - 28x - x^2 + 4x^3 - x^4 = 0$
3. $(3x - 1)^4(x + 1)^2 (x - 3) = 0$
4. $x(x - 2)^3 (3x + 2) (2x - 5)^2 = 0$
5. $(x^3 - 1)(x^2 - 6x + 9) = 0$

B. List all the possible rational roots/zeros of each equation/function.

1. $x^3 - 6x^2 - 8x + 24 = 0$
2. $f(x) = 2x^4 + 3x^2 - x + 15$
3. $6x^3 - 17x^2 - 4x + 3 = 0$
4. $(x) = 2x^4 + 3x^3 - 4x^2 - 3x + 2$
5. $f(x) = 2x^5 - x^4 - 10x^3 + 5x^2 + 8x - 4$

C. If the given complex number is a root of a polynomial equation, give the complex conjugate which is also a root of the polynomial equation.

1. $2i$
2. $12 + i$
3. $2i - 1$
4. $-6 - 3i$
5. $\sqrt{3} + 2i$

D. State the possible number of positive real zeros, negative real zeros and imaginary zeros of each equation/function.

Equation/Function	Number of Positive real zeros	Number of Negative Real Zeros	Number of Imaginary Zeros

1. $x^3 - 4x^2 - x + 4 = 0$			
2. $x^4 - 10x^2 + 9 = 0$			
3. $f(x) = 2x^3 + x^2 - 8x - 4$			
4. $f(x) = 2x^3 - 5x^2 - 11x - 4$			
5. $f(x) = 4x^5 + x^4 - 23x^3 + 22x^2 + 28x - 8$			

E. Determine if the given value of c is a root of the given equation.
1. $x^3 - 7x^2 + 17x - 15 = 0$; $c = 3$
2. $x^4 + x^3 - x^2 - x - 18 = 0$; $c = -2$
3. $6x^3 + 4x^2 - 14x + 4 = 0$; $c = -2$
4. $x^4 - 2x^3 - 3x^2 + 10x = 0$; $c = 0$
5. $x^3 + 8x^2 + 19x + 12 = 0$; $c = 4$

F. Find the rational roots/zeros of each polynomial equation/function.
1. $f(x) = 4x^4 - 17x^2 + 4$
2. $x^4 - 13x^2 + 36 = 0$
3. $f(x) = x^3 + 13x^2 + 10x - 60$
4. $81x^4 - 256 = 0$
5. $f(x) = 4x^4 + 12x^3 - 5x^2 - 21x + 10$

G. Find all the roots/zeros of each equation/function.
1. $f(x) = 3x^3 - 2x^2 + 8x + 5$
2. $4x^4 + 13x^3 - 16x^2 - 13x + 12 = 0$
3. $x^3 + 3x^2 - 10x - 24 = 0$
4. $f(x) = 3x^3 + x^2 - 12x - 4$
5. $f(x) = 4x^4 + 3x^3 - 17x^2 - 12x + 4$
6. $x^3 + 2x^2 - 25x - 50 = 0$
7. $2x^3 - 5x^2 - 14x + 8 = 0$
8. $x^4 - 5x^2 + 4 = 0$
9. $x^4 - x^3 - 7x^2 + 13x - 6 = 0$
10. $6x^4 + 11x^3 + 8x^2 - 6x - 4 = 0$

H. Find a polynomial equation of least degree with integral coefficients that have the given zeros.
1. -4, -2, 3
2. -3, 2, -3i
3. -1, 2, 3, 5
4. -2, 3, 4 – 3i
5. -3, -2, -5, 2i

END OF FIRM UP

In this section, the discussion was about the key concepts on Polynomial equations/functions. The use of rational zero theorem and the Descartes Rules of signs were also given emphasis.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

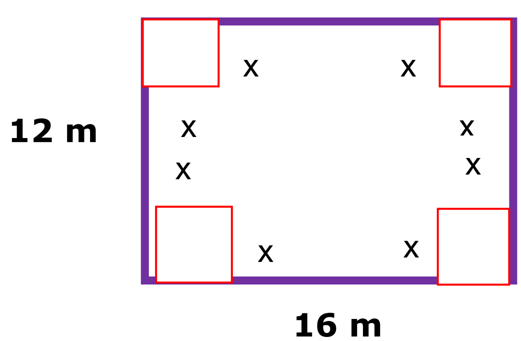
Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.

DEEPEN

Your goal in this section is to take a closer look at some aspects of the topic. This section gives emphasis on the deeper use of the concepts discussed earlier. I hope that you are now ready to answer the exercises given in this section using the modelling process to enhance your understanding of what has been learned.

ACTIVITY 12. Designing an Open Box

A manufacturer has a piece of tin 12 feet wide and 16 feet long. An open box with a volume of 160 ft^3 is to be made by cutting a square of the same size from each corner and folding up the edges of the piece of tin. To the nearest tenth of a foot, find the length of a side of a square piece that is removed.



Length of a side of a square	**x**
Length of the box	**16 – 2x**
Width of the box	**12 – 2x**

Questions:
1. What represents the height of the box?
2. What is the volume of the box?
3. What expressions will you multiply to get the volume of the box?
4. What polynomial equation did you get in item 3?

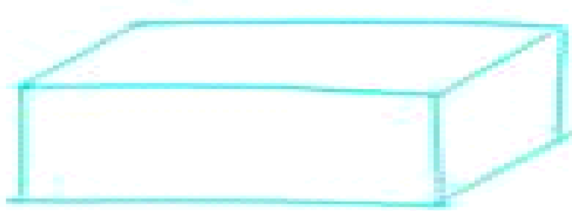
Since the shortest side of a piece of tin measures 12 feet, so the value of x must be between 0 and 6. Use synthetic substitution to find the solutions.

	1	-14	48	-40
1				
2				
3				
4				
5				

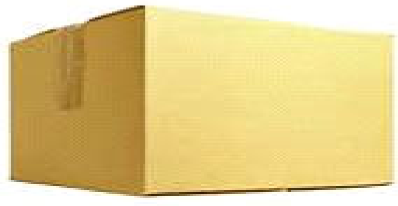
6. Is there any rational solutions?
7. Is there any real solutions? If any, use a calculator to approximate these solutions.
8. How will you interpret the results? What is the length of a side of a square piece that is removed?
9. How will you validate your answer?

ACTIVITY 13. Test of Understanding

1. Michael is building a computer desk with a separate compartment for the computer. The compartment for the computer is a rectangular prism and will be 8019 cubic inches. The compartment will be 24 inches longer than it is wide and the height will be 18 inches greater than the width. Find the dimensions of the computer compartment.



2. The volume of a packaging box is 1056 cubic centimeters. The length is 1 centimeter more than the width, and the height is 3 centimeters less than the width. Find the dimensions of the box.



3. A box is to be constructed by cutting out equal squares from the corners of a 30-cm square piece of card board and turning up the sides.
    a. Write a function V(x) for the volume of the box.
    b. For what vale of x will the volume of the box equal 1568 cubic centimeters?
    c. What will be the volume of the box if x = 6 centimeters?

Process Questions:

1. What is your difficulty in answering the three questions?
2. How is your understanding of solving polynomial equation helped solve the problems?
3. How do values of one variable behave in terms of the other in the polynomial
   function that models a real- world situation?

For more practice drills, visit the websites below.

http://regentsprep.org/Regents/math/algtrig/ATE13/HighPractice.htm
This website gives interactive practice on solving polynomial equation of higher degree.

http://www.mathsisfun.com/algebra/polynomials-solving.html
http://www.mathopolis.com/questions/q.php?id=2277&site=1&ref=/algebra/polynomials-solving.html&qs=466_467_468_469_1116_1117_2276_2277_2278_2279
This websites give an explanation and interactive quiz/practice at the end on so
lving polynomial equations

ACTIVITY 14. Try This!

1. A portion of the path of a certain roller coaster can be modelled by $f(t) = t^4 - 30t^3 + 291t^2 - 1070t + 1200$ where t represents the time in seconds and f(t) represents the height of the roller coaster.
   Use the Rational Zero Theorem to determine the four times at which the roller coaster is at ground level.

2. A restaurant orders spaghetti sauce in cylindrical metal cans. The volume of each can is about 160Π cubic inches, and the height of the can is 6 inches more than the radius.
   a. Write a polynomial equation that represents the volume of a can. Use the formula for the volume of a cylinder, $V = \Pi r^2 h$.
   b. What are the possible values of r? Which of these values are reasonable for this situation?
   c. Find the dimensions of the can.

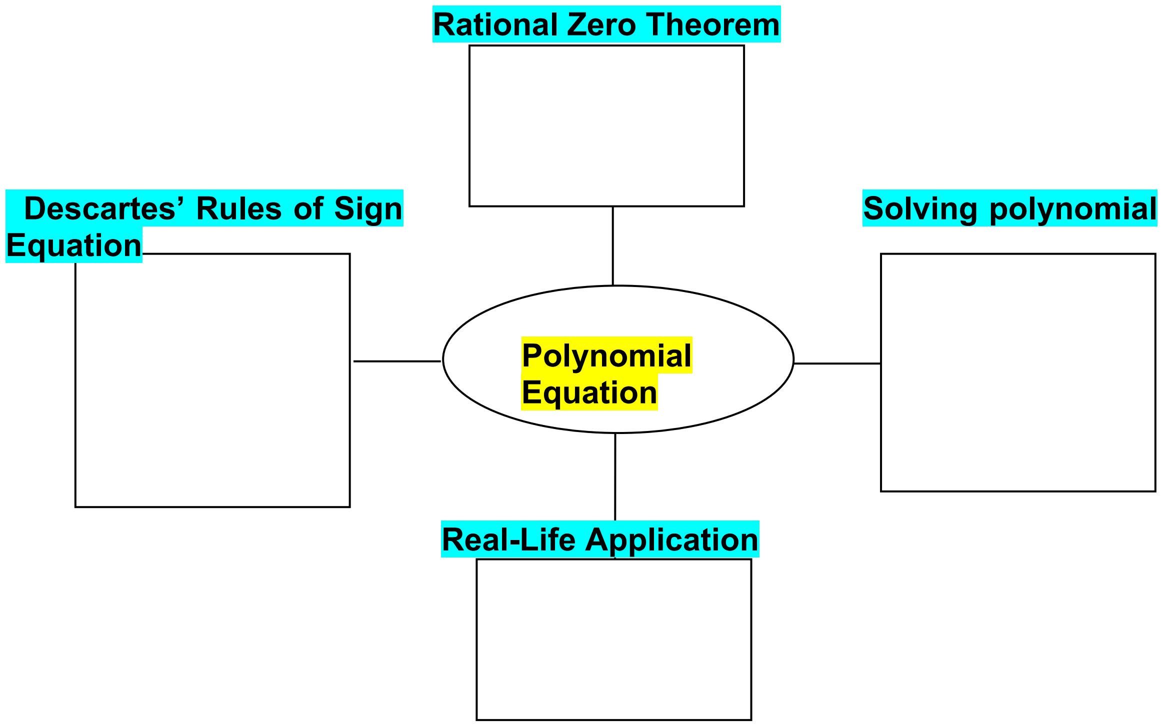
3. Annual sales of recorded music in the United States can be approximated by $f(t) = 30x^3 - 478x^2 + 1758x + 10092$, where f(t)is the total sales in millions of pesos and t is the number of years since 2000. You can use this function to estimate when music sales will be Php 9 billion.
   a. Write a polynomial equation that could be used to determine the year in which music sales would be about Php 9,000,000,000.
   b. List the possible whole number solutions for your equation in part a.
   c. Determine the approximate year in which music sales will be Php 9,000,000,000.

4. TRASH IT
   A trash can manufacturer designing a new can in the shape of a cylinder with a hemispherical top. The can must be 4 ft tall and hold a volume of 4 1/3 ft^3. What radius (r) should the designer plan to use for the trash can?

5. BOXING DAY
   Construct a box with an open top by cutting squares from the corners of a 12-inch by 16-inch piece of card board. Find the side length of the squares you can cut out to make a box with a volume of 180 in^3. Show that there are two possible dimensions for the box and explain how you found them.

ACTIVITY 15. CONCEPT MAPPING

Summarize the important concepts about of polynomial equation by completing the concept map below. You may do it in wise mapping by clicking this site: www.wisemapping.com



END OF DEEPEN

In this section, the discussion was about the deeper use of the basic properties of polynomial functions.

What new realizations do you have about the topic? What new connections have you made for yourself?

Now that you have enhanced your understanding of the topic, you are now ready to do the tasks in the next section.

TRANSFER

Your goal in this section is to apply your learning on the basic properties of polynomial functions to real life situation. You will be given a practical task which will demonstrate your understanding of the concepts.

ACTIVITY 16. Performance Task – All Boxed In (Scaffold Level 2)

Make a packaging box out of 8.5 inches by 11 inches sheet of illustration board. Write a function V(x) for the volume of the box. Find the dimensions, to the nearest tenth, of the box with the largest possible volume.

REPORT:
Non-Online: Submit your written report.
Online: Send your written report using the student dash board.

RUBRIC for Scaffold level 2				
CRITERIA	**EXCELLENT 4**	**SATISFACTORY 3**	**PROGRESSING 2**	**NEEDS IMPROVEMENT 1**
Accuracy of Compu-tation	Shows correct computation and solution with appro-priate explana-tion for the problem.	Shows correct computation and solution	Some solutions show incorrect computation.	Shows incorrect computation and solution.
Presentation of Output	Written output is well organized, detailed and complete	Written output is well organized.	Written output has missing parts somewhat disorganized.	Written output is disorganized.
Mathematical model justification	The recom-mendation shows sophisticated understanding of the relevant ideas and processes.	The recom-mendation shows solid understanding of the relevant ideas and processes.	The recom-mendation shows some-what limited understanding of the relevant ideas and processes.	The recommendation shows erroneous understanding of the relevant ideas and processes.

ACTIVITY 17. Lesson Closure – Reflection Organizer

You have accomplished the task- designing a box successfully. How did you find the performance task? How did the task help you see the real world use of the topic?

You learned concepts in this lesson which are very important for the next lesson. To end this meaningfully and to welcome the next lesson, I want you to accomplish the next activity.

In this lesson, I learned about…

____
____
____
____
____
____

These concepts can be used in…

____
____
____
____
____
____

I understand that…

____
____
____
____
____
____

One aspect of the lesson I find most difficult…., the reason of my difficulty is…,
I plan to remedy this by…

____
____
____
____
____
____

I can use these concepts in my life by…

____
____
____
____

END OF TRANSFER

You have completed the lesson. You have just learned the Rational Root Theorem, and solving polynomial equations. These concepts will help you in the next part of the module. Before you go to the next lesson, you have to answer the following lesson post-assessment.

LESSON: POST-ASSESSMENT

It's now time to evaluate your learning. Click on the letter of the answer that you think best answers the questions. If you do well, you may move on to the next lesson. If your score is not at the expected level, you have to go back and take the module again.

1. What is the leading coefficient of $f(x) = x^3 + 3x^2 - 9x + 5$?
   A. 1
   B. 8
   C. 12
   D. 19

2. Which of the following is a zero of $f(x) = x^3 + 6x^2 + 11x + 6$?
   A. 1
   B. -1
   C. -6
   D. 6

3. Which of the following is a zero of the function $f(x) = 2x^3 – x^2 - 13x -6$?
   A. – ½
   B. 1/3
   C. -3
   D. 2

4. What are the zeros of $f(x) = x^3 + 8x^2 –x – 5$?
   A. -5, -1, and 1
   B. 4 and 1 of multiplicity 2
   C. -5, 1 and 2
   D. 5, 1 and 2

5. What is the constant term of $f(x) = x^4 - 3x^3 - 11x^2 +3x + 10$?
   A. 3
   B. 4
   C. -11
   D. 10

6. What are the roots of $f(x) = x^4 - 3x^3 - 11x^2 +3x + 10$?
   A. {-1, 2, 1, 5}
   B. {-1, -2, 1-5}
   C. {-1, -2, 1, 5}
   D. {1of multiplicity 2, 2, 5}

7. What is the number of positive zeros of $f(x) = x^3 + 2x^2 –x - 2$?

A. 1
B. 2
C. 3
D. 4

8. What is the number of negative zeros of $f(x) = 4 + x^3 + 3x^2 + 3x + 2$?
   A. 2 or none
   B. 2 or 1
   C. 3 or 1
   D. 4 or 2

9. What are the possible rational zeros for $f(x) = x^3 – x^2 - 10x -$
   A. $\pm 1, \pm 2, \pm 3, \pm 8$
   B. $\pm 1, \pm 3, \pm 4, \pm 8$
   C. $\pm 1, \pm 2, \pm 6, \pm 8$
   D. $1, \pm 2, \pm 4, \pm 8$

10. A portion of the path of a certain roller coaster can be modelled by $f(t) = 2t^3 – 3t^2 + 136t - 160$ where t represents the time in seconds and f(t) represents the height of the roller coaster. Determine the four times at which the roller coaster is at ground level.
   A. ½, 3, 5
   B. ½, 6, 10
   C. ½, 4, 10
   D. 3, 6, 10

GLOSSARY OF TERMS USED IN THIS LESSON

Polynomial function/equation – is a function defined by an equation of the form $f(x) = a_n x^n + a_{n-1}$ xn-1 + … + $a_1 x + a_0$. The highest exponent is the degree and a_n is the leading coefficient of the polynomial.

Descartes' Rule of Signs:
Let f(x) be a polynomial function with real coefficients and a nonzero constant term and arranged in descending powers of the variable.

a. The number of positive zeros of f(x) is the number of variations in signs of f(-x) or is less than this number by an even number.
b. The number of negative zeros of f(x) is the number of variations in signs of f(x) or is less than this number by an even number.

Rational Zero Theorem:
Given a polynomial function defined by $F(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$ with integer coefficients and where n is a non-negative integer. If $\frac{p}{q}$ is a rational number in its simplest form and a zero of f(x), then p and q are factors of a_0 and a_n, respectively. Or if a polynomial function, written in descending order of the exponents, has integer coefficients, then any rational zero must be of the form ± *p*/ *q*, where *p* is a factor of the constant term and *q* is a factor of the leading coefficient.

REFERENCES AND WEBSITES LINKS USE IN THIS LESSON

References:

Ruivivar, Leonor A. (2011) Advanced Algebra,Trigonometry and Statistics:Sibs Publishing House, Inc.

Ogena, Ester, Diaz, Rosemarievic Fortes, Erminda, Balagtas, Marilyn. (2013) Our Math: Mc Graw Hill and Vibal

Onsones, Rigor B., Ocampo, Shirlee R.,Tresvalles, Regina M.(2013). Math Ideas and Life Applications. Abiva Publishing House, Inc.

Bernabe, Julieta G.,et al.(2013) Our World of Math. Vibal Publishing House, Inc.

Oronce, Orlando A. and Mendoza, Marilyn O. (2013). *E-Math: Intermediate Algebra*. Quezon City: Rex Book Store, Inc.

Websites:

http://regentsprep.org/Regents/math/algtrig/ATE13/HighPractice.htm
http://www.mathsisfun.com/algebra/polynomials-solving.html
http://www.mathopolis.com/questions/q.php?id=2277&site=1&ref=/algebra/polynomials-solving.html&qs=466_467_468_469
The websites above illustrate interactive practices on solving polynomial equations.

http://regentsprep.org/Regents/math/algtrig/ATE13/HighPractice.htm
This website gives interactive practice on solving polynomial equation of higher degree.
http://www.mathsisfun.com/algebra/polynomials-solving.html
http://www.mathopolis.com/questions/q.php?id=2277&site=1&ref=/algebra/polynomials-solving.html&qs=466_467_468_469_1116_1117_2276_2277_2278_2279
This websites give an explanation and interactive quiz/practice at the end on solving polynomial equations

Lesson 3: Graphs of Polynomial Functions

In this lesson, you will learn the following:

1. Illustrates polynomial functions – K
2. Graphs polynomial functions – S
3. Solves problems involving polynomial functions - S

EXPLORE

You have just finished learning how the roots of polynomial functions and equations are determined. In this lesson, you will learn how to sketch the graph of a polynomial function. Different theorems will also be discussed in relation to the graph of polynomial functions which can be used to model and solve problems.

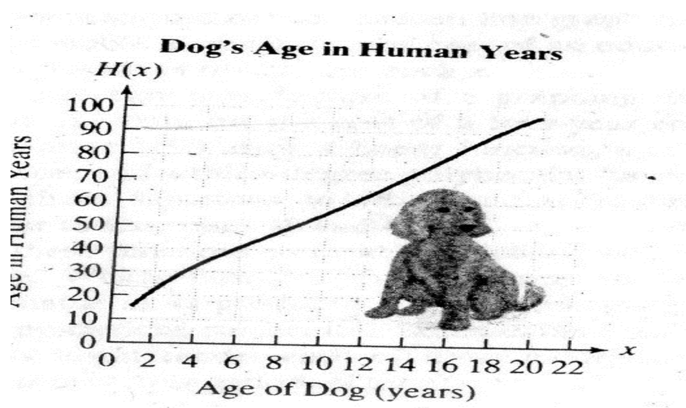
As we go through the lesson, keep in mind the question,

How do values of one variable or quantity behave in terms of the other in the polynomial function that models a real- world situation?

Let us start the lesson given the graph that models the age of human beings and of a dog by answering the questions below.

ACTIVITY 1. The Dog and I

The graph that models the age in human years, H(x), of a dog that is ***x*** years old, $H(x) = -0.001618x^4 + 0.077326x^3 - 1.2367x^2 + 11.460x + 2.914$ is given below:



Process Questions:

1. What kind of graph is illustrated above?

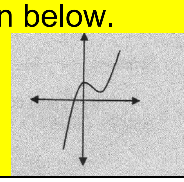
2. What two quantities are involved in the graph?

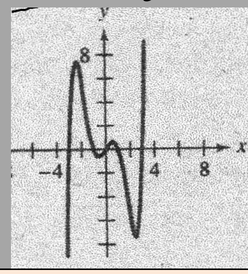
3. Is the given graph truly an application to real life? How?

4. **How do values of one quantity behave in terms of the other that models a real- world situation?**

ACTIVITY 2. Anticipation Reaction Guide: Agree or Disagree?

Answer the Before Discussion column of the ARG by clicking on the AGREE or DISAGREE column. Click A if you agree with the statement and click B if you disagree. Please answer all items. As you go through this module, look for the best correct answer to the statements included in this guide.

Before Discussion		Statements	After Discussion	
Agree	Disagree		Agree	Disagree
A	B	1. The graphs of polynomial functions are smooth and continuous.	A	B
A	B	2. The end behavior of the graph of a polynomial function depends on the constant term of the polynomial function.	A	B
A	B	3. If *f* is a polynomial of degree n, the graph of *f* has at most n -1 turning points.	A	B
A	B	4. If $f(x) = - x^3 + 4x$, then the graph of *f* falls to the left and to the right.	A	B
A	B	5. The graph the monomial function $f(x) = x^4$ touches the x-axis at the x-intercept.	A	B
A	B	6. The cubic function may have the graph given below. 	A	B

A	B	7. The graph of f(x) = $\frac{1}{5}x^5 - 2x^3 + \frac{9}{5}x$ is 	A	B
A	B	8. If the volume of a pyramid is given by V = x^3- $2x^2$ - 75 , the real value of x is 5.	A	B
A	B	9. The roots of polynomial functions are located between the upper and lower bounds of the polynomial graph.	A	B
A	B	10. The graph of even polynomial functions with positive leading coefficient rises to the left and to the right.	A	B

END OF EXPLORE

You have just finished answering a pre-assessment activity. What you will learn in the next sections will also enable you to do a final project which involves predicting an energy consumption for a certain period of time. We will start by doing the next activity.

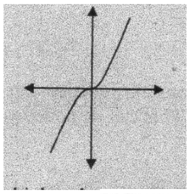
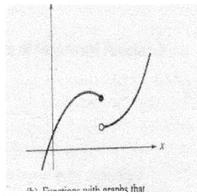
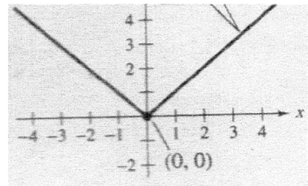
FIRM-UP

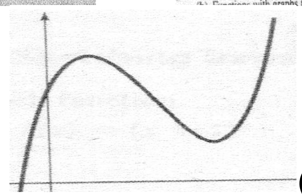
Your goal in this section is to have a good understanding of the graph of polynomial functions, its basic features and unique characteristics. Some theorems will also be introduced which will help you in sketching and graphing polynomial functions. Formative assessments will also be provided.

Start by performing the Activity No. 3 and learn the basic characteristics of a polynomial graph.

ACTIVITY 3. Am I Smooth and Continuous?

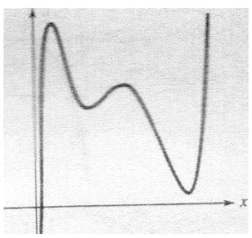
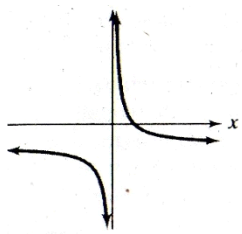
DESCRIPTION: In this activity you are to observe and describe the behavior of each graph given below.

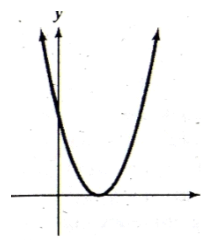




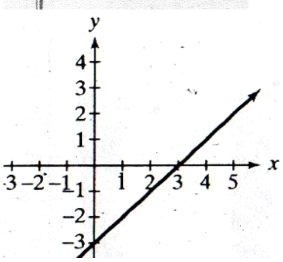


(a) (b) (c)

(d)





(e) (f) (g)

(h)

PROCESS QUESTIONS:

1. What did you observe from the given graphs?

2. Which graphs have rounded curves or no sharp corners?

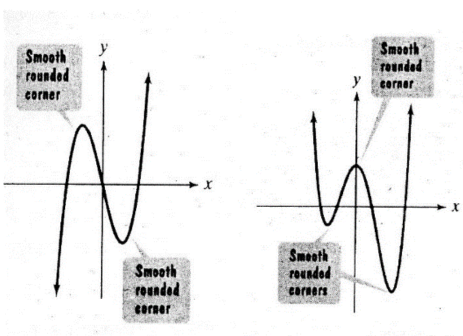
3. Which graphs have no breaks or continuous?

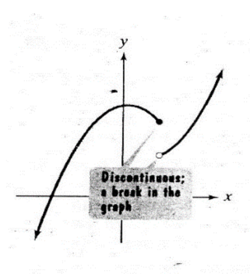
4. Do all the graphs represent a polynomial function? Why?

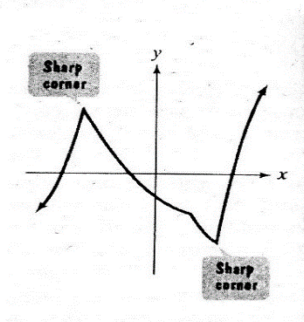
5. Which graphs are considered polynomial functions? Why?

After answering the questions above, read the text below and summarize the features of the graph of polynomial functions.

There are two important features of the graphs of polynomial functions. The *first feature* is that the graph of a polynomial function is continuous. Essentially, this means that the graph of a polynomial function has no breaks, holes, or gaps. It can be drawn without lifting your pencil from the rectangular coordinate system. The *second feature* is that the graph of a polynomial function has only smooth, and rounded turns. A polynomial function cannot have a sharp turn/corner. To understand these concepts look at the illustration below.







Graphs of Polynomial Functions

Not Graphs of Polynomial Functions

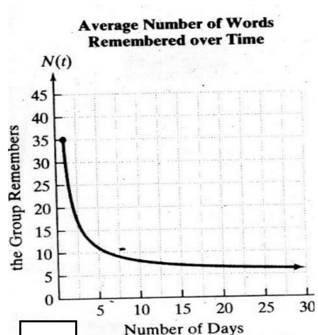
Write your summary inside the box.

To summarize the text read, complete the statements below.
The two important features of a graph of polynomial functions are ____________________ and _____________________. The first feature means that____ and the second feature means that _____________________________________.

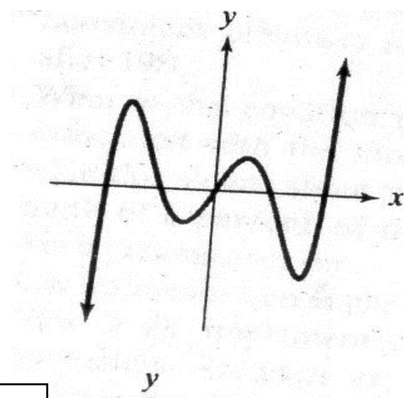
Now that you have an idea of the graphs of polynomial functions, perform the next Activity 4.

ACTIVITY 4. Which Graph is P(x)?

From the given graphs identify which is a polynomial function and which is not a polynomial function. In the box, write A if a polynomial function and B if not a polynomial function.

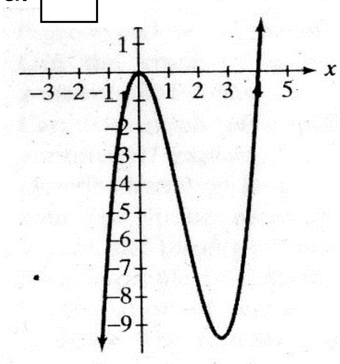


a. ☐



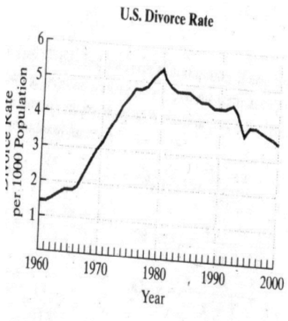
b ☐

☐

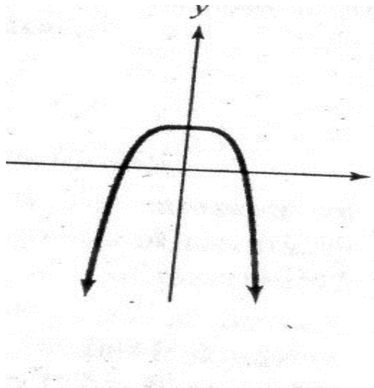


☐ ☐

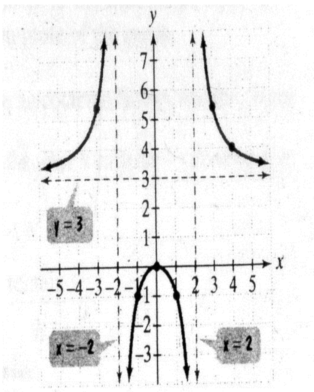
d. ☐

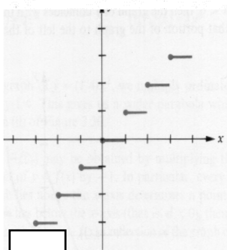


e.

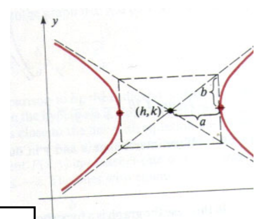


f.

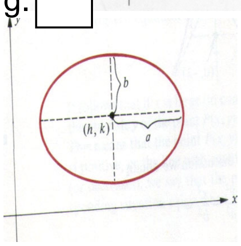




g.



i.



Process Questions:

1. How did you know that the graphs are polynomials or not?

2. What are your basis for identifying that the graphs are polynomials or not?

3. Did you find any difficulty in identifying which graph is polynomial function or not?

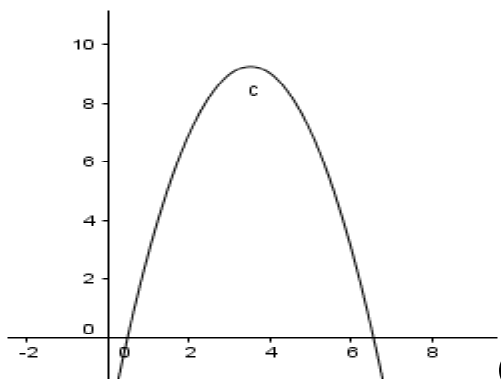
Explain.
Did you know that the graphs of polynomial functions are used in real life situations? In the next activity you will see some graphs of polynomial functions as applied in real life.

ACTIVITY 5. Graphs of P(x) in Real Life.

Below are graphs of polynomial functions as applied in real life.

1. The monthly profit P of a small business that sells bicycle stickers can be modeled by the function $P(x) = -x^2 + 7x - 3$, where x is the average price of a sticker . What range of selling prices will generate a monthly profit of at least Php 6000?

   (In thousand peso)

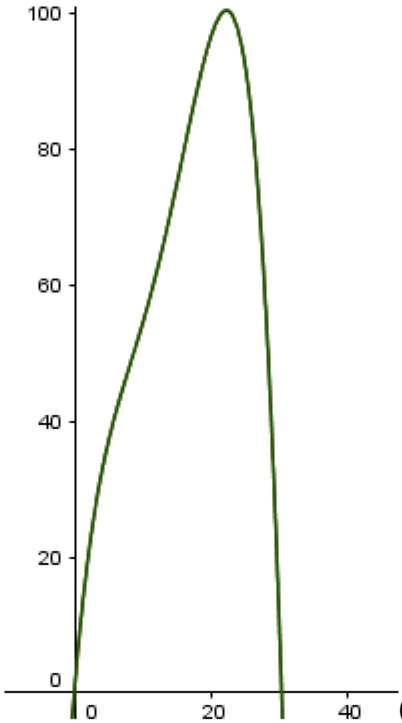


(In peso)

2. The graph of the function that models the age in human years, H(x), of a dog that is ***x*** years old,
   $H(x) = - 0.001618x^4 + 0.077326x^3 - 1.2367x^2 + 11.460x + 2.914.$

   Age in human years
   H(x)

   DOG'S AGE IN HUMAN YEARS



(Age of dogs in years)

Process Questions:

1. What kind of graphs are illustrated above?

2. What two quantities are involved in each graph?

3. Are the given graphs truly an application to real life? How?

4. **How do values of one quantity behave in terms of the other in the polynomial function that models areal- world situation?**

To initially sketch or graph a polynomial function, there are certain concepts and tests that must be taken into consideration. The following are needed information, concepts and test to initially understand the graph or the sketch of polynomial functions:

i. Leading Coefficient Test to determine the graphs end behavior

ii. The Multiplicities of Zeros and the X-intercepts
iii. The Y-intercepts
iv. Test for Symmetry
v. The Number of Turning Points
vi. The Upper and Lower Bounds

ACTIVITY 6. Sketch Me: The Leading Coefficient Test and Its End Behavior

Click the website
http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/col_alg_tut35_polyfun.htm
and read the given text. Focus on the first 4 examples to understand how to initially graph polynomial functions by understanding the leading Coefficient Test and its End Behavior. Take down notes in Evernote or in your notepad. This website gives a thorough discussion on how to sketch the graph of polynomial functions. Some important concepts are introduced and examples are provided to fully understand its graph. Answer the following questions;

1. From the given examples, what is the relation of the degree of the leading term and its coefficient in terms of the rising and falling of the polynomial curve?

2. When is the end behavior of the graph of a given polynomial rises to the right?

3. When is the end behavior of the graph of a given polynomial rises to the left?

4. When is the end behavior of the graph of a given polynomial falls to the right?

5. When is the end behavior of the graph of a given polynomial falls to the left? rises to the left and right?

6. When is the end behavior of the graph of a given polynomial rises to the left and right?

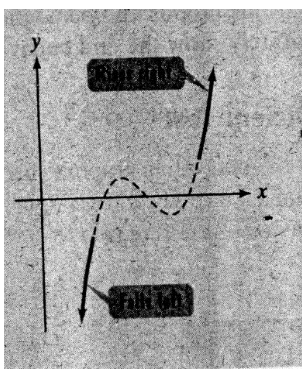
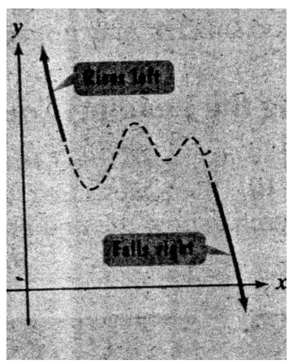
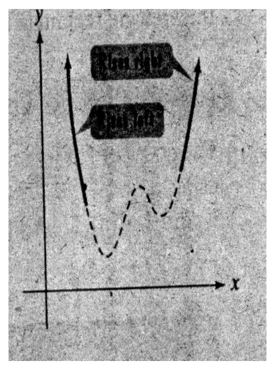
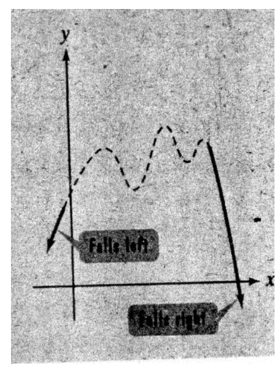
7. When is the end behavior of the graph of a given polynomial falls to the left and right?

8. If these graphs models a real-world situations, how do values of one variable behave in terms of the other in the polynomial function?

After reading the text from the given website and answering the questions above, summarize what you have understood by filling in the table with the correct data or information.

As x increases or decreases without bound, the graph of the polynomial function, $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0 \quad (a_n \neq 0)$ eventually rises or falls. In particular,			
For n is odd		For n is even	
Leading Coefficient is positive	Leading Coefficient is negative	Leading Coefficient is positive	Leading Coefficient is negative
The graph	The graph	The graph	The graph .
The sketch of the graph is	The sketch of the graph is	The sketch of the graph is	The sketch of the graph is

Answer:

<table>
<tr><td colspan="4">As x increases or decreases without bound, the graph of the polynomial function,
$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0 \qquad (a_n \neq 0)$$
eventually rises or falls. In particular,</td></tr>
<tr><td colspan="2">For n is odd</td><td colspan="2">For n is even</td></tr>
<tr><td>Leading Coefficient is positive</td><td>Leading Coefficient is negative</td><td>Leading Coefficient is positive</td><td>Leading Coefficient is negative</td></tr>
<tr><td>The graph falls to the left and rises to the right.</td><td>The graph rises to the left and falls to the right.</td><td>The graph rises to the left and to the right.</td><td>The graph falls to the left and to the right.</td></tr>
<tr><td>The sketch of the graph is
</td><td>The sketch of the graph is
</td><td>The sketch of the graph is
</td><td>The sketch of the graph is
</td></tr>
</table>

If the discussion in the given website is not enough, click the website below for additional information on the use of leading coefficient to determine the end behavior of the polynomial graph.

ACTIVITY 7. Sketch Me : The Leading Coefficient Test and Its End Behavior Video

Browse this additional website which further explains through a given video the graph of polynomial functions using the leading coefficient test.
http://my.hrw.com/math06_07/nsmedia/lesson_videos/alg2/player.html?contentSrc=6457/6457.xml

Process Questions:

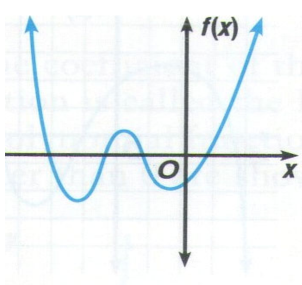
1. Did you find the video helpful in graphing polynomial function using the leading coefficient? Discuss.

2. Did you still find difficulty in graphing polynomial function using the leading coefficient test? Explain.

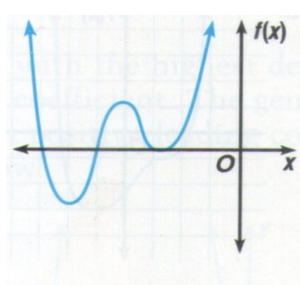
You have known earlier how to sketch the graph of the polynomial function considering the leading coefficient and its end behavior. In the next activity you will learn more how this end behavior of the graph is affected by either odd or even degree of the polynomial function.

ACTIVITY 8. Are You Odd or Even?

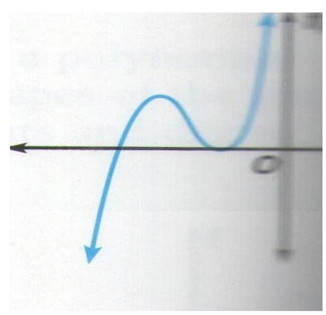
Determine whether the degree of the polynomial function represented by each graph is odd or even. Write Odd or Even on the space provided after each letter.



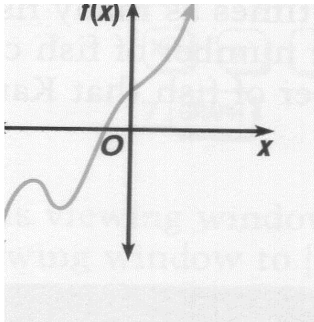
a. ______

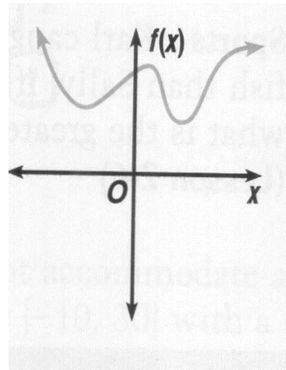


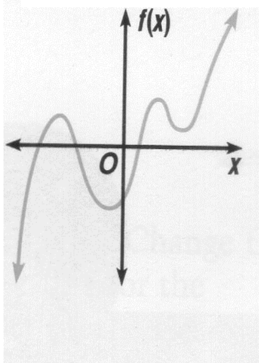
b. _______



c. ________

d.______ 

e.______ 

f.____ 

In the previous activities, you learned that the degree of the polynomial function and the leading coefficient determine the graph's end behavior. To further explore the graphing of the polynomial function, consider the succeeding activities.

ACTIVITY 9. Sketch Me: The Multiplicities of Zeros and X-Intercepts

Another strategy of graphing a polynomial function is finding the x-intercepts by setting P(x) = 0 and solving the resulting polynomial equation. For polynomial functions with multiplicities of roots, observe how these affects the behavior of the graph.
Visit the sites below.
http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/col_alg_tut35_polyfun.htm and http://www.purplemath.com/modules/polyends2.htm .
The websites illustrate behavior of graphs of polynomial functions with multiplicities of roots.
Read the text involving the graph of polynomial functions with multiplicity roots and study the given examples.
The simplest graph of polynomial functions in the form $f(x) = x^n$, where n is a positive number. The even degree functions intersect the x-axis once. An even degree function may or may not intersect the x-axis depending on its location in the coordinate plane. However, an odd degree function will always cross the x-axis

at least once. Remember that where the graph crosses the x-axis is called the zero of the function thus also called the x-intercept. On the coordinate plane, these zeros are real numbers.
Use Evernote or a notepad to take down notes on the important concepts involving graph of polynomial functions with multiplicity of roots. After reading and taking notes, answer the following questions and fill in the table given below.
Answer the following questions:

1. What do you mean by a zero has a multiplicity?

2. What is the relationship between the x-intercepts and the zeros of polynomial functions?

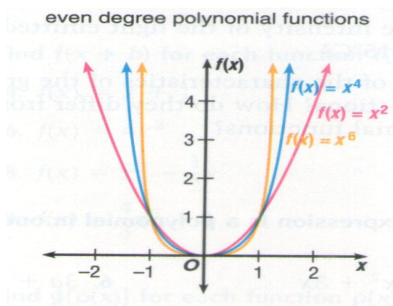
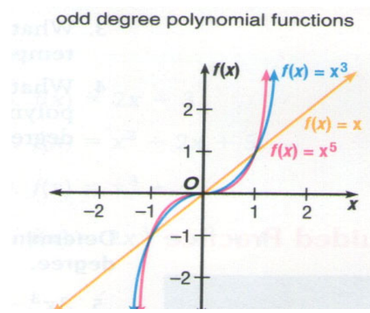
3. What is the graph of a polynomial function if it has an even-multiplicity zero?

4. What is the graph of a polynomial function if it has an odd-multiplicity zero?

Fill in the table given below with the correct term/terms

Given a polynomial function $P(x) = (x - r)^k$	
If k is even	If k is odd
the graph ______________________________ ______________________________ ______________________________.	the graph ______________________________ ______________________________ ______________________________.
The sample sketch of the graph is	The sample sketch of the graph is

Answer:

Given a polynomial function P(x) = $(x – r)^k$	
If k is even	If k is odd
the graph touches the x-axis at r and turns around.	the graph crosses the x-axis at (r ,0).
The sample sketch of the graph is 	The sample sketch of the graph is 

ACTIVITY 10. Sketch Me: More on Multiplicities of Zeros

For additional information regarding the graph with multiplicity of roots, click the websites below.
http://www.purplemath.com/modules/polyends3.htm .
http://www.purplemath.com/modules/polyends2.htm
These sites give further illustrative examples regarding graphs of polynomial functions with multiplicity of roots.

Process Questions:

1. What is the difference between the graph of f(x) = (x - 3) and f(x)= $(x – 3)^2$?

2. What is the difference between the graph of f(x) = $(x - 3)^4$ and f(x)= $(x – 3)^2$?

3. What is the difference between the graph of f(x) = (x - 3) and f(x)= $(x – 3)^3$?

4. Did you find the websites helpful in graphing polynomial function with even or odd multiplicities of roots? Discuss.

5. Did you still find difficulty in graphing polynomial function with even or odd multiplicities of roots? Explain.

ACTIVITY 11. Sketch Me: The Y-intercepts from TTW (Text, Table and Web)

You already learned the concept of y-intercept when you were in Grade 7. But let us recall the idea of y-intercept and how it is determined. Consider the given text, table and websites below to understand its concept.

The TEXT

The y-intercept of a polynomial function is where the graph crosses the y-axis on the real coordinate plane. Unlike with the x-intercepts, there can only be one y-intercept and there is always one y-intercept. No matter what polynomial you are given, there will be a y-intercept. The y-intercept is solved by simply setting x = 0 and simplify the equation. Whatever you end up with, it represents the y-intercept. Consider the table below on how the y-intercept is determined:

The TABLE

P(x)	If x = 0	y-intercept
1. $f(x) = 2x^4 - 9x^3 - 21x^2 + 88x + 48$	$f(0) = 2(0)^4 – 9(0)^3 - 21(0)^2 + 88(0) + 48 = 48$	(0, 48)
2. $f(x) = 9x^3 - 21x^2$	$f(0) = 9(0)^3 - 21(0)^2 = 0$	(0, 0)
3. $f(x) = 9x^2 + 4x - 7$	$f(0) = 9(0)^2 + 4(0) – 7 = - 7$	(0, -7)
4. $f(x) = (x – 5)(x + 5)(x + 3)$	$f(0) = (0 – 5)(0 + 5)(0 + 3) = - 75$	(0, - 75)

The WEB

Watch the given videos by browsing the websites https://www.youtube.com/watch?v=k92M4zzJk1s and https://www.youtube.com/watch?v=Ug5dayesjF0 .These videos show how the y-intercept is determined given polynomial functions.

Process Questions: Based on the given text, table, and websites above, answer the following questions:

1. What is the y-intercept given the polynomial function $f(x) = a_nx^n + a^{n-1}x^{n-1} + . . .+a_1x + a^0$?

2. What is the y-intercept of a polynomial function if the constant term (a^0) is zero?

3. Is it possible that a polynomial function has more than one intercepts? Why?

4. Is it possible that a polynomial function has no intercept? Why?

Complete the statements below.
I further conclude that the y-intercept is always ______________________ of a polynomial function. If there is no constant term in a given polynomial function, then the y-intercept is _________.

Answer:
Complete the statement below.
I further conclude that the y-intercept is always **the constant term** of a polynomial function. If there is no constant term in a given polynomial function, then the y-intercept is **0**.

ACTIVITY 12. Give Me My Y-Intercept, PLEASE?

The following websites give exercises on finding the x and y-intercepts of functions. Though the website focuses on rational function, but the concept and skills on finding the intercepts of polynomial and rational functions are of the same process.
cnx.org/contents/9b45e90c-b0ea-4fec-a6cc-576073b40b75@2/x_and_y-intercepts - This site gives exercises on finding the x and y-intercepts of given rational functions.

ACTIVITY 13. Sketch My Graph

Using the website http://apps.lonestar.edu/blogs/mapatetta/files/2013/09/notes-graphs-of-higher-degree-poly-funcs.pdf sketch the graph of polynomial functions. This site gives notes on graphing higher degree of polynomial functions. There are 4 exercises on graphing the polynomial functions and identifying the end behavior, x-intercepts and their multiplicity and the y-intercept of each function.

ACTIVITY 14. Sketch Me: Test for Symmetry

Browse the websites www.themathpage.com/aprecalc/symmetry.htm and http://www.analyzemath.com/function/even_odd.html. These websites further discuss even or odd function. Examples and problems are also provided to understand the concept. Read the text below regarding Test for Symmetry and underline or highlight the important word/s, or phrase/s that will lead to the understanding of the test for symmetry.

The words even and odd, when applied to a function describe the symmetry that exists for the graph of the function, *f*. A function is even, if and only if, whenever the point (x,y) is on the graph of *f*, then (-x, y) is also on the graph . In other words, a function *f* is even if, for every number *x* in its domain, the number *-x* is also in the domain: $f(-x) = f(x)$. A function *f* is odd, if and only if, whenever the point (x, y) is on the graph of *f* then the point (-x, -y) is also on the graph. In other words, a function *f* is odd if, for every number *x* in its domain, the number *-x* is also in the domain and $f(-x) = -f(x)$. Graphically, an even function is symmetric about the y-axis and an odd function is symmetric about the origin.

Answer the following process questions and summarize what you have understood by completing the given statements.

1. When are functions considered even?

2. When are functions considered odd?

3. In graphing an even function, its graph is symmetric about in what part of the x and y plane?

4. In graphing an odd function, its graph is symmetric about in what part of the x and y plane?

5. What is the importance of the Test of Symmetry in graphing polynomial functions?

If you were not able to answer correctly the questions above, go back to the given websites and take note of the answers to the given questions using the Evernote or your notepad. Then, summarize your understanding by completing the statements below.

The graph of a function is even when f(x) is equal to ___________and odd if f(x) is equal to _________. The graph of an even function is symmetric about the ______________and an odd function is symmetric about the _________. The importance of the test for symmetry is ___________________________________.

You just learned about when a graph of a polynomial function is either a graph of odd or even polynomial function. Furthermore a polynomial function can be easily and accurately graphed by finding the turning points. You will discover how is the degree of the polynomial function related to its number of turning points in the following websites.

ACTIVITY 15. Sketch Me: The Bumps, Bends or Turning Points of Polynomial Graph

Visit the websites https://www.mathsisfun.com/algebra/polynomials-behave.html - This website gives a brief description of the number of turning points of polynomial graph indicating the local minima and local maxima, http://www.purplemath.com/modules/polyends4.htm - This website discusses the number of bumps or turning points of polynomial graph and https://www.youtube.com/watch?v=9WW0EetLD4Q – This video focuses on locating and identifying the turning points given the polynomial function and its graph and also mentioned in the video is the x-intercept. Take down notes from these sites in Evernote or in your notepad. After reading the text and the examples, summarize what you have learned by completing the statements given below.

The graph of a polynomial function of degree n has ____________ turning points. If the degree of the polynomial is 7, then it has _____________ turning points.

Keep in mind also that a polynomial graph can have _________________ turning points, but it will never ______________ turning points.

Answer:

The graph of a polynomial function of degree n has ***at most n – 1*** turning points. If the degree of the polynomial is 7, then it has ***at most 6*** turning points.

Keep in mind also that a polynomial graph can have **fewer than *n* - 1** turning points, but it will never **exceed *n* - 1** turning points.

ACTIVITY 16. Count My Bumps!

Graphs of the polynomial functions have turning points. At each turning point, the graph changes direction from increasing to decreasing or vice versa. These points are called local maxima or local minima. Check your understanding by visiting the site below.
Click the site https://www.mathsisfun.com/algebra/polynomials-behave.html to answer the 10 questions as formative exercises about the turning points and its local maxima and local minima of a polynomial graph. These 10 questions can be found in the bottom portion of the page with an icon Your Turn.

ACTIVITY 17. The Upper and Lower Bounds

In Lesson 2, you have encountered The Descartes' Rule of Signs which gives us the possible number of zeros of a polynomial function without saying how large or how small any of them may be. In this activity we will be given information on the range in which the real zeros, if there are any, may lie. It will enable us to find a number that is greater than or equal to the largest real root of a polynomial equation (an upper bound of the zeros) and another number that is smaller than or equal to the least root of the equation (a lower bound of the zeros).

Determining the upper and lower bounds of the zeros of a polynomial function will greatly help in graphing the polynomial function in locating its real zeros.

To understand how the upper and lower bounds of a polynomial function are determined, visit the website http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/col_alg_tut39_zero2.htm. Read the given text and study the given example(Example 1) and solve the given problem in Practice Problem 1a. Click the website https://www.youtube.com/watch?v=XgGzm0d_MDY and watch the video. The video discusses how to determine the upper and lower bounds of a polynomial function.

ACTIVITY 18. Finally Graph Me!

Using all the knowledge and skills learned from previous activities, you can finally sketch the polynomial graph. To guide you on sketching the graph, click the following websites:

a. http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/col_alg_tut35_polyfun.htm - In this site, study the given problems in graphing polynomial function specifically Examples 7 and 8.

b. http://www.purplemath.com/modules/polyends5.htm - A sample problem on sketching the graph of a polynomial functions with multiplicity of roots.

c. https://www.youtube.com/watch?v=of2OG5NNNIo – A video on graphing polynomial functions and at the end it also discusses the leading coefficient test
d. https://www.youtube.com/watch?v=qi9ZlTiHIwY – A video on graphing polynomial functions.
e. https://www.youtube.com/watch?v=Op3OP_fHrRE – A video on graphing a cubic function.
f. www.analyzemath.com/Graphing/graphing_cubic_function.html - This site gives 4 examples on graphing cubic functions.

g. https://www.youtube.com/watch?v=WDuMkC6UFro – A video on graphing polynomial functions.
h. https://www.youtube.com/watch?v=en2ctMSVNEI – A video on graphing polynomial functions expressed in factored form.

After browsing all the given websites and understood the lessons on graphing polynomial functions, perform Activity 14as a formative assessment. Click the following websites and answer the given formative exercises.

ACTIVITY 19. Graph My Functions!

a. http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/col_alg_tut35_polyfun.htm - In this website, answer Practice Problems 1a - 1b found in the bottom page. Write your answers in your notebook. After answering the given problems, check your answers and solutions by clicking the Answer/ Discussion button.

b. http://www.classzone.com/etest/viewTestPractice.htm?testId=4387&seqNumber=4&testSessionId=null&startUrl=http://www.classzone.com/books/algebra_2/lessonquiz_national.cfm - This website contains five problems. You need to identify the required polynomial function given the polynomial graphs.

ACTIVITY 20. Graphing Polynomial Functions Using a Graphing Utility

Not all polynomial functions are easy to graph and determine some of its characteristics. To graph the polynomial functions accurately specifically the turning points, the location of the local minima and local maxima, the non- integer roots, a graphing utility is needed. The following websites are some graphing utilities to be used in graphing polynomial functions.

a. http://www.shodor.org/interactivate/activities/Graphit - This website creates graph of functions and sets of ordered pair on the same coordinate plane. It is like a graphing calculator with advance viewing path. This site is very helpful in determining the approximated roots of polynomial functions by graphing. A Java application is needed to use this site.

b. http://www.shodor.org/interactivate/activities/GraphSketcher/ - This create graph of functions by entering the formula similar to a graphing calculator. A Java application is needed to use this site.

c. http://www.shodor.org/interactivate/activities/FunctionFlyer/ - This will graph any function. Click the button Help to understand how to use the graphing utility.

d. https://www.desmos.com/calculator/hrujnpulw9 - This graphing utility will graph polynomial functions with degree of 4 or less. To graph the polynomial functions, adjust the values of a, b, c, d or f. You can use the slider, select the number and change it or “play” the animation. The local minima and local maxima can be determined by pointing the cursor on the highest or lowest points of the polynomial curve.

e. www.mathportal.org/calculators/polynomials-solvers/polynomials-graphing-calculator.php - A graphing calculator for a 4th degree polynomial functions. The roots, turning points, local maxima and minima can be obtained using this utility.

f. http://www.mathsisfun.com/data/function-grapher.php - This is graphing utility tool graph any functions.

ACTIVITY 21. Graphing Polynomial Functions

Using any of the above graphing utilities, graph the following polynomial functions. Present them in powerpoint. Then use www.powtoon.com to convert your created power point presentation into a video. :

a. $f(x) = x^4 - x^3 - 2x^2 + 4x - 6$
b. $f(x) = x^4 + x^3 + 3x^2 + 2x$
c. $f(x) = x^3 - 4x^2 + 3x + 1$
d. $f(x) = - x^3 - 4x^2 - 2x + 3$
e. $f(x) = -0.0017x^4 + 0.31x^3 - 0.277x^2 + 0.3005$
f. $f(x) = x^5 + x^4 - 3x^3 - 3x^2 – 4x – 4$
g. $f(x) = x^5 - x^4 + x^3 + 5x^2$
h. $f(x) = (x – 2)(x + 1)(x – 3)(x + 4)$

Process Questions:

1. How did you find graphing the polynomial functions using the graphing utilities?

2. What are the advantages of using the graphing utilities in graphing polynomial functions?

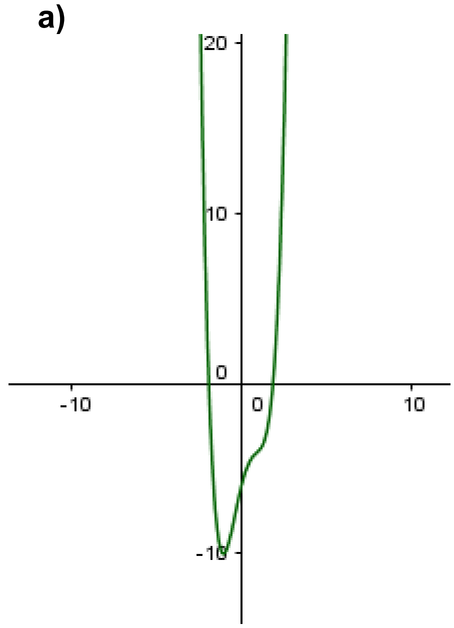
3. Are there disadvantages of using these graphing utilities? What are these?

4. Given the graph of a polynomial function using the graphing utility, what information can be obtained and be deduced?

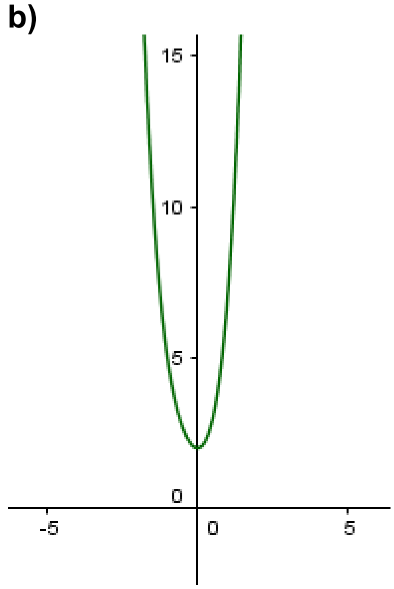
5. With the use of these graphing utilities, we were able to graph polynomial functions. From the given graph, how do values of one variable behave in terms of the other in the polynomial function?

Answer Key

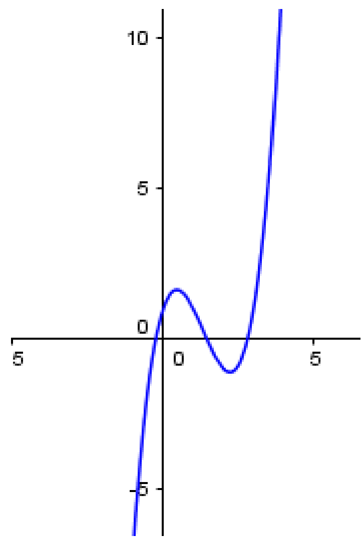
a)



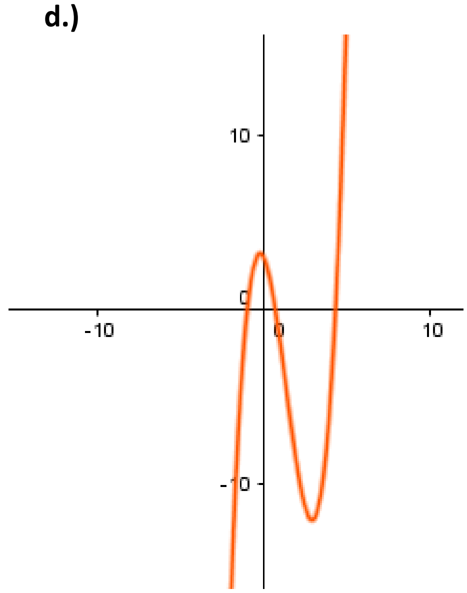
b)



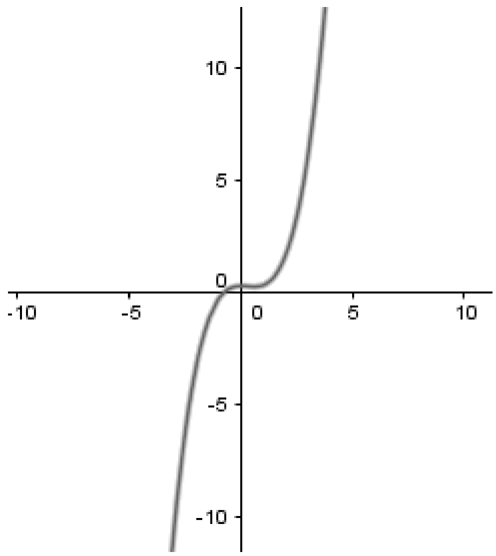
c.)



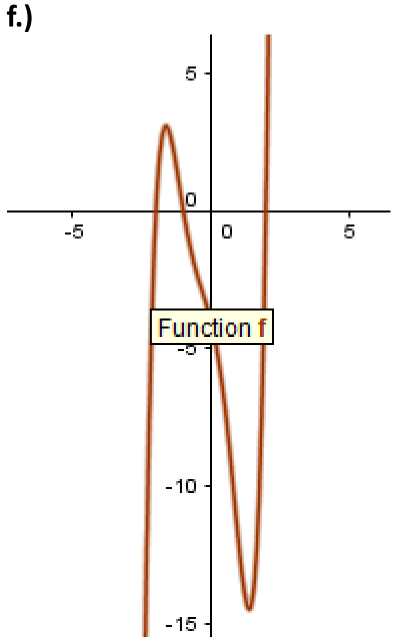
d.)



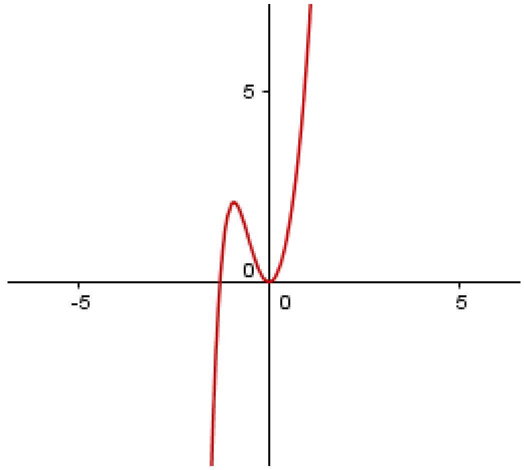
e.)



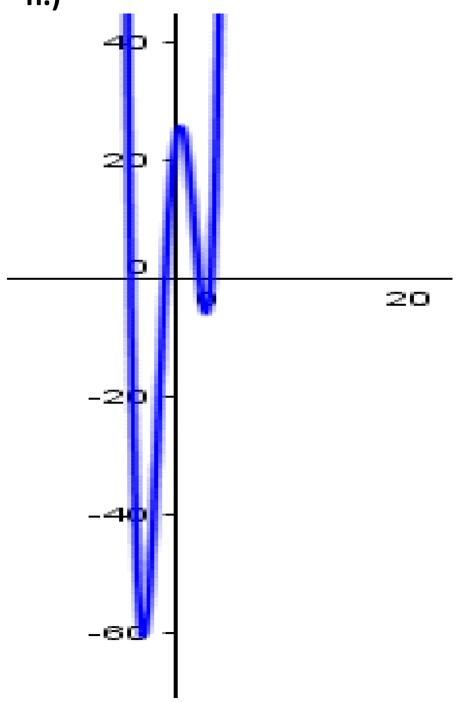
f.)



g.)



h.)



In the previous activity, you tried graphing polynomial functions using any of the graphing utilities. You learned that polynomial functions are a powerful tool that can help people make decisions everyday like designs of monuments, bridges and patterns. The following activity develops a better sense of how the polynomial functions relate to the real world.

PROBLEM:

As an environmentalist is it important not to waste material as well as getting as much out of the product as possible. You are being commissioned to develop a cardboard box, with no lid, that has the most volume. The problem is that the machine that produces the cardboard box has not been programed yet with a polynomial function for the 12 by 18 dm size of cardboard. It is up to you to conserve the material and create a box with no lid with the most possible amount of volume.

FORMULATE: (Write the concepts you need to answer the problem.)

COMPUTE: (Write your complete solution.)

INTERPRET: (Write a paragraph about the interpretation of your answer.)

VALIDATE: (Provide one more example to validate your answer.)

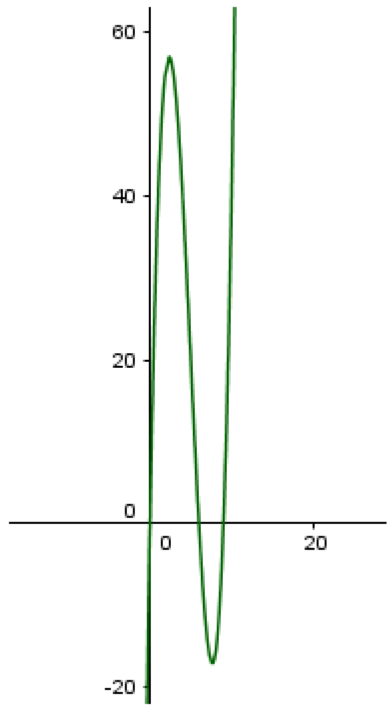
REPORT:

Non-Online: Submit your written report.

Online: Send your written report using the student dash board.

Answer Key

The width is 8 dm, the length is 14 dm and the height is 2 dm. The polynomial function is $F(x) = x^3 - 15x^2 + 54x$ and the graph is

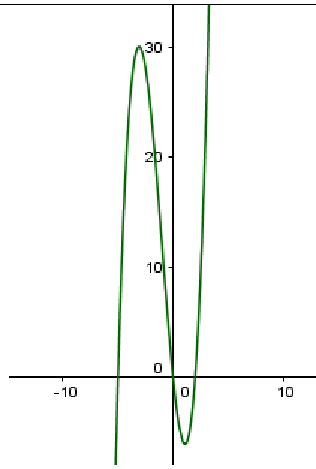
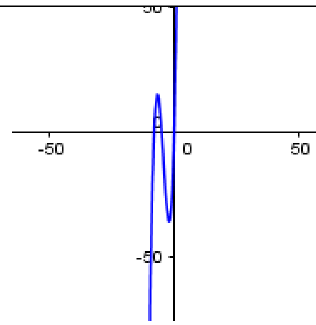
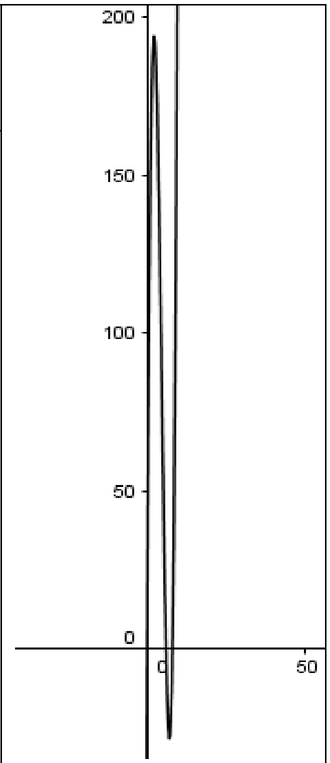


ACTIVITY 22. Graphs of PolynomiaL Functions SUM IT UP!

Put together in the table below your answers to the essential question that is asked for each problem.

	PROBLEM 1	PROBLEM 2	PROBLEM 3
ESSENTIAL QUESTION: **6. How do values of one variable behave in terms of**	The compartment for the printer is a rectangular prism and will be 400 cubic inches. The compartment will be 5 longer than it is wide and the height will be 2 inches less than the length. Find	The length of a pool is 3 m more than its width and the height is 5m less than its width. If the volume of the tank is 650 m^3, what are its dimensions?	An open box with a volume of 180 ft^3 can be made by cutting a square of the same size from each corner of a piece of metal 12 ft wide and 16 ft long on a side and folding up the

the other in the polynomial function that models a real- world situation?	the dimensions of the printer compartment. Answer: The width is 5 inches, the length is 10 inches and the height is 8 inches.	Answer: The width is 10 m, the length is 13 m and the height is 5 m.	edges. What is the length of a side of a square that is cut from each corner? Answer: The length of a side of a square that is cut for each corner is 3 ft.
	How dimensions are solved in the polynomial function that models a real-world situation? Answer: Make representations of the dimensions and write a polynomial function/equation and solve for the value of the variable that represents the unknown in the polynomial equation/function.	How dimensions are solved in the polynomial function that models a real-world situation? Answer: Make representations of the dimensions and write a polynomial function/equation and solve for the value of the variable that represents the unknown in the polynomial equation/function.	How dimensions are solved in the polynomial function that models a real-world situation? Answer: Make representations of the dimensions and write a polynomial function/equation and solve for the value of the variable that represents the unknown in the polynomial equation/function.
	Solution of a polynomial function that models real-world situation depends on ___________ Answer: the degree of the polynomial function	Solution of a polynomial function that models real-world situation depends on ___________ Answer: the degree of the polynomial function	Solution of a polynomial function that models real-world situation depends on ___________ Answer: the degree of the polynomial function
	Graph: Answer:	Graph: Answer:	Graph: Answer:

PROCESS QUESTIONS:

1. How are real world problems involving polynomial functions solved?
2. How do you graph polynomial functions? Was there only one way to graph a polynomial function?

   If yes, explain. If not, why are there many ways to graph a polynomial function that models real- world situation? Explain.

Answers:

1. Represent the unknown quantity by the variable and express other unknown quantities in terms of the variable. Write the corresponding polynomial equation of the problem and solve for the value of the variable in that polynomial equation.

2. You can graph a polynomial function manually or by using any graphing utility. There are many ways to graph a polynomial function because aside from graphing it manually, there are many graphing utilities that you can use as mentioned in the earlier part of this lesson.

ACTIVITY 23. Fill Me

To check if you understood the lesson on graphing polynomial functions, fill in the table below the different concepts, tests, theorems and others to be learned in order to graph polynomial functions.

Concepts Graphing	Tests of
Polynomial Theorems	Function Others

ACTIVITY 24. Rate Yourself!

How confident are you about graphing polynomial functions using the given test and other concepts? Check the box that applies.

	Not Confident	Confident	Very Confident
1. Using Leading Coefficient test to determine graph's behavior			
2. Determining the number of turning points of the polynomial function			

3. Determining local minima and local maxima of the graph of polynomial function			
4. Graphing polynomial function with multiplicity of roots			
5. Identifying x-intercepts			
6. Identifying y-intercepts			
7. Determining upper and lower bound to the roots of polynomial functions			

If you are still in the Not Confident and midway the Not Confident and Confident, ask for assistance from a resource person who is knowledgeable of the lesson or seek the help from your teacher in your face to face meeting. Give your reasons why you succeeded or not succeeded in this lesson.

END of FIRM UP

In this section, the discussion was about the Graphing Polynomial Functions. Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision? What new learning goal should you now try to achieve?

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.

DEEPEN

Your goal in this section is to take a closer look at some aspects of polynomial functions. Real- life problems involving polynomial functions will be given in this section and will be asked to solve related problems. Before proceeding to the next activity, consider the question: How do values of one variable behave in terms of the other in the polynomial function that models a real- world situation?

Perform the next activity, to find out how.

ACTIVITY 25. The Box in its Maximum Volume!

Click the sites below and watch the video and study the given sample problems.

a. http://my.hrw.com/math06_07/nsmedia/lesson_videos/alg2/player.html?contentSrc=8168/8168.xml. This video is an application of polynomial function solving for the dimensions of the box that will yield a maximum volume.

b. https://www.youtube.com/watch?v=OcjD6vYD7ic – This video discusses graphing polynomial functions and before the end of the video is an application problem on finding the maximum volume of a cylinder.

c. www.shelovesmath.com/algebra/advanced-algebra/graphing-polynomials/#APolynomialGraph – This site shows sample application problems on polynomial functions.

d. https://finitemathematics.wikispaces.hcpss.org/Unit+4+Polynomial+Functions – This site gives a sample application problem on polynomial functions. The sample is located at the last part of the site.

e. As part of art project, Carlos is designing a carton for repacking candy bars that the junior class will sell to raise money for the prom. The volume of the carton must be 120 in^3. To hold the correct number of candy bars, the carton must be 3 inches longer than its width. The height of the carton should be 2 inches less than its width. Using these restrictions, Carlos must determine the dimensions of the carton.

Solution: Let w be the width of the carton
w + 3 be the length of the carton
w – 2 be the height of the carton
Volume of the Carton = length x width x height
Substituting the corresponding dimensions and simplifying the equation

$$V(w) = (w+3)(w)(w-2)$$

$$120 = w^3 + w^2 - 6w$$
$$0 = w^3 + w^2 - 6w - 120$$
$$0 = (w - 5)(w^2 - 6w - 24)$$
$$w = 5 \quad w = 3 + i\sqrt{15} \quad w = 3 - i\sqrt{15}$$

From these values, since the we are finding for the dimensions of the carton, then the width is equal to 5 inches. The length is 8 inches and the height is equal to 3 inches.

f.  If the function that models the age in human years, H(x), of a dog that is ***x*** years old, $H(x) = -0.001618x^4 + 0.077326x^3 - 1.2367x^2 + 11.460x + 2.914$, how old is your dog if you are 16 years old?
To answer the question, we must substitute 16 for H(x) and solve for the resulting polynomial equation for x. The function becomes $16 = -0.001618x^4 + 0.077326x^3 - 1.2367x^2 + 11.460x + 2.914$ and finally becomes $0 = = -0.001618x^4 + 0.077326x^3 - 1.2367x^2 + 11.460x - 15.086$. Using a graphing utility, the graph of the function is given below.

g. In the 1980's, a rising trend in global surface temperature was observed, and then the term "global warming" was coined. Scientists are more convinced than ever that burning coal, oil and gas results in a buildup of gases and particles that trap heat and raise the planet's temperature. The average increase in global surface temperature, T(x), in degrees Centigrade, x years after 1980 can be modeled by the polynomial function $T(x) = \frac{21}{5{,}000{,}000}x^3 - \frac{127}{1{,}000{,}000}x^2 + \frac{1293}{50{,}000}x$ or $T(x) = 0.0000042x^3 - 0.000127x^2 + 0.02586x$. Use a graphing utility to graph the function.

- Can you predict the earth's temperature in the years to come? How?
- What do you observe about global warming through the year 2040?
- What is the average temperature increase in the year 2040? What does it imply?

How do values of one variable or quantity behave in terms of the other in the polynomial function that models a real- world situation?

Visit https://www.youtube.com/watch?v=3q42s4YEvyE for additional illustrative example of real-life application of polynomial function.

In the first and second lessons of this unit, you have been solving real-life problems involving polynomial equations. Solve the remaining problems below to enhance your understanding of how polynomial functions are applied in real life situations.

ACTIVITY 26. Problem Solving

Solve the following problems.

1. The largest pyramid at Gizeh, Egypt was built by Khufu, a king of the fourth dynasty. The pyramid has a square base and has a volume 0f 2645 cubic dekameters. The height of the pyramid is approximately 8 dekameters less than the length of a side of the square base. What are the dimensions of the pyramid?

   $V = (1/3)Bh$
   $2645 = (1/3)(s^2)(s-8)$
   $7935 = s^3-8s^2$
   $0= s^3-8s^2-7935$
   $0 = (s- 23)(s^2+15s +345)$

2. The space shuttle has an external tank for the fuel that the main engines need for the launch. About eight minutes into the flight, the fuel is gone and the tank is released. This tank is shaped like a capsule, a cylinder with a hemispherical dome at either end. The cylindrical part of the tank has a volume of 1170 cubic meters and a height of 17 meters more than the radius of the tank. What are the dimensions of the tank to the nearest tenth of a meter?

3. The number of eggs N, in a female moth as a function of her abdominal width, W, in millimeters modeled by $N(W) = 14W^3 – 17W^2 – 16W + 34$. How many eggs the moth have if her abdominal width is 10 mm.?

ACTIVITY 27.

http://my.hrw.com/math06_07/nsmedia/homework_help/alg2/alg2_ch06_07_homeworkhelp.html - this is for assignment before the post test is given

ACTIVITY 28. Test for Understanding: Scaffold 3 – The Toy Box for Kids

Use www.creately.com in the creation of your graph and in collaborating with your classmates or group mates in doing the task below.

With an abundant number of toys manufactured by the company where you work, the company executive officer considers the production of toy storage boxes. As a designer, you were assigned by your division manager to make a design for a rectangular toy storage box for kids. The box should have an open top and has a volume of 9 cubic feet. For design purposes, the length of the base is three times its width. Materials for the base costs Php 80/ft^2 and for the sides cost Php 0/ft^2. The materials should be practically chosen to ensure durability and quality. You may use foam board, thick poster board or anything that will hold its shape. A scaled model toy storage box will be presented to your division manager and the company executive officer. It will be evaluated according to accuracy of computation, presentation of output, usefulness of the model, and justification of the recommendation.

During the presentation, you need to include the following:

1. The graph of the created function by assigning values for the width (from 0.1 to 2 with an interval of 0.1) using available graphing utility. Give the characteristics of the graph,
2. Location of the maximum and the minimum values or the extrema of the graph.
3. The dimensions of the box if the cost of the production of a toy storage box is at most Php 1,500 to keep the volume at 9 ft^3.
4. The justification of the recommended dimensions that the manufacturer will use.

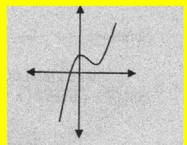
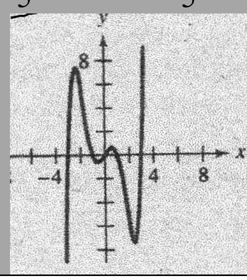
CRITERIA	EXCELLENT 4	SATIS-FACTORY 3	PROG-RESSING 2	NEEDS IMPROVE-MENT 1	RATING
Accuracy of Computation	Shows correct computation and solution with appropriate explanation.	Shows correct computa-tions and solutions.	Some solutions show incorrect computa-tions.	Shows solutions with incorrect computa-tions.	
Presentation of Output	Delivery is enthusiastic and projection is	Delivery and projection is	Delivery is confusing and	Delivery is weak and projection is inaudible.	

	loud and commanding.	clear to everyone.	projection is shaky.		
Usefulness of the model	The model explains in an in-depth way the relation-ships of different factors.	The model presents in a clear way the relation-ships of different factors.	The model explains in a limited way the relation-ships of different factors.	The model does not show the relationships of different factors.	
Justification of Recommen-dation	The recommen-dation shows sophisticated understand-ing of the relevant ideas and processes.	The recommen-dation shows solid understand-ing of the relevant ideas and processes.	The recommen-dation shows partially correct understandi ng of the relevant ideas and processes.	The recommen-dation shows erroneous understand-ing of the relevant ideas and processes.	
				TOTAL	

ACTIVITY 29. Anticipation Reaction Guide: Agree or Disagree? Revisited

In the earlier part of this lesson, you have an initial answers to the same activity by answering the Before Discussion column of the ARG. Note items which answers remain the same in the revisit and which ones differ.
Answer the After Discussion column of the ARG by clicking on the AGREE or DISAGREE column. Click A if you agree with the statement and click B if you disagree. Please answer all items.

Before Discussion		Statements	After Discussion	
Agree	Disagree		Agree	Disagree
A	B	1. The graphs of polynomial functions are smooth and continuous.	A	B

A	B	2. The end behavior of the graph of a polynomial function depends on the constant term of the polynomial function.	A	B
A	B	3. If *f* is a polynomial of degree n, the graph of *f* has at most n -1 turning points.	A	B
A	B	4. If f(x) = - x^3 + 4x, then the graph of *f* falls to the left and to the right.	A	B
A	B	5. The graph of the monomial function f(x) = x^4 touches the x-axis at the x-intercept.	A	B
A	B	6. The cubic function may have the graph given below. 	A	B
A	B	7. The graph of f(x) = $\frac{1}{5}x^5 - 2x^3 + \frac{9}{5}x$ is 	A	B

A	B	8. If the volume of a pyramid is given by $V = x^3 - 2x^2 - 75$, the real value of x is 5.	A	B
A	B	9. The roots of polynomial functions are located between the upper and lower bounds of the polynomial graph.	A	B
A	B	10. The graph of even polynomial functions with positive leading coefficient rises to the left and to the right.	A	B

END of DEEPEN

In this section, the discussion was about the real life application of polynomial functions. Revisit the problems you solved in Activity 23. If there will be changes in your answers, what would it be? Make your revision and give necessary justification If there are still difficulty encountered in the lesson, you can seek help from your mentor/teacher for further clarifications. Present your revised answer and justification in this link by creating your own video.

TRANSFER

Your goal in this section is apply your learning to real life situations. You will be given a practical task which will demonstrate your understanding.

ACTIVITY 30. The Performance Task

The energy provider wants to know the extent of use of electricity in the area to ensure enough power supply. As a Meralco inspector, you are tasked to predict energy consumption for the next three years and make the necessary recommendations. You are to present a video clip of a mathematical model illustrating household energy consumption to your Meralco area manager. Your output will be graded according to accuracy of computation, organization of data, representation of output, usefulness of the model, and justification of recommendation.

CRITERIA	EXCELLENT 4	SATIS-FACTORY 3	PROG-RESSING 2	NEEDS IMPROVE-MENT 1	RATING
Accuracy of Computa-tion	Shows correct computation and solution with approp-riate explana-tion.	Shows correct computa-tions and solutions.	Some solutions show incorrect computa-tions.	Shows solutions with incorrect computa-tions.	
Organiza-tion of Data	Flows smoothy and has logical and interesting connections	.Flow is generally smooth and logical.	Somewhat cluttered. Flow is not consistently smooth, appears disjointed.	Illogical and obscure. No logical connections of ideas. Difficult to determine the meaning.	
Presenta-tion of Output	Delivery is enthusiastic and projection is loud and commanding.	Delivery and projection is clear to everyone.	Delivery is confusing and projection is shaky.	Delivery is weak and projection is inaudible.	
Usefulness of the model	The model explains in an in-depth way the relation-ships of different factors.	The model presents in a clear way the relation-ships of different factors.	The model explains in a limited way the relation-ships of different factors.	The model does not show the relationships of different factors.	
Justifica-tion of Recom-mendation	The recommenda-tion shows sophisticated understanding of the relevant	The recommen-dation shows solid under-standing of	The recom-mendation shows partially correct under-	The recom-mendation shows erroneous under-standing of the relevant	

	ideas and processes.	the relevant ideas and processes.	standing of the relevant ideas and processes.	ideas and processes.	
				TOTAL	

ACTIVITY 31. Reflection Log

Answer the following questions after doing the final task.
Process Questions:

1. How did you find the performance task?

2. What are the important factors did you consider which contributed to the success of the Performance Task?

3. To what extent is your knowledge, skills and understanding of polynomial functions have helped you accomplish the task?

4. How did the task help you see the real world use of the topic? In what other real life situations can you apply the learnings you've gained in this module?

5. Which particular concepts did you find most difficult?

6. What would you want to be included in the lesson that could help minimize or eliminate your problems?

ACTIVITY 32. Synthesis Journal

Fill in the SYNTHESIS JOURNAL by completing the statements.
The lesson was about _______________. One key idea was __________. This is important because _____________. Another key idea was _________. This is also important because_________.
Regarding the question How do values of one quantity behave in terms of the other that models a real- world situation?I was able to think that ______________________. This further leads me to develop an essential understanding that ____________________.
Write your statements inside the box.

END of TRANSFER

You have completed this lesson. Before you go to the next module, you have to answer the following post-assessment.

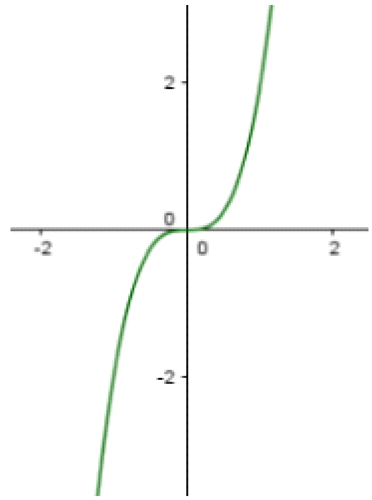
1. The graph of f(x) = (x+1)(2x+3)(x-2) crosses the x-axis ___________.
   a. once　　c. thrice
   b. Twice　　d. four times

2. Which of the following is an upper bound of $f(x) = x^4 - x^3 - 12x^2 - 2x + 3$?
   a. 2*　　c. 4
   b. 3　　d. 5

3. How many x-intercepts are on the graph of the polynomial $f(x) = 3x^3 - 9x^2$?
   a. 2　　c. 4
   b. 3*　　d. 5

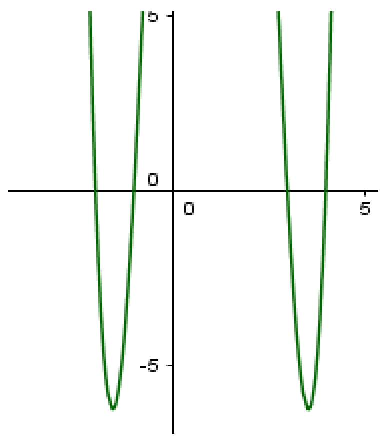
4. Which of the following is the lower bound for the real roots of the polynomial function $p(x) = x^3 - 19x + 30$?
   a. -4　　c. -6
   b. -5　　d. -7

5. Which of the following functions has the graph shown in the figure?



a. $= 2x^3$
b. Y = -24x4
c. y = -2x
d. $y = -2x^5$

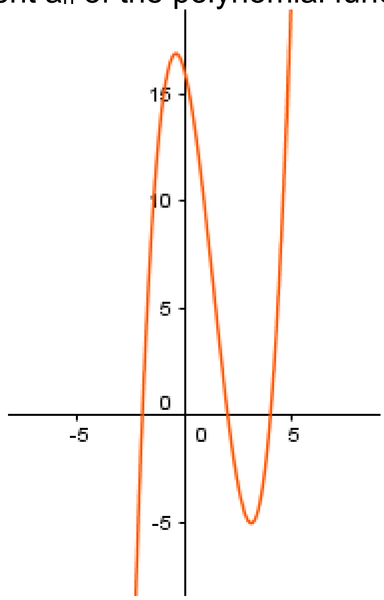
6. Which of the following statements about the degree n and leading coefficient a_n of the polynomial function, whose graph is given, is true?



a. n is even, $a_n > 0$
b. n is even, $n < 0$
c. n is odd, $a_n > 0$
d. n is odd, $a_n < 0$

7. Which of the following is an upper bound of $f(x) = x^4 - 6x^3 + 13x^2 - 12x + 4$?
   a. 3 c. 5
   b. 4 d. 6

8. Given the graph, which of the following statements about the degree n and leading coefficient a_n of the polynomial function y is true?



   a. n is even, $a_n > 0$ c. n is even, $a_n < 0$
   b. n is odd, $a_n > 0$ d. n is odd, $a_n < 0$

9. Which of the following is the lower bound to the roots of $f(x) = 3x^3 + 10x^2 - 2x - 4$?
   a. -3 c. -5
   b. -4 d. -6

10. If f is a polynomial of degree n, the graph of f has at most _____.
    a. n turning points c. n - 1 turning points
    b. n + 1 turning points d. n + 2 turning points

After answering these items, you still have to answer the unit post assessment items of the learning module. Your score will be deliberated by your teacher and inform you of the result.

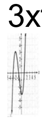
> You have completed this lesson. Before you proceed to the next module, you have to answer the following post assessment.

It's now time to evaluate your learning. Click on the letter of the answer that you think best answers the question. Your score will only appear after you answer all items. If you do well, you may move on to the next module. If your score in not at the expected level, you have to go back and take the module again.

MODULE POST ASSESSMENT

1. Which of the following is not a solution to the cubic equation below?
$x^3 + 3x^2 – 4x -12 = 0$
    A. -3
    B. -2
    C. 2
    D. 3

2. What is the remainder when $x^3 – 5x^2 – x + 5$ is divided by x - 2 ?
    A. -10
    B. – 9
    C. 9
    D. 10

3. Which of the following is a polynomial equation?
    A. $3x^4 - x^3 + 2x – 7 = 0$
    B.  $+ 2x^3 - 4x^2 + 5 = 0$
    C. $(3x – 4)^{-2} = 0$
    D. $x^{1/2} – 2x + 8$

4. Given a polynomial and one of its factors, what are the remaining factors of the polynomial?
$x^3 + 4x^2 - 4x – 16$; x + 2

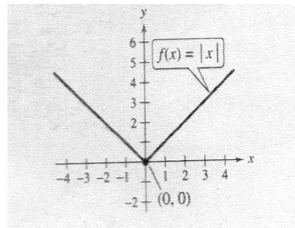
    A. (x – 2) (x – 4)
    B. (x – 2) (x + 4)
    C. (x + 2) (x + 4)
    D. (x + 2) (x - 4)

5. What are the zeros of the function
$f(x) = x^3 -4x^2 -19x -14$?
    A. -1, -2, 7
    B. -1, 2, -7
    C. -1, 2, 14
    D. -2, 7, -14

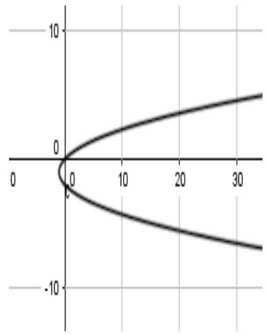
6. One zero of $f(x) = x^3 - 7x^2 -6x + 72$ is 4. What is the factored form of
$x^3 - 7x^2 -6x + 72$?
    A. (x-6)(x+3)(x+4)
    B. (x-6)(x+3)(x-4)
    C. (x+6)(x+3)(x-4)
    D. (x+12)(x-1)(x-4)

7. Which of the following illustrates the the graph of polynomial function?

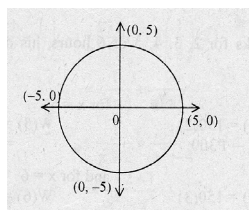
A.



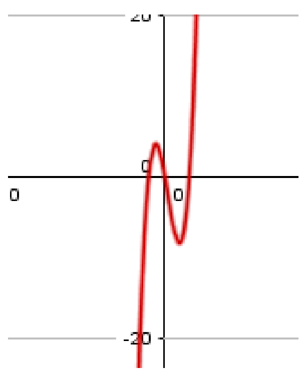
B.



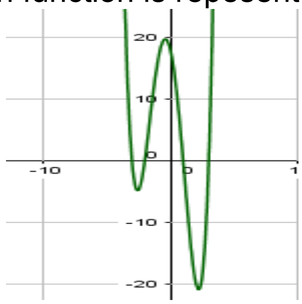
C.



D.



8. Which function is repesented by the graph?



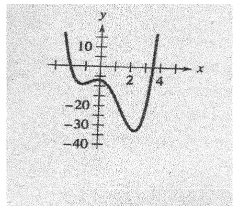
A. $f(x) = x^5 + 2x^4$
B. $f(x) = 2x^3 + 4x^2 – x + 8$
C. $f(x) = x^4 + x^3 – 11x^2 -9x + 18$
D. $f(x) =2x + 1$

9. A specific car's economy in miles per gallon can be approximated by $f(x) = 0.00000056x^4 – 0.000018x^3 – 0.016 x^2 + 1.38x – 0.38$, where x represents the car's speed in miles per hour. What is the fuel economy when the car is travelling 40 miles per hour?
A. 27.25 miles per gallon
B. 28.5 miles per gallon
C. 29.5 miles per gallon
D. 30 miles per gallon

10. The profit in hundreds of peso for selling c calculators per day can be modelled by $f(x) = -0.005c^4 + 0.25c^3 + 0.01c^2 – 2.5c + 100$. What is the meaning of zeros in this situation?
A. profit on sales
B. calculators sold
C. break even sales
D. sales shortages

11. The amount of money a certain foundation took in from 2006 to 2013 can be modeled by $M(x) = -2.03x^3 + 50.1x^2 -214x+4020$. What is the significance of each zero in the context of the situation?
A. The amount of money taken by the foundation.
B. The years where the foundation did not take any amount of money.
C. The years where the foundation took of money.
D. The years where the foundation have the most income.

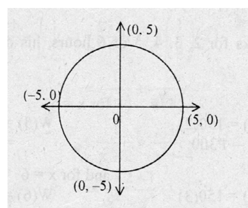
12. The graph of one of the following functions is shown below. Identify the function shown in the graph and explain why each of the others is not the correct function. You can use graphing utility to verify your result.



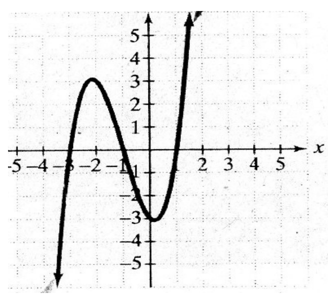
A. $f(x) = x^2(x + 2)(x – 3.5)$
B. $g(x) = (x + 2)(x – 3.5)$
C. $h(x) = (x + 2)(x – 3.5)(x^2 + 1)$
D. $k(x) = (x + 1)(x + 2)(x – 3.5)$

13. Which of the following CANNOT be a graph of polynomial function?

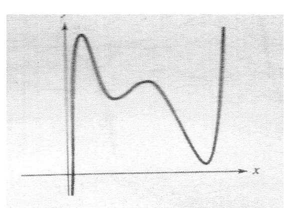
A.



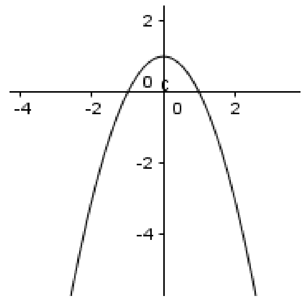
B.



C.



D.



14. 14. A packaging box with dimension 1 ft, 2 ft and 3 ft needs to be increased in size to hold four times as much bigger as the current size. (Assume each dimension is increased by the same amount). What is the function that represents the volume of the new packaging box? What are the dimensions of the new packaging box?
    A. $V(x) = x^3 + 6x^2 + 11x – 40$
       Dimensions: 2 ft x 3 ft x 4 ft
    B. $V(x) = x^3 - 6x^2 + 11x – 40$
       Dimensions: 4 ft x 6 ft x 7 ft
    C. $V(x) = x^3 + 6x^2 - 11x – 45$
       Dimensions: 5 ft x 6 ft x 4 ft
    D. $V(x) = x^3 - 9x^2 - 11x – 45$
       Dimensions: 3 ft x 5 ft x 4 ft

15. A computer store determines that the profit for producing x computer units per day is $P(x) = x – 7x + 8x + 12$. What possible number of computer units sold in a day for a break even?
    A. 1 and 2
    B. 2 and 3
    C. 3 and 4
    D. 5 and 6

16. The length of the rectangular tank is 3 ft more than its width and the height is 5 ft more than its width.
    How would you determine the dimensions of the tank if its volume is 100 ft^3?
    A. Determine the polynomial function for the volume and compute one real zero to represent the width of the tank
    B. Use guess and check in finding the width of the tank
    C. Determine the polynomial function for the volume and compute one real zero to represent the height of the tank
    D. Determine the polynomial function for the volume and compute one real zero to represent the length of the tank

17. The function $g(x) = 1.284x^3 – 0.004x^2 + 0.27x + 1.263$ can be used to model the average price of a gallon of diesel in a given year if x is the number of years since 2010. What is the easier way to determine the average price of diesel in year 2017?
    A. Find the zeros of g(x).
    B. Evaluate g(2017)
    C. Find the factors of g(x)
    D. Evaluate g(7)

18. The water refilling station owner wants to know the volume of water dispensed in the station in a day. As a station in charge you are tasked to predict water dispensed in the station for the next thirty days and present a mathematical equation for the data.
    Which of the following standards should the report be evaluated?
    A. Organization of data and justification of recommendation
    B. Accuracy of computation, organization of data and justification of recommendation
    C. Representation of output and organization of data
    D. Representation of output

19. You are a packaging designer of a certain company and you were tasked to create a tool for evaluating the product. The product is a box which will be used for storing the canned goods that will yield a maximum volume. Which of the following criteria should be considered the least as part of the rubric. for rating the performance task?
    A. Justification/Application of concept.
    B. Practicality of the Product
    C. Accuracy of Computations
    D. Delivery of Report

20. Which of the following product will best apply the concept of a cubic polynomial equations/functions?
    A. Volume of a box
    B. Surface area of a box
    C. Minimum cost for creating a box
    D. Maximum profit